RELATIVITY, COSMOS AND EXPERIMENTS

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Introduction and abstract

The special theory of relativity had a strong experimental background, among which the Michelson-Morley experiment was only a small element. This was not at all the case for the general relativity theory that is a synthesis of special relativity and gravitation. Among the possible hypotheses, Einstein has chosen some of the simplest but many other choices were also possible and various theories, among which that of Professor Logunov, are in competition with the general relativity.

Hence it is necessary to realize some test-experiment in order to clarify the situation. Space experiments are very expensive but have the advantage of an excellent accuracy.

Several experiments are presented are their accuracy is discussed.

1. The three classical tests of general relativity

The three classical tests of general relativity are very famous:

A) The relativistic advance of Mercury's perihelion.

B) The curvature of lights beams in the vicinity of the Sun.

C) The gravitational redshift.

Let us recognize that the two first tests are much more famous than the third, we will see the real reason...

These three tests are logical consequences of the solution of Hilbert-Einstein equations called ''*Schwarzchild* ds^2'' and expressing the curvature of space all around a spherical massive body of mass M :

$$c^2\, ds^2 = f(r)\, c^2\, dt^2 - g(r)\, dr^2 - r^2\, [d\phi^2 + \cos^2\phi\, dL^2] \qquad (1)$$

with:

| ***c*** = velocity of light = 299 792 458 m/s

| ***t*** = ''cosmic time'' = time used in the system of reference of Schwarzschild = time of | a clock at a very large distance r and without velocity ($dr/dt = d\phi/dt = dL/dt = 0$).

| ***r, ϕ, L*** : Schwarschild's spherical coordinates (distance, latitude, longitude) of the | test particle of interest.

| ***s*** = ''proper time'' of the moving test particle (the proper time s has an increment | equal to ds when the test particle covers dr, dϕ, dL during the interval of time dt).

| ***f(r)*** and ***g(r)*** : the two Schwarzschild functions related to the ''relativistic radius'' m | of the spherical mas sive body of interest :

| $f(r) = 1/g(r) = 1 - (2m/r)$; with $m = GM/c^2$; ***G*** = *constant* of Newton's law . (2)

Let us notice that :

A) If we take account of the cosmological constant λ the two functions f(r) and g(r) become :

$$f(r) = 1 / g(r) = 1 - (2m / r) - (\lambda\, r^2 / 3\, c^2) \qquad (3)$$

Notice that λ is very small and very often considered as equal to zero. Today its estimations give: $|\lambda| < 4 \times 10^{-36}\ s^{-2}$.

B) The relativistic radius of the Sun is m = 1.477 km, this length give the order of magnitude of the relativistic perturbations per revolution for the planetary motions.

C) The relativistic radius of the Earth is only 4.43 millimeters: it is hopeless to try to detect the relativistic effects on the motions of artificial satellites in the middle of considerably larger perturbations of all kinds... (but these relativistic effects can be detected in clock experiments).

The free motions have a simple definition: between two given points of space-time: r_o , ϕ_o , L_o , t_o and r_f , ϕ_f , L_f , t_f the motion with the largest proper time $s_f - s_o = \int ds$ is a free motion. If furthermore that motion is with $s_f - s_o = 0$, it is the motion of a light beam.

With these definitions it becomes easy to obtain:

A) The relativistic advance of the perihelion of a planet: its angular velocity is the following:

$$d\omega / dt = 3mn / p \qquad (4)$$

with: m = relativistic radius of the Sun = 1.477 km , n = mean orbital angular motion of the planet of interest (e.g. 360° or 2π radians per year for the Earth),

p = orbital ''semi-latus rectum'' of the planet of interest = $a(1 - e^2)$, where a is the semi-major axis and e the eccentricity.

This angular velocity is 43.0'' per century for the planet Mercury, but only 8.33''per century for the planet Venus and much less for the other planets. Since furthermore the orbital eccentricity is 0.205 6 for Mercury and 0.006 8 only for Venus the relativistic advance of perihelion is much more visible on the Mercury orbit…

B) The deflection of light beams is $(4m/r_o)$ radian where r_o is the smallest value of the distance r along the beam. With a Sun radius of R = 695 000 km the largest deflection (grazing beam) is only 1.75''.

C) The gravitational redshift is related to the ratio ds/dt of the proper time s at the surface of the Sun and the cosmic time t. Events, and especially physical and optical events, occur slower (i.e. have a slower proper time) at the surface of the Sun.

The corresponding gravitational redshift of a wavelength of length ℓ is:

$$\delta\ell / \ell = m / R \text{ , that is } 2.125 \times 10^{-6} \text{ in the case of the Sun} \qquad (5)$$

This redshift is equivalent to a Doppler-Fizeau redshift of 637m/s. The surprising feature is that this gravitational redshift can be well verified in the observations along the edges of the Sun but is not at all verified for observations near the center…There it corresponds to a Doppler-Fizeau redshift of 246 m/s only!

Several hypotheses have been developed for the explanation of this surprising phenomenon (fast vertical motions, magnetic effects, etc.) but none is satisfactory. Of course this situation is the real reason of the difference of fame between the two first classical tests of general relativity and the gravitational redshift…

2. Experiments on the equivalence principle

The equivalence principle between the inertial mass, that resists to accelerations, and the gravitational mass, that attracts neighbouring bodies, is one of the main pillars of general relativity.

The first verification of that principle was the famous Galileo experiment from the leaning tower of Pisa: heavy and light bodies fall together at the same speed and with the same acceleration. Of course the accuracy of that experiment was small, only a few percent.

A few decades later Newton, using a series of pendulums, improved the accuracy to about 10^{-3} and, with the same method, Bessel obtained a few 10^{-5} in the first half of the nineteenth century.

A decisive improvement was made by Eötvös in 1922 with his special torsion pendulums. The accuracy reached then about 10^{-8} [Ref. 1].

Going near the possible limits of a terrestrial laboratory, essentially because of the small and continuous seismic noise, Dicke reached 10^{-11} in 1964 [Ref. 2], and Braginsky 10^{-12} in 1971 [Ref. 3]. Any further improvement requires a space experiment and several projects have been prepared [Ref. 4-8]. Most of them use a ''drag free system'' (fig. 1): a central proof mass is protected from the surface forces (air drag , radiation pressure , solar wind, etc.) by a surrounding shield , and a suitable propulsion system moves the shield back each times it approaches the proof mass.

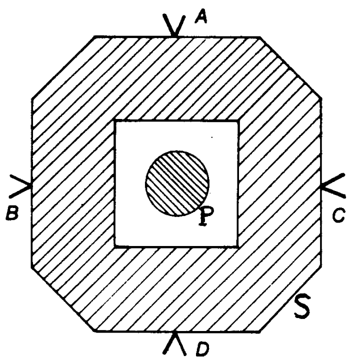

Fig. 1. The principle of drag free systems. The proof mass P is protected from the surface forces by the shield S that keeps a distance to P with the suitable propulsion system ABCD.

The interest of drag free systems is to give to their proof mass a pure geodetic motion , with a high degree of accuracy, it becomes then easy to compare the orbital motions of two neighbouring proof masses at to detect any difference of acceleration : this is the experiment of the leaning tower of Pisa in much more accurate conditions.

Of course there are many sources of small perturbations (magnetic effects, gravitational attraction between the shield and the proof mass, influence of the system of detection of the position of the proof mass with respect to the shield, etc.) and the accuracy remains limited. Nevertheless the expected improvements with respect to the ground experiments of Dicke and Braginsky are impressive and the accuracy can climb to about 10^{-1}

Notice that the experiment ''Galileo Galilei'' [Ref. 6] can only reach 10^{-17} : it don't use a drag free system and don't need very cold temperatures. Its super-rotation is considered as sufficient for very accurate measures and the compensating advantage is of course a much lower price.

3. The orbital gyroscope

This experiment proposed by Francis Everitt [Ref. 9] is extremely difficult, at the limit of today's possibilities. The main elements are a set of four extremely sensible gyroscopes near the heart of the satellite (fig. 2) and a drag-free system at the center.

The reason of the experiment is the curvature of space in the vicinity of a massive body: that curvature implies a precession of the gyroscopes with the rotation vector $\mathbf{\Omega}$:

$$\mathbf{\Omega} = 3\,GM\,(\,\mathbf{r} \times \mathbf{V}\,) / 2\,c^2\,r^3 \qquad (6)$$

with: $GM / c^2 = m$ = relativistic radius of the massive body of interest = 4.43 millimeters for the Earth.

and : $\mathbf{r}$ = radius vector of the satellite and its gyroscopes.

$r = |\mathbf{r}|$

$\mathbf{V} = \mathbf{dr} / dt$ = velocity vector of the satellite.

There is also a smaller precession $\mathbf{\Omega_2}$ related to the Earth rotation, its rotation vector $\boldsymbol{\omega}$ and its polar moment of inertia I :

$$\mathbf{\Omega_2} = GI\,[\,3\,\mathbf{r}\,(\,\boldsymbol{\omega}.\,\mathbf{r}\,)\, - \boldsymbol{\omega}\,r^2\,] / c^2\,r^5 \qquad (7)$$

Of course these two precession rates are very small. If we consider at an altitude of 500 km a circular polar orbit (that gives to $\mathbf{\Omega}$ and $\mathbf{\Omega_2}$ two normal directions), we obtain 6.96'' per year for $\mathbf{\Omega}$ and 0.044'' per year for $\mathbf{\Omega_2}$ only.

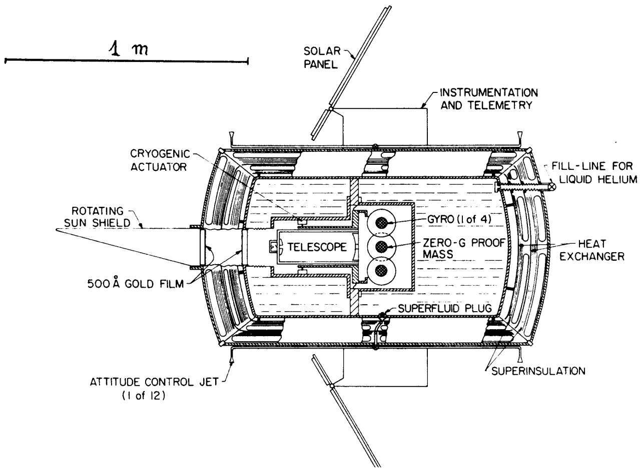

Fig. 2. The orbital gyroscope experimental.

These two very small quantities emphasize the extreme difficulty of the orbital gyroscope experiment. The heart of the satellite is at the temperature of liquid helium in order to minimize the parasite effects and the central drag-free system is a requirement. But let us also emphasize the interest of this experiment: in some general relativity theories using the Mach principle these two effects are related to the whole Universe, to its average density and its general properties.

Francis Everitt hopes to reach an accuracy of 0.001'' per year…

4. The post-newtonian parameters

Let us consider the Schwarschild ds^2 and the corresponding Robertson ds^2 .

The Schwarzschild ds^2 has already been presented in the equations (1)-(3):

$$c^2\,ds^2 = f(r)\,c^2\,dt^2 - g(r)\,dr^2 - r^2\,[d\phi^2 + \cos^2\phi\,dL^2\,] \qquad (8)$$

with in the Einsteinian case without cosmological constant:

$$f(r) = 1 / g(r) = 1 - (2m / r). \qquad (9)$$

Robertson uses three Cartesian coordinates x, y, z and the corresponding distance ρ:

$$\rho = [\,x^2 + y^2 + z^2\,]^{0.5} \;. \qquad (10)$$

These four parameters are related to the latitude and the longitude of Schwarzschild by the usual Euclidean expressions:

$$x = \rho\cos\phi\,\cos L\,; \qquad y = \rho\cos\phi\,\sin L\,; \qquad z = \rho\sin\phi\,. \qquad (11)$$

The distance ρ is chosen by Robertson in order that the proper time s becomes :

$$c^2\,ds^2 = F(\rho)\,c^2\,dt^2 - G(\rho)\,[\,dx^2 + dy^2 + dz^2\,] \qquad (12)$$

since $dx^2 + dy^2 + dz^2 = d\rho^2 + \rho^2\,[d\phi^2 + \cos^2\phi\,dL^2\,]$, this requires of course:

$$F(\rho) = f(r)\,; \qquad G(\rho)\,d\rho^2 = g(r)\,dr^2\,; \qquad \rho^2\,G(\rho) = r^2 \qquad (13)$$

that is in the Einsteinian case (9) :

$$r = \rho + m + (\,m^2 / 4\rho\,)\,, \qquad (14)$$

$$F(\rho) = (\,2\rho - m\,)^2 / (\,2\rho + m\,)^2 = 1 - (\,2m / \rho\,) + (\,2m^2 / \rho^2\,) - (\,3m^3 / 2\rho^3\,) + \ldots \qquad (15)$$

$$G(\rho) = [\,1 + (\,m / 2\rho)\,]^4 = 1 + (\,2m / \rho\,) + (\,3m^2 / 2\rho^2\,) + \ldots \qquad (16)$$

The other theories of gravitation (Logunov, Ni, Jordan, Thiry, Brans-Dicke, etc… Ref. 10-11) are based on various ideas on the equivalence principle, the Mach principle, the ''covariance'', the ''invariance'' the notions of space-time, matter and energy. They lead to various functions F(ρ) and G(ρ) that were originally approximated by :

$$F(\rho) = 1 - (2m\alpha / \rho) + (2m^2\beta / \rho^2) + O(m^3 / \rho^3), \qquad (17)$$

$$G(\rho) = 1 + (2m\gamma / \rho) + O(m^2 / \rho^2) \qquad (18)$$

with $\alpha = \beta = \gamma = 1$ in the Einsteinian case (and also in Mr Logunov's relativistic theory of gravity) but with for instance $\gamma \sim 0.85$ in the Brans-Dicke theory.

Notice that the coefficient α is useless. With a twice smaller m the case $\alpha = 2$, $\beta = 4$, $\gamma = 2$ is obviously equivalent to the case $\alpha = \beta = \gamma = 1$. The products αm, βm^2 , γm remain the same and αm remains equal to the relativistic radius GM/c^2 : the first order motions of surrounding particles are given by the Newtonian approximation and are directly related to that relativistic radius. On the other hand the mass M is the gravitational mass that is known by its gravitational effects only.

Hence it has been decided that the coefficient α will systematically be chosen equal to unity and for this reason the two first ''post-Newtonian parameters'' are called β and γ.

For some theories (Yilmaz, Papapetrou, etc…), the spherical symmetry is no more respected, there are some small effects related to the velocity of the Sun with respect to either the Galaxy or the reference of the cosmic background radiation and the analysis becomes more complex but remains similar to that developed below.

The comparison of the different theories of general relativity, also called theories of gravitation, is based on all the ''post-Newtonian parameters'': $\beta, \gamma, a_1, a_2, a_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4$, etc. (all, save β and γ, equal to zero for Einstein and also for Logunov and for Dicke). These parameters modelize the different relativistic effects; for instance the relativistic advance of the perihelion of Mercury is proportional to the sum $2 + 2\gamma - \beta$.

5. Motions in a central field. The effects of the parameters β and γ

The relativistic effects related to β and γ allow the measures of these two parameters.

The free motions of tests bodies and light beams lead to the best examples of relativistic effects.

Let us recall that these free motions have a very simple definition : let us consider in space-time an initial and a final point: t_o, r_o, ϕ_o, L_o and t_f, r_f, ϕ_f, L_f (or t_o, x_o, y_o, z_o and t_f, x_f, y_f, z_f); a test body that goes from the first point to the second with the largest possible proper time $s_f - s_o$ follows a trajectory of free motion. If, furthermore, that proper time $s_f - s_o$ is zero, the trajectory is that of a light beam.

The free motion problem is thus an ordinary optimization problem. Let us consider it with the Robertson coordinates t, x, y, z .

The equation of evolution is then:

$$ds^2 = F(\rho)\, dt^2 - G(\rho)\, c^{-2} (dx^2 + dy^2 + dz^2) \qquad (19)$$

with:

$$\boldsymbol{\rho} = \text{vector} (x, y, z) \; ; \qquad \rho = |\boldsymbol{\rho}| = [x^2 + y^2 + z^2]^{1/2} . \qquad (20)$$

The parameter of description will be the cosmic time t , the state parameters will be s and $\boldsymbol{\rho}$ and the corresponding conjugate parameters will be p_s and the vector $\mathbf{p}_\rho$. The control parameter will then be the velocity vector $d\boldsymbol{\rho} / dt = \mathbf{V}$, and the Hamiltonian H of the control will be :

$$H = p_s (ds / dt) + \mathbf{p}_\rho \,.\, d\boldsymbol{\rho} / dt = p_s [F(\rho) - G(\rho)\, c^{-2} V^2]^{1/2} + \mathbf{p}_\rho . \mathbf{V} . \qquad (21)$$

The optimal Hamiltonian H* is the least upper bound of H with respect to all possible controls $\mathbf{V}$:

$$H^* = H^* (p_s , \mathbf{p}_\rho , \boldsymbol{\rho}) = \{ F(\rho)\, p_s^2 + [F(\rho)\, c^2\, \mathbf{p}_\rho^2 / G(\rho)] \}^{1/2} \qquad (22)$$

with the optimal velocity vector $\mathbf{V}^*$:

$$\mathbf{V}^* = \mathbf{p}_\rho\, c^2 \sqrt{F}\, [G^2 p_s^2 + G\, c^2 p_\rho^2]^{-1/2} = \mathbf{p}_\rho F(\rho)\, c^2 / G(\rho)\, H^* . \qquad (23)$$

The usual Pontryagin optimality equations give then:

$$ds / dt = \partial H^* / \partial p_s = p_s F(\rho) / H^* , \qquad (24)$$

$$dp_s /dt = - \partial H^* / \partial s = 0 , \qquad (25)$$

$$d\boldsymbol{\rho} /dt = \partial H^* / \partial \mathbf{p}_\rho = \mathbf{V}^* = \mathbf{p}_\rho F(\rho)\, c^2 / G(\rho)\, H^* , \qquad (26)$$

$d\mathbf{p}_\rho /dt = - \partial H^* / \partial \boldsymbol{\rho}$, hence, H* being a not function of $\boldsymbol{\rho}$ but of ρ only ,

the vector $d\mathbf{p}_\rho /dt$ is parallel to $\boldsymbol{\rho}$ and the vector product $\boldsymbol{\rho} \times \mathbf{p}_\rho$ is constant, (27)

$$dH^*/dt = \partial H^* / \partial t = 0 . \qquad (28)$$

Thus p_s , $\boldsymbol{\rho} \times \mathbf{p}_\rho$ and H* are constant and the corresponding integrals of motion are the following:

$F(\rho)\,.\,(dt/ds) = H^*/p_s = k_1 = \text{constant},$ (29)

$\boldsymbol{\rho}\times\mathbf{V}\,.\,G(\rho)/F(\rho) = \mathbf{k_2}$ = constant vector ; with of course $\mathbf{V} = \mathbf{d\rho}/dt$, (30)

$[F(\rho)/k_1^2] + [G(\rho)V^2/F(\rho)c^2] = 1.$ (31)

With the Schwarzschild parameters these integrals of motion become:

$f(r)\,.\,(dt/ds) = k_1$ = constant , (32)

$\mathbf{r}\times(\mathbf{dr}/ds) = \mathbf{k_3}$ = constant vector ; $\mathbf{k_3} = k\,\mathbf{k_2}$, (33)

$(dr/ds)^2 = \{[k_1^2c^2/f(r)] - c^2 - (k_3/r)^2\}/g(r)$. (34)

Hence the problem of the free motions in a Robertson ds^2, or in a Schwarzschild ds^2, is integrable for any functions $F(\rho)$, $G(\rho)$, or any functions $f(r)$, $g(r)$.

Notice that in the Einsteinian case the equation (34) gives:

$(dr/ds)^2 = k_1^2c^2 + [c^2 + (k_3/r)^2].[(2m/r) - 1]$ (35)

and thus a test body falling into the sphere $r < 2m$ cannot stop its fall: the radial velocity dr/ds remains forever negative and goes to minus infinity when r approaches zero. This is the famous ''black hole'' effect, an effect that is essentially dependent on the evolution of the functions f(r) and g(r) in the vicinity of zero.

For a slow body far from the central mass its free motion is almost Keplerian and we will express it with the help of a suitable Keplerian motion and its usual parameters, the constants n, a, e, p and the anomalies v, E, M :

n = mean angular motion ,

a = semi-major axis ,

$n^2a^3 = GM = mc^2$ = gravitational constant (= $1.3271\times10^{20}\ m^3/s^2$ for the Sun) ,

e = eccentricity ,

$p = a(1-e^2)$ = semi-latus rectum ,

v and E : true and eccentric anomalies ; $\tan(E/2) = [(1-e)/(1+e)]^{1/2}\,.\,\tan(v/2)$,

$M = E - e\sin E$ = mean anomaly .

This auxiliary Keplerian motion is related to the integrals of motion (29)-(30) by the following complex expressions that allow a simple description of the neighbouring free motion

$k_1^2 = 1 - (m/a) + [(1+\gamma)m^2/a^2]$, (36)

$k_2^2 = mc^2[p + m(4 + 3\gamma + \gamma e - 2\beta)]$. (37)

The neighbouring free motion is plane and we will assume that it is in the equatorial plane $\phi = 0$ (or $z = 0$). The parameters ρ, L (longitude), s and t are then given by the following:

$\rho = a(1 - e\cos E) = p/(1 + e\cos v)$, (38)

$L = L_o + v\{1 + [(2 + 2\gamma - \beta)m/p]\}$, (39)

$s = s_o + (M/n).\{1 + [(1 + 5\gamma)m/2a]\} + (2\gamma\, me\sin E/na)$, (40)

$t - s = t_o - s_o + (3Mm/2na) + (2me\sin E/na)$, (41)

L_o , s_o , t_o , are three constants of integration and M/n is the proper time of the auxiliary keplerian motion.

These first order expressions (in m) are valid for any eccentricity and, with the spherical symmetry, for any inclination.

Let us note that:

A) The neglected second and upper order terms give errors of the order of (m^2/ρ) that is always less than one centimeter in the solar system.

B) During a revolution, the relativistic perturbations are of the order of the relativistic radius m , we must then expect effects of a few kilometers.

C) The difference $t - s$ is independent of β and γ , hence a clock experiment cannot give these two parameters.

D) The only long period visible effect is related to the expression of L. It is the secular advance of perihelion the angular velocity of which is:

$$(2 + 2\gamma - \beta)\,m\,n/p\,. \quad (42)$$

Notice the agreement with (4) in the Einsteinian case, where $\beta = \gamma = 1$.

E) This advance of perihelion is the only relativistic effect on motions of test bodies that contain the parameter β .

We also need the motion of a light beam, for instance a light beam starting at the instant $t = 0$ and at the point $(0, y_o, 0)$, with $y_o > 0$, in the direction of Ox.

The trajectory of the photon is then given by:

$$x = ct - m(1+\gamma)\,\mathrm{Ln}\{[ct + \sqrt{(c^2t^2 + y_o^2)}]/y_o\} + O(ctm^2/y_o^2)\,, \quad (43)$$

$$y = y_o - m(1+\gamma) \quad \{ [\sqrt{(c^2 t^2 + y_o^2)} - y_o] / y_o \} + O(ctm^2/y_o^2), \qquad (44)$$

$$z = 0. \qquad (45)$$

Hence the total deflection of the light beam is $2m(1+\gamma)/y_o$ and thus a light beam arriving at 90° from the Sun into the astrometric satellite Hipparcos has already a deflection of $(1+\gamma)0.002''$ that must be taken into account !

6. The Shapiro experiment

Let us consider now the transit time between the Earth and a space probe (Fig 3).

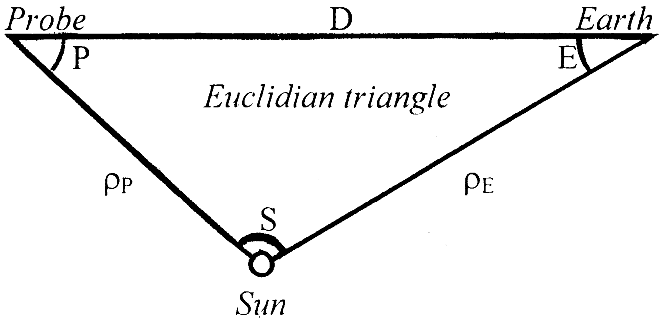

Fig. 3. Analysis of a light beam. *The Shapiro experiment.*

The transit time $(t_f - t_o)$ is given by the following expression that can easily be deduced from (43)-(45):

$$c(t_f - t_o) = D - m(1+\gamma)\ \mathrm{Ln}[\tan(E/2) . \tan(P/2)] + O(m^2 D/\rho_E^2 \sin^2 E) \qquad (46)$$

with :

$$D = \text{``Euclidian'' Earth-probe distance} = [\rho_E^2 + \rho_P^2 - 2\rho_E \rho_P \cos S]^{1/2} \qquad (47)$$

E, P, S : angles of the Euclidian triangle based on the Earth-Sun-probe angle S and the Robertson radial distances ρ_E and ρ_P .

$$E + P + S = 180° ; \quad \sin E / \rho_P = \sin P / \rho_E = \sin S / D . \qquad (48)$$

The fast variation of the factor $\mathrm{Ln}[\tan(E/2) . \tan(P/2)]$ when the angles E and P are both small (probe beyond the Sun) gives an easy way for an accurate measure of the parameter γ . This experiment is the ''Shapiro experiment''.

The relativistic term: $m(1+\gamma)\ \mathrm{Ln}[\tan(E/2) . \tan(P/2)]$ of equation (46) can reach about 30 km for a grazing beam (with an accuracy of measure that is better than one meter), nevertheless the experiment is not as simple as it may appear :

A) Grazing beams interfere with the solar corona that is a source of small variable delays depending on the wavelength…These effects are fought either by laser beams or by double radar frequencies.

B) The motion of a space probe is not simple, it undergoes many perturbations. The planetary perturbations are accurately modelizable but the surface forces (radiation pressure, solar wind, Poynting-Robertson effect, etc.) have many unknown variations related to the variations of the surface of the probe.

The first Shapiro's experiments on Mariner 6 and 7 in 1969-70 were very inaccurate. The use of radar echos on planets, without transponder, was also difficult. This experiment has been improved with a radar transponder on planet Mars (the rotation of the planet must be modelized…) and two other improvements are proposed: the use of a drag-free probe as in figure 1, [Ref. 12] and the use of a radar transponder on a small asteroid as Icarus [Ref. 13].

Today we know that the parameter γ is within 1% of its Einsteinian value and the equipment presented in the reference 12 would improve that measure to perhaps the accuracy 10^{-6} or 10^{-7}. However the knowledge of the parameter β remains weak : its measure needs a long experiment, several revolutions, on an eccentric orbit close to the Sun and with well controlled perturbations. The asteroid Icarus is sufficiently large : its diameter is 1.5 km and the surface forces have only a very small effect on its orbit, its semi-major axis is not the best : 1.0777 UA but its eccentricity is excellent : 0.8266 ; equipped with a radar transponder it will probably be the best test body in the solar system, even better than the planet Mercury, and will give a measure of β with an accuracy that could be 0.1% [Ref. 13].

7. Detection of the gravitational waves

The detection of gravitational waves will be one of the major test of relativity.

Two experiments on ground are prepared: LIGO [in USA, ref. 14] and VIRGO [in Italy, ref. 15] but these experiments are looking for very minute effects and are extremely difficult.

The corresponding space experiments SAGITTARIUS and LISA [Ref. 16-17] are easier because they can use very large bases (1million km for SAGITTARIUS, 10 million for LISA…). Their principle is the following (Fig.4).

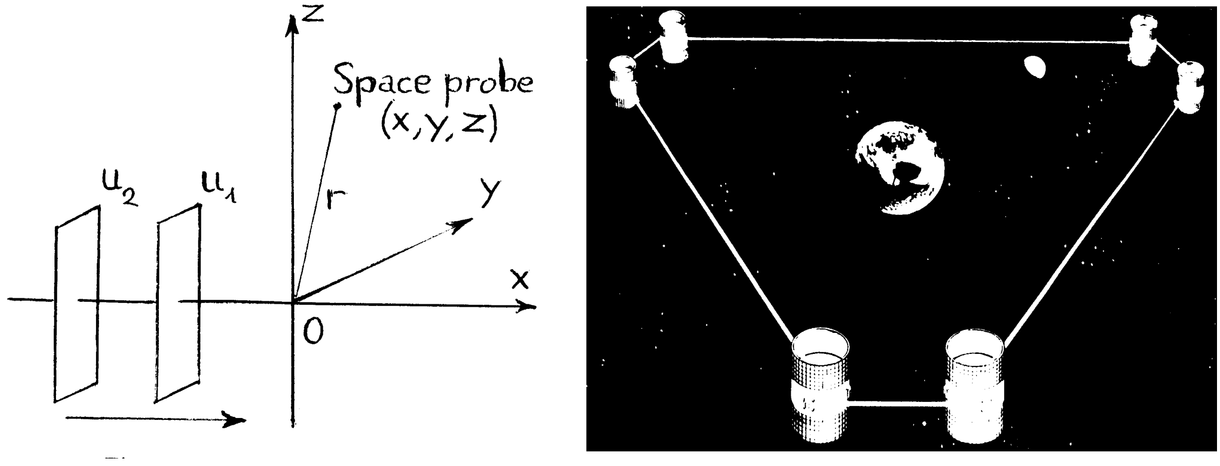

Fig. 4. The space detection of gravitational waves. *The Sagittarius experiment.*

Several space probes at large distances from each other measure very accurately their mutual distances and the corresponding time delay, with suitable radar or laser rangings.

The passages of gravitational waves introduce fast variations of these time delays, variations the ''signature'' of which can be related to various very energetic astronomical events (supernovae, collapse of a black hole, etc.) and the presence of many space probe allow to detect the direction of these events, as shown below.

The general first order solution of Hilbert-Einstein equations in vacuum is the following:

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu \qquad (49)$$

with: $$g_{\mu\nu} = M_{\mu\nu} + \partial \varepsilon_\mu / \partial x^\nu + \partial \varepsilon_\nu / \partial x^\mu + \text{term } j_{\mu\nu} \text{ (function of A,B,C)} \qquad (50)$$

an with :

A) The $M_{\mu\nu}$ are the usual components of the Minkowski matrix, with +1, –1, –1, –1 for the diagonal and 0 for the other terms (the velocity of light being chosen as unity).

B) ε_o , ε_1 , ε_2 , ε_3 , A , B , C are seven small first order functions of the four parameters $x^o = t$, $x^1 = x$, $x^2 = y$, $x^3 = z$.

The four ε_μ functions are related to infinitesimal modifications of the referential, they are sufficiently differentiable and arbitrary (in the general relativity theory). We will see below the restrictions introduced by the relativistic theory of gravity of Mr. Logunov.

Finally the $j_{\mu\nu}$ terms are zero for the diagonal (i.e. when $\mu = \nu$), and they are elsewhere :

$$j_{01} = j_{10} = \partial^2 A / \partial x^2 \partial x^3 \;;\; j_{02} = j_{20} = \partial^2 B / \partial x^1 \partial x^3 \;;\; j_{03} = j_{30} = \partial^2 C / \partial x^1 \partial x^2$$

$$j_{12} = j_{21} = -\partial^2 C / \partial x^o \partial x^3 \;;\; j_{13} = j_{31} = -\partial^2 B / \partial x^o \partial x^2 \;;\; j_{23} = j_{32} = -\partial^2 A / \partial x^o \partial x^1 \;. \qquad (51)$$

The three functions A, B, C are not arbitrary; we can impose them the ''hard conditions'' (where $\square$ is the usual Dalembertian symbol, $\square A = \partial^2 A / \partial t^2 - \partial^2 A / \partial x^2 - \partial^2 A / \partial y^2 - \partial^2 A / \partial z^2$) :

$$A + B + C = 0 \;;\quad \square A = 0 \;;\quad \square B = 0 \;;\quad \square C = 0 \qquad (52)$$

or, with other suitable functions ε_μ , the equivalent ''soft conditions'' :

A + B + C is a polynomial of t, x, y, z of degree three or less.

$$\square A_{,23} = \square A_{,01} = \square B_{,13} = \square B_{,02} = \square C_{,12} = \square C_{,03} = 0 \;. \qquad (53)$$

If we add the classical Logunov condition:

$$D_\mu \{ \sqrt{(-g)} \,.\, g^{\mu\nu} \} = 0 \qquad (54)$$

that is presently :

$$\partial \{ \sqrt{(-g)} \, . \, g^{\mu\nu} \} / \partial x^{\mu} = 0 \qquad (55)$$

we obtain the following supplementary conditions :

$$\text{For all } \mu : \quad \Box \, \varepsilon_{\mu} = 0 \, . \qquad (56)$$

The simplest ''plane waves'' going into the Ox direction are then given by the following, with $u = t - x$, and with the two infinitesimal functions h(u) and k(u):

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 + h(u) [dy^2 - dz^2] + 2 k(u) dy.dz + \text{second order terms.} \qquad (57)$$

These plane waves correspond to $\varepsilon_o = \varepsilon_1 = 0$; $\varepsilon_2 = y h(u) / 2$; $\varepsilon_3 = - z h(u) / 2$; A=A(u) ; B = yz h(u) / 2; C = – A – B ; $k(u) = d^2A / du^2$.

With this ds^2 , bodies at rest before the arrival of the wave remain at rest during its passage and their proper time s (with s = t + constant) is unperturbed.

We need to know the duration necessary, for a light beam, to go from one space probe to another, for instance to go from a space probe at rest at the origin to another at rest at the point x, y, z and at the distance $r = [x^2 + y^2 + z^2]^{0.5}$ (Fig. 4).

If t_o and t_f are the departure and arrival time their difference $t_f - t_o$ is given (to the first order) by :

$$t_f - t_o = r + [(z^2 - y^2) / 2r(r - x)] \int_{u_o}^{u_f} h(u) \, du + [yz / r(r - x)] \int_{u_o}^{u_f} k(u) \, du \qquad (58)$$

with $u_o = t_o$ and $u_f = t_f - x$ (and with the velocity of light as unity).

The variations of the durations $t_f - t_o$ for several space probes at suitable positions will allow to know both the functions h(u) and k(u) and also the direction of arrival of the wave. But notice that:

A) (58) is valid for the Einstein and the Logunov theories only. The other competing theories have their own expressions and the detection of gravitational waves will perhaps allow to choose.
B) The difficulty of this experiment is in the extreme smallness of the functions h(u) and k(u). Their estimations give at most about 10^{-21} .

Also notice that some waves are exact solutions of Hilbert-Einstein equations, for instance, with a twice continuously differentiable function f(u, y, z) that can be large :

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 + f(u, y, z).[dt - dx]^2 \qquad (59)$$

with the only condition : $\Box f = 0$, that is here : $\partial^2 f / \partial y^2 + \partial^2 f / \partial z^2 = 0$.

It is even possible to give the corresponding exact solution when the cosmological constant λ is not zero [Ref. 18].

Let us notice that the wave (57) is a particular case of the wave (59), with a suitable modification of the referential, however the analysis of the effects of the wave (59) is not as simple as the analysis (57), (58) because a probe at rest before the arrival of the wave don't remain at rest in the system (59) during the passage of the wave.

The gravitational waves are not related to the post-Newtonian parameters as simply as the other effects and, if detected, they will give much more information.

For the other effects we have the following proportionalities:

Relativistic advance of planetary and test body perihelion :	Proportional to $(2 + 2\gamma - \beta)$.
Curvature of light beams; Shapiro effect :	Proportional to $(1 + \gamma)$.
Gravitational redshift; clock experiments :	Independent of β and γ.
Deviation of orbital gyroscopes :	Proportional to $(1 + 2\gamma)$.

8. The nordvedt effect

In 1968 Nordvedt has shown [Ref. 19] that the Brans-Dicke theory implies a small supplementary perturbation in the motion of the Moon (amplitude: 3m), with the synodic period of 29.53 days.

A similar perturbation appear in most gravitation theories, but not in Einstein general relativity.

The radar lunar ranging allow to measure the distance of the Moon with an accuracy of a few centimeters and the Nordvedt effect has not been detected; but let us recognize the difficulty of this observation that imply:

A) The analysis of the motion of the Moon with all its perturbations including those given by the planetary attraction, by the Earth oblateness and by the tides.

B) The analysis of the rotation of the Moon with all its irregularities.
C) The analysis of the rotation of the Earth, with its precession, its nutation, the inequalities of polar motion and the tides of the Earth crust that move and lift up and down the observatories.
D) And finally the analysis of the curvature and the delay of radar beams along their path in the Earth atmosphere…

9. Other relativistic space experiments

It is possible to write that all accurate astronomical observations or experiments have a relation to relativity.

The measures and the analyses of the cosmic background radiation by the satellites COBE and ISO are strongly dependent of cosmology and of the corresponding theory.

The accurate astrometric satellite Hipparchos and its successor Gaïa, the Hubble space telescope and its successor NGST (Next Generation Space Telescope) are very dependent of the analyses of the curvature of light beams. They reveal gravitational mirages that are of course typical relativistic effects.

Many futuristic experiments are prepared [Ref. 20].

A) The mission Omega: an improvement of the Sagittarius mission.
B) The mission ODIE: Orbiting Drag-free International Experiment.
C) The mission Grace: Gravity Recovery And Climate Experiment.
D) The mission CHAMP: CHAllenging Micro-satellite Payload for geophysical researches and applications.
E) The missions Sirte, First, Planck, Darwin, Imex, Twinq, Sort, the Russian radiotelescop Spectre-Radio Astron etc.

All these very accurate missions have a relativistic part and many are used for fundamental physics and quantum theory, that will also help to understand the phenomenon of gravitation.

Conclusion

The strange astronomical phenomena discovered in the second half of the twentieth century : quasars, pulsars, neutron stars, black hole, gravitational mirages and Einstein cross, etc. are the real reason of the bloom of so many new theories of gravitation. Unable to understand directly these phenomena the theoreticians have put some doubts on the old theories and have expressed a large variety of new ideas.

However ''an experiment of general relativity is the measure of the very small difference between two very large numbers'' and it was classical to add ''relativity is the paradise of theoreticians and the hell of experimenters''. Fortunately the variety and the remarkable accuracy of space experiments on relativity will remove this curse and will lead us in the near future to a better understanding. We have now the reasonable hope to eliminate all theories but one, but also the fear that no present theory will survive and that the truth still escape…

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