CONSEQUENCES OF THE CAUSALITY PRINCIPLE IN THE RELATIVISTIC THEORY OF GRAVITATION

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This paper illustrates the strength of the Causality Principle in the Relativistic Theory of Gravitation. Some classical problems like Bogorodskii's homogeneous gravitational field, Taub's empty universe and Gödel's rotating universe are considered in RTG's framework. The obtained results differ from those given in Einstein's General Relativity Theory.

1. Introduction

In this paper, we present some consequences of the Causality Principle (CP) in the Relativistic Theory of Gravitation (RTG), for some classical problems analysed long time ago in Einstein's General Relativity Theory (GRT) .

The effective Riemannian space -time which appears in this new theory is determined unambigously by using the four supplementary Eqs. of RTG. CP permits the selection of those solutions of RTG's Eqs. which can have physical sense .

In Section 3, we study the gravitational field produced by a system of masses uniformly distributed on a plane. This problem was analysed by Bogorodskii (see[3], Section 17) in the framework of GRT. We'll show that the real homogeneous gravitational field in Bogorodskii's sense can't be accepted because it doesn't fulfill CP. So, it remains open the problem of finding this field according to RTG.

In Section 4, we present Taub's "empty" universe which is not Minkowskian ([4], [5]). Considering this problem in RTG, we'll see that using CP, Taub's universe is in fact reduced to Minkowski's universe.

In Section 5, we deal with another universe with strange properties in GTR, Gödel's rotating universe ([6]). Analysing Gödel's metric in RTG's framework, we conclude that it can't correspond to an acceptable gravitational field.

2. The Causality Principle in RTG

RTG was constructed within the framework of special relativity theory (SRT) like the theories of other physical fields ([1], sections 6–8, [2]). In an arbitrary admissible coordinate system x^m, m=1,2,3,4 the interval of the Minkowski space-time has the form:

$$d\sigma^2 = \gamma_{mn}(x)dx^m dx^n, \tag{2..1}$$

where $\gamma_{mn}(x)$ are the components of the Minkowskian metric in the assumed coordinate system.

Though the Minkowskian geometry of space-time is the basis of RTG, it does not mean that in the presence of a gravitational field the effective physical space-time will be also Minkowskian.

One of the basic assumption of RTG tells us that the behaviour of matter in the Minkowskian's space -time with metric $\gamma_{mn}(x)$, under the influence of the gravitational field, described by a symmetric tensor $\phi^{mn}(x)$, is identical to its behaviour in the effective Riemannian space-time with metric $g_{mn}(x)$, determined according to the rules:

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$$\tilde{g}^{mn} = \sqrt{-g}\; g^{mn} = \sqrt{-\gamma}\;\gamma^{mn} + \sqrt{-\gamma}\;\phi^{mn}\;,\; g = \det(g_{mn})\;,\; \gamma = \det(\gamma_{mn}). \tag{2..2}$$

The above sentence represents the geometrisation principle of RTG.

The behaviour of the gravitational field is governed by the following differential laws of RTG:

$$R^m_n - \frac{1}{2}\delta^m_n R = 8\pi T^m_n\;, \quad R = R^m_m, \tag{2..3}$$

$$\tilde{g}^{mn}{}_{,m} + \gamma^n_{mp}\tilde{g}^{mp} = 0\;, \quad m,n,p = 1,2,3,4. \tag{2..4}$$

Here R^m_n is Ricci's tensor corresponding to g_{mn} , δ^m_n are Kronecker's symbols and T^m_n denotes the energy-momentum tensor of the sources of the gravitational field. In (2.4) γ^n_{mp} are the components of the metric connection generated by γ_{mn} and the comma is the derivation relative to the involved coordinate. Eq. (2.4) tells us that a gravitational field can have only the spin states 0 and 2. In the monograph [1] this represents one of the basic assumption of RTG. In the work [2], these Eqs. which determine the polarization states of the field, are the consequence of the fact that the source of the gravitational field is the universal conserved density of the energy-momentum tensor of the entire matter including the gravitational field.

We use in all Eqs. relativistic units.

The causality principle (CP) in RTG is presented and analysed by Logunov in [2] section 6.

According to CP any motion of a pointlike test body must have place within the causality light cone of Minkowski's space-time. According to Logunov's analysis CP will be satisfied if and only if for any isotropic Minkowskian vector u^m , i.e. for any vector u^m satisfying the condition:

$$\gamma_{mn}u^m u^n = 0, \tag{2..5}$$

the metric of the effective Riemannian space-time satisfies the restriction:

$$g_{mn}u^m u^n \leq 0. \tag{2..6}$$

According to CP of RTG only those solutions of the system (2.3), (2.4) can have physical meaning which satisfies the above restriction.

We stress the fact that CP in the above form can be formulated only in RTG, because only in this theory, the space-time is Minkowskian and the gravitational field is described by a second order symmetric tensor field $\phi_{mn}(x)$, x^m being the admissible coordinates in the underlying Minkowskian space-time, x^1, x^2, x^3 being the space-like variables and x^4 being the time-like variable.

In the following we'll analyse some consequences of RTG and its CP, taking into account some classical problems analysed already in the framework of Einstein's GRT.

3. Bogorodskii's homogeneous gravitational field

In the classical mechanics (CM) a homogeneous gravitational field is generated by a system of masses uniformly distributed on a plane. The connection between the surface density σ of the mass and the acceleration $\mathcal{G}$ due to the produced gravitational field is given by the relation $\mathcal{G} = 2\pi\sigma k > 0$, k being Newton's gravitational constant.

We choose the Cartesian axes x and y in the mentioned plane and the axis z perpendicular to this plane. According to Bogorodskii, in CM the motion of a free test particle in this gravitational field is governed by Eqs.:

$$\frac{d^2x}{dt^2} = 0\;,\; \frac{d^2y}{dt^2} = 0 \tag{3..1}$$

$$\frac{d^2z}{dt^2} + \mathcal{G} = 0. \tag{3..2}$$

Later on, we'll return to this system.

We also notice that the motion of a particle relative to a non-inertial frame moving with constant proper acceleration $\mathcal{G}$ is governed by the same system (3.1), (3.2).

Bogoradskii's problem is the following (see [3], Section 17): There exists a homogeneous gravitational field in GTR ?

Taking into account the classical results, the author looks for the solutions of Einstein's homogeneous Eqs., for any $z \neq 0$, in the form:

$$ds^2 = -A\ dx^2 - A\ dy^2 - C\ dz^2 + D\ dt^2, \tag{3..3}$$

where A, C, D are positive functions depending only on z.

In this way he gets two solutions:

$$A = 1\ , \quad C = aD^{-1}D^{'\ 2}, \tag{3..4}$$

$$A = D^{-2}\ , C = bD^{-5}D'^{\ 2}. \tag{3..5}$$

Here a and b are real constants, D is an arbitrary function on z.

We observe that Einstein's Eqs. are not sufficient for finding in an unique manner the field produced by the considered distribution of mass.

With the view of finding D(z), Bogorodskii observes that the motion of a free test particle in the produced gravitational field is determined by Eqs. of geodesics:

$$\frac{d^2x}{dt^2} + \left(\frac{A'}{A} - \frac{D'}{D}\right)\frac{dx}{dt}\frac{dz}{dt} = 0\ ,\ \frac{d^2y}{dt^2} + \left(\frac{A'}{A} - \frac{D'}{D}\right)\frac{dy}{dt}\frac{dz}{dt} = 0\ , \tag{3..6}$$

$$\frac{d^2z}{dt^2} - \frac{A'}{2C}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right] + \left(\frac{C'}{2C} - \frac{D'}{D}\right)\left(\frac{dz}{dt}\right)^2 + \frac{D'}{2C} = 0. \tag{3..7}$$

From the above system it can be seen that the vertical motion is described by Eq. (see [3], Section 17):

$$\frac{d^2z}{dt^2} + \left(\frac{C'}{2C} - \frac{D'}{D}\right)\left(\frac{dz}{dt}\right)^2 + \frac{D'}{2C} = 0. \tag{3..8}$$

In the case of slow motion Eq. (3.8) becomes:

$$\frac{d^2z}{dt^2} + \frac{D'}{2C} = 0. \tag{3..9}$$

Now, Bogorodskii looks for those solutions (3.4), (3.5) for which classical Eq. (3.2) and Eq. (3.9) are the same. Hence, he requires:

$$\frac{D'}{2C} = \mathcal{G}. \tag{3..10}$$

In this way, finally Bogoradskii obtains the solutions:

$$A = 1\ ,\ C = e^{2\mathcal{G}z}\ ,\ D = e^{2\mathcal{G}z}\ , \tag{3..11}$$

$$A = (1 - 8\mathcal{G}z)^{1/2}\ ,\ C = (1 - 8\mathcal{G}z)^{-5/4}\ ,\ D = (1 - 8\mathcal{G}z)^{-1/4}. \tag{3..12}$$

For the first solution (3.11), the Riemann-Christoffel curvature tensor is identically zero. Thus, the autor concludes that (3.11) does not represent a real gravitational field. It's easy to see that by the transformations:

$$X = x\ ,\ Y = y\ ,\ Z = \frac{1}{\mathcal{G}}[e^{\mathcal{G}z}ch(\mathcal{G}t) - 1]\ ,\ T = \frac{1}{\mathcal{G}}e^{\mathcal{G}z}sh(\mathcal{G}t)\ , \tag{3..13}$$

the fundamental invariant (3.3) becomes the Minkowskian one:

$$d\sigma^2 = -dX^2 - dY^2 - dZ^2 + dT^2. \tag{3..14}$$

The properties and the singularities of the non-inertial frame characterized by the relations (3.13) are studied in detail in the monograph of Jukov [4], Section15.

Consequently, the first solution (3.11) corresponds to a non-inertial frame whose origine moves with the constant proper acceleration $\mathcal{G}$ along the pozitive axis Z of an inertial frame.

The Riemann-Christoffel curvature tensor corresponding to the second solution (3.12) is not zero. According to Bogorodskii, this solution represents the real homogeneous gravitational field in GRT, produced by the considered distribution of mass.

First of all, we observe that the solution (3.12) has a strange singularity in $z = \frac{1}{8\mathcal{G}}$, which is difficult to be explained. Now, it's the moment to return to the system (3.1), (3.2). We notice that even in CM, there is a difference between gravitational forces and forces of inertia and in the first case Eq. (3.2) must be replaced with:

$$\frac{d^2z}{d^2t} + \mathcal{G} = 0 \ , \text{ for } z > 0 \text{ and } \frac{d^2z}{dt^2} - \mathcal{G} = 0 \ , \text{ for } z < 0. \tag{3..15}$$

Thus, Bogorodskii's solution (3.12) must be replaced with:

$$A = (1 \mp 8\mathcal{G}z)^{1/2} \ , \ C = (1 \mp 8\mathcal{G}z)^{-5/4} \ , \ D = (1 \mp 8\mathcal{G}z)^{-1/4} \ , \tag{3..16}$$

the sign + corresponds to $z < 0$, the sign - corresponds to $z > 0$.

The solution (3.16) can be accepted only for $-\frac{1}{8\mathcal{G}} < z < \frac{1}{8\mathcal{G}}$ and the singularities in $z = \pm\frac{1}{8\mathcal{G}}$ remain without physical explanation.

Now, we verify if the solution (3.16) can be an admissible solution in the framework of RTG. For this purpose, we must verify if Eqs. (2.4) are fulfilled by (3.16). An elementary calculus shows that these Eqs. are not fulfilled by this solution.

Keeping the same point of departure as Bogorodskii, we look now for the solution of this problem in RTG, in the form (3.3). From Eqs. (2.3), with $T_n^m = 0$, for $z \neq 0$,we get the solutions (3.4), (3.5). We determine the function $D(z)$ using Eqs. (2.4). We consider x, y, z, t the Galilean coordinates of an inertial frame. So, Eqs. (2.4) take the simple forme:

$$\tilde{g}^{mn}{}_{,m} = 0 \ . \tag{3..17}$$

Taking into account (3.4), (3.5), from (3.17) we get:

$$D(z) = pe^{qz} \ , \tag{3..18}$$

where p and q are constants.

Thus, from (3.4) we obtain the first solution according to RTG:

$$A = 1 \ , \ C = ape^{qz} \ , \ D = pe^{qz} \ , \text{ for } z \neq 0 \tag{3..19}$$

and from (3.5) the second solution:

$$A = p^{-2}e^{-2qz} \ , \ C = bp^{-3}e^{-3qz} \ , \ D = pe^{qz} \ , \text{ for } z \neq 0 \ . \tag{3..20}$$

The constants a, b, p, q must be determined from the Correspondence Principle: after switching off the gravitational field, the curvature of space disappears and we find ourselves in the Minkowski space-time in the chosen reference system. So, Eqs. of motion become classical Eqs. of motion in the chosen reference system. From this principle we must have the relation (3.10) for the first solution (3.19) and taking into account (3.15), for the second solution (3.20) we must have:

$$\frac{D'}{2C} = \begin{cases} \mathcal{G} & , z > 0 \\ -\mathcal{G} & , z < 0 \end{cases} . \tag{3..21}$$

For the same principle, the metric must tend to the Galilean metric for $\mathcal{G}$ converges to zero. Thus, we get:

$$A = 1 \ , \ C = e^{2\mathcal{G}z} \ , \ D = e^{2\mathcal{G}z} \ , \text{ for } z \neq 0, \tag{3..22}$$

$$A = e^{\mp 4\mathcal{G}z} \ , \ C = e^{\mp 6\mathcal{G}z} \ , \ D = e^{\pm 2\mathcal{G}z} \ , \text{ for } z \neq 0 \ . \tag{3..23}$$

Our first solution is identical with that obtained by Bogorodskii. Hence, its physical significance was clarified. Concerning the second solution, we observe that the conditions (3.21) can be only approximately fulfilled. Hence in RTG o homogeneous gravitational field in Bogorodskii's sense can exist only in this approximative manner.

All details concerning the analysed problem will be presented in a future paper.

Obviously, even in the above approximative manner, our second solution can be acceptable only if it satisfies CP of RTG. We consider the isotropic vector u=(1,0,0,1) in the underlying Minkowskian space-time. The condition (2.6) is fulfilled if:

$$e^{2\mathcal{G}z} \leq e^{-4\mathcal{G}z} \ , \text{ for } z > 0 \quad \text{and} \quad e^{-2\mathcal{G}z} \leq e^{4\mathcal{G}z} \ , \text{ for } z < 0. \tag{3..24}$$

These conditions can be satisfied only if

$$\mathcal{G} = 0. \tag{3..25}$$

From (3.25) the solution (3.23) reduces to the Minkowskian metric in the Galilean coordinates.

We conclude that in accordance with RTG it can't exist a real gravitational field, anyway in Bogorodskii's sense. Thus, it remains open the problem : Which is the real gravitational field produced by a distribution of mass concentrated in an infinite plane, according to RTG ?

4. Taub's empty universe and Mach Principle

A famous example of an "empty" physical universe, which is not Minkowski's universe is delivered in the framework or GTR by Taub in 1951 ([5], see also the monograph of Arifov [6], Section 4). The metric of the Riemannian universe is given by the expression:

$$\begin{aligned} ds^2 \ &= \ e^{(C_1+C_2)x^4} ch\xi (dx^4)^2 - e^{C_2 x^4} ch\xi (dx^1)^2 - \frac{1}{ch\xi}(dx^2)^2 + \\ &+2\frac{\sqrt{C_1 C_2}}{ch\xi} x^1 dx^2 dx^3 - \left[\frac{C_1 C_2}{ch\xi}(x^1)^2 + e^{C_1 x^4} ch\xi \right] (dx^3)^2 \ , \end{aligned} \tag{4..1}$$

where $\xi = \sqrt{C_1 C_2}(x^4 + const)$, $C_1 = const$, $C_2 = const$.

The metric (4.1) is defined for any reals x^1, x^2, x^3, x^4; C_1, C_2 are arbitrary real constants. We can also notice that Taub's metric has the signature 2 and it is regular in its entire domaine of definition. But the remarkable property of this metric is the fact that it satisfies Einstein's fields Eqs. with the identical null energy-momentum tensor of the sources of the gravitational field , i.e. $T_n^m \equiv 0$. This is the reason for what we said that Taub's universe is an "empty" one.

Moreover, the Riemann-Christoffel curvature tensor corresponding to the metric (4.1) is not identically null. For example the components:

$$R^1_{441} \ = \ -\frac{1}{2}\frac{C_1 C_2}{ch^2\xi}\left(1 - \frac{1}{2}ch^2\xi - \frac{1}{2}\sqrt{\frac{C_1}{C_2}} sh\xi ch\xi\right)$$

$$
\begin{aligned}
R^2_{442} &= \frac{C_1C_2}{ch^2\xi}\left[1-\frac{1}{2}ch^2\xi-\frac{1}{4}\left(\sqrt{\frac{C_1}{C_2}}+\sqrt{\frac{C_2}{C_1}}\right)sh\xi ch\xi\right]\\
R^3_{443} &= -\frac{1}{2}\frac{C_1C_2}{ch^2\xi}\left(1-\frac{1}{2}ch^2\xi-\frac{1}{2}\sqrt{\frac{C_2}{C_1}}sh\xi ch\xi\right) ,
\end{aligned}
$$

don't vanish simultaneously if $C_1 \neq C_2$. Consequently, if $C_1 \neq C_2$ the "empty" Taub's universe is a Riemannian universe.

So, we can see once more that Eistein's field Eqs. are not enough to obtain an unique gravitational field produced by a well precised system of field sources. Indeed, the Minkowskian metric:

$$d\sigma^2 = (dx^4)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 \tag{4..2}$$

satisfies also Einstein's field Eqs. with $T^m_n \equiv 0$.

In this way, Taub's metric shows clearly that Mach's principle in Einstein's acception: "the G-field is fully determined by the masses of bodies", is not included in GTR. For more details and for various formulations of Mach's principle see [2], Section 7.

We consider now this Taub's "empty"universe in RTG. Like coordinates in the underlying Minkowski space-time, we select the Galilean coordinates of an inertial reference frame x^1, x^2, x^3, x^4 in which the metric has the form (4.2). This hypotesis is justified because Taub's metric (4.1) becomes the Minkowskian metric (4.2) if $C_1 = C_2 = 0$. In the chosen coordinates, Eqs. (2.4) become:

$$\tilde{g}^{mn}{}_{,m} = 0 \ . \tag{4..3}$$

From (4.1) the nonzero components of the tensor g^{mn} are:

$$g^{11} = -\frac{e^{-C_2x^4}}{ch\xi} \ , \ g^{22} = -\left(\frac{C_1C_2(x^1)^2}{ch\xi} + e^{C_1x^4}ch\xi\right)e^{-C_1x^4} \ ,$$

$$g^{23} = -\frac{\sqrt{C_1C_2}x^1e^{-C_1x^4}}{ch\xi} \ , \ g^{33} = -\frac{e^{-C_1x^4}}{ch\xi} \ , \ g^{44} = \frac{e^{-(C_1+C_2)x^4}}{ch\xi} \ , \tag{4..4}$$

and the determinant of the metric tensor g_{mn} is given as follows:

$$g = \det g_{mn} = -e^{2(C_1+C_2)x^4}ch^2\xi. \tag{4..5}$$

From (4.4), (4.5), (2.2), we obtain the nonzero components of $\tilde{g}^{mn}$:

$$\tilde{g}^{11} = -e^{C_1x^4} \ , \ \tilde{g}^{22} = -e^{(C_1+C_2)x^4}\left[C_1C_2(x^1)^2e^{-C_1x^4} + ch^2\xi\right] \ ,$$

$$\tilde{g}^{23} = -\sqrt{C_1C_2}x^1e^{C_2x^4} \ , \ \tilde{g}^{33} = -e^{C_2x^4} \ , \ \tilde{g}^{44} = 1. \tag{4..6}$$

Now, it can be verified easily that the system (4.3) is satisfied identically for Taub's metric. It remains to be seen if it satisfies or not CP of RTG. For this purpose, we consider the Minkowskian isotropic vector u=(1,0,0,1). Then, from (2.6) the following inequality must be hold:

$$e^{C_1x^4} \leq 1, \text{for any } x^4. \tag{4..7}$$

Obviously, this restrction is fulfilled if and only if:

$$C_1 = 0. \tag{4..8}$$

Taking into account (4.8) and choosing the Minkowskian isotropic vector u=(0,0,1,1), we obtain for (2.6):

$$e^{C_2 x^4} \leq 1, \text{ for any } x^4. \tag{4..9}$$

This implies:

$$C_2 = 0. \tag{4..10}$$

According to (4.8), (4.10), the Taub "empty" universe is in fact the Minkowski empty universe.

In this way and in this case we can conclude that RTG contains Mach's principle in its Einstein's acception: the matter of the universe should determine uniquely the geometry of the universe, i.e. in RTG language the effective Riemannian space is a Minkowskian space.

This example shows clearly the importance of the causality principle, the complet system of Eqs. (2.3), (2.3) being not sufficient for the selection of the physical acceptable solution.

5. Gödel's rotating universe

In a paper published in 1949 [7], Gödel showed that the following metric is compatible with an incoherent matter distribution if the cosmological constant is taken into account in Einstein's field Eqs. of GTR:

$$ds^2 = (dx^4 + e^{\alpha x^1} dx^2)^2 - (dx^1)^2 - \frac{1}{2} e^{2\alpha x^1} (dx^2)^2 - (dx^3)^2, \tag{5..1}$$

where α is a constant with the dimension of an inverse length.

In his monograph [8], Chapter VIII, Section 4, Synge has shown that Gödel's metric is also compatible with the metric obtained from Einstein's field Eqs. without the cosmological constant, if we assume that the involved gravitational field is produced by an ideal fluid at rest in the considered reference frame in which the metric (5.1) is given.

More exactly the field Eqs. (2.3) can be satisfied identically by Gödel's metric if we assume that the energy-momentum tensor of the sources has the form:

$$T^{mn} = (p + \rho) u^m u^n - p g^{mn}, \tag{5..2}$$

where $\rho > 0$, $p > 0$, and u^m represent the density of the fluid, the hydrostatic pressure in the fluid and its 4-vector velocity. Since the fluid is at rest in the considered frame (the co-moving frame), we have:

$$u^m = \delta^m_4. \tag{5..3}$$

Using the results given in the monograph of Adler, Bazin and Schiffer [10], Chapter 12, Section 4, it can be seen that Eistein's field Eqs. are satisfied identically by Gödel's metric (5.1), if the density ρ, the pressure p and the Gödel's parameter α satisfy the following conditions:

$$8\pi\rho = 8\pi p = \frac{\alpha^2}{2}. \tag{5..4}$$

From physical point of view, the condition (5.4) is somehow disappointed, because in real fluid usually $\frac{p}{\rho} \ll 1$.

But not this fact represents the only strange property of Gödel's rotating universe. Some of curious physical properties of Gödel's metric are analysed in detail in [8], Chapter 12, Section 4 and in [9], Chapter 3. We don't mention in this paper these aspects of the problem.

Now, we analyse Gödel's model in RTG framework. We suppose that x^1, x^2, x^3, x^4 represent an admissible system of coordinates in the underlying Minkowski space-time. This hypothesis is justified because in the absence of the source of the gravitational field:

$$p = \rho = 0 \tag{5..5}$$

from (5.4), we get:

$$\alpha = 0, \tag{5..6}$$

and in this case the Riemannian metric (5.1) of Gödel's rotating universe gets the Minkowskian form:

$$d\sigma^2 = (dx^4 + dx^2)^2 - (dx^1)^2 - \frac{1}{2}(dx^2)^2 - (dx^3)^2. \tag{5..7}$$

By the following change of variables:

$$x^1 = X \ , \ x^2 = \sqrt{2}Y \ , \ x^3 = Z \ , \ x^4 = T - \sqrt{2}Y \tag{5..8}$$

which leaves the considered frame unchanged, the Minkowskian metric takes the Galilean form:

$$d\sigma^2 = dT^2 - dX^2 - dY^2 - dZ^2. \tag{5..9}$$

So that, the frame in which the Minkowskian metric has the forme (5.7) is an inertial frame. The fluid, the source of the Gödel's gravitational field, is at rest exactly in this frame, according to GTR. The relation $(5.8)_4$ shows the way of introduction of the temporal coordinate x^4, which depends on the position, in this frame.

To study any problem in RTG framework one must solve Eqs. (2.3), (2.4) in termes of the coordinates of the underlying Minkowskian space-time. For the system of coordinates x^1, x^2, x^3, x^4 chosen above, the Gödel's metric (5.1) does not satisfy Eqs. (2.4) if $\alpha \neq 0$.

Indeed, in these coordinates the components of the metrc connexion $\gamma^n_{mp}(x)$ are identically nulls. More over, since the components of the metric (5.1) depend only on the coordinate x^1 the system (2.4) takes the form:

$$\tilde{g}^{1n}{}_{,1} = 0. \tag{5..10}$$

From the metric (5.1), the expression of $\tilde{g}^{1n}$ are:

$$\tilde{g}^{11} = -\frac{1}{\sqrt{2}} e^{\alpha x^1} \ , \ \tilde{g}^{12} = \tilde{g}^{13} = \tilde{g}^{14} = 0. \tag{5..11}$$

It is easy to see that Eqs. (5.10) are not fulfilled if $\alpha \neq 0$.

For finding the solution of the complete RTG system of Eqs., we'll use the same procedure as Logunov and Mestverishvili in [1], Chapter 13. Thus we are looking for a system of coordinates y^i in which Eqs. (2.3) are fulfilled, the system of Eqs. (2.4) establishing a one to one relationship between the sets of coordinates y^i and x^i, in the Minowskian space-time.

We make this change remaining in the inertial frame X, Y, Z, T, with Galilean metric (5.9), so $\gamma^n_{mp}(y) \equiv 0$.We write the system of Eqs. (2.4) in a somewhat different form (see the relation (13.17), (13.22) from [1], Chapter 13):

$$\frac{\partial}{\partial x^m}\left(\sqrt{-g(x)} g^{mn}(x) \frac{\partial y^p}{\partial x^n}\right) = 0. \tag{5..12}$$

Because the components $g_{mn}(x)$ of the metric (5.1) depend only on x^1, we shift from the variables x^i to the variables y^i, assuming that:

$$y^1 = y^1\left(x^1\right) \ , \ y^2 = x^2 \ , \ y^3 = x^3 \ , \ y^4 = x^4. \tag{5..13}$$

It is easy to see that the system (5.12) becomes:

$$\frac{d}{dx^1}\left(\sqrt{-g\left(x\right)}g^{11}\left(x\right)\frac{dy^1}{dx^1}\right)=0. \tag{5..14}$$

From (5.11), Eq. (5.14) takes the form:

$$\frac{d}{dx^1}\left(e^{\alpha x^1}\frac{dy^1}{dx^1}\right)=0. \tag{5..15}$$

Integrating this equation and choosing the constants of intergration in such way that for α tends to zero, y^1 tends to x^1, we get:

$$y^1=\frac{1}{\alpha}\left(1-e^{-\alpha x^1}\right). \tag{5..16}$$

After an elementary calculus we find the components of Gödel's metric (5.1) in the system of coordinates (5.13), (5.16) :

$$ds^2=-\frac{1}{\left(1-\alpha y^1\right)^2}\left(dy^1\right)^2+\frac{1}{2\left(1-\alpha y^1\right)^2}\left(dy^2\right)^2-\left(dy^3\right)^2+\left(dy^4\right)^2+\frac{2}{\left(1-\alpha y^1\right)}dy^2dy^4. \tag{5..17}$$

This is the solution which satisfies the complet system of Eqs. (2.3), (2.4) in the system of coordinates y^1 , y^2 , y^3 , y^4, the coordinate y^1 being submited to the condition:

$$1-\alpha y^1>0. \tag{5..18}$$

We notice that an unaccountable singularity appears here.

Now, we submit this solution to CP of RTG. We consider the Minkowskian isotropic vector u=(0, $\sqrt{2}$, 0, 1-$\sqrt{2}$). The causality condition (2.6) becomes:

$$\frac{1}{\left(1-\alpha y^1\right)^2}+\frac{2\sqrt{2}\left(1-\sqrt{2}\right)}{\left(1-\alpha y^1\right)}+\left(1-\sqrt{2}\right)^2\leq 0, \tag{5..19}$$

for any y^1 which fulfilled (5.18).

It's easy to see that for $1-\alpha y^1=\frac{1}{2}$ we obtain a contradiction, since the left member of (5.19) becomes $2\sqrt{2}-1$ which is positiv. This contradiction can be eliminated only if

$$\alpha=0. \tag{5..20}$$

Consequently, Gödel's metric, for $\alpha\neq 0$, $p\neq 0$, $\rho\neq 0$, doesn't satisfie CP, so this effective Riemannian space-time can't correspond to a physical acceptable gravitational field.

A similar conclusion can be given off from the paper of Panov [11], if in the restriction (21) presented in this paper we go up to the end in the establishment of the consequences.

Thus we can see that if RTG is true, Gödel's rotating universe and its strange properties, one of them being the possibility of travel into the past and even of influence it, can not exist.

Conclusions

We can conclude that CP in RTG plays a very important role in finding the real gravitational fields which can have physical sense. We stress again that CP, in its adopted form, can be formulated only in RTG, because only in this theory we dispose of the metric $\gamma_{mn}(x)$ of the underlying Minkowski space-time.

As we have seen, considering the universes of Taub and Gödel in the framework of RTG, CP allowed us to reduce these universes to the Minkowskian one (see the relations (4.8), (4.10) and respectively (5.20)). So, in RTG the Mach Principle is realized-an inertial reference frame is determined by the distribution of matter. We have also shown that if the metric which represents, according to RTG, Bogoroskii's homogeneous gravitational field, is submitted to CP it becomes the Minkowskian one (see (3.25)). It can be shown (we'll present this in a future paper) that in the obtained space-time (3.23) , the velocity of a free test particle can overpass the velocity of light in vacuum. The problem of finding in RTG the gravitational field produced by a uniform distribution of mass concentrated on an infinite plane is a very interesting problem which remains to be solved.

Acknowledgements. We would like to thank A. A. Logunov (Chairman), V. A. Petrov (Vice-Chairman), A. P. Samokhin, S. M. Klishevich (Secretary) and all the other organizers of the XXIII International Workshop on the Fundamental Problems of High Energy Physics and Field Theory for inviting us to this Conference and for their warm hospitality during the stay at Institute of High Energy Physics. We would also like to thank M.Arminjon for his attention and critisism during the preparation of the work.

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