

The Synchrotron—A Proposed High Energy Particle Accelerator

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One of the most successful methods for accelerating charged particles to very high energies involves the repeated application of an oscillating electric field, as in the cyclotron. If a very large number of individual accelerations is required, there may be difficulty in keeping the particles in step with the electric field. In the case of the cyclotron this difficulty appears when the relativistic mass change causes an appreciable variation in the angular velocity of the particles.

The device proposed here makes use of a “phase stability” possessed by certain orbits in a cyclotron. Consider, for example, a particle whose energy is such that its angular velocity is just right to match the frequency of the electric field. This will be called the equilibrium energy. Suppose further that the particle crosses the accelerating gaps just as the electric field passes through zero, changing in such a sense that an earlier arrival of the particle would result in an acceleration. This orbit is obviously stationary. To show that it is stable, suppose that a displacement in phase is made such that the particle arrives at the gaps too early. It is then accelerated; the increase in energy causes a decrease in angular velocity, which makes the time of arrival tend to become later. A similar argument shows that a change of energy from the equilibrium value tends to correct itself. These displaced orbits will continue to oscillate, with both phase and energy varying about their equilibrium values.

In order to accelerate the particles it is now necessary to change the value of the equilibrium energy, which can be done by varying either the

magnetic field or the frequency. While the equilibrium energy is changing, the phase of the motion will shift ahead just enough to provide the necessary accelerating force; the similarity of this behavior to that of a synchronous motor suggested the name of the device.

The equations describing the phase and energy variations have been derived by taking into account time variation of both magnetic field and frequency, acceleration by the “betatron effect” (rate of change of flux), variation of the latter with orbit radius during the oscillations, and energy losses by ionization or radiation. It was assumed that the period of the phase oscillations is long compared to the period of orbital motion. The charge was taken to be one electronic charge. Equation (1) defines the equilibrium energy; (2) gives the instantaneous energy in terms of the equilibrium value and the phase variation, and (3) is the “equation of motion” for the phase. Equation (4) determines the radius of the orbit.

$$E_0 = (300cH)/(2\pi f), \quad (1)$$

$$E = E_0[1 - (d\phi)/(d\theta)], \quad (2)$$

$$2\pi \frac{d}{d\theta} \left(E_0 \frac{d\phi}{d\theta} \right) + V \sin \phi = \left[\frac{1}{f} \frac{dE_0}{dt} - \frac{300}{e} \frac{dF_0}{dt} + L \right] + \left[\frac{E_0}{f^2} \frac{df}{dt} \right] \frac{d\phi}{d\theta} \quad (3)$$

$$R = (E^1 - E_r^2)^{1/2}/300H. \quad (4)$$

The symbols are:

- E – total energy of particle (kinetic plus rest energy),
- E_0 – equilibrium value of E ,
- E_r – rest energy,
- V – energy gain per turn from electric field,
at most favorable phase for acceleration,
- L – loss of energy per turn from ionization and radiation,
- H – magnetic field at orbit,
- F_0 – magnetic flux through equilibrium orbit,
- ϕ – phase of particle (angular position with respect to gap
when electric field = 0),
- θ – angular displacement of particle,
- f – frequency of electric field,
- c – light velocity,
- R – radius of orbit.

(Energies are in electron volts, magnetic quantities in e.m.u., angles in radians, other quantities in c.g.s. units.)

Equation (3) is seen to be identical with the equation of motion of a pendulum of unrestricted amplitude, the terms on the right representing a constant torque and a damping force. The phase variation is, therefore, oscillatory so long as the amplitude is not too great, the allowable amplitude being $\pm\pi$ when the first bracket on the right is zero, and vanishing when that bracket is equal to V . According to the adiabatic theorem, the amplitude will diminish as the inverse fourth root of E_0 , since E_0 occupies the role of a slowly varying mass in the first term of the equation; if the frequency is diminished, the last term on the right furnishes additional damping.

The application of the method will depend on the type of particles to be accelerated, since the initial energy will in any case be near the rest energy. In the case of electrons, E_0 will vary during the acceleration by a large factor. It is not practical at present to vary the frequency by such a large factor, so one would choose to vary H , which has the additional advantage that the orbit approaches a constant radius. In the case of heavy particles E_0 will vary much less; for example, in the acceleration of protons to 300 Mev it changes by 30 percent. Thus it may be practical to vary the frequency for heavy particle acceleration.

A possible design for a 300 Mev electron accelerator is outlined below:

peak $H = 10,000$ gauss,

final radius of orbit = 100 cm.

frequency = 48 megacycles/sec.,

injection energy = 300 kv. initial radius of orbit = 78 cm.

Since the radius expands 22 cm during the acceleration, the magnetic field needs to cover only a ring of this width, with of course some additional width to shape the field properly. The field should decrease with radius slightly in order to give radial and axial stability to the orbits. The total magnetic flux is about 1/2 of what would be needed to satisfy the betatron flux condition for the same final energy.

The voltage needed on the accelerating electrodes depends on the rate of change of the magnetic field. If the magnet is excited at 60 cycles, the peak value of $(l/f)(dE_0/dt)$ is 2300 volts. (The betatron term containing dE_0/dt is about 1/2 of this and will be neglected.) If we let $V = 10,000$ volts, the greatest phase shift will be 13° . The number of turns per phase oscillation will vary from 22 to 440 during the acceleration. The relative variation of E_0 during one period of the phase oscillation will be 6.3 percent at the time of injection, and will then diminish. Therefore, the assumptions of slow variation during a period used in deriving the equations are valid.

The energy loss by radiation is discussed in the letter following this, and is shown not to be serious in the above case.

The application to heavy particles will not be discussed in detail, but it seems probable that the best method will be the variation of frequency. Since this variation does not have to be extremely rapid, it could be accomplished by means of motor-driven mechanical turning devices.

The synchrotron offers the possibility of reaching energies in the billion-volt range with either electrons or heavy particles; in the former case, it will accomplish this end at a smaller cost in materials and power than the betatron; in the latter, it tacks the relativistic energy limit of the cyclotron.

Construction of a 300-Mev electron accelerator using the above principle at the Radiation Laboratory of the University of California at Berkeley is now being planned.