G. Gamow, Phys. Rev., **70** 572 **1946**

Expanding Universe and the Origin of Elements

G. GAMOW
The George Washington University, Washington, D.C,
September 13, 1946

It is generally agreed at present that the relative abundances of various chemical elements were determined by physical conditions existing in the universe during the early stages of its expansion, when the temperature and density were sufficiently high to secure appreciable reaction-rates for the light as well as for the heavy nuclei.

In all the so-far published attempts in this direction the observed abundance-curve is supposed to represent some equilibrium state determined by nuclear binding energies at some very high temperature and density [1] [2] [3]. This point of view encounters, however, serious difficulties in the comparison with empirical facts. Indeed, since binding energy is, in a first approximation, a linear function of atomic weight, any such equilibrium theory would necessarily lead to a rapid exponential decrease of abundance through the entire natural sequence of elements. It is known, however, that whereas such a rapid decrease actually takes place for the first half of chemical elements, the abundance of heavier nuclei remains nearly constant [4]. Attempts have been made[2] to explain this discrepancy by the assumption that heavy elements were formed at higher temperatures, and that their abundances were already "frozen" when the adjustment of lighter elements was taking place. Such an explanation, however, can be easily ruled out if one remembers that at the temperatures and densities in question (about 10^{10}°K, and 10^6 g/cm^3) nuclear transformations are mostly caused by the processes of absorption and re-evaporation of free neutrons so that their rates are essentially the same for the light and for the heavy elements. Thus it appears that the only way of explaining the observed abundance-curve lies in the assumption of some kind of unequilibrium process taking place during a limited interval of time.

The above conclusion finds a strong support in the study of the expansion process itself. According to the general theory of expanding universe [5], the time dependence of any linear dimension l it is given by the formula

$$\frac{dl}{dt} = \left(\frac{8\pi G}{3}\rho l^2 - \frac{G^2}{R^2}\right)^{1/2} \tag{1}$$

where G is the Newton constant, ρ the mean density, and R (real or imaginary) a constant describing the curvature of space. It may be noticed that the above expression represents a relativistic analog of the familiar classical formula

$$v = \left(2 \cdot \frac{4\pi l^2}{3}\rho \cdot \frac{G}{l} - 2E\right)^{1/2} \tag{2}$$

for the inertial expansion-velocity of a gravitating dust sphere with the total energy E per unit mass. The imaginary and real values of R correspond to an unlimited expansion (in case of superescape velocity), and to the expansion which will be ultimately turned into a contraction by the forces of gravity (subescape velocity). To use some definite numbers, let us consider in the present state of the universe (considered as quite uniform) a cube containing, say, 1 g of matter. Since the present mean density of the universe is $\rho_{\text{present}} \approx 10^{-30}$ g/cm^3, the side of our cube will be; $l_{\text{present}} \approx 10^{10}$ cm. According to Hubble [6], the present expansion-rate of the universe is 1.8×10^{-17} cm/sec, per cm, so that $(dl/dt)_{\text{present}} \approx 1.8 \times 10^{-7}$ cm/sec. Substituting the numerical values in (1) we obtain

$$1.8 \times 10^{-7} = (5.7 \times 10^{-17} - G^2/R^2)^{1/2}, \tag{3}$$

showing that at the present stage of expansion the first term under the radical (corresponding to the potential energy of gravity) is negligibly small as compared with the second one. For the numerical value of the (constant) radius of curvature we get from (3): $R = 1.7 \times 10^{17}\sqrt{-1}$ cm or about 0.2 imaginary light year.

In the past history of the universe, when l was considerably smaller, and ρ correspondingly larger, the first term in (1) was playing an important role corresponding physically to the slowing-down effect of gravity on the original expansion. The transition from the slowed down to the free expansion took place at the epoch when the two terms were comparable, i.e., when I was about one thousandth of its present value. At this epoch the gravitational clustering of matter into stars, stellar clusters, and galaxies, probably must have taken place [7].

Applying our formula (2) with $G^2/R^2 = -3.3 \times 10^{-14}$ to the earlier epoch when the average density of masses in the universe was of the order of 10^5 g/cm^3 (as required by the conditions for the formation of elements), we find that at that time $l \approx 10^{-2}$ cm, and $dl/dt \approx 0.01$ cm/sec. This means that at *the epoch when the mean density of the universe was of the order of* 10^5 *g/cm*^{3}*, the expansion must have been proceeding at such a high rate, that this high density was reduced by an order of magnitude in only about one second.* It goes without saying that one must be very careful in extrapolating the expansion formula to such an early epoch, but, on the other hand, this formula represents nothing more than the statement of the law of conservation of energy in the inertial expansion against the forces of gravity.

Returning to our problem of the formation of elements, we sec that *the conditions necessary for rapid nuclear reactions were existing only for a very short time,* so that it may be quite dangerous to speak about an equilibrium-state which must have been established during this period. It is also interesting to notice that the calculated time-period during which rapid nuclear transformations could have taken place is considerably shorter than the β-decay period of free neutrons which is presumably of the order of magnitude of one hour. Thus if free neutrons were present in large quantities in the beginning of the expansion, the mean density and temperature of expanding matter must have dropped to comparatively low values *before* these neutrons had time to turn into protons. We can anticipate that neutrons forming this comparatively cold cloud were gradually coagulating into larger and larger neutral complexes which later turned into various atomic species by subsequent processes of β-emission. From this point of view the decrease of relative abundance along the natural sequence of elements must be understood as being caused by the longer time which was required for the formation of heavy neutronic complexes by the successive processes of radiative capture. The present high abundance of hydrogen must have resulted from the competition between the β-decay of original neutrons which was turning them into inactive protons, and the coagulation-process through which these neutrons were being incorporated into heavier nuclear units.

It is hoped that the further more detailed development of the ideas presented above will permit us to understand the observed abundance-curve of chemical elements giving at the same time valuable information concerning the early stages of the expanding universe.

References

[1] v. Weizaäcker, Physik. Zeits., **39. 633** (1938).

[2] Chandrasekhar and Henrich, Astrophys. J. **95,** 288 (1942).

[3] G. Wataghin, Phys. Rev. **66**, 149 (1944).

[4] Goldschmidt, Verleilung der Elemente (Oslo, 1938).

[5] R. Tolman, Relativity, Thermodynamics and Cosmotoly (Oxford Press, New York. 1934).

[6] Hubble, The Realm of the Nebulos (Yale University Press. New Haven, 1936).

[7] G. Gamow and E. Teller, Phys. Rev. **55**, 654 (1939).