Meson Corrections in the Theory of Beta Decay

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(Submitted to JETP editor June 8, 1955)
(Received November, 1955)

In a recent note, Finkelstein and Moszkowski[1] discuss the effect of strong coupling between nucleons and pions, on the beta decay of nucleons.

Using the language of Feynman diagrams these authors consider, in addition to the fundamental process (Fig. 1 a), another process involving the virtual emission of one π^0 meson (Fig. 1 b). The calculation is carried out on the basis of the hypotheses under which Chew[2] discusses nuclear forces and the creation and scattering of mesons, and Friedman[3] discusses the anomalous magnetic moment of the nucleon: the nucleon is assumed to be infinitely heavy, and integrals over the momenta of virtual mesons are cut off at a specified value $p_{\max}$. From comparison with experiment it is found that $p_{\max}$ is close to M_C. A system with charge symmetry is considered so that the operator τ_3 enters into the expression for the coupling of nucleons to π^0.

Our notation will be very similar to that of Sachs[4]. Let P_1 be the probability that there is a virtual π^0 meson around the nucleon, compared with the probability that the nucleon is "bare", i.e., has no mesons around it.[2] The beta–decay coupling constants of a bare nucleon are denoted by g'_F (Fermi interaction with S and V interaction types) and g'_T (Gamow-Teller interaction with T and A interaction types). In the case of a real nucleon surrounded by a meson cloud, the same constants as obtained experimentally are denoted by g_F and g_T, and the ratio g_T^2/g_F^2 equals R; from experiment[5] $R = 1.75$, i.e., $R > 1$. The results of Finkelstein and Moszkowski[1] then

[1] Translated from russian ZETF 29, 698 (1955)
[2] In the notation of Finkelstein and Moszkowski[1], $P_1 = 3\delta$.

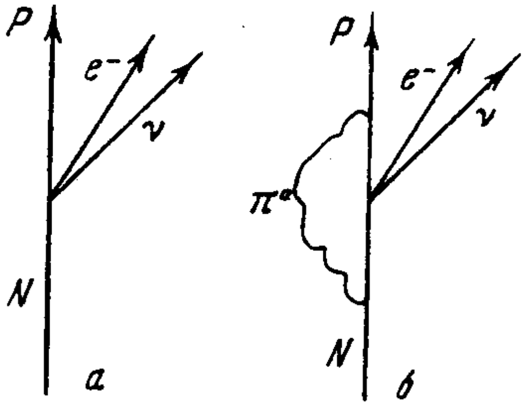

Рис. 1:

appear as follows:

$$g_F = g'_F\,(1 - P_1)\,; \qquad g_T = g'_T\left(1 + \frac{1}{3}\,P_1\right),$$

$$\frac{g'_T}{g'_F} = \frac{1 - P_1}{1 + \frac{1}{3}P_1} \cdot \frac{g_T}{g_F} = \frac{1 - P_1}{1 + \frac{1}{3}P_1} \cdot \sqrt{R} \cong 1.$$

While agreeing with their principal conclusion, which is that $g'_T/g'_F \cong 1$, we wish to make a few comments regarding the calculation.

1) The calculation does not follow for renormalization of the nucleon wave function as a result of the possibility of creating virtual mesons. By Feynman's[6] method renormalization is represented as an added self–energy contribution at the free ends of the diagrams (Figs. 2a and 2b). In these diagrams it is necessary to consider not only neutral but also charged pions (a charged meson in the vertex part of Fig. 1b obviously gives a vanishing result). It is easily seen that on the basis of the hypotheses which were adopted by Finkelstein and Moszkowski, and taking renormalization into account, the correct result is

$$g_F = g'_F\;\frac{1 - P_1}{1 + 3P_1}; \qquad g_T = g'_T\;\frac{1 + ^1/_3 P_1}{1 + 3P_1}.$$

Thus the correction does not affect the ratio g'_T/g'_F. However, the correction may be significant when comparing the absolute value of $g_\beta = g'_F = g'_T$

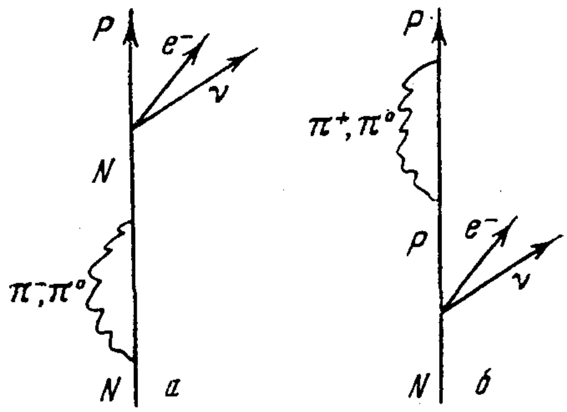

Рис. 2:

with the coupling constant g_μ that determines the probability of $\mu^\pm = e^\pm + 2\nu$ decay; Finkelstein and Moszkowski[1] had this comparison in mind. A numerical result in accordance with both the latter authors and Chew is that $1 + 3P_1 = 1.7$.

2) Finkelstein and Moszkowski[1] do not consider the possibility of beta conversion of mesons, i.e., processes such as $\pi^\pm = \pi^0 + e^\pm + \nu$, which may be represented by diagrams like Fig. 3.

If the probability of $\pi^\pm \to \pi^0$ beta decay is the same as for mirror nuclei, the contribution from the process represented by the diagram of Fig. 3 is of the same order as the meson correction in diagram 1b, the absence of experimental indication of the $\pi^\pm = \pi^0 + e^\pm + \nu$ process does not contradict the hypothesis, since as a result of strong competition from $\pi^\pm = \mu^\pm + \nu$ decay, the former process may occur in only the small fraction 10^{-10} of the decays of free charged pions.[3]

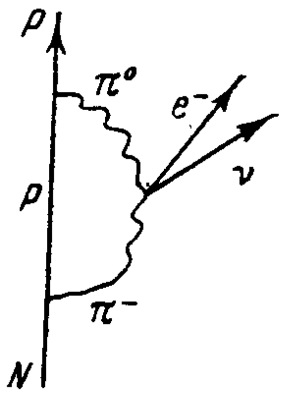

Рис. 3:

The question of the beta decay of pions has been thoroughly examined

[3]In the case of virtual mesons (Fig. 3) $\pi \to \mu\nu$ decay is obviously forbidden by energy considerations.

by one of the authors of the present note[7]. It was shown that the S and T beta–interaction types do not give beta decay of a pion in the approximation to which the theory of isotopic invariance is valid. Thus the possibility of neglecting diagrams such as Fig. 3, as is done by Finkelstein and Moszkowski[1], is actually associated with the representation of the Fermi beta interaction by the scalar S rather than by the vector type V, which is in accordance with the latest experimental findings[8].

It is of no practical significance but only of theoretical interest that in the case of the vector interaction type V we should expect the equality

$$g_{F(V)} \equiv g'_{F(V)}$$

to any order of the meson–nucleon coupling constant, taking nucleon recoil into account and allowing also for interaction of the nucleon with the electromagnetic field, etc. This result might be forseen by analogy with Ward's identity for the interaction of a charged particle with the electromagnetic field; in this case virtual processes involving particles (self–energy and vertex parts) do not lead to charge renormalization of the particle.

3) We have calculated the meson corrections by invariant perturbation theory, using pseudoscalar coupling between pion and nucleon (coupling constant g).

In the expression for the self–energy and vertex parts a convergence factor $C(k^2)$ was introduced, where k is the momentum 4–vector of a virtual meson:

$$\sum = \int \tau_i \gamma_5 \left(\hat{p} - \hat{k} - m\right)^{-1} \gamma_5 \tau_i \left(k^2 - \mu^2\right)^{-1} C\left(k^2\right)\ d^4k,$$

$$\Gamma_0 = \int \tau_i \gamma_5 \left(\hat{p} - \hat{k} - m\right)^{-1} \tau_+ \hat{O}\ \left(\hat{p} - \hat{k} - m\right)^{-1} \gamma_5 \tau_i\ \left(k^2 - \mu^2\right)^{-1} C\left(k^2\right)\ d^4k,$$

$$C\ \left(k^2\right) = \lambda^2 / \left(\lambda^2 - k^2\right).$$

In addition to integration over momentum space (d^4k), a summation was carried out over the index i of meson isotopic spin. The beta process operator was represented as the product of the operator τ_+ which transforms a neutron into a proton, and the operator $\hat{O}$ which consists of the γ matrix ($\hat{O} = 1$ for S; $\hat{O} = \gamma_i \gamma_k$ for the T interaction type). The meson mass renormalization term was calculated from Σ in the usual way: m is the mass of the nucleon, μ is the mass of the meson, and terms of the order of μ/m are neglected. Taking renormalization of the wave functions into account, the result becomes

$$g_{F(S)} = g'_{F(S)}\ \left[1 - \frac{g^2}{32\pi^2}\ \left(5 \ln\left(\frac{\lambda^2}{m^2}\right) - \frac{1}{2}\right)\right],$$

$$g_{GT(T)} = g'_{GT(T)} \left[1 - \frac{g^2}{32\pi^2} \left(3\ln\left(\frac{\lambda^2}{m^2}\right) + \frac{1}{2}\right)\right].$$

For small g and large λ a relativistic calculation also gives a decrease of g'_{GT}/g'_F compared with g_{GT}/g_F

In the present state of the theory of interactions of pions with nucleons one cannot give preference to a relativistic perturbation theory calculation of Finkelstein and Moszkowski[1], who employ coupling constants derived from experimental data.

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