NONSYMMETRIC GRAVITATIONAL THEORY IN MINKOWSKI SPACE

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We suggest a theory of nonsymmetrical tensor field in the Minkowski space. This theory generalizes the relativistic theory of gravity with a symmetrical potential and preserves all its basic postulates. The connection of the effective space-time in wich the gravitational field equations can be represented possesses a torsion and nonmeticity.

1. The relativistic theory of gravity (RTG) [1] can be regarded as a gauge theory of group of Lie variations for dynamic variables. The related transformations are variations of the form of function for generally covariant transformations, and they form the internal symmetry group. That the action be invariant for this group under the transformations of the dynamic variables alone requires replasing the "nondynamic" density $\tilde{\gamma}^{ik} = \sqrt{-\gamma}\gamma ik$ of the Minkowski metric in the matter Lagrangian with $\tilde{g}^{ik} = \sqrt{-g}g^{ik} = \sqrt{-\gamma}(\gamma^{ik} + k\psi^{ik})$, where $\gamma = det\gamma_{ik}, g = detg_{ik}$ and k^2 is the Einstein constant, and thus introducing the gauge gravitational potential ψ^{ik}. The expression g^{ik} is interpreted here as the metric of the effective space-time from which the connection, the Christoffel bracket, can be uniquely constructed. On the order hand, the gauge invariancepermits using a more general form of the efffective metric,

$$\sqrt{-g}g^{ik} = \sqrt{-\gamma}(\gamma^{ik} + kh^{ik}), \tag{1}$$

where h^{ik} is a nonsymmetrical potential. Accordingly, we conjecture that the gravitational field is described by a nonsymmetrical tensor potential. In this paper, we elaborate a theory of such field that generalizes the RTG and investigate the sructure of the resulting effective space-time. As is known, Einstein already used a nonsymmetrical metric in the attempts to construct a unified theory of gravitation and electromagnetism. It was introduced in the Moffat theory [2] in the framework of the geometrical description of purely gravitational interaction. The consideration in the present paper is close to the latest version of this theory [3], but the field equations here differ from those in [3]. Although the presently existing experimental basis of gravitational theories gives no direct indications for the necessity of generalizing Riemannian geometry, these theoretical schemes have been much investigated in the literature beginning Einstein-Cartan theory. As can be seen in our further consideration, the only way to go beyond the scope of the effective Riemannian geometry in describing gravitational interaction in the Minkowski space in the framework oe tensor theory is just exactly introducing a nonsymmetrical potential.

2. We construct the theory of the nonlinear field h^{ik} proceeding from the following postulates accepted in the RTG:

1. The theory of a massless field must be gauge invariant.
2. The field equations must have the form of equations with a linear identically preserved left-hand side and nonlinear universal source.
3. The field must be described by states with helicity two and zero.
4. The interaction between the gravitational field and matter must be minimal, which means the absance oe the effective connection in the matter Lagrangian.

By the requirement of gauge invariance, the matter Lagrangian L^M has the form

$$L^M(\tilde{h}^{ik}, \tilde{\gamma}^{ik}, Q^A) = L^M(\tilde{g}^{ik}, Q^A), \tag{2}$$

where Q^A are the dynamic matter variables. It is follows from (1) and (2) that the field source in the field equations $\frac{\delta L}{\delta \tilde{h}^{ik}} = 0$ that relates to matter is

$$\tilde{A}^M_{ik} = -\frac{\delta L^M}{\delta \tilde{f}^{ik}}_{|f^{ik}=\gamma^{ik}}, \tag{3}$$

where f^{ik} is hte Minkowski metric and is assumed to be nonsymmetrical under the variation and equal to γ^{ik} after the variation. To simplify the representation in what follows, we omit the notation relating to the transition from f^{ik} to γ^{ik} in the corresponding formulas. To obtain the RTG equations under the transition to the symmetrical potential, it is necessary to pass to a linear combination of this equations that involves

$$A^{ik}_M = -\frac{2}{\sqrt{-\gamma}} \frac{\delta L^M}{\delta f_{ik}}, \tag{4}$$

as a source, where f_{ik} is the inverse tensor of f^{ik}. As in the RTG, we now assume that the full matter and gravitational field source is universal and find its form,

$$A^{ik} = -\frac{2}{\sqrt{-\gamma}} \frac{\delta L}{\delta f_{ik}}, \tag{5}$$

where $L = L^g + L^M$ is the Lagrangian of the entire system. The symmetrical part A^{ik} of the source is the Hilbert energy-momentum tensor

$$A^{(ik)} = t^{ik} = -\frac{2}{\sqrt{-\gamma}} \frac{\delta L}{\delta \gamma_{ik}}, \tag{6}$$

and the antisymmetrical part can be written as

$$A^{[ik]} = -\frac{2}{\sqrt{-\gamma}} \frac{\partial L}{\partial f_{ik}} + (S^{ikl} + S^{kli} + S^{lik})_{|l}. \tag{7}$$

The vertical bar in (7) symbolizes covariant differentation with respect to the Minkowski metric, and S^{ikl} is the spin moment tensor, which can be expressed in terms of the generators of the generally covariant transformations Ω^{kB}_{iA} for Q_A under an infinitesimal displacement $\xi^i(\delta Q_A = \Omega^{kB}_{iA} Q_B \xi^i_{,k})$ and has the form

$$S^{ikl} = \frac{\partial L}{\partial Q^A_{|i}} \Omega^{[kl]A}{}_B Q^B. \tag{8}$$

Using defenition (8), the formula

$$Q^A_{|i} = Q^A_{,i} - \Omega^{lA}_{kB} \ddot{\Gamma}^k_{li} Q^B \tag{9}$$

for the covariant derivate, and the expression for the Minkowski space connection, which under the replacement γ_{ik} with f_{ik} becomes $\ddot{\Gamma}^k_{li} = \frac{1}{2} f^{kn}(f_{ni,l} + f_{nl,i} + f_{li,n})$, we can easily show that (7) holds.

Because the matter Lagrangian does not contain the coordinate connection, the totally antisymmetrized spin moment in (7) enters only the source specified by the gravitational field itself.

According to the second postulate, we seek the equations of the free massless field in the form

$$L^{il} = h^{il}{}_{|m}{}^{m} - h^{im}{}_{|m}{}^{l} - h^{ml}{}_{|m}{}^{i} + h^{rs}{}_{|rs}\gamma^{il}, = kA^{il} \;, L^{il}_{|l} = L^{il}_{|i} = 0\;. \tag{10}$$

System (10) can now be split into the equations for the symmetrical fields ψ^{ik} and antisymmetrical fields ϕ^{ik}

$$J^{ik}(\psi) = \psi^{ik}{}_{|m}{}^{m} - \psi^{im}{}_{|m}{}^{k} - \psi^{mk}{}_{|m}{}^{i} + \psi^{rs}{}_{|rs}\gamma^{ik} = kt^{ik}(h) \ , \tag{11}$$

$$F^{ik}(\phi) = \phi^{ik}{}_{|m}{}^{m} - \phi^{im}{}_{|m}{}^{k} - \phi^{mk}{}_{|m}{}^{i} = kA^{[ik]}(h) \ . \tag{12}$$

Equation (11) has the same form as in the RTG with the distinction that the Hilbrt tensor of the entire field plays the role of the source. Equation (12) is a nonlinear generalization of the equation for the antisymmetrical tensor field that was first considered in [4], where the object with helicity zero was called "notohp" ("photon" backwards).

We note that if the source of the symmetrical field is preseved in the equations for any generally covariant Lagrangian, then the source $A^{[ik]}$ is preseved only because of the identity $F^{ik}{}_{|k} = 0$. This situation relates to the absence of the antisymmetrical Noether current of the gauge group.

The free field Lagrangian density with first derivatives and a gauge- invariant massless part can be written in the general form

$$L^g = \frac{1}{k^2}[\tilde{R}(g,\Gamma) - div] + L^{QN} + L^m \ , \tag{13}$$

where $\tilde{R} = \sqrt{-g}R$ is the scalar curvature density (constructed from the metric and the connection Γ^i_{kn})

$$R = g^{ik}R_{ik} = g^{ik}(\Gamma^l_{ik,l} - \Gamma^l_{il,k} + \Gamma^l_{ik}\Gamma^m_{lm} - \Gamma^m_{il}\Gamma^l_{mk}) \ . \tag{14}$$

The connection can possess a torsion Q^i_{kn} and a nonmetricity $\tilde{N}^{kn}_i$

$$Q^i_{kn} = \Gamma^i_{[kn]} \ , \tilde{N}^{kn}_i = \nabla_i \tilde{g}^{kn} = \tilde{g}^{kn}_{,i} + \Gamma^k_{ri}\tilde{g}^{rn} + \Gamma^n_{ri}\tilde{g}^{kr} - \Gamma^r_{ir}\tilde{g}^{kn} \ . \tag{15}$$

The Lagrangian L^{QN} can be constructed using different combinations of the torsion and nonmetrisity tensors. Finally, the Lagrangian L^m describing the field mass and violating the gauge invariance has the form

$$L^m = -\frac{m^2}{k^2}(\frac{1}{2}\gamma_{ik}\tilde{g}^{ik} - \sqrt{-g} - \sqrt{-\gamma}) \ , \tag{16}$$

as in the RTG. Lagrangian (16) contains the antisymmetrical part of the field only in the term $\sqrt{-g}$, and the linear term $\frac{1}{k}m^2\phi^{ik}$ in the field equations therefore enters the source $A^{[ik]}$.

We note, that the part of Lagrangian related to the curvature can be represented in the more general form

$$L^g_1 = \frac{1}{k^2}(a\tilde{g}^{ik} + b\tilde{g}^{ki})R_{ik} \ , \tag{17}$$

where a and b are coefficients satisfying the condition $a + b = 1$. However, it is easy to see that the Lagrangians L^g and L^g_1 are equivalent because the transition between them simly reduces to redefining the potential ϕ^{ik} .

Discarding the scalar density in $\tilde{R}$, which form the divergence and involves second derivatives, we obtain a Lagrangian that contains first derivatives and is expressible in terms of the affine deformation tensor $D^l_{ik} = \Gamma^l_{ik} - \ddot{\Gamma}^l_{ik}$

$$L^g = \frac{1}{k^2}[\tilde{g}^{ik}(D^l_{im}D^m_{kl} - D^l_{ik}D^m_{lm}) + D^l_{il}\tilde{N}^{ik}_k - D^l_{ik}\tilde{N}^{ik}_l] + L^{QN} + L^m \ . \tag{18}$$

We now consider the field equations. We assume that the only relation between the metric(potential) and the connection must follow from the field equations $\frac{\delta L}{\delta \Gamma^l_{ik}} = 0$. This approach corresponds to the well-known Palatini method in General Relativity. The equation relating the metric to the connection cannot be resolved explicity with respect to the connection in the case of a nonsymmetrical metric, and it therefore becomes necessary to use the first-order formalism here. The relation between the metric and the connection must be differential, therefore, to obtain these equations, the Lagrangian L^R , involving second derivatives,

$$L^R = \frac{1}{k^2}\tilde{R}(g, \Gamma) + L^{QN} + L^m \tag{19}$$

must be used. On the other hand, to obtain equations of form (10), the Lagrangian L^g with first derivatives must be used because the source for Lagrangian (19) vanishes by virtue of the field equations.

We consider the variational derivative $\frac{\delta L^g}{\delta f_{ik}}$. The Minkowski metric enters Lagrangian (18) implicitly via the metric $\tilde{g}^{ik}$ and explicity via the coordinate connection $\ddot{\Gamma}^l_{ik}$. Consequently,

$$\frac{\delta L^g}{\delta f_{ik}} = \frac{\delta^\star L^g}{\delta f_{ik}} + \frac{\delta L^g}{\delta \tilde{g}^{rs}}\frac{\partial \tilde{g}^{rs}}{\partial f_{ik}} \ , \tag{20}$$

where $\delta^\star$ symbolizes the variation with respect to the expression f_{ik} entering the coordinate connection. By (20) and the field equations $\frac{\delta L^g}{\delta \tilde{g}^{rs}} = 0$, these equations assume the form

$$-\frac{2}{\sqrt{-\gamma}}\frac{\delta^\star L^g}{\delta f_{ik}} = A^{ik} \ . \tag{21}$$

If the chosen Lagrangian is such that

$$-\frac{2}{\sqrt{-\gamma}}\frac{\delta^\star L^g}{\delta f_{ik}} \equiv L^{ik} - m^2\psi^{ik} \ , \tag{22}$$

then the equations for the potential h^{ik} take the desired form (10). Eor the field ϕ^{ik} to describe only the state with helicity zero in the linear approximation, it is necessary (see [4]) that the condition

$$\phi^{ik}{}_{|k} = 0 \ . \tag{23}$$

hold. We recuire that this condition hold for the exact solutions as well. Then the simplest Lagrangian simultaneously resulting in identity (22) and condition (23) is the sum of the scalar curvature density and the massive term. to prove this, we find the relation between the metric and the connection inplied by the field equations $\frac{\delta L^R}{\delta \Gamma^l_{ik}} = 0$. This relation is written as

$$\nabla_l \tilde{g}^{ik} = 2Q^k_{ml}\tilde{g}^{im} - \frac{2}{3}Q_m\tilde{g}^{im}\delta^k_l \ . \tag{24}$$

Formula (24) implies

$$\nabla_l \tilde{g}^{il} = -\frac{2}{3}Q_l\tilde{g}^{il} \ , \nabla_l \tilde{g}^{li} = 2Q^i_{mn}\tilde{g}^{nm} - \frac{2}{3}Q_l\tilde{g}^{il} \ . \tag{25}$$

Combining relations (25), we obtain condition (23).

Substituting (25) in (18) (for $L^{QN} = 0$), we obtain

$$L^g = \frac{1}{k^2}\tilde{g}^{ik}(D^l_{im}D^m_{lk} - D^l_{ik}D^m_{lm} + 2D^l_{ik}Q_k) + L^m \ . \tag{26}$$

We note that substituting part of the field equations into the Lagrangian is legitimate in this case becouse Γ^l_{ik} and f_{ik} are inderpendent and f_{ik} is not a field variable. We vary relation (26) with respect to the metric f_{ik}. Passing to the Cartesian coordinates after the variation, using the formulas Переходя после варьироваения к декартовым координатам и используя

$$\frac{\partial \ddot{\Gamma}^l_{nm}}{\partial f_{ik}} = 0\ , \frac{\partial \ddot{\Gamma}^l_{nm}}{\partial f_{ik,s}} = \frac{1}{2}(\gamma^{li}\delta^s_n\delta^k_m + \gamma^{lk}\delta^i_n\delta^s_m - \gamma^{ls}\delta^i_n\delta^k_m)\ , \tag{27}$$

and taking (23) into account, we obtain the field equations

$$L^{ik} - m^2\psi^{ik} + 2\gamma^{ik}(Q_n\tilde{g}^{mn})_{|m} = kA^{ik}\ . \tag{28}$$

We now take into consideration that Eq. (24) has some additional invariance with respect to the connection transformation

$$'\Gamma^l_{ik} = \Gamma^l_{ik} + \delta^l_i v_k\ , \tag{29}$$

where v_k is an arbitrary vector field. By the existence of this invariance, the torsion trace Q_k remains arbitrary. We shall use the gauge $Q_n = 0$, which totally violates the indicated invariance.

An analysis of all possible scalar densities L^{QN} shows that there is a unique Lagrangian

$$L^{QN} = -Q_i\nabla_k\tilde{g}^{ik} - \frac{1}{3}\tilde{g}^{ik}Q_iQ_k\ . \tag{30}$$

leading to condition (23) and to a relation invariant with respect to the more general gauge transformation

$$'\Gamma^l_{ik} = \Gamma^l_{ik} + \delta^l_i v_k - \delta^l_k v_i\ . \tag{31}$$

Under the condition $Q_n = 0$, we obtain field equations in form (21), which, however, coincide with the equations for the Lagrangian constructed from the scalar curvature. Hence, we ultimately conclude that the Lagrangian

$$L^R = -\frac{1}{k^2}\tilde{R} + L^m + L^M \tag{32}$$

leads to the field equations in the effective space with the nonsymmetrical metric g^{ik} and the cnnection Γ^l_{ik}

$$R_{ik}(g,\Gamma) - \frac{1}{2}m^2(g_{ik} - \gamma_{ik}) = -k^2\frac{\delta L^M}{\delta\tilde{g}^{ik}}\ , \tag{33}$$

$$\nabla_k\tilde{g}^{mn} = 2Q^n_{lk}\tilde{g}^{ml}\ , \tag{34}$$

$$Q_n = 0\ , \tag{35}$$

A consequence of Eqs. (33)–(35) is the conditions

$$\tilde{g}^{(ik)}_{|k} = \tilde{g}^{[ik]}_{|k} = 0\ . \tag{36}$$

Equations (33)–(35) and conditions (36) become

$$\Box\psi^{ik} - m^2\psi^{ik} = kt^{ik}\ , \tag{37}$$

$$\Box\phi^{ik} = kA^{[ik]}\ , \tag{38}$$

$$\psi^{ik}{}_{|k} = \phi^{ik}{}_{|k} = 0 \tag{39}$$

in the Minkowski space, where $\Box$ is the d'Alembert operator in this space.

3. We consider some matter systems interacting with the gravitational field. As a consequence of the interaction minimality, the connection of the effective space-time in which the matter motion occurs is determined by the metric alone. Regarding the matter as a field source, we can easily see that a field ϕ^{ik} for matter systems with a zero source A^{ik}_M is nonzero begining with the first nonlinear approximation. These systems include point masses, wich can be charged and possess an "innate" dipole moment, and also scalar fields. The fields ψ^{ik} and ϕ^{ik} are the source of an antisymmetrical field for these systems in nonlinear approximations. At the same time, the source A^{ik}_M is nonzero for a rotating particle and an electromagnetic field, and an antisymmetrical field already appears here in the linear approximation.

The action for a moment-free test particle of a mass m moving in a external field has the form

$$S = -mc \int \sqrt{g_{ik} dx^i dx^k} \; , \tag{40}$$

whence it follows that the particle moves in the effective Riemannian space determined by the symmetrical part of the metric g_{ik}. Therefore, the particle does not interact with the antisymmetrical field in the linear approximation.

Clearly, the scalar field with the Lagrangian

$$L = \phi_{,i}\phi_{,k}\tilde{g}^{ik} - m^2\phi^2 \tag{41}$$

does not interact with the antisymmetrical part of the external field at all.

The Lagrangian of a spinor field involves only the symmetrical part of the metric because the tetradic coefficients h^i_a which are used to determine the spinor connection, are found from the condition

$$h^i_a h^k_b \eta^{ab} = g^{ik} \tag{42}$$

As in the case of test particle, the spinor field equations are defined in the effective Riemannian space with the metric $g_{(ik)}$, and the interaction with the antisymmetrical gravitational field appears begining with the first nonlinear approximation.

The situation is different in the case of an electromagnetic field because the Lagrangian

$$L = -\frac{1}{4} F_{ik} F_{mn} (\tilde{g}^{(im)} g^{(kn)} + \tilde{g}^{[im]} g^{[kn]}) \; , F_{ik} = A_{k,i} - A_{i,k} \tag{43}$$

involves the antisymmetrical part of the metric. The equation for the vector potential A_k

$$(F_{ik}\tilde{g}^{im} g^{kn})_{,n} = 0 \tag{44}$$

is distinct from the corresponding RTG equation even in the linear approximation. In particular, a result is that a light does not propogate in a week field h^{ik} along the isotropic geodesics of the Riemannian space with the metric $g^{(ik)}$.

Comparing the above theory with the geometric generalization of General Relativity, we indicate their fundamental difference in describing the geometric characteristics of the space-time. The torsion and nonmetricity are new field potentials in the geometric theories and are algebraically related to their sources. In the presented theory, these quantities play the role of "intensity" for the field ϕ^{ik}, which changes their physical meaning. Generally speaking, the field ϕ^{ik} and the torsion do not dissapear in the vacuum, whereas, for instance, the torsion in the Einstein-Cartan theory is nonzero only in the presence of spinning matter [5],[6]. The torsion and the nonmetricity in the Moffat nonsymmetrical theory are also determined by the derivatives of the metric, but the Lagrangian in this theory is different from that in (32), and the field equations cannot be brought to the form with the universal source in the Minkowski space.

In conclusion, we note that the postulated additional interaction realized by the antisymmetrical tensor field can by defined in different ways. However, the requirements of gauge invariance, interaction minimality, and universality of the field source lead exactly to Lagrangian (32) and an effective space-time with a nonsymmetrical metric, torsion and nonmetricity.

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