

## The Acceleration of Electrons by Magnetic Induction

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### Abstract

Apparatus with which electrons have been accelerated to an energy of 2.3 Mev by means of the electric field accompanying a changing magnetic field is described. Stable circular orbits are formed in a magnetic field, and the changing flux within the orbits accelerates the electrons. As the magnetic field reaches its peak value, saturation of the iron supplying flux through the orbit causes the electrons to spiral inward toward a tungsten target. The  $x$ -rays produced have an intensity approximately equal to that of the gamma-rays from one gram of radium; and, because of the tendency of the  $x$ -rays to proceed in the direction of the electrons, a pronounced beam is formed.

In the past the acceleration of electrons to very high voltage has required the generation of the full voltage and the application of that voltage to an accelerating tube containing the electron beam. No convenient method for repeated acceleration through a small potential difference has been available for electrons, although the method has been highly successful in the cyclotron for the heavier positive ions at velocities much less than the velocity of light.

Several investigators<sup>2 3 4 5</sup> have considered the possibility of using the electric field associated with a time-varying magnetic field as an accelerating force. This is a very attractive possibility because the magnetic field can be used to cause a circular or spiral orbit for the electron while the magnetic

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<sup>1</sup>On leave at the General Electric Company Research Laboratory.

<sup>2</sup>G. Breit and M. A. Tuve, Carnegie Institution Year Book (1927-28) No. **27**, p. 209.

<sup>3</sup>R. Wideröe, Arch. f. Electrotechnik **21**, 400 (1928).

<sup>4</sup>E. T. S. Walton, Proc. Camb. Phil. Soc. **25**, 469-81 (1929).

<sup>5</sup>W. W. Jassinsky, Arch. f. Electrotechnik **30**, 500 (1936).

flux within the orbit increases and causes a tangential electric field along the orbit. The energy gained by the electron in one revolution is about equal to the instantaneous voltage induced in one turn of a wire placed at the position of the orbit. Since the electron can make many revolutions in a short time, it can gain much energy. The comparatively small momentum of a high energy electron requires correspondingly small values of  $Hr$  for high energy orbits. For example, the energy of an electron when  $v \sim c$  is  $KE = 3 \times 10^{-4} Hr - 0.51$  million electron volt. Thus with  $H = 3000$  oersteds and  $r = 5$  cm, the energy of the electron would be about 4 Mev, and the orbit could be held between the poles of a small magnet.

Because of the experimental experiences of previous investigators<sup>1-3</sup> with this method of acceleration, a rather detailed study of the focusing to be expected was made, and it is presented in the paper immediately following this one. With the results of this theoretical investigation to guide the design, it was possible to make an induction accelerator which produced  $x$ -rays of 2.3 Mev<sup>6 7</sup>. Briefly, in the focusing theory it is shown that:

1. The electrons have a stable orbit, "equilibrium orbit," where

$$\phi_0 = 2\pi r_0^2 H_0. \quad (1)$$

$\phi_0$  is the flux within the orbit at  $r_0$ , and  $H_0$  is the magnetic field at  $r_0$ . Both  $\phi_0$  and  $H_0$  are increased during the acceleration process. This flux condition holds for all velocities, of the electrons, and it shows that if a maximum flux density of 10,000 gauss is allowed in the iron then 5000 oersteds is the maximum magnetic field which can be used at the orbit.

2. In the plane of their orbits the electrons oscillate about their instantaneous circles, circles for which  $p = eHr/c$ , with an increasing frequency

$$\omega_r = \Omega(1 - n)^{1/2}, \quad (2)$$

where  $\Omega$  is the angular velocity of the electron in its orbit, and  $\omega_r$  is  $2\pi$  times the radial focusing frequency. The number  $n$  is determined by the radial dependence of the magnetic field, which we take to be of the form  $H \sim 1/r^n$ . For radial focusing  $n$  must be less than unity.

3. Axial oscillations, oscillations perpendicular to the plane of the orbit, have

$$\omega_A = \Omega n^{1/2}. \quad (3)$$

For axial stability  $n$  must be greater than zero. If the beam is to be smaller in an axial direction than it is in a radial direction then  $n > 1/2$ .

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<sup>6</sup>D. W. Kerst, Phys. Rev. **58**, 841 (1940).

<sup>7</sup>D. W. Kerst, Phys. Rev. **59**, 110 (1941).

4. Decrease of the amplitude of both axial and radial vibration occurs because of the increase of the restoring force with increasing magnetic field.

At nonrelativistic velocities the damping is

$$da/a = -dE/4E, \quad (4)$$

where  $dE/E$  is the fractional increase of the kinetic energy of an electron and  $da/a$  is the fractional decrease in amplitude of the oscillation about the instantaneous circle. This holds for both axial and radial oscillations.

5. Instantaneous circles not coincident with the equilibrium orbit shrink or expand toward coincidence:

$$dx/x = -dE/2E, \quad (5)$$

where  $x$  is the displacement of the instantaneous circle from the equilibrium orbit and  $dx$  is the shift of the circle toward the equilibrium orbit while the electron's energy increases by the fraction  $dE/E$ .

Because of the shrinking or expansion of the instantaneous circle of a displaced electron toward the equilibrium orbit and the decrease of the amplitude of oscillation of an electron about its instantaneous circle, it was expected that a cathode or an electron injector placed outside of the equilibrium orbit could shoot in electrons which would miss the injector on successive revolutions around the magnet. Furthermore, since an instantaneous circle for electrons with a constant injection energy exists within the acceleration chamber for a small but finite interval of time while the magnetic field is increasing, a finite amount of charge should be captured in orbits not striking the walls. With constant potential on an injector the electrons first would hit the outer wall of the chamber before the magnetic field had grown large enough to give the electrons a radius of curvature less than the radius of the wall. Then as the field increased it would reach values which give instantaneous circles within the vacuum tube. Eventually the field would be so large that the electrons coming out of the injector strike the walls or spiral around in small circles.

From Eq. (2) it can be seen that the spreading rays from the injector will form injector images  $\pi(1-n)^{-1/2}$  radian apart, since if each ray of the beam oscillates about the instantaneous circle, it also oscillates about the central ray from the injector.

Equations (4) and (5) indicate that for large  $da$  or  $dx$  it is desirable to use a small injection voltage  $E$ , and a high voltage per turn  $dE$ . A shift  $da$  in two revolutions or  $dx$  in one revolution of about 1 mm when the displacement of the injector from the equilibrium orbit is  $x = a = 1.5$  cm

would require  $dE/E \simeq \frac{1}{8}$ . It is not possible to decrease the injection voltage  $E$  indefinitely since scattering increases when this is done, and it may be that the beam would be lost by scattering through an angle greater than that which magnetic focusing can handle. Conservative estimates on the scattering out of a cone of  $7^\circ$  half-angle, which is about the focusing limit, showed that at  $10^{-6}$  mm of Hg air pressure in the tube, a maximum  $Hr = 15,000$  gauss cm, and  $f = 600$  cycles/sec, for the frequency of oscillation of the magnetic field, the injection voltage must exceed 100 volts for less than a 20-percent loss of beam. This then requires that  $dE$  or the voltage per turn at the equilibrium orbit must be at least 12 volts at the time of injection. In the magnet which was built this condition was easily satisfied, for 25 volts per revolution could be attained. There is no trouble with energy loss by the electrons in their long paths through the residual gas in the vacuum tube. Wideröe<sup>2</sup> was aware of the flux requirement (1), but apparently he did not realize that the voltage gain per revolution could not be too small compared with the injection voltage, or that it would be necessary to inject the electrons nearly tangent to the equilibrium orbit.

After the injected electrons have orbits approximately coincident with the equilibrium orbit, they should remain close to  $r_0$ . A simple method was used to shift the equilibrium orbit so that the electrons struck a thin tungsten target.

A portion of the flux through the center of the orbit passed through disks cemented to the center of the pole faces and made of material which saturated easily. As saturation progressed, the electrons had to move inward to a smaller radius of curvature in order that  $\phi = 2\pi r^2 H$ . Eventually the electrons grazed past the target producing thin target  $x$ -radiation.

## Space Charge Effect

It is difficult to estimate the effect of space charge within the beam on the formation of images, since there are two focusing oscillations, axial and radial, in general having different frequencies. However, an estimate can be made of the upper limit of current which the magnetic forces can hold.

At injection the magnetic forces are small so that the estimate should be made with the magnetic field at this time. The electrons at the edge of the beam, which is assumed to be a cylinder of radius  $\Delta$  determined by the distance between the equilibrium orbit and the injector, will experience a repulsive force of

$$F_e = (cQe)/(10\pi r_0 \Delta) \quad (6)$$

dynes.  $Q$  is the number of coulombs in the orbit, and  $r_0$  is the radius of the orbit.

For radial unbalanced magnetic forces we get

$$F_m = e^2 H^2 (1 - n) \Delta / mc^2 \quad (7)$$

dynes acting on the electrons toward  $r_0$ . If  $F_e = F_m$  at the edge of the beam, then this balance will hold within the beam for all other distances  $\delta$  smaller than  $\Delta$  because the  $Q$  of (6) is proportional to  $\delta^2$ . Thus

$$I = 10\pi f e r_0 \Delta^2 H^2 (1 - n) / mc^3. \quad (8)$$

Or in terms of the injection voltage  $E$ ,

$$I = \pi \Delta^2 f E (1 - n) / (15 r_0 c). \quad (9)$$

$I$  is the target current in amperes,  $f$  the frequency of oscillation of the magnetic field, and  $n$  gives the dependence of the magnetic field on  $r$ . Since the value of  $E$  which can be used depends upon the voltage per turn  $dE$ , which is also dependent on the peak magnetic field reached or the output energy, and also upon the orbit shift  $da$  which is required to miss the injector, we get

$$I = 2\pi^3 \Delta^3 f^2 (KE + \frac{1}{2}) (1 - n) / (45 c da \times 10^4), \quad (10)$$

where  $KE$  is the final energy of the electrons in Mev. Inserting into (10) the constants of the machine which was built, we have  $\Delta = 1.5$  cm,  $f = 600$  cycles/sec.,  $KE = 2.3$  Mev, and  $da \simeq 0.1$  cm. The target current should be about 0.03 microampere. This is fairly close to the lower limit of current as estimated from the  $x$ -ray output of the accelerator. This lower limit was found by using the thick target yield of Van Atta and Northrup<sup>8</sup> in the direction of the electron beam for comparison with the  $x$ -ray yield from the induction accelerator. It is found that the target current in the induction accelerator must be greater than 0.02 microampere. It is only possible to find the lower limit of this current since the output is thin target radiation.

A curious behavior which was immediately noticed when the apparatus began to work was that injection voltages much greater than the largest voltage which would allow the orbit to miss the injector could be used. In fact the yield increased greatly as this voltage was raised. This seems understandable on the basis of space charge forces spreading the beam away from the central ray of the injector.

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<sup>8</sup>L. C. Van Atta and D. L. Northrup, Am. J. Roentgen. Rad. Ther. **41**, 633 (1939).

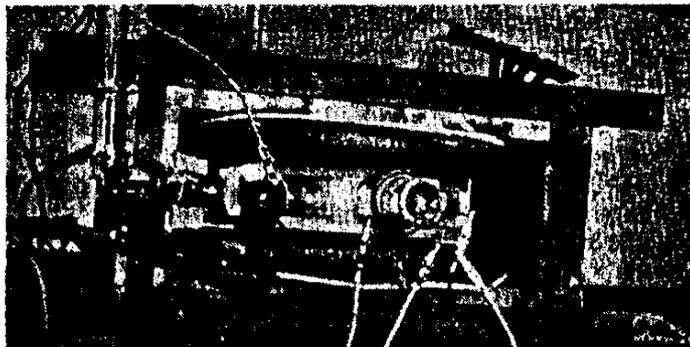


Figure 1: The induction accelerator. The glass doughnut is between pole faces which are held apart by eccentric wedges.

At relativistic energies space charge forces are completely balanced by magnetic self-focusing of the beam, for the electric force on a stray electron at a distance  $\Delta$  from the beam center is

$$e\mathcal{E} = 2\sigma e/\Delta, \quad (11)$$

where  $\sigma$  is the linear charge density in e.s.u./cm. The magnetic attraction due to the main current in the beam is

$$evH/c = (v/c)^2 2\sigma e/\Delta. \quad (12)$$

Thus it is evident that when  $v \rightarrow c$ , the magnetic pull of the beam for a stray electron just equals the electrostatic repulsion. Or, from the point of view of an observer on the electron, the spacing of the fixed number of electrons around the orbit will increase, since as  $v \rightarrow c$  his yardstick becomes a smaller fraction of the circumference of the orbit.

A photograph of the apparatus is shown in Fig. 1.

For the production of the rapidly changing magnetic field, with the proper dependence on the radius, finely laminated iron pole faces were made from 0.003-inch silicon steel sheets. The return magnetic circuit made from the same material was interleaved for mechanical strength; and the roughly circular central pole pieces were formed by stacking with different widths of laminations as shown in Fig. 2. Each pole piece was capped by a disk of radially arranged laminations so that perfect circular symmetry was achieved at the pole surfaces ( $B$ , Fig. 2). The whole pole face was held together by

a thick Transite or asbestos board ring about its perimeter, with cement of water glass and flint dust filling the cracks between the laminations and hardened by baking. The pole caps were held against the pole pieces by eccentric wedges between the Transite rings.

To supply the flux within the orbit which was necessary to hold the electrons out at the equilibrium orbit, two disks about two inches in diameter, each made of pressed iron dust of permeability about eight, were cemented onto the flat center portions of the pole faces. The thickness of these disks was chosen so that the equilibrium orbit was formed at about 7.5 centimeters radius, and the final adjustment of the position of the equilibrium orbit was made by painting a mixture of water glass and iron filings, or the turnings from the compressed iron dust disks, onto the surface of the iron that the reluctance of the gap was correct. Since the iron particles in these disks are separated, the flux density through them is greater than the average flux density through the disks. This causes the saturation which shrinks the equilibrium orbit down to the target.

Because of the large leakage flux the main coil of the magnet had to be highly subdivided. Two hundred strands of No. 20 double enameled wire were made into a cable approximately 10 yards long which was twisted about ten times and which had its inside wires interchanged with its outside wires several times. This was formed into a 10-turn coil with about  $\frac{1}{32}$  inch of insulation between turns. Two such 10-turn coils were made and fitted with large lugs of 1-inch copper tubing. The coils were wrapped with cotton tape, dipped in Bakelite varnish and baked. Figure 3 shows the circuit with the coils connected to a total of eighty 5-microfarad Pyranol condensers which were rated at 660 volts a.c. Energy was supplied to this resonating circuit by a 2-turn primary of stranded enameled wire around the pole pieces. The r.m.s. electromotive force in this primary could be run up above 100 volts, but 60 to 80 volts were usually used in operation since this was sufficient to saturate the powdered iron and to collapse the orbit. Power was supplied by a 4-kilowatt 600-cycle alternator driven by a direct-current motor with an adjustable speed.

To determine how the magnetic field decreased with  $r$  a small search coil was arranged so that it could be held between the poles at different radial distances from the center. The e.m.f. from this search coil was bucked against the e.m.f. from a voltage divider connected across a 1-turn coil about the leg of the magnet, equality of e.m.f. being determined by no deflection of the beam of an oscillograph. Since the wave form of the voltage on the search coil differed slightly from the wave form from the coil around the core, balance could be obtained only at one phase, and the phase of

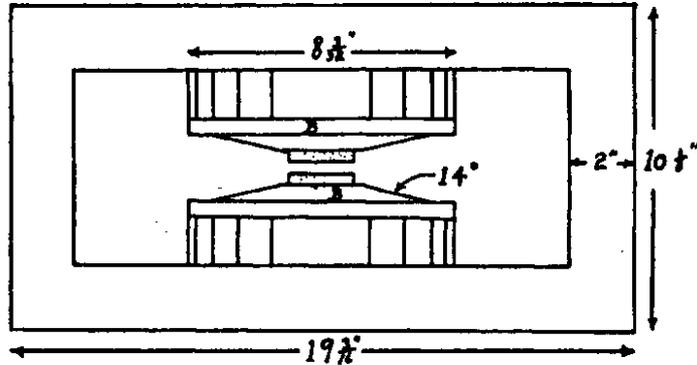


Figure 2: Dimensions of the magnet. Parts *B* are the pole caps made with all laminations placed in a radial direction. The iron dust disks supply the central flux. Outside of the  $14^\circ$  conical surface a flat rim on the pole face tends to prevent too rapid decrease of the field at the edge of the gap.

interest was that of zero magnetic field. The use of the oscillograph as a null instrument in this way proved to be sufficiently accurate. The pole faces gave a field following the  $1/r^{2/3}$  law between a radius of 4.5 centimeters and 9.25 centimeters when the separation between the flat central portions of the pole faces was 2.8 centimeters. At  $r = 10$  cm the law was  $1/r$ .

To determine the position of the equilibrium orbit use was made of a concentric system of seven 1-turn coils set in grooves in a Bakelite disk which fit snugly around one of the iron dust disks. Relative values for the e.m.f.'s induced in these coils were determined by the same null method which was used for determining the shape of the field. These data could be used in several ways to find  $r_0$ . The simplest method is to find the radius of minimum electric field, since this will be  $r_0$ .

## The Acceleration Chamber

It was desirable to have the volume available for electron orbits as large as possible. This required a doughnut-shaped glass vessel with walls parallel to the conical pole faces. The outer diameter was 20 cm, and the center of the doughnut was 7 cm in diameter. The glass work for this vessel required

great skill<sup>9</sup>. A 20-cm spherical bulb with a wall thickness of 2.5 mm was flattened and dished conically on both sides so that it would fit between the magnet poles. The central 7-cm hole of the doughnut was formed by pushing the centers of the dished faces together and picking out the glass.

The inside of this bulb was coated with a thin conducting layer by chemical silvering. This coating is necessary to prevent stray charges from building up potentials on the walls. Contact is made to the silver coating which is then grounded outside of the tube. Resistances of the silver coat between

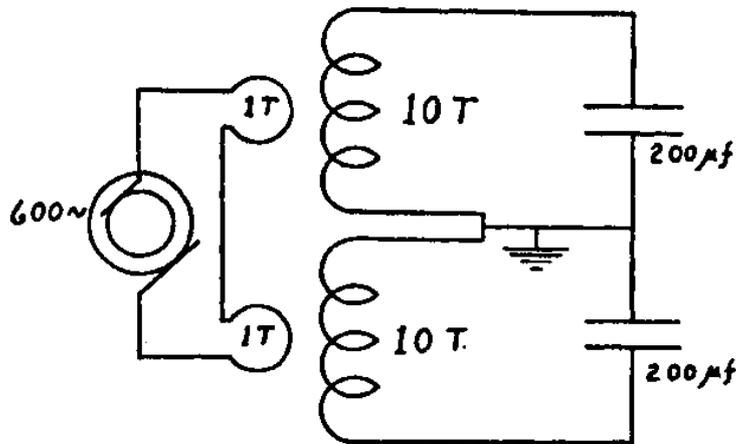


Figure 3: The resonant circuit which energizes the magnet. Losses are supplied by a 4-kilowatt 600-cycle-per-second generator.

20 ohms and 300 ohms between test probes put in the side arms were used.

Both target and injector were mounted on the glass pinch seal which was waxed to a flare on the doughnut. The target was a piece of 0.015-inch tungsten sheet folded to present an edge of large radius of curvature to the electron orbit which shrinks inward toward it. Thin tungsten is used so that eddy currents will not be excessive. The injector was made of thin molybdenum sheets and a cylindrically spiraled tungsten wire for the filament. The whole assembly is shown in Fig. 4.

Fortunately the presence of this much metal near the orbit does not seem to disturb the local magnetic field too much. However, the orbits are destroyed if a piece of metal is brought up to the side of the glass doughnut

<sup>9</sup>I am indebted to the Vacuum Tube Department of the General Electric X-Ray Corporation for producing a Correctly shaped tube.

while the machine is running. This was observed when a block of beryllium was put near the tube for experiments with photo-disintegration. The  $x$ -ray yield disappeared when the beryllium was too close. The original injector was arranged to send a beam of electrons in both directions around the magnet so that both directions of field could be used. Thus two beams of  $x$ -rays oppositely directed were produced.

## Operation

Although the focusing theory shows that for the present design of the accelerator electrons should be injected at approximately 200 volts, it was found that the yield increased with voltage and that it was still increasing at 600 volts. Likewise the negative voltage on the small focusing plates  $G$  beside the filament gave an increasing yield as it was raised to 200 volts.

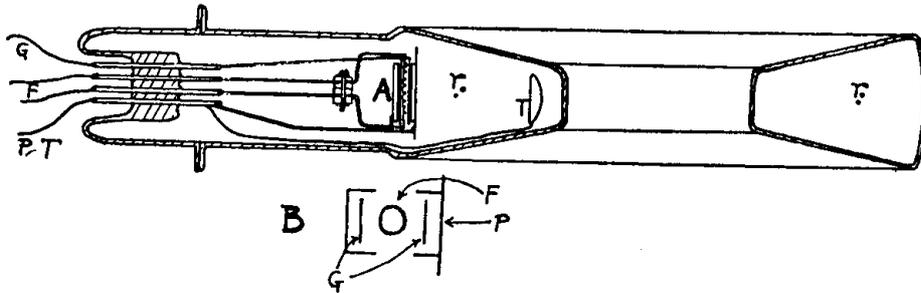


Figure 4: Cross section of the doughnut-shaped acceleration chamber. The equilibrium orbit is at  $r_0$ .  $T$  is the tungsten target,  $A$  is the injector,  $B$  is a top view of the injector. A ribbon beam of electrons from the filament  $F$  is shot out through slots in the positive plates  $P$ .  $G$  are negative focusing electrodes.

The beam from the injector was allowed to pour into the vacuum tube continuously.

Initially the yield obtained was approximately equivalent to 10 millicuries of radium when measured in a direction from the target at right angles to the electron beam as it strikes the target, and the yield was approximately ten times greater in the direction of the electron beam. Later the yield in the direction of the beam was increased to the equivalent of one gram of radium.

Figure 5 shows an absorption curve of the radiation as it passes through

lead, and the comparison with two-million-volt  $x$ -rays from the M. I. T. electrostatic machine<sup>7</sup> shows that the energy reached in the induction accelerator is approximately 2.0 million electron volts and that the radiation produced is thin target radiation. With other iron dust disks cemented on the pole faces, absorption coefficients indicating 2.3-Mev  $x$ -rays have been obtained. By means of a collimated Geiger-Müller counter the source of the  $x$ -rays was shown to be the target. A large lead block with a small hole in it let an abundant amount of  $x$ -rays through to the Geiger-Müller tube only when it was pointed at the target and not when it was pointed at the injector or other portions of the vacuum tube. By connecting the vertical deflection plates of an oscillograph to the Geiger-Müller counter and the horizontal deflection plates to a 1-turn coil around one leg of the magnet, the phase of the magnetic field at which electrons struck the target could be determined. As was expected it was found that the greater the excitation of the dynamo which energizes the magnet, the earlier in the cycle the iron dust saturated and brought the beam in to the target. Lowering the primary voltage postponed sufficient saturation in the iron dust until the peak of the magnetic field was reached. The Geiger-Müller counter then gave  $x$ -ray pulses at the center of the oscillograph screen. This indicated a path length of about sixty miles from injector to target. If the primary voltage was lowered beyond this point, the yield disappeared, for the electrons were not drawn in to the target but were slowed down by the decreasing magnetic field. Fortunately the operation of the accelerator is not sensitive to the alignment of the pole faces. No difference in the output can be detected when the pole faces are placed off axis as far as a thirty-second of an inch. It is also surprising that vacuum requirements are not as severe as was expected. No rigorous outgassing is necessary and the apparatus has been run with a vacuum as poor as  $10^{-5}$  mm Hg. The tube can be opened for changes and operated three-quarters of an hour after sealing shut.

At present, low flux densities have been used at the orbit. When these are increased, it should be possible to go to 5 million volts even with this small model. One of the promising possibilities for the induction accelerator as a research tool is that the electrons from the beam can come out through the glass walls of the doughnut after they strike the target. They should be fairly homogeneous in energy provided that the target has a high atomic number. The great increase in bremsstrahlung production with rising electron energy in addition to the concentration of this radiation in a cone of solid angle  $mc^2/E$ , about the original electron direction gives the induction accelerator the possibility of providing an intense source of  $x$ -radiation for nuclear investigations. Since there is no evident limit on the energy which

can be reached by induction acceleration, it may soon be possible to produce some small scale cosmic-ray phenomena in the laboratory.

I am indebted to Professor H. M. Mott-Smith and Professor R. Serber for many discussions of the theoretical aspects of this problem and to Mr. R. P. Jones for assistance in the construction of the magnet.

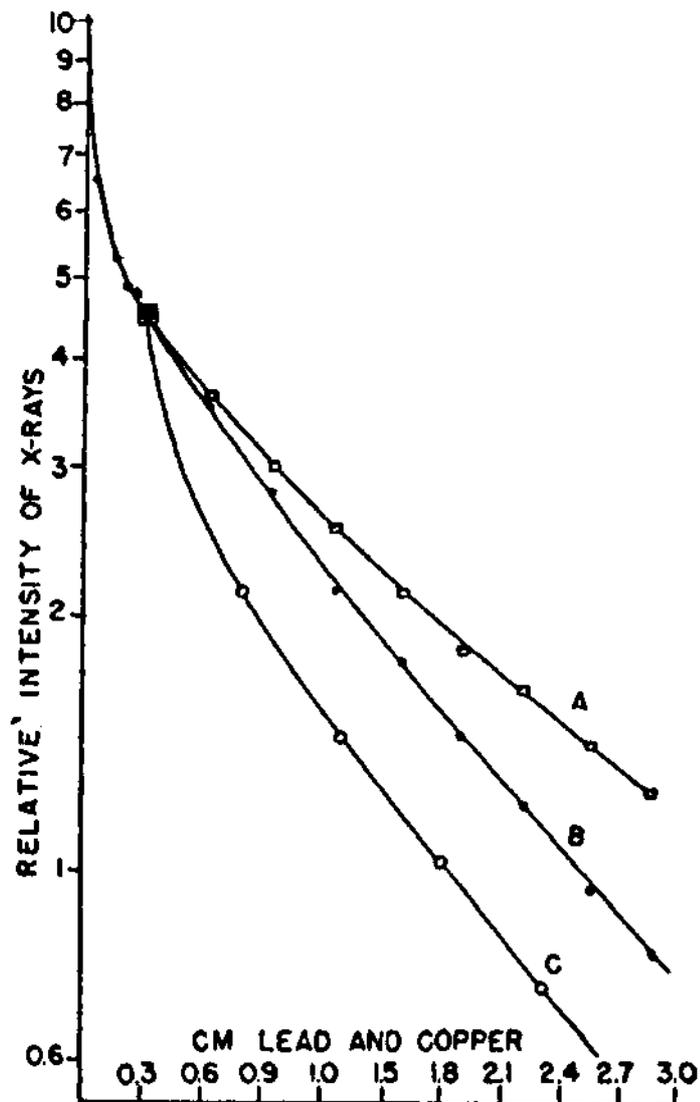


Figure 5: The relative intensity of  $x$ -rays from the induction accelerator as a function of absorber thickness. A is absorption in copper. The absorption coefficient is  $0.454 \text{ cm}^{-1}$  which gives a monochromatic equivalent of 1.35 Mev for the  $x$ -rays. B is the curve for lead. The coefficient is  $0.62 \text{ cm}^{-1}$ , which corresponds to 1.4 Mev monochromatic equivalent. C is the lead absorption curve of Van Atta and Northrup for  $x$ -rays produced by 2.0-Mev electrons striking a thick target.