YANG–MILLS THEORY WITH NONSTANDARD LAGRANGIAN AND FINITE-ENERGY CLUSTER OBJECTS

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Using an effective field theory approach we try to show that quantum fluctuations and vacuum polarization effects lead to generation of finite-energy objects in QCD.

Introduction

A topic of my talk is some possible way of generation of finite-energy compact objects in non-abelian quantum field theory. The basis of our approach are singular solutions of classical Yang-Mills theory, and we try to show that vacuum polarization effects play the role of some regularization of singularity of classical solutions. Such approach is analogous to the regularization of self–energy of Coulomb potential (problem of singular self–energy of electron).

I would like to begin with a small remark about the searching of classical solutions of Yang–Mills (YM) theory. The crucial point of such analysis stems from the fact that classical YM theory *has no* solutions with such properties. This is a mathematical theorem [1] that is a consequence of the scale invariance of YM theory. From methodological point of view this theorem is a modification of the well–known Derrick theorem [2]. After this theorem the searching of classical solutions of YM theory had separated into three branches. First of all, this is the searching of nontrivial topological structure of vacuum sector of YM theory and the problem of ground state of this theory. Such problem is connected to the well-known instanton physics. Second branch is the searching of singular solutions of YM theory. The first example of such solutions was proposed by Wu and Yang [3]. Wu–Yang monopole has singular self–energy due to the singularity in origin. This solution plays the very important role in physics of singular solutions of YM theory because the singular Wu–Yang monopole is a prototype, in YM theory plus Higgs fields of the finite-energy 't Hooft-Polyakov monopole [4] in modified theory. This example is the basis of modern point of view on singular solution of classical YM equations as some prototype of finite-energy objects in modified theories.

The searching of solutions of such modified field theories is a third branch of modern development of classical solutions of YM theory. First of all, these are very interesting solutions of YM+Higgs theory that have nontrivial topological nature: monopoles [4], dyons [5] and sphalerons [6]. There are many interesting solutions had been founded in YM theory with external sources [7].

In our approach another type of singular solutions of YM equations will be studied. In contrast with Wu–Yang monopole that has singularity at some point (at origin), our solutions have a singular behavior on some two–dimensional surface: on sphere [8, 9], on cylinder [10], on torus [11] and so on.

In the present work spherically–symmetrical solutions with singularity on a finite-radius sphere will be considered. This is a simple example of such type of singular solutions.

Let us consider the classical field theory with a pure YM lagrangian

$$\mathcal{L}^{\varepsilon}_{YM} = -\frac{1}{4}(F_{\mu\nu})^a(F^{\mu\nu})^a\,, \qquad (1)$$

where $(F_{\mu\nu})^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc}A^b_\mu A^c_\nu$ and $(D_\mu)^{ab} = \delta^{ab}\partial_\mu + \epsilon^{abc}A^c_\mu$. Here we deal with a $SU(2)$ Yang–Mills field.

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We consider the spherically symmetrical chromomagnetic field configuration and substituting the well-known [3] Wu–Yang ansatz

$$A_0^a = 0, \qquad A_i^a = \epsilon_{aij} n_j \frac{1 - H(r)}{r}, \qquad n_i = x_i/r \qquad r = \sqrt{x_i^2}, \tag{2}$$

in Yang–Mills equations, we get the following Wu–Yang equation for amplitude $H(r)$

$$r^2 H(r)'' = H(r)\Big(H(r)^2 - 1\Big). \tag{3}$$

This equation has solutions with singularity at the finite-radius sphere that had been obtained in the different approaches. [12, 13, 8, 9] (see Fig. 1)

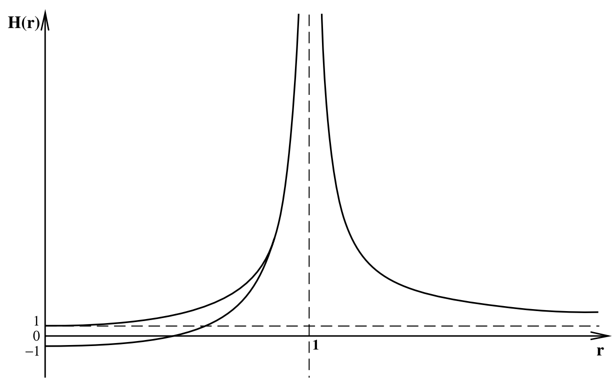

Figure 1: Singular solutions of Wu-Yang equation.

Recently a gauge-invariant approach to the YM theory had been proposed [14]. In this approach YM theory can be reformulated in terms of the bimetrical gravity and one get that typical spherically-symmetrical solution is a black-hole-like solution with Schwarzschild singularity on a finite-radius sphere [8, 9] (see Fig. 1). Because of such singular behavior, this solution has infinite self-energy and physical interpretation of such solution is a very difficult problem, some regularization procedure should be proposed. But on the other hand, on the basis of these solutions one can construct a model of hadron gluonic bag where such solutions play the role of confinement potential for color particle [9].

But how the vacuum polarization effects can be taken into account? One can find an answer to this question in the classical work of J. Schwinger [15]. Using the proper-time technique, Schwinger has shown that in case of quantum electrodynamics corrections to the static electro-magnetic potential from vacuum polarization effects can be taken into account by means of some effective c-number field theory. Lagrangian of such theory consists of a standard lagrangian of QED and some additional part that correspond to the polarization of electron-positron vacuum. Using such effective theory, one can obtain corrections to the Coulomb law from polarization process. Of course, such corrections are absolutely identical to the well-known loop results [16], but such approach seems very useful in cases where perturbation expansion is invalid and nonperturbative effects dominate.

In the present work a very similar approach of consideration of vacuum polarization effects is used in the case of nonabilian field theory. There are many approaches of finding such effective field theory for nonabelian gauge theories. First of all, the quasiclassical effective lagrangian was obtained by Diakonov, Petrov and Yung [17]. A very similar lagrangian had been introduced in the series of papers [18]-[26]. As a result, in leading approximation for gluon field one obtaineds the theory with a new lagrangian $\mathcal{L} = \mathcal{L}_{YM} + \Delta\mathcal{L}$ for the c-number field A_{eff} and this classical Yang–Mills field is an average of the initial gluon field A_0 over quantum fluctuations: $A_{eff} = \langle A_0 \rangle$. In [20, 21, 22] the investigation of classical solutions in such effective theories was discussed in context of the color confinement problem. A very similar problem is discussed in paper [27] also.

In the present work we try to find compact finite-energy objects by using of an effective approach to the QCD that was discussed above.

1. Finite-energy gluon clusters

In this section we investigate the classical Yang-Mills theory with a nonstandard modified lagrangian

$$\mathcal{L}^{\varepsilon}_{YM} = -\frac{1}{4}(F_{\mu\nu})^a(F^{\mu\nu})^a - \frac{\varepsilon^2}{6}\epsilon^{abc}(F_{\mu\nu})^a(F^{\nu}{}_{\rho})^b(F^{\rho\mu})^c\,, \tag{4}$$

where effective coupling constant $\varepsilon = 1/M$ is an inverse mass dimensional parameter characterizing the intensity of quantum fluctuation and polarization effects.

Such form of modification of the Yang-Mills lagrangian is chosen due to the fact that the theory obtained contains only second-order derivative terms. Thus the dynamics of this field theory can be studied in detail.

Using the variation principle, we get the equation of motion

$$D^{ab}_{\mu}(F^{\mu\nu} - \varepsilon^2 G^{\mu\nu})^b = 0. \tag{5}$$

where $(G^{\mu\nu})^a = \epsilon^{abc}(F^{\nu}{}_{\rho})^b(F^{\rho\mu})^c$.

Adding the divergence

$$\partial_\rho[(F^{\nu\rho} - \varepsilon^2 G^{\nu\rho})^a A^a_\mu], \tag{6}$$

to the energy-momentum tensor

$$T^{\nu}{}_{\mu} = \partial_\mu A^a_\rho \frac{\partial \mathcal{L}^{\varepsilon}_{YM}}{\partial(\partial_\nu A^a_\rho)} - \delta^{\nu}{}_{\mu}\mathcal{L}^{\varepsilon}_{YM} = -(F^{\nu\rho} - \varepsilon^2 G^{\nu\rho})^a \partial_\mu A^a_\rho - \delta^{\nu}{}_{\mu}\mathcal{L}^{\varepsilon}_{YM}, \tag{7}$$

we obtain the symmetrical form of this tensor

$$T^{\nu}{}_{\mu} = -(F^{\nu\rho} - \varepsilon G^{\nu\rho})^a (F_{\mu\rho})^a - \delta^{\nu}{}_{\mu}\mathcal{L}^{\varepsilon}_{YM}. \tag{8}$$

Now we consider the spherically symmetrical chromomagnetic field configuration. Substituting the ansatz (2) in (5), we get the following equation on amplitude $H(r)$:

$$\Big(1 - \frac{\varepsilon^2}{r^2}(H(r)^2 - 1)\Big) r^2 H(r)'' = H(r)\Big(H(r)^2 - 1\Big) +$$

$$+ \frac{\varepsilon^2}{r^2}\Big((rH(r)')^2 H(r) - 2rH(r)'(H(r)^2 - 1)\Big). \tag{9}$$

The energy of field configuration generated by the solution of equation (9) $H(r)$ is the functional

$$E^{\varepsilon}[H] = \int T^{00} d^3x =$$

$$= 4\pi \int\limits_0^\infty \left[\Big(1 - \frac{\varepsilon^2}{r^2}(H(r)^2 - 1)\Big)(H(r)')^2 + \frac{(H(r)^2 - 1)^2}{2r^2}\right] dr = \int\limits_0^\infty E(r)dr. \tag{10}$$

The next aim of our investigation is finding the solutions of equation (9). Notice that only finite-energy solutions are interesting for us. Hence the functional $E^{\varepsilon}[H]$ (10) should be finite on such solutions.

Equation (9) is a very complicated nonlinear differential equation. In order to solve it only numerical or approximation methods seem applicable. The crucial point of such analysis is that the leading derivative term in this equation contains the factor

$$\Phi[H](r) = \Big(r^2 - \varepsilon^2(H(r)^2 - 1)\Big). \tag{11}$$

If $H_s(r)$ is a solution of equation (9) and there is a point $r = R$ such that $\Phi[H_s](R) = 0$, then this solution H_s has singular behavior in a neighborhood of this point $r = R$ due to smallness of the factor $\Phi[H_s]$. Using the standard procedure, one obtains the asymptotic behavior near this point

$$H_s(r) \overset{r\to R\pm 0}{\longrightarrow} \pm\sqrt{1+R^2/\varepsilon^2} - C\,(R-r)^{2/3} + \underline{O}(R-r), \tag{12}$$

where C is a constant. Of course, the $H_s(R)$ finite but its derivative at the point $r = R$ is singular. Indeed,

$$H'_s(r) \overset{r\to R\pm 0}{\longrightarrow} \frac{2}{3}C\,(R-r)^{-1/3} + \underline{O}(1) \longrightarrow \infty. \tag{13}$$

Such singular behavior is analogous to the singular behavior on finite sphere of solutions of pure Yang–Mills field [9] discussed above but there is a principal difference. Energy of such solutions in the pure Yang–Mills case is infinite but in the modified Yang–Mills case energy (and other physical characteristics) of solution with singular behavior (13) should be *finite*:

$$E(r)|_{r=R} \sim 4\pi\left(\pm\frac{8\varepsilon^2}{9R^2}\sqrt{1+R^2/\varepsilon^2}\,C^3 + \frac{R^2}{2\varepsilon^4}\right) < \infty. \tag{14}$$

Therefore such solutions are *physical*.

Now we should discuss the numerical investigation of solutions of equation (9) that have the asymptotic behavior (13) at some point $r = R$.

We should choose this asymptotics at origin ($r \to 0$)

$$H(r) \simeq -1 + a_1\,r^2 + a_1^2\,\frac{2\varepsilon^2 a_1 - 3}{10(1+2a_1\varepsilon^2)}\,r^4 + \underline{O}(r^6), \tag{15}$$

and at infinity ($r \to \infty$)

$$H(\rho) \simeq 1 + a_2\,\rho + \frac{3}{4}a_2^2\,\rho^2 + \frac{11}{20}a_2^3\,\rho^3 + \frac{193a_2^2 - 240\varepsilon^2}{480}a_2^2\rho^4 + $$
$$+\frac{329a_2^2 - 1280\varepsilon^2}{1120}a_2^3\rho^5 + \underline{O}(\rho^6), \qquad \rho = 1/r, \tag{16}$$

where $a_1 > 0$ and $a_2 > 0$ are constants.

Notice that equation (9) has very useful symmetries. First of all, this equation is symmetrical with respect to the changes $H \leftrightarrow -H$. So, if we have a solution $H(r)$, then $-H(r)$ is a solution too.

Now let $\varepsilon = \varepsilon_1$ and we have a set of solutions $\{H_{\varepsilon_1}(r)\}$. If we perform the change of variable

$$r \to \frac{\varepsilon_1}{\varepsilon_2}r, \qquad \{H_{\varepsilon_1}(r)\} \overset{r\to\varepsilon_1 r/\varepsilon_2}{\Longrightarrow} \{H_{\varepsilon_2}(r) = H_{\varepsilon_1}(\frac{\varepsilon_1}{\varepsilon_2}r)\}, \tag{17}$$

we get equation (9) again but with new $\varepsilon = \varepsilon_2$. If we know a solution for some $\varepsilon > 0$, say, $\varepsilon = 1$, then using (17) we can obtain a solution for any other $\varepsilon_1 > 0$.

The numerical investigation of equation (9) is presented in Fig. 2. (the solution with $\varepsilon = 1$). In Fig. 3 we can see the energy density (10) corresponding to this solution.

The solution $H_<$ starting at the origin (or *internal*) increases monotonically and its energy density increases too. Evidently, as energy density grows, the role of quantum fluctuations grows too. At the point $r = R$, the energy density attains its critical value E^{cr} and H(r) becomes singular (12). The solution $H_>$ starting at the infinity (or *external*) demonstrates an absolutely similar behavior. Now, a vary essential questions arises: How to connect these two sets of solutions and how to determine such solutions on the whole space?

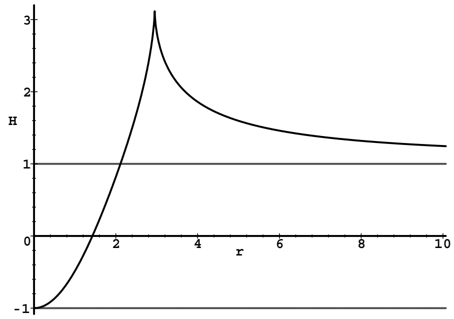

Figure 2: Gluon cluster object. Amplitude $H(r)$ ($\varepsilon = 1$, $a_1 = 0.544$ and $R = 1.425$).

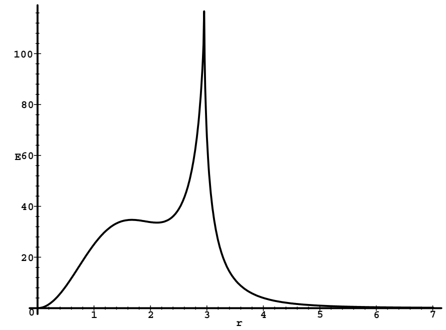

Figure 3: Gluon cluster object. Density of energy.

These questions have no mathematical answer because in this case we deal with the solutions that can not be extended to the right (to the left) because the point of singularity $r = R$ is essential.

Obviously, this nonuniqueness of solution in the whole space is due to underdetermination of our effective model. It is necessary to introduce some additional physical condition that would allow to choose a physically reasonable solution from the broad class of solutions described above.

Since this solution of the model (4) has to be an effective approximation to an existing gluon object, the general properties of the latter should be represented by the former. Thus, if energy density of this gluon object is continuous everywhere, then it should be continuous for the approximating solution of equation (9) as well. We show below that the condition of continuity of energy density is sufficient for the construction of a unique solution and investigation of its properties.

According to mathematical structure of solutions of this model, the condition of continuity of energy density can be formulated as follows: *There exists a critical density of the energy for classical solutions in our effective Yang–Mills theory and the value of this critical density E^{cr} is a physical property of the theory. Therefore E^{cr} shouldn't depend on kind of solution (internal or external).* It follows that

$$E(r)_<|_{r=R} = E(r)_>|_{r=R} \qquad \Longrightarrow \qquad C_< = C_>. \tag{18}$$

It is easily shown that condition (18) uniquely determines our solution and its properties (a_1, a_2 and R) for any ε. This solution is shown in Fig. 2 (if $\varepsilon = 1$, $a_1 = 0.544$ and $R = 1.425$). This solution looks like a shell with radius R.

Using (17), one obtains the following expression for energy of such gluon cluster

$$E^\varepsilon = \frac{1}{\varepsilon}\, E^{\varepsilon=1} = M\, E^{\varepsilon=1}, \tag{19}$$

where $E^{\varepsilon=1} = 110.75$ is the energy of field configuration if $\varepsilon = 1$. Expression (19) is intuitively clear. Indeed, the pure Yang–Mills theory is scale invariant and has no mass-dimensional parameters. Modified Yang–Mills theory (4) has such parameter $\varepsilon = 1/M$ and the mass of gluon objects under investigation is proportional to this parameter.

Now to predict the physical mass $M_{cluster}$ and effective radius we should have a prediction of the value of parameter $\varepsilon = 1/M$. In this paper, following [21], we proposed that $M \simeq 0.59\pi GeV$, and our model gives the following prediction of the mass and effective radius of investigated gluon clusters:

$$M = 1/\varepsilon \simeq 0.59\pi\, GeV, \qquad M_{cluster} \simeq 205\, GeV, \qquad R \simeq 0.15 fm \tag{20}$$

In the next section we give some conclusions and perspectives of such investigations are discussed.

Conclusions

The aim of this paper is to show that quantum fluctuations of nonabelian Yang-Mills field can lead to generation of the cluster finite-energy solution. In our work we used the gauge-invariant approach [18, 19, 20, 25, 26] in which such quantum fluctuation should be taken into account by adding high-derivative terms to the pure Yang–Mills lagrangian. In the present work, we investigated the effective $SU(2)$ Yang–Mills theory and chromomagnetic spherically symmetrical field configurations.

One of the interesting consequences of this effective theory is a fact that for the investigated field configuration there exists a critical value of energy density. This fact is due to the physical condition of continuity of energy density. Such condition allowed us to construct the cluster solution for all space points. We predicted that the mass of such object should be about two hundred GeV and effective radius should be about 0.2 fm.

Of course, we do not give a comprehensive investigation of this effective Yang–Mills theory. The questions about dyon solution or about a role of contributions from other high derivative modified term in pure Yang–Mills lagrangian are clear now. But maybe the most important question in such investigation is about physical consequences of existence of such gluon cluster objects and about their experimental status. All of these questions should be the themes for future investigation.

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