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# Capture of Slow Neutrons

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**Abstract**

Current theories of the large cross sections of slow neutrons are contradicted by frequent absence of strong scattering in good absorbers as well as the existence of resonance bands. These facts can be accounted for by supposing that in addition to the usual effect there exist transitions to virtual excitation states of the nucleus in which not only the captured neutron but, in addition to this, one of the particles of the original nucleus is in an excited state. Radiation damping due to the emission of $\gamma$-rays broadens the resonance and reduces scattering in comparison with absorption by a large factor. Interaction with the nucleus is most probable through the $s$ part of the incident wave. The higher the resonance region, the smaller will be the absorption. For a resonance region at 50 volts the cross section at resonance may be as high as $l0^{-19}$ cm$^2$ and $0.5 \times 10^{20}$ cm$^2$ at thermal energy. The estimated probability of having a nuclear level in the low energy region is sufficiently high to make the explanation reasonable. Temperature effects and absorption of filtered radiation point to the existence of bands which fit in with the present theory.

## 1 Introduction

Bethe,[1] Fermi,[2] Perrin and Elsasser,[3] Beck and Horsley[4] gave theories of the anomalously large cross sections of nuclei for the capture of slow

[1]H. A. Bethe, Phys. Rev. **47,** 747 (1935). We refer to this paper as H. B. in the text.
[2]E. Amaldi, 0. d'Agostino, E. Fermi, B. Pontecorvo, F. Rasetti, E. Segre, Proc. Roy. Soc. **A149,** 522 (1935).
[3]Perrin and Elsasser, Comptes rendus **200,** 450 (1935).
[4]Beck and Horsley, Phys. Rev. **47,** 510 (1935).

neutrons. These theories are essentially alike and explain the anomalously large capture cross sections as a sort of resonance of the $s$ states of the incident particle. Resonance is usually helpful in causing a large scattering as well as a large probability of capture and it has been shown [H. B. Eq. (35)] that large scattering is to be expected by nuclei showing anomalously large capture at thermal energies. This consequence of the current theories is apparently in contradiction with experiment, there being no evidence of a large scattering in good absorbers. If also follows from current theories that with very few exceptions the capture cross section should vary inversely as the velocity of the slow neutrons. Experiments on selective absorption recently performed[5] indicate that there are absorption bands characteristic of different nuclei and it appears from the experiments of Szilard[6] that these bands have fairly well-defined edges. It has been pointed out by Van Vleck[7] that it is hard and probably impossible to reconcile the difference in internal phase required by the Bethe-Fermi theory with reasonable pictures of the structure of the nucleus. The combined evidence of experimental results and theoretical expectation is thus against a literal acceptance of the current theories and it is our purpose to outline an extension which is capable of explaining the above facts by a mechanism similar to that used for the inverse of the Auger effect by Polanyi and Wigner.[8]

It will be supposed that there exist quasi-stationary (virtual) energy levels of the system nucleus+neutron which happen to fall in the region of thermal energies as well as somewhat above that region. The incident neutron will be supposed to pass from its incident state into the quasi-stationary level. The excited system formed by the nucleus and neutron will then jump into a lower level through the emission of $\gamma$-radiation or perhaps at times in some other fashion. The presence of the quasi-stationary level, $Q$, will also affect scattering because the neutron can be returned to its free condition during the mean life of $Q$. If the probability of $\gamma$-ray emission from Q were negligible there would be in fact strong scattering at the resonance, the scattering cross section being then of the order of the square of the wave-length. Estimates of order of magnitude show that it is reasonable to assign

[5] Moon and Tillman, Nature **135,** 904 (1935); Bjerge and Westcott, Proc. Roy. Soc. **A150,** 709 (1935); Arsimovitch, Kourtschatow, Miccovskii and Palibin, Comptes rendus **200,** 2159 (1935); Ridenour and Yost, Phys. Rev. **48,** 383 (1935); Pontecorvo, Ricerca scientifica **6-7,** 145 (1935).

[6] L. Szilard, Nature **136,** 950 (1935).

[7] J. H. Van Vleck, Phys. Rev. **48,** 367 (1935).

[8] O. K. Rice, Phys. Rev. **33,** 748 (1929); **35,** 1551 (1930); **38,** 1943 (1931); J. Chem. Phys. **1,** 375 (1933). A similar process was used by M. Polanyi and E. Wigner, Zeits. f. Physik **33,** 429 (1925).

12 volts to the "half-value breadth" of $Q$ due to radiation damping and that the "half-value breadth" due to passing back into the free state is about one-fortieth of the above amount. This means that when the system passes into the state $Q$ it radiates practically immediately and the neutron has no time to be rescattered. It will, in fact, be seen from the calculations that follow that the ratio of scattering to absorption is essentially the ratio of the corresponding half value breadths. The hardness of the emitted $\gamma$-rays is of primary importance for the small ratio of scattering to absorption because it makes the probability of $\gamma$-ray emission sufficiently high. Inasmuch as the interesting phenomena occur for low energies we may suppose that in most cases the coupling of the incident state occurs through its $s$ state, i.e., in virtue of head on collisions. It will be seen, however, that the possibility of obtaining observable effects by means of $p$ states is not excluded even though it is less probable and leads to smaller cross sections. Calculation shows that with resonances of the type considered here one may obtain appreciable probability of capture at energies of the order of 1000 volts. It is possible to have at such energies cross sections of roughly $10^{-22}$ cm$^2$ with a half-value breadth of about 20 volts. It is therefore not necessary to ascribe all large cross sections to neutrons of thermal velocities and the probability of finding a quasi-stationary level in a suitable region is not so small as to make the process improbable.

We are presenting below the theory of capture on this basis in some detail not because we believe it to be a final theory but because further development may be helped by having the preparatory structure well cemented.

## 2 Theory of Damping

The process of absorption from the continuum into a quasi-stationary level and a subsequent reemission of a photon is related to the phenomena of predissociation discussed by O. K. Rice[8] who made the first application of quantum mechanics to this type of process since Dirac's first approach.[9] It is essential for us to consider two continua and in this respect the present problem is more general. It resembles closely the problem of absorption of light from a level $a$ to a level $c$ which is strongly damped by radiation in jumps to a third level $b$. The absorption from $a$ to $c$ corresponds to the transition of the neutron into the quasi-stationary level and the jumps from $c$ to $b$ correspond to the emission of $\gamma$-rays in a transition to a more stable level of the nucleus. The absorption probabilities can be obtained by

[9] P. A. M. Dirac, Zeits. f. Physik **44,** 594 (1927)

using the principle of detailed balance from the solution which represents emission[10] from the level $c$ to the levels $a$, $b$ or else by a direct application of the theory of absorption.[11] The usual theory as developed for either process is not accurate enough to represent the effect of the variation of matrix elements with velocity which is essential for our purpose, inasmuch as it is responsible for the existence of two regions of large absorption. The usual type of calculation will now be generalized so as to take the variation into account.

### (a) Calculation of the absorption and scattering process

Let $a_s$ denote the probability amplitudes of states in which the neutron is free and in a state $s$. Similarly let $b_r$ stand for the probability amplitude of a state in which the neutron is captured and there is a photon $r$ emitted and let $c$ be the probability amplitude of the quasi-stationary state having energy $h\nu$. The states $r$, $s$ are here considered to be discrete but very closely spaced in energy. The average spacing of the levels $r$, $s$ are written $\Delta E_r = h\Delta\nu_r$, $E_s = h\Delta\nu_s$ so that the number of levels $s$ per unit energy range is $1/\Delta E_s$. The matrix element of the interaction energy responsible for transitions from $a_s$ to $c$, $c$ to $b_r$ will be written, respectively,

$$M_s = hA_s, \quad M_r = hB_r. \tag{1}$$

The damping constants for $c$ due, respectively, to the possibility of emitting $a_s$ or $b_r$ are then[12]

$$(4\pi\tau_a)^{-1} = \Gamma_s = [\pi\overline{|A_s|^2}/\Delta\nu_s]_{[\nu_s=\nu_0};\tag{2}$$

$$(4\pi\tau_b)^{-1} = \Gamma_r = [\pi\overline{|B_r|^2}/\Delta\nu_r]_{\nu_r=\nu_0}; \quad \Gamma = \Gamma_s + \Gamma_r,$$

where $\tau_a$ , $\tau_b$, are respective mean lives of $c$ due to emission of $a_s$, and $b_r$. The quantities $\Gamma$ represent one-half of the “half-value breadth” measured in frequency. In discussing line emission and absorption10, 11 the directional averages of $|A_s|^2$ and $|B_r|^2$ can be taken for any energy within the breadth of the line because the line can be usually considered to be sharp. In the present case it will be necessary to distinguish among directional averages of $|A_s|^2$ for different energies.

[10]V. Weisskopf and E. Wigner, Zeits. f. Physik **63,** 54 (1930).
[11]V. Weisakopf, Ann. d. Physik **9,** 23 (1931).
[12]G. Breit, Rev. Mod. Phys. **5,** 91, 104, 117 (1933).

The states $s$ will be thought of as plane waves modified by a central field due to the nucleus and satisfying boundary conditions at the surface of a fundamental cube of volume $V$. The equations satisfied by $a_s,\ b_r,\ c$ are

$$\left(\frac{d}{2\pi i dt}+\nu_s\right)a_s = A_s c;\quad \left(\frac{d}{2\pi i dt}+\nu_r\right)b_r = B_r c, \tag{3}$$

$$\left(\frac{d}{2\pi i dt}+\nu\right)c = \Sigma A_s^* c_s + \Sigma B_r^* b_r.$$

In these equations the influence of only one quasi-stationary level is taken into account and for this reason they are not quite accurate. They are sufficiently good for the present purpose because it will be supposed that different quasi-stationary levels do not fall closely together. At $t = 0$ it will be supposed that

$$a_s = \delta_{ss_0},\quad b_r = 0,\quad c = 0,\quad (t = 0). \tag{4}$$

An approximate solution of (3) satisfying this initial condition can be obtained by forming a linear combination of

$$\begin{aligned} c &= e^{-2\pi i(\nu - i\Gamma')t}; \\ a_s &= A_s[e^{-2\pi i(\nu-i\Gamma')t} - e^{-2\pi i\nu_s t}]/(\nu_s - \nu + i\Gamma'), \\ b_r &= B_r[e^{-2\pi i(\nu-i\Gamma')t} - e^{-2\pi i\nu_s t}]/(\nu_r - \nu + i\Gamma'), \end{aligned} \tag{5}$$

with

$$\Gamma' = [\pi\overline{|A_s|^2}/\Delta\nu_s + \pi\overline{|B_r|^2}/\Delta\nu_r] \text{ resonance region}, \tag{5$'$}$$

and

$$\begin{aligned} a_{s_0} &= e^{-2\pi i(\nu_0 - i\gamma)t};\quad c = As_0{}^* e^{-2\pi i(\nu_0-i\gamma)t}/(\nu-\nu_0-i\Gamma), \\ a_s &= A_s As_0{}^*[e^{-2\pi i(\nu_0-i\gamma)t} - e^{-2\pi i\nu_s t}]/(\nu_s-\nu_0+i\gamma)(\nu-\nu_0-i\Gamma), \\ b_r &= B_r As_0{}^*[e^{-2\pi i(\nu_0-i\gamma)t} - e^{-2\pi i\nu_r t}]/(\nu_r-\nu_0+i\gamma)(\nu-\nu_0-i\Gamma), \\ \Gamma &= [\pi\overline{|A_s|^2}/\Delta\nu_s + \pi\overline{|B_r|^2}/\Delta\nu_r]_{\nu_s=\nu_0}. \end{aligned} \tag{6}$$

In Eq, (6) $s \neq s_0$. The quantities $\gamma$ and $\nu_0 - \nu_{s_0}$, are small compared with $\Gamma$; they will go to zero with increasing volume. From (3), one finds for them the equation:

$$(\nu_{s_0} - \nu_0 + i\gamma)(\nu - \nu_0 - i\Gamma) = |A_{s_0}|^2, \tag{7}$$

so that

$$\gamma = |A_{s_0}|^2\Gamma/[(\nu - \nu_0)^2 + \Gamma^2]; \quad \nu_0 = \nu_{s_0} + (\nu_0 - \nu)(\gamma/\Gamma). \tag{8}$$

In obtaining Eq. (7) the approximations

$$\Sigma'_s |A_s|^2 \frac{1 - e^{2\pi i(\nu_0 - \nu_s - i\gamma)t}}{\nu_s - \nu_0 + i\gamma} = \pi i \overline{|A_s|^2}/\Delta\nu_s \tag{9}$$

are made. These correspond to replacing the sums by integrals and extending the range of integration from $\nu_s = -\infty$ to $\nu_s + \infty$ and similarly for $\nu_r$. In addition it is supposed that $\overline{|A_s|^2}$, $\overline{|B_r|^2}$ vary so slowly through the region in which the integrand is large that they may be taken outside the integral sign. These approximations are, therefore, valid only if the contributions to the sums (9a), (9b) are localized in a sharp maximum. Such a maximum exists for $\nu_s \cong \nu_0$ because: (1) $\gamma$ vanishes as the fundamental volume is increased and therefore one may consider $\gamma t \ll 1$ and (2) for any $\nu_s - \nu_0$ it is possible to choose $t$ sufficiently large to make $|\nu_s - \nu_0|t \gg 1$. For such times the most important part of the integrand oscillates rapidly with $\nu_s$. However for $|\nu_s - \nu_0| \sim \gamma$, the values of $t$ which satisfy $\gamma t \ll 1$ are always such that $|\nu_s - \nu_0|t \ll 1$. The integrand is thus not oscillatory for $\nu_s = \nu_0 \pm \gamma$ and the values of $\overline{|A_s|^2}$, $\overline{|B_r|^2}$ on the right side of (9) are to be understood as corresponding to $\nu_s = \nu_0$ with an uncertainty of the order $\gamma$. It can be verified by calculation that the contribution to (9) due to a finite region at a distance $|\nu_s - \nu_0| \gg \gamma$ contributes imaginary quantities decreasing exponentially with $2\pi|\nu_s - \nu_0|t$ and real quantities which contribute to a frequency shift[9] of $\nu$. For the present this shift will be neglected. Eqs. (6) are thus approximate solutions which become increasingly better as $t$ increases, provided $\gamma t \ll 1$. In our application $\Gamma$ is mostly due to the radiation damping $\Gamma_r$. The directional averages of $|B_r|^2$ vary smoothly since the energy of the $\gamma$-ray is of the order of several million volts and is large compared to $\Gamma$.

The quantity $\Gamma'$ which enters (5) is not determined accurately by the present method because $\overline{|A_s|^2}$ which enters in this case is some sort of average over the resonance width. This complication causes no trouble because: (a) for times $t \gg 1/4\pi\Gamma'$ the rates of emission of states $a_s, b_r$ are, respectively, $4\pi\gamma\Gamma_s/\Gamma$, $4\pi\gamma\Gamma_r/\Gamma$ and depend[13] only on $\Gamma$ and not on $\Gamma'$; (b) the largeness of $\Gamma_r$ in comparison with $\Gamma_s$ makes $|\Gamma' - \Gamma| \ll \Gamma$. Thus $\Gamma'$ is of importance only in determining the initial transients but not the steady rate of absorption. This can be expected from the fact that the solutions (6)

[13] Appendix I.

represent a condition in which $s_0$ is absorbed at the rate $4\pi\gamma$. The addition of the "emission solution" (5) is only needed to enforce the condition $c = 0$ at $t = 0$; it modifies the emission of states $b_r, a_s$ during times comparable with the mean life of the nucleus but leaves them unchanged over longer periods very similarly to the way in which analogous transient conditions are of no importance in the absorption of monochromatic radiation by classical vibrating systems.

The total cross section $\sigma$ which corresponds to the disappearance of the incident states $s_0$ is given by

$$\sigma = 4\pi\gamma V/v, \tag{10}$$

where $v$ is the neutron velocity because the modified plane waves denoted by $s$ were normalized in the volume $V$ and thus represent states of density $1/V$.

The number of possible plane waves in $V$ per unit frequency range is

$$1/\Delta\nu_s = 4\pi V/v\Lambda^2, \tag{11}$$

where $\Lambda$ is the de Broglic wave-length. From (2), (10), (11) we have

$$\sigma = \gamma\Lambda^2/\Delta\nu_s = \frac{\Lambda^2}{\pi} S \frac{\Gamma_s \Gamma}{(\nu - \nu_0)^2 + \Gamma^2}. \tag{12}$$

Here the statistical factor $S$ takes account of the fact that the state $s_0$ may be more or less effective in its coupling to the quasi-stationary level than the average modified plane wave in the same energy region. If the quasi-stationary level has an orbital angular momentum $L\hbar$ and if there is no spin orbit interaction then $|A_{s_0}|^2 = (2L+1)\overline{|A_s|^2}$ because coupling to c can take place only through $1/(2L+1)$ of the total number of states. Thus.

$$S = 2L + 1 \tag{13}$$

in these special circumstances. For $s$ terms $S = 1$. The total cross section

$$\sigma = \sigma_c + \sigma_s,$$

where $\sigma_s$ is the cross section due to scattering and $\sigma_c$ is the cross section due to capture. We have

$$\sigma_c = \frac{\Lambda^2}{\pi} S \frac{\Gamma_s \Gamma_r}{(\nu - \nu_0)^2 + \Gamma^2}; \quad \sigma_s = \frac{\Lambda^2}{\pi} S \frac{\Gamma_s^2}{(\nu - \nu_0)^2 + \Gamma^2}. \tag{14}$$

The above value of $\sigma_s$ corresponds to the value $\Sigma|a_s|^2$ and does not take into account the fact that there is scattering in the abscence of the quasi-stationary level. If this is strong one must correct $\sigma_s$ for interference of the states $s$ with the spherical wave present in $s_0$. In the applications made below the scattering effect due to either cause will be small and the correction need not be considered. According to (14) the extra scattering can be expected to be of the order $\Gamma_s/\Gamma_r$ times the capture and is quite small for small $\Gamma_s$.

It should be noted that the order of magnitude of $\sigma_c$ at resonance is changed by taking into account the radiation damping. If this were neglected and if one were to calculate simply by using Einstein's emission probability for the stationary states of matter then one would obtain an incorrect value,

$$\sigma_c' = \frac{\Lambda^2}{\pi} S \frac{\Gamma_s \Gamma_r}{(\nu - \nu_0)^2 + \Gamma_s^2}. \tag{14'}$$

For resonance $\sigma_c/\sigma_c' = \Gamma_s^2/\Gamma^2$ and approximately $\int \sigma_c dE / \int \sigma_c' dE$ is $\Gamma_s/\Gamma_r$. No paradox is involved here because it is not legitimate to apply Einstein's emission probability formula to levels separated by less than their breadth due to radiation damping. Eq. (14$'$) gives too high values to the cross section. If $\nu - \nu_0 \gg \Gamma$ there is no difference between $\sigma_c'$ and $\sigma_c$. For sufficiently large values of $\nu - \nu_0$ the discussion which led to Eqs. (13), (14) will break down because Dirac's frequency shift[9] is neglected in these formulas. A more complete formal discussion including the frequency shift is given in Appendix I. The calculation shows that one should change the frequency of the quasi-stationary level $\nu$ by

$$\nu \to \nu - \int \frac{|A_s|^2}{\Delta\nu_s} \frac{d\nu_s}{\nu_s - \nu_0} - \int \frac{|B_r|^2}{\Delta\nu_r} \frac{d\nu_r}{\nu_r - \nu_0} \tag{15}$$

where the integrations are extended over the complete range of states $s, r$ and where the principal values of the integrals are to be taken. The last part of (15) represents the frequency shift due to electromagnetic radiation and can be incorporated in $\nu$ as a constant because $\nu_0$ need be varied only in a range small in comparison with the frequency of the $\gamma$-ray. It is dangerous to take this shift into account on account of the well known inconsistency of quantum electrodynamics. The second term on the right side of Eq. (15) is due to interactions between free neutron states and the quasi-stationary state. It is physically correct and it is necessary in order to bring about agreement between (14) and calculations away from resonance by means of the Einstein emission probabilities. The shift is large in the applications. Nevertheless changes in it are small in the relatively small range of values which need be considered and its effect is therefore primarily that of displacing the resonance frequency by a constant amount.

## (b) Resonance of one-body systems

The above discussion cannot be applied directly to cases in which resonance consists simply in a sharp increase of the wave function of one neutron to a maximum inside the nucleus because there is no intermediate state $c$ under such conditions in the same sense as in the previous section. For low velocity neutrons such resonance can be sharp for states with $L \geq 1$. Formally one could try to apply the discussion already given by starting with wave functions which are solutions of the wave equation for an infinitely high barrier somewhat outside the nucleus. The difference between the actual height of the barrier and $\infty$ can be then treated as a perturbation essentially responsible for the matrix elements $hA_s$. Such a procedure leads apparently to correct results which can be verified by other methods. It is troublesome to justify it completely because the region where the infinite barrier must be erected should be such that the wave functions within are small for all energies. It is preferable to use a more direct calculation for such a case. We consider a plane wave of neutrons incident on the nucleus. Resonance takes place to the wave functions of angular momentum $L\hbar$. We surround the nucleus by a large perfectly reflecting sphere of radius $R$ and we calculate the rate at which states of angular momentum $L\hbar$ disappear by radiation. There is no essential restriction on the possibility of forming wave packets out of the plane waves if We admit only those states $L$ which satisfy the boundary conditions on the sphere. The radius will be made finally infinitely large and the spacing between the levels infinitely small. This provides the necessary flexibility for the formation of the wave packets.

The spacing between successive possible neutron levels is given by

$$\Delta\nu = v/2R. \tag{16}$$

The radial function will be expressed as $F/r$ where $F$ will be by definition a sine wave with unit amplitude at a large distance from the nucleus. The normalized wave function is then $Y_L(F/r)(2/R)^{1/2}$ where $Y_L$ is a spherical harmonic normalized so as to have $\int |Y_L|^2 d\Omega = 1$. The wave function for the bound state will be written

$$Y_{L\pm 1} f/r; \quad \int_0^\infty f^2 dr = 1. \tag{17}$$

The damping constant which corresponds to the emission of radiation from the state $F$ is obtained by using the formula for Einstein's emission probability and is

$$\gamma E = (C/R)|\int_0^\infty F f r dr|^2, \tag{18}$$

where

$$C = \frac{32\pi^3 e'^2 \nu^3}{3hc^2} \frac{L + 1/2 \pm 1/2}{2L+1}, \tag{18'}$$

the upper sign applying to jumps $L \to L+1$ and $e' \sim e/2$ is the effective charge of the neutron nucleus system. As $R \to \infty$, both $\Delta\nu$ and $\gamma E$ decrease towards zero but their ratio remains constant. The cross section due to capture computed directly from the emission probability is

$$\sigma_{C'} = (2L+1)\Lambda^2 \gamma E/\Delta\nu. \tag{19}$$

If this expression approaches $\Lambda^2$ then $\gamma E/\Delta\nu$ becomes comparable with unity and the levels are close enough together to make Eq. (19) meaningless. It is then necessary to take into account the mutual influence of neighboring levels. This can be done by means of the damping matrix.[14] The successive states of angular momentum $L\hbar$ will be denoted by indices $j, l$ and their probability amplitudes by $a_j$. These satisfy

$$\left(\frac{d}{2\pi i dt} + \nu_j\right) a_j = i\Sigma\gamma^{jl} a_l, \tag{20}$$

where

$$\gamma^{jl} = CJj^* J_l/R; \quad J_j = \int F_j f r dr. \tag{20'}$$

In our case only states with the same magnetic quantum number can interact so that a complete specification of the states is obtained through their energy. Solutions of (20) in which all quantities vary as $\exp\{-2\pi i(\nu_0 - i\gamma)t\}$ correspond as closely as possible to the notion of a stationary state decaying under influence of radiation damping. From (20) one obtains

$$(\nu_j - \nu_0 + i\gamma) a_j = i\Sigma\gamma^{jl} a_l. \tag{20''}$$

These equations with the complex eigenwert $\nu_0 - i\gamma$ can be reduced making use o( the fact that $\gamma^{jl}$ is a matrix of rank 1. Thus eliminating the $a_l$ one finds

$$1 = \frac{iC}{R} \Sigma \frac{|J_j|^2}{\nu_j - \nu_0 + i\gamma} \tag{21}$$

for the secular equation which determines $\nu_0$ and $\gamma$. This equation will be solved approximately for the case of sharp resonance. The resonance will be supposed to take place at an energy $h\nu_F$ and to have a "half-value breadth" $2h\Gamma_F$. Close to resonance

$$|J_j|^2 = \frac{\Gamma_{F^2} |I|^2}{(\nu_j - \nu_F)^3 + \Gamma_{F^2}}, \tag{22}$$

[14] G. Breit, Rev. Mod. Phys. **5**, 117 (1933); G. Breit and I. S. Lowen, Phys. Rev. **46**, 590 (1934).

where $|I^2|$ is the maximum value of $|J|^2$. This approximation will usually apply only in a region of a few $\Gamma_F$. The value of $\Gamma_F$ can be estimated using[15]

$$4\pi\Gamma_F = v_r / \int \overline{G}^2 dr, \tag{23}$$

where $\overline{G}$ is $F$ for resonance, $v_r$ is the velocity at resonance, and the integration is to be carried through the range of large values of $\overline{G}$. The quantity $\Gamma_F$ is analogous to $\Gamma_s$, of section (a). The state represented by $\overline{G}$ is analogous to the quasi- stationary state of section (a). In order to bring out the analogy we introduce a damping constant similar to the previous $\Gamma_r$

$$\Gamma_R = C|I|^2/2 \int \overline{G}^2 dr = 2\pi C|I|^2\Gamma_F/v_r, \tag{23$'$}$$

which is the damping constant of the state represented by $\overline{G}$ when that state is normalized within the nucleus and its immediate vicinity. Substituting (23′) into (21), replacing the sum by an integral everywhere except in the vicinity of $\nu_0$ and performing the summation in that region on the assumption that the $\Delta\nu$ can be considered as equal to each other in that region gives

$$1 = i\{a \cot\left[\frac{\pi(\nu_{j0} - \nu_0 + i\gamma)}{\Delta\nu_{j0}}\right] + ia + \int_0^\infty \frac{v_r|J_j|^2\Gamma_R d\nu_j}{\pi v_j|I^2|\Gamma_F(\nu_j - \nu_0 + i\gamma)} \tag{24}$$

$$a = \left[\frac{v_r|J_j|^2\Gamma_R}{v_j|I|^2\Gamma_F}\right]_{\nu_0},$$

where the integral must be extended over all $\nu_j$ and the region around $\nu_0$ is integrated over the real axis. The quantity $\nu_{j0}$ is any one of the $\nu_j$ located so close to $\nu_0$ that the variation in $\Delta\nu$ in between can be neglected. In the approximation of Eq. (22) the integration over the resonance region $\Gamma_F$ leads to an equation which to within a sufficient approximation reduces to

$$1 + itTh = (Th + it)(a + ib); \quad b = \frac{v_{j0}|I|^2 q}{v_r|J_{j0}|^2(1 + q_2)} \tag{25}$$

with

$$\nu_0 - \nu_F = q\Gamma_F; \quad Th = \tan h\frac{\pi\gamma}{\Delta\nu_{j0}}; \quad t = \tan\frac{\pi(\nu_0 - \nu_{j0})}{\Delta\nu_{j0}}. \tag{25$'$}$$

By eliminating $t$

$$Th + 1/Th = a + 1/a + b^2/a, \tag{25$''$}$$

which has the approximate solution

$$1/Th = a + 1/a + b^2/a. \tag{26}$$

[15] G. Breit and F. L. Yost, Phys. Rev. **48**, 203 (1935). See also Eq. (32′).

For values of $\nu_0$ which lie in the region where Eq.. (22) applies and where $\Delta\nu_{j0} \sim \Delta\nu_r$ we have approximately

$$\frac{\pi\gamma}{\Delta\nu} = \frac{\Gamma_R\Gamma_F}{\Gamma_{R^2} + \Gamma_{F^2}(1+q^2)}, \tag{26'}$$

where it is supposed that $\Gamma_r \gg \Gamma_F$. If, however, $\Gamma_F \gg \Gamma_R$ then

$$\frac{\pi\gamma}{\Delta\nu} = \frac{\Gamma_R}{\Gamma_F(1+q^2)}, \tag{26''}$$

which is equivalent to using the $\gamma E$ of Eq. (18), (19); in this case one may compute using emission probabilities. If one is so far away from resonance that $b^2/a < a,\ 1/a$ Eq. (25") gives

$$\gamma = \gamma^{j0j0}, \tag{26'''}$$

provided the right side is $\ll 1$. Here again the simple emission point of view applies. For $\Gamma_R \gg \Gamma_F$ all regions are approximated by

$$\frac{\pi\gamma}{\Delta\nu_{j0}} = \frac{v_r|J_{j0}|^2\Gamma_R\Gamma_F(1+q^2)}{v_{j0}|I|^2[\Gamma_{R^2} + \Gamma_{F^2}(1+q^2)]} \tag{27}$$

The treatment of scattering by means of the damping matrix is somewhat involved and will not be reproduced here. The phase shift due to $\nu_0 - \nu_{j0}$ when added to the phase shift already present in $F_{j0}$ gives the phase shift required. The scattering is diminished by $\Gamma_R$ in much the same way as it was diminished by it in section (a). By comparing (27) with (19)

$$\sigma_C = (2L+1)\frac{\Lambda^2 v_r|J_{j0}|^2\Gamma_R\Gamma_F(1+q^2)}{\pi v_{j0}|I|^2[\Gamma_{R^2} + \Gamma_{F^2}(1+q^2)]}, \tag{28}$$

which is similar to Eq. (14), close to resonance. The factors $|J|^2/|I|^2$ and $v_r/v_{j0}$ take into account the deviations from the dependence of $|J|^2$ on $\nu$ given by (22). In (14) this is analogous to the dependence of $\Gamma_s$ on $\nu_{j0}$ combined with Dirac's frequency shift.

## (c) Sharpness of resonance for single-body problem

The upper limit of integration in Eq. (23) has been left indefinite. By Green's theorem

$$\frac{d}{dr}\left[F_1\frac{dF_2}{dr} - F_2\frac{dF_1}{dr}\right] + \frac{2\mu}{\hbar^2}(E_2 - E_1)F_1F_2 = 0,$$

where $F_1, F_2$ correspond to energies $E_1, E_2$ and need not be regular at $r = 0$. Hence[16]

$$\frac{\partial}{\partial r}\left[F^2\frac{\partial}{\partial E}\frac{\partial F}{F\partial r}\right] + \frac{2\mu}{\hbar^2}F^2 = 0. \tag{29}$$

[16] J. A. Wheeler. We are indebted to Dr. Wheeler for communicating to us other applications of this relation.

In this section let $F_i$ be the function inside the nucleus, and let $F$ stand for the regular solution of the wave equation for $r\times$ radial function on the absence of the nuclear field. The normalization of $F$ is such as to make it a sine wave $\sin(kr+\varphi)$ of unit amplitude at $\infty$. Similarly $G$ is defined as satisfying the same differential equation as $F$ but it is to be $90^{circ}$ out of phase with $F$ at $\infty$ i.e. $\cos(kr+\varphi)$. The regular solution of the differential equation in the presence of the nuclear field, normalized in the same way as $F$ and $G$, will be called $\bar{F}$. At the nuclear radius $r_0$

$$F^2 = G^2 / \left\{ \left[ G^2 \left( \frac{G'}{G} - \frac{F_1'}{F_i} \right) \right]^2 + \left[ FG \left( \frac{F'}{F} - \frac{F_i'}{F_i} \right) \right]^2 \right\}. \tag{30}$$

Here the accent stands for differentiation with respect to $kr$. At resonance $F_i'/F_i = G'/G$ and the second term in the curly bracket is then 1, while the first term is zero. As $E$ changes to either side of the resonance value $E_r$ the first term may become 1 for $E = E_r \pm \Delta E$ where $\Delta E$ is properly chosen. The half-value breadth is then $2\Delta E$ and $\Delta E = h\Gamma_F$. The value of $\Delta E$ can be estimated by

$$\Delta E \frac{\partial}{\partial E} \left[ G^2 \left( \frac{G'}{G} - \frac{F_i'}{F_i} \right) \right]_{r_0} = 1. \tag{31}$$

Using Eq. (29) and calculation the $\partial/\partial E$ for $E = E_r$, one obtains a result which can be expressed in terms of integrals up to $R$ where $R$ is any value of $r$ which is greater than $r_0$. The function which is $G$ for $r > r_0$ and $F_i(G/F_i)_{r_0}$, for $r < r_0$ is continuous at $r_0$ and at resonance its derivative with respect to $r$ is also continuous. The function will be called $\bar{G}$ for $0 < r < \infty$. We have then[17]

$$\frac{E}{\Delta E} = k \int\limits_0^R \bar{G}^2 dr + \left[ \frac{G^2 E\partial}{k\partial E} \frac{\partial G}{G\partial r} \right]_{r=k} ; \quad \Delta E = h\Gamma_F. \tag{32}$$

The right side of this result is independent of $R$ and is finite. The term outside the integral should be included in Eq. (23) changing

$$\int G^2 dr \to \int\limits_0^R \bar{G}^2 dr + \left[ \frac{G^2 E\partial}{k^2\partial E} \frac{\partial G}{G\partial r} \right]_{r=k} . \tag{32'}$$

Eq. (32) has a well-defined meaning only if resonance is sharp. Otherwise the $\partial/\partial E$ entering in Eq. (31) cannot be supposed to be sufficiently constant through the half-breadth $2h\Gamma_F$. It cannot be expected to hold for the broad $S$ resonance discussed by Bethe.

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[17] Cf. Eq. (22) reference 15. In calculations with Coulombian fields it is sometimes convenient to transform Eq. (32) of the text into

$$\frac{E}{\Delta E} = G^2 \left[ \frac{k}{F_j^2} \int\limits_0^{r_0} F_i^2 dr - \frac{k}{F^2} \int^{r_0} F^2 dr - \frac{EG^2\partial}{kr\partial E} \left( \frac{kr}{FG} \right) \right]$$

all quantities outside the integrals being taken for $r = r_0$.

### (d) Capture by $p$ states

For a potential well of constant depth

$$F_i = \sin z/z - \cos z; \quad F = \sin\rho/\rho - \cos\rho;$$

$$G = \cos\rho/\rho + \sin\rho, \tag{33}$$

where

$$z = Kr; \quad \rho = kr; \quad K = \mu v_i/\hbar; \quad k = \mu v/\hbar \tag{33$'$}$$

$v_i, v$ being, respectively, the velocities inside and outside the nucleus. The resonance condition is

$$z\sin z/[\sin z/z - \cos z] = \rho\cos\rho/\rho[\cos\rho/\rho + \sin\rho].$$

For slow neutrons $\rho \ll 1$ the right side is $\ll \rho^2$ and therefore very small. The first resonance point is obtained for $z = \pi - \epsilon$, $\epsilon \sim \rho^2/\pi$. It will suffice to take $z = \pi$. By substituting into Eq. (32) it follows that

$$\Delta E/E = 2\rho/3 = 4\pi r_0/3\Lambda. \tag{33$''$}$$

For $E = (1/40)$ volt, $\Lambda = 1.8 \times 10^{-8}$ cm, $h\Gamma_f = 5.8 \times 10^{-6}$ volt. For 3-MEV $\gamma$-rays a reasonable value of $h\Gamma_R$ is 5.8 volts. The cross section at resonance is by Eq. (28) $3\Lambda\Gamma_F/\pi\Gamma_R = 300 \times 10^{-24}$ cm$^2$. Since scattering is of the order $\Gamma_F/\Gamma_R$ times capture the scattering cross section is small. According to Eq. (33$''$) the cross section at resonance for $p$ terms with $\Gamma_R \gg \Gamma_F$ can be expected to vary as $v$ and $h\Gamma_F$ as $v^2$. The range in which $p$ terms can be expected to give large capture cross sections and small scattering is therefore roughly from 1/40 volt to 1 volt. At higher velocities $h\Gamma_F$ is likely to be higher than $h\Gamma_R$. In the absence of an apparent reason for nuclear $p$ levels to fall in this narrow velocity range, an explanation in terms of $p$ terms although possible is improbable on account of the small range of neutron velocities required.

## 3 Capture Through $s$ Wave

### (a)

This section will contain the calculation of the $A_s$ used in 2a. It is supposed that the system "nucleus+neutron" can be treated in first approximation by means of an effective central field acting on the neutron. The difference between the Hamiltonian of the system and the Hamiltonian corresponding to the central field will be called $H'$. On account of this difference there exist transitions from the $s$ wave of the incident state to quasi-stationary excited states of the "nucleus +neutron" system. Normalizing the $s$ waves within a sphere of radius $R$ the wave function inside the nucleus is

$$C \sin Kr/r; \quad C^{-2}[1 + (U/E)\cos^2 Kr_0]2\pi R,$$

where

$$K^2/k^2 = (U + E)/E$$

and $U$ is the depth of the potential hole. The interaction energy $H'$ involves besides $r$ also internal coordinates $x$. The wave function of the whole system in the incident state may be written $C\psi_0(x)\sin Kr/r$ and in the quasi-stationary state $\psi_q(r, x)$. The matrix element $M_s$ of Eq. (1) is then

$$M_s = \int \psi_Q(r, x) H' C \sin Kr \psi_0(x) dv/r, \tag{34}$$

where $dv$ is the volume element of the whole system. The state $Q$ is by definition such that the integral of $|\psi_Q|^2$ through nuclear dimensions is unity. The order of magnitude of $M_s$ is therefore

$$M_s = C\bar{H} r_0^{1/2}, \tag{34$'$}$$

where $\bar{H}$ is an average of $H'$ through the nucleus and may have reasonably a value of 0.5 MEV. It cannot be specified further without detailed calculation which would probably be unsatisfactory in the present state of nuclear theory. Since $\Delta E = hv/2R$,

$$h\Gamma_s \cong \frac{\bar{H}^2 r_0}{2\Lambda U \cos^2 Kr_0}. \tag{34$''$}$$

According to Eq. (14)

$$\sigma_c = \frac{\Lambda r_0}{2\pi} \frac{\bar{H}}{U\cos^2 Kr_0} \frac{\bar{H} h \Gamma_r}{h^2(\Gamma_r + \Gamma_s)^2 + (E - E_r)^2,} \tag{35}$$

where $E_r$ is the value of $E$ for resonance; According to this formula there are two maxima

TABLE I. *Calculated cross sections for neutron capture.*

| Position of Resonance (volts) | $\bar{H}$ (MEV) | $h\Gamma_r$ (volts) | $h\Gamma_s$ at resonance (volts) | $10^{24}\sigma$ (cm$^2$) Resonance | Thermal Energies |
|---|---|---|---|---|---|
| 1/40 | 0.1 | 10 | 0.01 | 90000 | |
| | .1 | 1 | .01 | 900000 | |
| 1 | .1 | 10 | .05 | 14000 | |
| | .1 | 1 | .05 | 140000 | |
| 50 | .1 | 10 | .37 | 2000 | 3500 |
| | .5 | 10 | 9 | 13400 | 80000 |
| | .1 | 1 | 0.37 | 11000 | 350 |
| | .5 | 1 | 9 | 4800 | 9000 |
| 1000 | .1 | 10 | 1.6 | 320 | 9 |
| | .5 | 10 | 40 | 420 | 200 |
| 10000 | .1 | 10 | 5 | 60 | 0.09 |
| | .5 | 10 | 125 | 18 | 2 |

for $\sigma_c$, one for $E = E_r$ and one for $E = 0$. The expected cross sections are given in Table I to about ten percent accuracy. The numbers correspond to $\Lambda(kT) = 1.8 \times 10^{-8}$ cm; $\Lambda(1 \text{ volt}) = 2.9 \times 10^{-9}$ cm; $r_0 = 3 \times 10^{-13}$ cm; $U \cos^2 Kr_0 = 10^7$ volts. For $E_r = 140$ 1 volt the table shows large cross sections at thermal energies and above. The condition is similar to Bethe's except for a relatively sharper resonance determined by $h\Gamma_r$. For 50 volts one sees the development of two maxima one at resonance and one at thermal energies. For $E = 1000$ and 10,000 volts the maximum at thermal energies decreases as $E_r^{-2}$ and the maximum at resonance roughly as $E_r^{-1/2}$. For such high values of $E_r$ scattering has a chance of becoming comparable with absorption or even greater than the absorption at resonance. In the thermal energy region the $1/v$ law is obeyed for high values of $E_r$; for low $E_r$r the maximum at $E_r$ interferes with the $1/v$ law and the region of its validity is displaced below thermal energies.

In Table I only the effect of a quasi-stationary level at $E_r$ is considered. In addition there may be effects of other levels as well as radiation jumps of the kind considered by Bethe and Fermi which do not depend on the existence of virtual levels. It is thus probable that in most cases there is a region with a $1/v$ dependence although it may be at times masked by a resonance region.

### (b) Dirac's frequency shift

In the above estimates the effect of Dirac's frequency shift was neglected. This is given by

$$(h\Delta\nu)_D = \int_0^\infty \frac{h\Gamma_s}{\pi}\frac{dE}{E-E_0} = \frac{\bar{H}^2 r_0}{\pi\Lambda_0 x_0}\int_0^\infty \frac{x^2 dx}{(x^2-x_0^2)(x^2+a^2)},$$

where $x = E^{1/2}$, $a^2 = U\cos^2 Kr_0$ and the value of $h\Gamma_s$ was substituted by means of Eq. (34″). Here the subscript 0 refers to the neutron energy $E_0$ and the principal value of the integral is understood. Evaluating the expression

$$(h\Delta\nu)_D = \frac{\bar{H}^2 r_0 U^{1/2}\cos Kr_0}{2\Lambda_0(E_0+U\cos^2 Kr_0)E_0^{1/2}}. \tag{36}$$

The shift is seen to be of the order of 3000 times $h\Gamma_r$ for $E_0 = l$ volt. The shift is nearly independent of the velocity. In the approximation of Eq. (36)

$$\frac{d(h\Delta\nu)_D}{dE_0} = \frac{h\Gamma_s}{E_0}\frac{E_0^{1/2}}{U^{1/2}|\cos Kr_0|}, \tag{36′}$$

which shows that the variation in the shift is small and of the order of $2\times 10^{-5}(E-E_r)$ for $\bar{H} = 0.1$ MEV.

## 4 Discussion

### (a) Absence of scattering

According to Dunning, Pegram, Fink and Mitchell[18] the elastic scattering of slow neutrons by Cd is less than one percent of the number captured. According to A. C. G. Mitchell and E. J. Murphy[19] scattering as detected by silver is about the same as absorption for Fe, Pb, Cu, Zn, Sn while for Hg scattering is about 1/80 of the absorption. In the later communication of Mitchell and Murphy[19] it is also found that Ag, Hg, Cd are poor scatterers of slow neutrons detected by silver. It is interesting that Ag shows small scattering in these experiments because the detection took place by means of silver and that Hg and Cd show small scattering because they

[18] J. R. Dunning, G. B. Pegram, G. A. Fink and D. P. Mitchell, Phys. Rev. **48,** 265 (1935).

[19] A. C. G. Mitchell and E. J. Murphy, Phys. Rev. **48,** 653 (1935). Cf. also Bull. Am. Phys. Soc. **11,** paper 27, Feb. 4, 1936.

have large absorption cross sections.[18] The observation of scattering by a material having large absorption is difficult because the neutrons entering the material are absorbed before they can be scattered and it is possible that to some extent the failure to observe scattering in good absorbers is due to this cause. The absence of observed scattering in the region of strong absorption is therefore not a surprise, particularly in view of the relatively small numbers of neutrons available for experimentation. It seems more significant, however, that strong absorbers do not show, so far, strong scattering in any velocity region because, according to the Fermi-Bethe theory, the scattering cross section should be large in a wide range of energies. The experimental evidence says little about the ratio of scattering to absorption near resonance. It indicates that this ratio is less than 1/10 in most cases. It is impossible, therefore, to ascertain definitely the ratio $\Gamma_s/\Gamma_r$ until more detailed experimental data are available. According to Table I the condition $\Gamma_s/\Gamma_r < 1/10$ can be satisfied in many ways up to velocities of over 1000 ev.

### (b) Magnitude of interaction with internal states and probability of internal state in required region

In Table I arbitrary assignments of values of $\Gamma_s, \Gamma_r$ were made. It will be noted that at low neutron velocities the desired large capture cross sections are easily obtained through relatively wide bands having a half-value breadth $2\Gamma_r$. Keeping $\Gamma_r$ fixed one can decrease the interaction energy $\bar{H}$ to 10,000 ev for $h\Gamma_r = 1$ volt, $E_r = 1$ volt and still have a cross section of $1000 \times 10^{-24}$ cm$^2$ in an energy range up to 2 volts. In some cases relatively weak radiative transitions will come into consideration leading to smaller $\Gamma_r$. For such transitions $\bar{H}$ need not be as large as 10,000 ev in order to have cross sections of $1000 \times 10^{-24}$ cm$^2$ in the resonance region. For the large energies involved in nuclear structure it is reasonable to expect interaction energies of the order of 10,000 volts between practically any pair of levels not isolated by a selection rule and interaction energies of the order 100,000 volts between a great many levels.

There are about ten elements among 72 observed that show cross sections of more than $500 \times 10^{-24}$ cm$^2$. Allowing for the fact that there are more isotopes than elements it appears fair to say that the chance of such an anomalous cross section is about 1/20. One can try to account for these solely by the low velocity regions which exist for any resonance level, thus probably overestimating the necessary number of levels. In order that $\sigma_c > 500 \times 10^{-24}$ cm$^2$ at 1/40 volt for a nucleus having $r_0 = 10^{-12}$ cm and $h\Gamma_r = 10$ volts the resonance region must be not farther than at $|E_r| = \bar{H}/420$ from thermal energies by Eq. (35). We do not wish $h\Gamma_s$. at thermal energies to be greater than 0.1 volt so as not to have too much scattering and therefore $\bar{H}$ should be below $2 \times 10^5$ ev at the higher $E_r$. Thus $E_r$ should be kept below about 460 volts in order to give the large capture cross sections for $E = 1/40$ volt together with small scattering. A level below ionization will also be effective in producing an increased absorption. The observed number of large absorptions corresponds in this way to one level in 900 volts for 1/20 of nuclei or one level every 18,000 volts for a single nucleus. In addition some cross sections will be caused by direct resonance. Just how many is uncertain but it is clear that such effects exist in Cd, Ag, Au, Rh, In.

The average spacing between the $\gamma$ ray levels of Th C$''$ as given in Gamow's book is about 100,000 volts and this is apparently the order of magnitude usual for $\gamma$-ray levels of radioactive nuclei. There appears to be no reason why the energy levels found through the analysis of $\gamma$-ray spectra should include all the nuclear levels and there may be as many as one level in 20,000 of a land that may be responsible for coupling to incident neutrons. It should be remembered here that some of the levels may be active even though the coupling is weak so that more possibilities are likely to matter than for the $\gamma$-rays of radioactive nuclei.

For a complicated configuration of particles it seems reasonable to consider a total number of 100 possible levels per configuration because protons and neutrons can be combined separately to give different states. On this basis we deal with an average spacing between configurations of about 2 MEV which is not excessively small. It is, of course, impossible to prove anything definitely without calculating the levels; this appears to be premature at present on account of uncertainties in nuclear theories.

### (c) Existence of two maxima

According to the calculation given above it is expected that there will be in general two maxima one of which should be at resonance and another at $v = 0$. According to the experiments of Rasetti, Segre, Fink, Dunning and Pegram[20] the $1/v$ law is not obeyed by Cd but is obeyed by Ag. Cadmium has therefore a resonance region close to thermal velocities. In the classification of Fermi and Amaldi[21] this region must be affected by the $C$ group since absorption measurements by the Li ionization chamber which

[20] F. Rasetti, E. Segre, G. Fink, J. R. Dunning and G. B. Pegram, Phys. Rev. **49,** 103 (1936).

[21] E. Amaldi and E. Fermi, Ricerca scientifica **2,** 9 (1936); E. Fermi and E. Amaldi, Recerca scientifica **2,** 1 (1936).

was used in these experiments agree for most elements with the measurements of Fermi and Amaldi on the $C$ group.[22] The verification of the $1/v$ law for Ag by the rotating wheel indicates that in Ag the resonance band is located above thermal energies. This conclusion is in agreement with the smallness of the temperature effect for the $A$ neutrons detected by silver which was recently established by Rasetti and Fink.[23] Since Rh behaves similarly to Ag in these temperature experiments Rh also has a resonance region above thermal velocities. Fermi and Amaldi have evidence that $D$ neutrons, which affect Rh, are different from $A$ neutrons which affect Ag. It is very probable that both of these groups lie above the thermal region and they may reasonably cover a range of 30 volts inasmuch as the $B$ group overlaps weakly with both $A$ and $D$.

According to Szilard[6] In shows strong selective effects outside the $C$ group and according to Fermi and Amaldi[21] the same period of In (54 min.) detects the $D$ group. The number of neutrons in the groups is presumably in the ratios $C/80 = B/20 = D/15 = A/1$. One could try to conclude that the order of increasing energies is $C, B, D, A$ on the assumption that the number of neutrons increases towards low energies. Such a conclusion is dangerous because little is known about the velocity distribution, because within each group there may be several bands at different velocities, and also because the number of expected neutrons in a group should depend on its width. Temperature effects show that practically all captures increase as the energy is lowered. The effects are strongest[24] for Cu, V are smaller for Ag, Dy weaker for Rh and weakest for I. The absorption coefficient for $C$ neutrons is, however, larger for Rh than for Ag indicating that the smaller temperature effect in Rh is due to a relatively greater importance in it of a band above thermal energies. All temperature effects agree in indicating the presence of a region in which the $l/v$ law is followed approximately but again no definite conclusion about the order of bands is possible. The low temperature effect in I would tend to indicate that its absorption region is high and detection-absorption experiments on I and Br tend to indicate that their bands are isolated from the others discussed here; perhaps these isotopes have resonance bands at higher energies. A new band was recently discovered in Au by Frisch, Hevesy and McKay[25] which represents strong absorption on a weaker background. The large number of selective effects

[22]Unpublished results of F. Rasetti. We are very grateful to Professor Rasetti for informing us of these results.

[23]F. Rasetti and George A. Fink, Bull. Am. Phys. Soc. **11,** Paper 28, Feb. 4, 1936.

[24]P. B. Moon and R. R. Tillman, Proc. Roy. Soc. **A153,** 476 (1936).

[25]0. R. Frisch, G. Hevesy and H. A. C. McKay, Nature **137,** 149 (1936).

observed makes the present explanation reasonable and the existence of a region of low energies in which the absorption decreases with energy is seen to fit in well with expectation.

### (d) Other possibilities

One may consider weak long range forces as a possible explanation of the same phenomenon. Potentials of the order of neutron energies in a region comparable with the neutron wave-length would produce strong effects on absorption and scattering. For thermal energies the wave-length is of atomic dimensions and one would therefore expect the binding energy of deuterium compounds to be different from that of hydrogen compounds by an amount comparable to 1/40 volt if such potentials were present. Such energy differences do not exist. It would be possible to devise potentials which fall off sufficiently rapidly with distance to make the interaction potential negligible for chemical binding and which would cover a total region appreciably larger than the nucleus. Such hypotheses seem improbable without additional argument. Besides special relations between the phase integrals through the nuclear interior and the part of the range of force outside the nucleus would have to be set up in order to make absorption large and scattering small. It is improbable that the large number of bands could be accounted for by any single particle picture.

Forces between electrons and neutrons even though they may exist are not likely to have much to do with the bands. Thus it has been shown by Condon[26] that electron neutron interactions would give rise to scattering cross sections varying roughly as the square of the atomic number $Z$ on the assumption that the electron-neutron forces alone are responsible for the scattering. Forces inside the nucleus must also be supposed to contribute to the phase shifts responsible for scattering. Since these forces also vary with $Z$ one could obtain a more complicated dependence of the scattering cross section by suitably adjusting the nucleus-neutron and electron-neutron potentials. On such a picture one could try to account for sharp resonances by making the electron neutron interaction repulsive. However, Condon's calculation shows that isotope shifts would be also produced by these interactions. It is improbable that the isotope shift is due solely to neutron-electron interaction because the deviation from the inverse square law inside the nucleus due to smearing out of protons produces a considerably larger effect than the observed shift. But it would also be unreasonable to try to

[26] E. U. Condon, in press. We are indebted to Professor Condon for showing us his manuscript before publication.

combine the proton and neutron effects in the nucleus so as to have each large but their difference small. It is therefore probable that the electron-neutron interaction is not much larger than that which corresponds to the observed isotope shift. Since the density of the Fermi-Thomas distribution varies for small $r$ as $r^{-3/2}$ the effective potential acting on the neutron will become high for small $r$. However, calculation shows that it is not high in a wide enough region to account for sharp resonances if the limitation due to the isotope shift is considered.

Bombardment of light nuclei with charged particles has also shown the existence of resonances. Thus there are resonances[27] for the emission of $\gamma$-rays in proton bombardment of Li, C, F and similarly there are the well known resonances in disintegrations produced by $\alpha$ particles. Experimental methods have not been very suitable so far for the detection of resonance regions on account of the scarcity of monochromatic sources and the necessity of using thin films. In Li protons are apparently able to produce $\gamma$-rays in two ways; by resonance at 450 kv and by another process at higher energies. In fluorine there are several peaks. In carbon there was an indication of the main resonance peak being double. It appears possible that many more levels will be detected inasmuch as neutron experiments indicate a high density of levels. Calculations on the radiative capture of carbon under proton bombardment[15] lead to a higher yield than is observed by a factor of several thousand. In these calculations the capture was supposed to occur by a jump from the $p$ state of the incident wave to an $s$ state of the $N^{13}$ nucleus. The calculated half-value breadth due to proton escape from the quasi-stationary $p$ level was of the order $h\Gamma_F \sim 10,000$ ev and thus much larger than the width due to radiation damping. The yield in thick targets under these conditions is nearly independent of the special value of $h\Gamma_F$. It is clear from the formulas given here for neutron capture that one can decrease the theoretically expected yield either by ascribing the capture to a transition having a small probability of radiation (small $\Gamma_r$) such as would correspond to quadruple or other forbidden transitions or else by using an intermediate state of excitation of the nucleus with a small transition probability to the incident state of the proton (small $\Gamma_s$). In the latter case this transition probability would have to be made so small as to have $\Gamma_s < \Gamma_r$ and the observed width of resonance would have to be ascribed to experimental effects. If $\Gamma_s < \Gamma_r$ the thick target yield depends on $\Gamma_s$ and is proportional to it for small $\Gamma_s$. The apparent disagreement between theory

[27] L. R. Hafstad and M. A. Tuve, Phys. Rev. **48,** 306 (1935); P. Savel, Comptes rendus **198,** 1404 (1934), Ann. de physique **4,** 88 (1935).

and experiment previously found for carbon is thus not alarming from the many-body point of view and supports the belief that excitation states of the nucleus have often to do with the simultaneous excitation of more than one particle.

The excitation states responsible for the neutron absorption bands make it possible for a fast neutron to lose energy by inelastic impact with the nucleus. Estimates show that the cross sections for such processes are likely to be small when energy losses are high. The cross section is estimated to be

$$\frac{\Lambda_1}{4\pi\Lambda_2}\frac{\bar{H}^2 r_0^2}{U^2\cos^4 Kr_0},$$

where $\Lambda_1, \Lambda_2$ are, respectively, neutron wavelengths in the incident and final states. For large energy losses $\Lambda_2 \gg \Lambda_1$ and only a small effect need be expected. The excitation levels responsible for neutron capture will give small values $\Lambda_1/\Lambda_2$. Excitation levels located lower are more favorable and probably the excitation of Pb to about 1.5 MEV has to do with such a possibility.[28]

We are very grateful to Professors R. Ladenburg and F. Rasetti for interesting discussions of the experimental material.

# Appendix I

## Variation of damping constant with energy and Dirac's frequency shift

Eq. (6) of the text lead to [cf. Eqs. (126′) to (129') of reference in footnote (12)]

$$(\nu_{s0}-\nu_0+i\gamma)(\nu-\nu_0+i\gamma) = |A_{s0}|^2 + (\nu_{s0}-\nu_0+i\gamma)[\Sigma'|A_s|^2\frac{1-e^{2\pi(\nu_0-\nu_s-i\gamma t}}{\nu_s-\nu_0+i\gamma}$$

$$+\Sigma|B_r|^2\frac{1-e^{2\pi(\nu_0-\nu_r-i\gamma)t}}{\nu_r-\nu_0+i\gamma}\Bigg] \qquad (28)$$

which determines $\Gamma$ by comparison with (7) $[\Gamma \gg \gamma]$.

It is by no means natural that this equation can be satisfied because the right side depends on $t$. If the $A_s$ as well as the $B_r$ were all essentially equal and if $\gamma$ were great in comparison with the frequency differences of consecutive levels the sums could be transformed, into integrals in the well-known way[10–12] so that (9) as well as (6) would follow. We shall attempt here a more exact procedure.

Consider the $\Sigma'_s$ in the square brackets. It is natural to divide the range of $\nu_s$ into two parts: one for which $|\nu_s-\nu_0| > a \gg \gamma$ and one for which $|\nu_s-\nu_0| \leq a$.

[28] J. Chadwick and M. Goldhaber, Proc. Roy. Soc. **A151,** 479 (1935).

Since $|A_s|^2/\Delta\nu_s$. changes slowly this quantity will be replaced by a constant in $|\nu_s - \nu_0| \leq a$ and its value may be taken to be that at $\nu_0$ for the evaluation of the contribution of this region. We have then to consider

$$(\overline{|A_s|^2})_{r0} \sum_{\nu_0 - a}^{\nu_0 + a} \frac{1 - e^{2\pi i(\nu_0 - \nu_s - i\gamma)t}}{\nu_s - \nu_0 + i\gamma}. \tag{39$a$}$$

An exact evaluation of this sum is not simple because $\gamma$ and $\Delta\nu$ are of the same order of magnitude and the replacement of (39a) by an integral is somewhat objectionable. This point has never been completely cleared up and we have only qualitative arguments in favor of the correctness of the replacement of (39a) by an integral. For $\nu_s - \nu_0$ of the order of a few $\gamma$ such a replacement is indeed meaningless but fortunately this region is not vital for $t \ll 1/\gamma$ since the numerator of (39a) is then small. For larger $|\nu_s - \nu_0|$ the terms of (39a) vary more smoothly and finally they become rapidly oscillating for $|\nu_s - \nu_0|t \gg 1$ which can be satisfied simultaneously with $t \ll 1/\gamma$ provided $a \gg \gamma$. The smallness of $\gamma$ is thus not as serious as might appear from the fact that $\gamma/\Delta\nu \sim 1$. It should also be observed that the treatment of Rice[8] using real eigenwerte for a single one-dimensional continuum is in agreement with replacing (39a) by an integral. The result of doing so is given by (9).

In addition one has the contribution of $|\nu_s - \nu_0| > a$. This integral can be treated neglecting $\gamma$ because it is of interest to evaluate $\gamma$ only to quantities of order $\gamma/(\nu - \nu_0)$ and because the discussion is supposed to apply only to $\gamma t \ll 1$. This integration gives

$$\left( \int_0^{\nu_0 - a} + \int_{\nu_0 + a}^{\infty} \right) \frac{\overline{|A_s|}^2}{\Delta\nu_s} \frac{d\nu_s}{\nu_s - \nu_0} \tag{39$b$}$$

which means that the principal value of the $\int$ is understood. Similarly one obtains a contribution due to $|B - r|^2$. These two integrals give the Dirac frequency shift which is included in Eq. (15).

As stated in the text the difference between $\Gamma'$ and $\Gamma$ does not affect the absorption for $t \gg 1/\Gamma$. Thus for the initial condition given by Eq. (4)

$$|b_r|^2 = \frac{|B_r|^2 |A_{s0}|^2}{(\nu - \nu_0)^2 + \Gamma^2} \left\{ \frac{1 + e^{-4\pi\gamma t} - 2e^{-2\pi\gamma t} \cos 2\pi(\nu_r - \nu_0)t}{(\nu_r - \nu_0)^2 + \gamma^2} \right.$$
$$\left. + \frac{1 + e^{-4\pi\Gamma' t} - 2e^{-2\pi\Gamma' t} \cos 2\pi(\nu_r - \nu_0)t}{(\nu_r - \nu) + \Gamma'^2} + \text{cross product term} \right\}.$$

Only the first fraction in the curly brackets contributes to the steady increase of $\Sigma|b_r|^2$ in times $\gg 1/\Gamma$. Its contribution is

$$\frac{\pi |A_{s0}|^2 \overline{|B_r|^2}}{\gamma[(\nu - \nu_0)^2 + \Gamma^2]\Delta\nu_r} (1 - e^{-4\pi\gamma t}).$$

The last factor is for practical purposes $4\pi\gamma t$. The second and third terms in the curly bracket give terms $\exp(-4\pi\Gamma' t)$, $\exp(-2\pi\Gamma' t)$ and constants. The first two kinds die off and the last kind represents the effect of transients which do not matter in the long run, so that for times not too large as compared with $1/\gamma$ and yet great as compared with $1/\Gamma$ one may consider the rates of change of $\Sigma|b_r|^2$ and of $\Sigma'|a_s|^2$ to be $4\pi\gamma\Gamma_r/\Gamma$ and $4\pi\gamma\Gamma_s/\Gamma$. These are the results used in the text.