

On Quantum-Electrodynamics and the Magnetic Moment of the Electron

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Attempts to evaluate radiative corrections to electron phenomena have heretofore been beset by divergence difficulties, attributable to self-energy and vacuum polarization effects. Electrodynamics unquestionably requires revision at ultra-relativistic energies, but is presumably accurate at moderate relativistic energies. It would be desirable, therefore, to isolate those aspects of the current theory that essentially involve high energies, and are subject to modification by a more satisfactory theory, from aspects that involve only moderate energies and are thus relatively trustworthy. This goal has been achieved by transforming the Hamiltonian of current hole theory electrodynamics to exhibit explicitly the logarithmically divergent self-energy of a free electron, which arises from the virtual emission and absorption of light quanta. The electromagnetic self-energy of a free electron can be ascribed to an electromagnetic mass, which must be added to the mechanical mass of the electron. Indeed, the only meaningful statements of the theory involve this combination of masses, which is the experimental mass of a free electron. It might appear, from this point of view, that the divergence of the electromagnetic mass is unobjectionable, since the individual contributions to the experimental mass are unobservable. However, the transformation of the Hamiltonian is based on the assumption of a weak

interaction between matter and radiation, which requires that the electromagnetic mass be a small correction ($\sim (e^2/hc)m_0$) to the mechanical mass m_0 .

The new Hamiltonian is superior to the original one in essentially three ways: it involves the experimental electron mass, rather than the unobservable mechanical mass; an electron now interacts with the radiation field only in the presence of an external field, that is, only an accelerated electron can emit or absorb a light quantum;¹ the interaction energy of an electron with an external field is now subject to a *finite* radiative correction. In connection with the last point, it is important to note that the inclusion of the electromagnetic mass with the mechanical mass does not avoid all divergences; the polarization of the vacuum produces a logarithmically divergent term proportional to the interaction energy of the electron in an external field. However, it has long been recognized that such a term is equivalent to altering the value of the electron charge by a constant factor, only the final value being properly identified with the experimental charge. Thus the interaction between matter and radiation produces a renormalization of the electron charge and mass, all divergences being contained in the renormalization factors.

The simplest example of a radiative correction is that for the energy of an electron in an external magnetic field. The detailed application of the theory shows that the radiative correction to the magnetic interaction energy corresponds to an additional magnetic moment associated with the electron spin, of magnitude $\delta\mu/\mu = (1/2\pi)e^2/hc = 0.001162$. It is indeed gratifying that recently acquired experimental data confirm this prediction. Measurements on the hyperfine splitting of the ground states of atomic hydrogen and deuterium² have yielded values that are definitely larger than those to be expected from the directly measured nuclear moments and an electron moment of one Bohr magneton. These discrepancies can be accounted for by a small additional electron spin magnetic moment.³ Recalling that the nuclear moments have been calibrated in terms of the electron moment, we find the additional moment necessary to account for the measured hydro-

¹A classical non-relativistic theory of this type was discussed by H. A. Kramers at the Shelter Island Conference, held in June 1947 under the auspices of the National Academy of Sciences.

²J. E. Nafe, E. B. Nelson, and I. I. Rabi, Phys. Rev. **71**, 914 (1947); D. E. Nagel, R. S. Julian, and J. R. Zachariad, Phys. Rev. **72**, 971 (1947).

³G. Breit, Phys. Rev. **71**, 984 (1947). However, Breit has not correctly drawn the consequences of his empirical hypothesis. The effects of a nuclear magnetic field and a constant magnetic field do not involve different combinations of μ and $\delta\mu$.

gen and deuterium hyperfine structures to be $\delta\mu/\mu = 0.00126 \pm 0.00019$ and $\delta\mu/\mu = 0.00131 \pm 0.00025$, respectively. These values are not in disagreement with the theoretical prediction. More precise conformation is provided by measurement of the g values for the ${}^2S_{1/2}$, ${}^2P_{1/2}$, and ${}^2P_{3/2}$ states of sodium and gallium.⁴ To account for these results, it is necessary to ascribe the following additional spin magnetic moment to the electron, $\delta\mu/\mu = 0.00118 \pm 0.00003$.

The radiative correction to the energy of an electron in a Coulomb field will produce a shift in the energy levels of hydrogen-like atoms, and modify the scattering of electrons in a Coulomb field. Such energy level displacements have recently been observed in the fine structures of hydrogen,⁵ deuterium, and ionized helium.⁶ The values yielded by our theory differ only slightly from those conjectured by Bethe⁷ on the basis of a non-relativistic calculation, and are, thus, in good accord with experiment. Finally, the finite radiative correction to the elastic scattering of electrons by a Coulomb field provides a satisfactory termination to a subject that has been beset with much confusion.

A paper dealing with the details of this theory and its applications is in course of preparation.

⁴P. Kusch and H. M. Foley, Phys. Rev. **72**, 1256 (1947), and further unpublished work.

⁵W. E. Lamb, Jr. and R. C. Retherford, Phys. Rev. **72**, 241 (1947).

⁶J. E. Mack and N. Austern, Phys. Rev. **72**, 972 (1947)

⁷H. A. Bethe. Phys. Rev. **72**, 339 (1947).