MODELS OF HYBRID MESONS

Yu.S. Kalashnikova[1]

Institute of Theoretical and Experimental Physics, Moscow, Russia

Theoretical models for hybrid mesons are discussed, and the possible ways to establish the appropriate degrees of freedom for constituent glue are reviewed.

Introduction

The existence of gluonic hadrons remains one of the most challenging problems in the nonperturbative QCD. If one assumes that a kind of string is developed between quark and antiquark at large distances, then it is natural to identify the $q\bar{q}$-system connected by the string in its ground state with conventional $q\bar{q}$-meson, while the string vibrations are responsible for gluonic (hybrid) excitations. Such picture does not follow directly from the QCD Lagrangian, and the effective degrees of freedom for constituent glue should be introduced to construct models describing hybrid mesonic excitations.

There are two main ideas on how to introduce constituent (valence) glue. One is to consider string phonons [1], and another is to consider pointlike gluons confined by some potential [2] or string-like force [3]. In what follows I discuss the motivations for such models, as well as consequences for the spectrum of low-lying hybrids, with special attention paid to hybrids with heavy quarks. There exist lattice calculations of hybrid adiabatic potentials [4], which are accurate enough to check the model predictions. It will be demonstrated, in particular, that the behaviour of the adiabatic hybrid potentials could shed light on the nature of appropriate degrees of freedom for constituent glue.

1. Area law and confining force

The area law asymptotics for the Wilson loop conveniently provides the action of the string. Still the QCD is not a string theory, and a question arises of what degrees of freedom should enter the effective string theory describing the QCD in the nonperturbative region.

The Wilson loop expectation value is defined as an integral along some closed contour C averaged over gluonic vacuum configurations:

$$\langle W(C)\rangle = Tr\langle P\exp\ ig\oint_C A_\mu dz_\mu\rangle, \tag{1}$$

where trace is taken over colour indices. The area law asymptotics implies that

$$\langle W(C)\rangle \to N_C \exp(-\sigma S), \tag{2}$$

where N_C is the number of colours, σ is the string tension and S is the surface bound by the closed contour C. Quite obviously, as the initial expression (1) depends on the contour, the area in (2) should depend only on the contour too, and should be the minimal area. In other words, the area law asymptotics leads to the minimal string configurations.

Placing quarks at the ends of the string we have new degrees of freedom, and the $q\bar{q}$ mesonic system is described by the Lagrangian

$$L = -m_1\sqrt{\dot{x}_1^2} - m_2\sqrt{\dot{x}_2^2} + L_{\rm string}. \tag{3}$$

[1]e-mail: yulia@heron.itep.ru.

The system (3) was extensively studied in [5] in the so-called straight-line approximation for the minimal string. In the heavy-quark limit the Lagrangian (3) gives rise to the effective quasi-potential Hamiltonian for the $q\bar{q}$ system,

$$H = \sqrt{\vec{p}^2 + m_1^2} + \sqrt{\vec{p}^2 + m_2^2} + \sigma r, \tag{4}$$

and for high excitations the Regge trajectory takes the form

$$M^2 = 2\pi\sigma(L + n_r). \tag{5}$$

The most important point here is that the area law yields the linear confining potential only in the case of heavy quarks. In general case the corrections due to the string become sizeable, and, in the quasiclassical case (5), the spectrum becomes completely string-like.

2. Vibrating string

It is natural to identify the $q\bar{q}$ system connected by the minimal string with conventional $q\bar{q}$ meson. The effective string model should be arranged to allow the extra degrees of freedom to populate the string and to be responsible for more complicated string configurations.

2.1. Flux-tube model

The flux-tube model [1] is motivated by strong-coupling expansion of lattice QCD, and assumes that in the nonperturbative region, i.e.at large $q\bar{q}$ distances, a flux tube is produced with phonon-type effective degrees of freedom. As a flux line in the strong-coupling limit is extended only in the transverse direction, one allows only transverse motion of the flux tube. This model approximates the confining region by a chain of N pointlike masses, the so-called "beads" confined together by the linear potential.

First studies [1] were performed in the adiabatic approximation and assuming small string oscillations. In the large-N limit one obtains the excitation energy gap π/R between ground state and first hybrid potential.

The numerical results for adiabatic potentials without small oscillation approximation were obtained in [6] for the fixed-N flux tube. The single-bead adiabatic potential is the solution of the equation

$$-\frac{1}{2m_b}\left(\frac{d^2\psi}{d\rho^2} + \frac{1}{\rho}\frac{d\psi}{d\rho}\right) + \left(2\sigma\sqrt{\rho^2 + \frac{R^2}{4}} + \frac{\Lambda^2}{2m_b\rho^2}\right) = E_\Lambda(R)\psi, \tag{6}$$

where Λ is the orbital momentum component along the $q\bar{q}$ axis, and $m_b = 0.2\ GeV$ is the mass of the bead. The latter quantity is a new parameter of the single-bead flux- tube model [6]. The potential gap between $\Lambda = 0$ and $\Lambda = 1$ behaves at large distances as $2\sqrt{\sigma/m_b R}$ instead of π/R of [1]. This is due to the $N = 1$ approximation and the necessity to introduce the mass of the bead by hand.

The adiabatic potentials are to be inserted into $q\bar{q}$ Schroedinger equation in order to obtain spectrum of a hybrid with heavy quarks. This procedure was applied in [1] to the case of light quarks too. The word "adiabatic", however, can be removed from the single-bead version of the flux-tube model [6]. It appears that adiabatic approximation overestimates the orbitally excited levels but underestimates the hybrid energy [6]. For example, if the model fits the experimental D-levels, then the hybrid mass is underestimated by 200 MeV for the case of light quarks.

The quantum numbers of lightest hybrids in the flux tube model are

$$J^{PC} = 0^{\mp\pm}, 1^{\mp\pm}, 2^{\mp\pm}, 1^{\mp\mp}. \tag{7}$$

In the realistic mass calculations the long range confining force is augmented by a short range Coulomb interaction. In the flux tube model the string phonons do not carry colour quantum number, so that the $q\bar{q}$ pair is in colour singlet state, and Coulomb interaction is attractive,

$$V_c = -\frac{4}{3}\frac{\alpha_s}{R}. \tag{8}$$

The lightest hybrid meson masses in the flux tube model are found to be about 1.8–1.9 GeV, and the lightest charmonium hybrid has the mass of about 4.1–4.2 GeV.

2.2. Constituent gluon model

Constituent gluon models have a long history. The possibility for mesons with gluonic lump and $q\bar{q}$ pair in colour octet state to exist was first considered in [7]. The bag models [8] and potential models [9] deal with pointlike gluons. The naive version of potential model for hybrids is given by the Hamiltonian

$$H = \frac{p_q^2}{2m_q} + \frac{p_{\bar{q}}^2}{2m_{\bar{q}}} + \frac{p_g^2}{2m_g} + \sigma r_{qg} + \sigma r_{\bar{q}g} + V_c, \quad V_c = \frac{\alpha_s}{6r_{q\bar{q}}} - \frac{3}{2}\frac{\alpha_s}{r_{qg}} - \frac{3}{2}\frac{\alpha_s}{r_{\bar{q}g}}. \tag{9}$$

The coefficients in the Coulomb potential (9) are in accordance with the colour content of the $q\bar{q}g$ state. Note that the quark-antiquark Coulomb force is repulsive in contrast to flux tube attraction (8).

The masses entering the Hamiltonian (9) are the constituent masses. In the light quark sector the values m_q about 0.3 GeV and m_g about 0.8 GeV are usually used.

The departure from simple constituent cartoon (9) is made in [2]. The Dynamical Quark Model employed there is based on the field theoretic Hamiltonian of QCD. The interaction potential is given by

$$V(r) = \frac{\alpha_s}{r} - \frac{2\sigma N_C}{N_C^2 - 1} r(1 - e^{-\Lambda_{UV} r}). \tag{10}$$

The quark and gluon masses entering the model Hamiltonian are the current masses; the constituent masses are generated in the nontrivial QCD vacuum [10, 11]. The constituent mass of light quark is about 200 MeV. The dispersion law for a gluon is given by

$$\omega(k) = \sqrt{k^2 + m_g^2 e^{-k/\kappa}} \tag{11}$$

with $\kappa = 6.5$ GeV and $m_g = 800$ MeV. The glueball spectrum was calculated in this model [10], and the results are in remarkable agreement with lattice data.

There is no full hybrid calculations with dynamical quarks in this model; only adiabatic potentials are calculated in [2] and compared with the lattice data [4]. The general conclusion is rather confusing: the model agrees with the lattice data at short and intermediate interquark distances, if the intrinsic gluon parity is taken to be positive. At large interquark distances the model fails. It is argued in [2] that the soft glue becomes string-like at large distances, with number of effective gluons growing and producing a flux tube. However, as it will be discussed below, the reason might be more simple: the confining part of the interaction (10) corresponds to the linear potential. It does not yield area law asymptotics, and cannot reproduce a string.

2.3. QCD string model

The QCD string model is based on the Vacuum Background Correlators method [12] and deals with quarks and pointlike gluons propagating in the confining QCD vacuum. The latter is given by the set of gauge invariant field strength correlators responsible for the area law. For example the lowest bilocal correlator takes the form

$$\langle F_{\mu\nu}(x)F_{\lambda\rho}(y)\rangle = (\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda})D(x-y).$$

Only this Lorentz structure produces area law for the Wilson loop, if the scalar function $D(z)$ decreases with z.

The QCD string model for gluons is derived from the perturbation theory in the nonperturbative confining background developed in [13]. The main feature of this approach is that here one is able to distinguish clearly between confining gluonic field configurations and confined valence gluons. The hybrid mesons [14], glueballs [15] and gluelump [16] were described in this model.

The starting point is the Green's function of quark, antiquark and gluon propagating in the given background field. Using Feynman-Schwinger representation for these Green functions one averages over the background, expressing the interaction in terms of Wilson loop averages. The main assumption of the QCD string model is the minimal area law for the Wilson loop. There exists the approximate einbein field method [5], which allows to deal with relativistic kinematics in a simple way.

Within this method the Lagrangian of the $q\bar{q}g$ system takes the form

$$L = -\frac{\mu_q+\mu_{\bar{q}}+\mu_g}{2} + \frac{\mu_q\dot{r}_q^2+\mu_{\bar{q}}\dot{r}_{\bar{q}}^2+\mu_g\dot{r}_g^2}{2} - \frac{m_q^2}{2\mu_q} - \frac{m_{\bar{q}}^2}{2\mu_{\bar{q}}} - \sigma r_{qg}\int_0^1 d\beta_1\sqrt{1-l_1^2} \tag{12}$$

$$-\sigma r_{\bar{q}g}\int_0^1 d\beta_2\sqrt{1-l_2^2},$$

where

$$l_1^2 = (\beta_1\dot{\vec{r}}_q + (1-\beta_1)\dot{\vec{r}}_g)^2 - \frac{1}{r_{qg}^2}(\beta_1(\dot{\vec{r}}_q\vec{r}_{qg}) + (1-\beta_1)(\dot{\vec{r}}_g\vec{r}_{qg}))^2, \tag{13}$$

$$l_2^2 = (\beta_2\dot{\vec{r}}_{\bar{q}} + (1-\beta_2)\dot{\vec{r}}_g)^2 - \frac{1}{r_{\bar{q}g}^2}(\beta_2(\dot{\vec{r}}_{\bar{q}}\vec{r}_{\bar{q}g}) + (1-\beta_2)(\dot{\vec{r}}_g\vec{r}_{\bar{q}g}))^2. \tag{14}$$

The quantities μ_q, $\mu_{\bar{q}}$ and μ_g are the eibein fields; one is to integrate over these fields in the path integral representation.

The terms with square roots (last two terms in (12) are the Lagrangians of the minimal strings between quark and gluon and antiquark and gluon. For low values of relative angular momenta these square roots can be expanded up to the second order in the transverse velocities. In this approximation the Hamiltonian for low-lying hybrids takes the form [14]

$$H = \frac{m_q^2}{2\mu_q} + \frac{m_{\bar{q}}^2}{2\mu_{barq}} + \frac{\mu_q+\mu_{\bar{q}}+\mu_g}{2} + \frac{p^2}{2\mu_p} + \frac{Q^2}{2\mu_Q} + \sigma r_{qg} + \sigma r_{\bar{q}g}, \tag{15}$$

where the Jacobi momenta p and Q of the $q\bar{q}g$ three-body system and reduced masses μ_p and μ_Q are introduced. To obtain the spectrum one finds the eigenvalues of the Hamiltonian (15) and minimize them with respect to the einbein fields. Obviously the eibein fields play the role of constituent quark and gluon masses; in contrast to the model ([9]) they are not introduced by hand, but are calculated.

The numerical estimates of lowest hybrid masses were obtained in [14] with short range Coulomb interaction of the form (9). The lightest hybrids in the light quark sector are placed at about 1.7-1.8 GeV, and the quantum numbers are

$$J^{PC} = 0^{\mp+}, 1^{\mp+}, 2^{\mp+}, 1^{\mp-}. \tag{16}$$

Note the important difference between (16) and (7), which comes from the fact that valence gluon carries quantum numbers like spin and C-parity.

The mass range 1.7–1.8 GeV is close to the mass range given by the flux tube model, and both are rather close to the region where hybrid candidates are believed to be placed [17].

3. Hybrid adiabatic potentials

Lattice calculations are now able to provide reliable data on the properties of soft glue. The measurements [4] of the adiabatic hybrid potentials are of particular interest here.

The adiabatic potential are measured in [4] for quark-antiquark separations R ranging from $0.1fm$ to $4fm$. The region of small and intermediate R is relevant for heavy hybrid mass estimations. The large R limit is interesting *per se*, as the formation of confining string is expected at large distances, and direct measurements of string fluctuations become available. In this string limit one expects the lowest excitations to be the Goldstone modes associated with spontaneously-broken transverse translational symmetry.

The main results of studies [4] are as follows. The spectrum does not exhibit the universal π/R gaps even for R as large as $4fm$. For separations smaller than $2fm$ the measured energies lie much below universal Nambu–Goto curves. As it is stated in [4]: "These results are rather surprising. Our results cast serious doubts on the validity of treating glue in terms of a fluctuating string for quark-antiquark separations less than $2fm$".

The analysis of adiabatic potentials was performed in the framework of the QCD string model in [3]. These first studies were made in neglection of gluon spin, which, in principle, could be treated as perturbation [12]. For small R, $R \ll 1/\sqrt{\sigma}$, one has

$$E_n(R) = \sqrt{8\sigma\left(n+\frac{3}{2}\right)} + \frac{\sigma^2 R^2}{\sqrt{8\sigma\left(n+\frac{3}{2}\right)}} \tag{17}$$

featuring oscillator adiabatic potential, n is the number of oscillator quanta. This regime corresponds to the neglect of string inertia (orbital velocities in (12), which is justified at small interquark distances.

In the case of large R, $R \gg 1/\sqrt{\sigma}$, the situation changes drastically. It is the limit of small string oscillations, and one cannot neglect here the contributions from string inertia. There are two different kinds of excitations, along the $Q\bar{Q}$ axis and in the transverse direction. These oscillations are decoupled for large R, and one has

$$E(R) = \sigma R + 3\left(\frac{\sigma}{2R}\right)^{1/3}(n_z+1/2)^{2/3} + \frac{2\sqrt{3}}{R}(n_\rho+\Lambda+1). \tag{18}$$

There is no QCD string calculation with proper account of gluon spin yet, so the direct comparison with lattice results is premature. Nevertheless, some preliminary conclusions can be drawn. The behaviour (18) displays the most pronounced difference between QCD string and other models for constituent glue. In contrast to flux tube model, the string vibrations are caused by pointlike valence gluon, but, in contrast to constituent models with linear potential the confining force follows from minimal area law.

Small oscillation approximation of the flux tube model predicts

$$E(R) = \sigma R + \frac{\pi\Lambda}{R}. \tag{19}$$

The potential model gives

$$E(R) = \sigma R + 3\left(\frac{\sigma}{2R}\right)^{1/3}(n+3/2)^{2/3}. \tag{20}$$

Clearly the regime (18) is the combinations of both. There are longitudinal vibrations with potential-type $(\sigma/R)^{1/3}$ subleading behaviour and transverse string-type Λ/R subleading contributions. For quasiclassicaly large Λ the QCD string model gives

$$E(R) = \sigma R + 2\sqrt{3}\frac{\Lambda}{R}, \tag{21}$$

which is very close to the flux tube predictions. The coefficients π and $2\sqrt{3}$ differ of course, mainly due to the fact that string configurations differ: there are two straight-line strings in the QCD string model, and one continuous string in the Nambu-Goto case. There is nothing like single-bead $2\sqrt{\frac{\sigma}{m_b R}}$ behaviour, which seems to be an artifact of single-bead approximation.

The dominant subleading behaviour is defined by longitudinal motion: even if no longitudinal quanta are excited, there is the contribution of zero longitudinal oscillations in (18). Still, if the distances are not asymptotically large, the potential regime is substantionally contaminated by string-type transverse vibrations.

There are several important messages from the small and intermediate R region too. First, one should expect that in the case of very heavy quarks hybrid resides in the minimum of potential well. If only confining force is taken into account, both the QCD string model and potential model predicts the oscillator potential with the minimum at $R = 0$. The same takes place in the single-bead version of flux tube model. The inclusion of repulsive Coulomb force which is demanded in the models with pointlike gluons shifts this minimum, while the attractive Coulomb force of the flux tube model does not.

The lattice results [4] are not very accurate for small R, but the message is quite clear: the bottom of potential well is somewhere around $0.25 fm$ for lowest curves, which is completely excluded in the single-bead flux tube model. As the gluon energies of [4] lie well below the curves (19), the simple Nambu-Goto regime is excluded too. The minima of potential wells in the sophisticated model [2] also do not seem to fit the lattice curves, and, in addition, the modified gluon parity is needed. It is an open question of the ability of the QCD string model to do the job.

Another very interesting point concerning the small distance region is the following. As R decreases below $0.5 fm$, the excited levels become unstable [4]: the gaps above the ground state levels exceed the mass of the lightest glueball. The phenomenological consequences of this are not very clear at present.

Conclusions

There is a lot of experimental indications that hybrid mesons are already found, but the conclusive evidences have never been presented, nor have alternative explanations been completely excluded [17]. The existing models of constituent glue are still in their infancy, and more sophisticated versions of these models should appear, with more pronounced motivation from QCD, to establish the appropriate degrees of freedom for soft glue and to describe hybrid mesonic excitations. A lot of joint efforts of experimentalists and phenomenologists are needed for successful "hybrid hunting", with continuous feedback from lattice QCD.

I gratefully acknowledge the support from RFFI grant offered to me by the Organizing Committee, which made possible my participation in the Workshop.

References

[1] N. Isgur and J. Paton, Phys. Rev. **D31** (1985) 2910.

[2] E.S. Swanson and A.P. Szczepanyak, Phys. Rev. **D59** (1999) 014035.

[3] Yu.S. Kalashnikova and D.S. Kuzmenko, hep-ph/0006073, Yad.Fiz. (in press).

[4] K.J. Juge, J. Kuti and C. Morningstar, in Proc. Third Int. Conf. on Quark Confinement and the Hadron Spectrum, Jefferson Lab (1998), hep-lat/9809015.

[5] A.Yu. Dubin, A.B. Kaidalov, Yu.A. Simonov, Phys. Lett. **B323** (1994) 41.
V.L. Morgunov, A.V. Nefediev, Yu.A. Simonov, Phys. Lett. **B459** (1999) 653.

[6] T. Barnes, F.E. Close, E.S. Swanson, Phys. Rev. **D52** (1995) 5242.

[7] A.I. Vainstein and L.B. Okun, Sov. Journ. Nucl. Phys. **28**(1976) 716.

[8] M. Chanowitz and S. Sharpe, Nucl. Phys. **B222** (1983) 211.
T. Barnes, F.E. Close and F. de Viron, Nucl. Phys. **B224** (1983) 241.

[9] D. Horn and J. Mandula, Phys. Rev. **D17** (19780 898.

[10] A.P. Szczepaniak and E.S. Swanson, Phys. Rev. **D55** (1997) 1578.

[11] A.P. Szczepaniak, E.S. Swanson, C.-J. Ji and S.R. Cotanch, Phys. Rev. Lett. **76** (1996) 2011.

[12] Yu.A. Simonov, Lectures at the XVII Int. School of Physics, Lisbon (1999), hep-ph/9910514.

[13] Yu.A. Simonov, Phys. At. Nucl. **58** (1995) 107.

[14] Yu.S. Kalashnikova and Yu.B. Yufryakov, Phys. Lett. **359** (1995) 175.
Yu.S. Kalashnikova and Yu.B. Yufryakov, Phys. At. Nucl. **60** (1997) 307.

[15] A.B. Kaidalov and Yu.A. Simonov, Phys. Lett. **B477** (2000) 163.

[16] Yu.A. Simonov, Nucl. Phys. **B592** (2001) 350.

[17] A. Donnachie and Yu.S. Kalashnikova, Phys. Rev. **D60** (1999) 114011.