TEN ARGUMENTS FOR THE ESSENCE
OF SPECIAL RELATIVITY

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In searching for the essence of special relativity, we have been gradually accumulating ten arguments focusing on one fundamental postulate based on quantum mechanics. A particle is always not pure. It always contain two contradictory fields, $\varphi(\vec{x}, t)$ and $\chi(\vec{x}, t)$, which are coupled together with the symmetry $\varphi(-\vec{x}, -t) \longrightarrow \chi(\vec{x}, t)$ and $\chi(-\vec{x}, -t) \longrightarrow \varphi(\vec{x}, t)$. In a particle state φ dominates χ as $|\varphi| > |\chi|$. But the enhancement of hiding χ ingredient in accompanying with the increase of particle velocity precisely accounts for the various strange effects of special relativity. After newly defined space-time inversion($\vec{x} \longrightarrow -\vec{x}, t \longrightarrow -t$), $\varphi(\vec{x}, t) \longrightarrow \varphi(-\vec{x}, -t) = \chi_c(\vec{x}, t), \chi(\vec{x}, t) \longrightarrow \chi(-\vec{x}, -t) = \varphi_c(\vec{x}, t), (|\chi_c| > |\varphi_c|)$, the particle transforms into its antiparticle with the same momentum and (positive) energy. The above symmetry should be regarded as a starting point to construct the theory of special relativity, the relativistic quantum mechanics, the quantum field theory and the particle physics, including the very interesting superluminal theory for neutrino [1].

1. Motivation and belief

It was not until 1959 (J. Terrell, Phys. Rev. 116 (1959)) did physicists realize that the visual image of so-called Lorentz contraction is not simply a contraction along the motin direction. The misunderstanding lasting for 54 years should be regarded as a lesson that the deduction method could be far overwhelming the induction method and concrete analysis. However, we should also raise a further question :

(a) Why there is a Lorentz contraction?

Besides the Lorentz contraction, various strange effects in special relativity(SR) have been exhibiting themselves as mysteries of nature for nearly 100 years. They are:

(b) Why there is a limit $c = 3 \times 10^8 m/s$ (speed of light in vacuum) for the velocity v of any particle? (see, however, [25, 26])

(c) Why the inertial mass m of a particle increases with its velocity v and without a limit [$m = m_0(1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$]?

(d) Why a moving clock accompanying the particle runs slower and slower when the particle velocity increases [clock frequency $f = f_0(1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$]?

We should not be satisfied with the existing derivation of the above four SR effects. Rather, we have been insisting on searching for a deeper explanation for these effects, i. e. , for the essence of SR. Since 1905 till the recent years, there were many scholars trying to derive SR by merely one relativistic postulate — the principle of relativity (A) and abandoning the other relativistic postulate — postulate of constancy of the speed of light (B). Eventually, all of them failed. They did not realize that in 1905 Einstein had to establish the kinematics of SR before the dynamics. So Eienstein proposed B for establishing the Lorentz transformation, the latter at that time was no more than the mutual definition between the coordinates of two inertial frames. If without B, it would be meaningless to talk about A. Hence the meaning and value of c must be fixed in advance by B.

We should learn from the failure of many attempts and extract the second lesson that "A scientist should go beyond the 'mere think', one should raise the 'intelligent questions' via experiments" (by Galileo, quoted from [2]).

Einstein did the best work in 20th century. But we may go further beyond him because of the enormous accumulation of new experiments since 1905.

As is well known, the combination of SR with quantum mechanics (QM) leads to relativistic quantum mechanics (RQM), quantum field theory(QFT) and particle physics(PP) successfully. Could it be a consequence of "complementarity" of two kinds of theory with different essence (SR being clasical whereas QM being quantum)?

No, let's look at the fact in biology. The combination of living beings of different species can not reproduce their descendants. It would be interesting to mention an exotic example. The mating of a horse and a donkey gives birth of a mule, but the latter can no longer have descendant.

Now we know that the genetic factor in inheritance is DNA. So our problems becomes the following:

What is the DNA of RQM, QFT and PP inherited from the SR and QM respectively?

Further evidence about the conformity between SR and QM came from de'Broglie. How could he derive his famous relation $\lambda = \frac{h}{p}$?

First, he assumed that Einstein's relation $E = h\nu$ does hold for the electron. However, on the other hand, the relation $E = m_0 c^2 (1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$ in SR implies that E increases with the increase of velocity v, so does ν, a serious contradiction to the prediction of SR that the frequency of moving clock should decrease (question d in section 1).

For finding the way out of the paradox, de'Broglie assumed next that ν is not the frequency of a clock moving with the particle but the frequency of a wave accompanying the particle (The frequency of wave is measured at a fixed point in space). So the velocity of the particle, v, should be identified with the group velocity of wave:

$$v_g = \frac{dw}{dk} = v.$$

(1)

Then after combination with the Lorentz transformation, the de'Broglie relation

$$\lambda = \frac{h}{p}, \qquad or \qquad p = \hbar k$$

(2)

was derived.

In short, de'Broglie derived the "half" of quantum theory $p = \hbar k$ from another "half" of quantum theory, $E = \hbar \omega$, in combination with the whole theory of SR. His thinking clearly showed that the SR and QM do have the common essence.

The above two enlightenments are focusing on one belief that we should strive to find the essence of SR on the basis of QM. The problem is: where is the breakthrough point?

2. The investigations on C, P, T problems

There are three discrete transformations discussed in QM:

2.1. Space inversion (P)

The sign change of space coordinates ($\vec{x} \longrightarrow -\vec{x}$) in the wavefunction (WF) of QM may lead to two eigenstates:

$$\psi_{\pm}(\vec{x}, t) \longrightarrow \psi_{\pm}(-\vec{x}, t) = \pm \psi_{\pm}(\vec{x}, t)$$

(3)

with eigenvalues 1 or -1 being even or odd parity.

2.2. Time reversal (T)

Actually, the so-called time reversal (T) implies the reversal of motion, which ascribes to the demand of invariance of Schrodinger equation as follows:

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{x},t) = H\psi(\vec{x},t) \tag{4}$$

$$i\hbar\frac{\partial}{\partial t}[\psi^*(\vec{x},-t)] = H[\psi^*(\vec{x},-t)] \tag{5}$$

In other words, the T invariance implies a relation of equivalence:

$$\psi(\vec{x},t) \sim \psi^*(\vec{x},-t) \tag{6}$$

2.3. Charge conjugation transformation

Aimming at changing the sign of charge(q) for a particle, e. g. , from $q = -e$ for an electron to $q = e$ for the positron, one realize the C transform in QM by demanding

$$\vec{p} + e\vec{A} = -i\hbar \bigtriangledown +e\vec{A} \longrightarrow -i\hbar \bigtriangledown -e\vec{A}, \tag{7}$$

which implies a complex conjugation on the WF:

$$\psi(\vec{x},t) \longrightarrow \psi^*(\vec{x},t). \tag{8}$$

2.4. CPT combined transform

If one considers the product of C, P, and T transformations together, the complex conjugation contained in the T and C transforms will cancel each other, yielding

$$\psi(\vec{x},t) \longrightarrow \text{CPT}\psi(\vec{x},t) = \psi_{\text{CPT}}(\vec{x},t) \sim \psi(-\vec{x},-t). \tag{9}$$

In the right side, the WF should be understood to describe the positron (the antiparticle of electron). But it differs from the original WF only in the sign change of x and t. What does it mean?

2.5. Parity violation, CP(or T) violation and CPT theorem

The historical discovery of parity violation in 1956([3, 4]) reveals that both P and C symmetries are violated. Since 1964, it is found that CP symmetry is also violated whereas CPT theorem remains valid, which in turn implies the violation of T reversal symmetry, as further verified by recent experiments [5].

Therefore, the relation between a particle $|a >$ and its antiparticle $|\bar{a} >$ is not $|\bar{a} >= C|a >$ but [6]

$$|\bar{a} >= \text{CPT}|a >, \tag{10}$$

which means nothing but precisely the Eq. (9).

For an electron in free motion, its WF reads:

$$\psi_{e^-}(\vec{x},t) \sim \exp[\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - Et)] \tag{11}$$

Then Eq. (9) [or (10)] gives the WF of a positron as:

$$\psi_{e^+}(\vec{x},t) \sim \exp[-\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - Et)] \tag{12}$$

with the momentum $\vec{p}$ and energy $E > 0$ precisely as that in the particle state (11).

The relation between Eqs. (11) and (12) should be viewed as a new symmetry: " The (newly defined) space-time inversion ($\vec{x} \longrightarrow -\vec{x}, t \longrightarrow -t$) is equivalent to particle-antiparticle transformation". In other words, the CPT theorem has already exhibited itself as a basic postulate. The transformation of a particle to its antiparticle ($\mathcal{C}$) is not something which can be defined independently but a direct consequence of (newly defined) space-time inversion $\mathcal{PT}$ ($\vec{x} \longrightarrow -\vec{x}, t \longrightarrow -t$) [7]:

$$\mathcal{PT} = \mathcal{C}. \tag{13}$$

2.6. Three understandings in physics [8]

(a) The observableness of a physical quantity must be related to some symmetry or conservation law. Once it fails to do so, it will cease to be an observable. The same is true for a transformation in physics. Therefore, though C, P, and T are all clearcut transformations in mathematics, they cease to be meaningful in physics.

(b) There is an important difference between a "theorem" and a "law". The various quantities contained in a theorem must be defined clearly and unambiguously in advance before the theorem can be proved. On the other hand, a law can often (not always) accommodate a definition of a physical quantity which can only be defined unambiguously after the law is verified by experiments.

As a comparison, though Newton's equation $F = ma$ could be derived from Lagrange variational principle or Hamilton principle, it is a law rather than a theorem. The definition of inertial mass m is given by the equation $m = \frac{F}{a}$ which should be verified by experiments.

(c) A concept in physics should be expressed in terms of mathematical language rigorously. If some concept can only be described by ordinary language, it would be likely incorrect or at least not a deep one.

For instance, the concepts like "The positron carries the opposite charge to that of electron" or " In the vacuum all the (infinite) negative states of electron are filled. Once a 'hole' is created in the 'sea' , it would correspond to a positron", etc. are all incorrect. By contrast, the correct concept of charge number Q ($= q/e$ as the substitution of charge q) now finds its expression as reflected in the sign of phase of WF in Eqs. (11) and (12) , $Q = -1$ and 1 respectively.

3. Einstein–Podolsky–Rosen paradox and antiparticle

In 1935, Einstein, Podolsky and Rosen(EPR) in a paper titled "Can the description of physical reality by quantum mechanics be considered complete?" [9] raised a very strange question about two spinless particles. Then after the seminar work by Bohm[10] and Bell [11], physicists have been turning their attention to the puzzle of entangled state, e. g. , the nonlocally correlated two-photon quantum state over long distance, which was verified again and again by experiments, especially in recent years [12, 13]. However, another puzzle in the original version of EPR paper was overlooked by the majority of physicists but reemphasized by Guan [14] as follows:

Consider two spinless particle in one-dimensional space with positions $x_i(i = 1, 2)$ and momentum operators $\hat{p}_i = -ih\frac{\partial}{\partial x_i}$. Then the commutation relation

$$[x_1 - x_2, \hat{p}_1 + \hat{p}_2] = 0 \tag{14}$$

implies that they have a common eigenstate with eigenvalues

$$x_1 - x_2 = a = const., \tag{15}$$

$$p_1 + p_2 = 0, p_2 = -p_1. \tag{16}$$

How strange the state is !? Two particles are moving in the opposite momentum directions while keeping their distance unchanged. Incredible! As stressed in Ref. [14], "no one can figure out how to realize it." Now we propose the following answer [15].

If the WF of particle 1 is written as Eq. (11):

$$\psi(x_1, t) = \exp[\frac{i}{\hbar}(p_1 x_1 - E_1 t)] \tag{17}$$

with $p_1 > 0$ and $E_1 > 0$, then the particle 2 must be an antiparticle:

$$\psi_c(x_2, t) = \exp[\frac{i}{\hbar}(p_2 x_2 - E_2 t)] = \exp[-\frac{i}{\hbar}(p_1 x_2 - E_1 t)] \tag{18}$$

with $p_2 = -p_1 < 0, E_2 = -E_1 < 0$. But as shown in Eq. (12), we should view the WF of "negative energy state" of particle directly as the WF of its antiparticle with corresponding operators:

$$\hat{p}_c = i\frac{\partial}{\partial x}, \hat{E}_c = -i\hbar\frac{\partial}{\partial t}, \tag{19}$$

which are the counterparts of that for the particle:

$$\hat{p} = -i\hbar\frac{\partial}{\partial x}, \hat{E} = i\hbar\frac{\partial}{\partial t}. \tag{20}$$

So the observed momentum and energy of antiparticle in state (18) are p_1 and E_1 respectively, precisely the same as that of the particle state (17). Now every thing is reasonable.

If instead of Eq. (14), we consider the commutation relation:

$$[x_1 + x_2, \hat{p}_1 - \hat{p}_2] = 0. \tag{21}$$

Then the correct answer turns out to be a particle and its antiparticle moving in the opposite directions with momentum p_1 vs $p_1^c = -p_1$ and positions x_1 vs $x_2 = -x_1$. Such kind of experiments have been performed many times, say, in the process of $e^+ e^-$ pair creation by a high-energy photon in the vicinity of heavy neucleus. Especially, a recent experiment reveals an entangled state of $K^0 - \bar{K}^0$ system[16] and provide a beautiful realization of relation (21).

4. Klien paradox in Klien-Gordon equation and antiparticle

In 1929, Klien discovered his famous paradox in Dirac equation, resorting to the concept of "hole" for introducing the antiparticle [17]. Now we will discuss this paradox in Klien-Gordon(KG) equation with much more simplicity and clarity [15]. As first shown in Ref. [18], the KG Eq.

$$[i\hbar\frac{\partial}{\partial t} - V(\vec{x})]^2 \psi = -c^2\hbar^2\bigtriangledown^2\psi + m^2 c^4\psi \tag{22}$$

can be recast into two coupled Schrodinger equations of φ andχ:

$$
\begin{aligned}
(i\hbar\frac{\partial}{\partial t} - V)\varphi &= mc^2\varphi - \frac{\hbar^2}{2m}\bigtriangledown^2(\varphi + \chi), \\
(i\hbar\frac{\partial}{\partial t} - V)\chi &= -mc^2\chi + \frac{\hbar^2}{2m}\bigtriangledown^2(\chi + \varphi),
\end{aligned}
\tag{23}
$$

$$
\varphi = \frac{1}{2}[(1 - \frac{V}{mc^2})\psi + i\frac{\hbar}{mc^2}\dot{\psi}],
$$

$$
\chi = \frac{1}{2}[(1 + \frac{V}{mc^2})\psi - i\frac{\hbar}{mc^2}\dot{\psi}].
\tag{24}
$$

Eq. (23) is invariant under the (newly dedined) space-time inversion ($\vec{x} \longrightarrow -\vec{x}, t \longrightarrow -t$) and transformation:

$$\varphi(-\vec{x}, -t) \longrightarrow \chi(\vec{x}, t), \chi(-\vec{x}, -t) \longrightarrow \varphi(\vec{x}, t), \tag{25}$$

$$V(-\vec{x}, -t) \longrightarrow -V(\vec{x}, t). \tag{26}$$

Let us consider a potential barrier in one dimensional space:

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0, & x \geq 0 \end{cases}. \tag{27}$$

The incident wave, reflected wave and transmitted wave are , respectively:

$$\psi_i = a \exp[\frac{i}{\hbar}(px - Et)], \qquad (x \leq 0), \tag{28}$$

$$\psi_r = b \exp[\frac{i}{\hbar}(-px - Et)], \qquad (x \leq 0), \tag{29}$$

$$\psi_t = b' \exp[\frac{i}{\hbar}(p'x - Et)], \qquad (x \geq 0) \tag{30}$$

with $p > 0, p'^2 = \frac{(E-V_0)^2}{c^2} - m^2c^2$. The continuation condition leads to

$$\frac{b}{a} = \frac{p - p'}{p + p'}, \frac{b'}{a} = \frac{2p}{p + p'}. \tag{31}$$

The Klien paradox emerges when $V_0 > E + mc$. Since $p' = \pm\sqrt{\frac{(V_0-E)^2}{c^2} - m^2c^2}$ remains real, the transmitting wave is oscillatory while the reflectivity of incident wave reads:

$$R = \left|\frac{b}{a}\right|^2 = \frac{|p - p'|^2}{|p + p'|^2},$$

$$R < 1, \text{if} \quad p' > 0, \tag{32}$$

$$R > 1, \text{if} \quad p' < 0. \tag{33}$$

While the choice (32) is obviously unreasonable, the choice (33) seems quite attractive. Then we have to realize why $p' < 0$ and what happens in this situation?

For this purpose, we look back at Eqs. (22)–(24) and the accompanying continuity equation [19]

$$\frac{\partial \rho}{\partial t} + \triangledown \cdot \vec{j} = 0, \tag{34}$$

$$\rho = \frac{i\hbar}{2mc^2}(\psi^*\dot{\psi} - \psi\dot{\psi}^*) - \frac{V}{mc^2}\psi^*\psi = \varphi^*\varphi - \chi^*\chi; \tag{35}$$

$$\vec{j} = \frac{i\hbar}{2m}(\psi \triangledown \psi^* - \psi^* \triangledown \psi) = \frac{i\hbar}{2m}[(\varphi \triangledown \varphi^* - \varphi^* \triangledown \varphi) , nonumber \tag{36}$$

$$+ (\chi \triangledown \chi^* - \chi^* \triangledown \chi) + (\varphi \triangledown \chi^* - \chi^* \triangledown \varphi) + (\chi \triangledown \varphi^* - \varphi^* \triangledown \chi)]. \tag{37}$$

Combining them with Eqs. (27)–(33), we find

$$\rho_i = |\varphi_i|^2 - |\chi_i|^2 = \frac{E}{mc^2}|a|^2 > 0, j_i = \frac{p}{m}|a|^2 > 0, \tag{38}$$

$$\rho_r = \frac{E}{mc^2}|b|^2 > 0, j_r = -\frac{p}{m}|b|^2 < 0, \tag{39}$$

$$\rho_t = |\varphi_t|^2 - |\chi_t|^2 = \frac{(E - V_0)}{mc^2}|b'|^2 < 0, j_t = \frac{p'}{m}|b'|^2. \tag{40}$$

So we should demand $p' < 0$ to get $j_t < 0$ in conformity with $\rho_t < 0$ and to meet the requirement of Eq. (34) $(j_i + j_r = j_t)$ with $|j_r| > j_i(|b| > |a|)$ and reflectivity $R > 1$.

The reason is clear. For an observer located at $x > 0$, the energy of particle in the transmitted wave should be measured with respect to the local potential V. So it has a negative energy $E' = E - V_0 < 0$ locally and behaves as an antiparticle with its WF redefined as:

$$\psi_t \longrightarrow \psi_t^c = b' \exp[\frac{i}{\hbar}(p'x - E't)] = b' \exp[-\frac{i}{\hbar}(|p'|x - |E'|t)], \qquad (x > 0). \qquad (41)$$

According to Eq. (19), the energy and momentum of this antiparticle are $|E'| > 0$ and $|p'| > 0$ respectively. It moves to the right though $p' < 0$ and $j_t < 0$. Therefore, the KG Eq. is self-consistent even at one-particle level to explain qualitatively the phenomenon of the $\pi^+\pi^-$ pair creation at strong field barrier bombarded by incident π^- beam.

5. The hidden antiparticle ingredient of a KG particle or Dirac particle

5.1. Klien–Gordon particle

Consider a freely moving wave packet for KG particle [19]:

$$\psi(x, t) = (4\sigma\pi^3)^{-\frac{1}{4}} \int_{-\infty}^{\infty} e^{-\frac{k^2}{2\sigma}} e^{i(kz - wt)} dk$$

$$\simeq \frac{\left(\frac{\sigma}{\pi}\right)^{\frac{1}{4}}}{(1 + \frac{i\sigma\hbar t}{m})^{\frac{1}{2}}} \exp\{\frac{-\sigma z^2}{2(1 + \frac{i\sigma\hbar t}{m})} - i\frac{mc^2 t}{\hbar}\}. \qquad (42)$$

Assuming $\frac{\sigma\hbar t}{m} \ll 1$ to ignore the spreading of wave packet in low speed $(v << c)$ case, we perform a "boost" transformation, i. e., to push the wave packet to high speed $(v \longrightarrow c)$ case. Thus we see from the figures in [19] that:

(a) The width of packet shrinks——Lorentz contraction;

(b) The amplitude of ρ increases——"boost" effect;

(c) The new observation is that both $|\varphi|^2$ and $|\chi|^2$ in ρ increase even more sharply while keeping $|\varphi| > |\chi|$ to preserve $|\varphi + \chi| = |\psi|$ invariant.

The ratio of hidden ingredient of χ to that of φ is defined as

$$R_{free}^{KG} = \frac{\int_{-\infty}^{\infty} |\chi|^2 dz}{\int_{-\infty}^{\infty} |\varphi|^2 dz} = \left[\frac{1 - \sqrt{1 - \frac{v^2}{c^2}}}{1 + \sqrt{1 - \frac{v^2}{c^2}}}\right]^2 \xrightarrow{v \to c} 1. \qquad (43)$$

We are now in a position to answer the question (a) in section 1. It is the enhancement of hidden χ field accompanying with the increase of particle velocity v and the increasing coupling between χ and φ fields lead to the upper bound for $v(v < c)$ and the appearance of Lorentz contraction together with the boost effect.

Next, we consider a π^- atom. The π^- meson in the Coulomb field of neuclus has potential energy $V(r) = -\frac{z\alpha}{r}$ and its $1S$ state energy being a function of Z:

$$E_{1s}^{KG} = mc^2 \left[\frac{1}{2} + \sqrt{\frac{1}{4} - z^2\alpha^2}\right]^{\frac{1}{2}} \xrightarrow{z \to \frac{1}{2\alpha}} \frac{1}{\sqrt{2}} mc^2. \qquad (44)$$

while the relevant ratio reads: $(y = \sqrt{\frac{1}{4} - z^2\alpha^2})$

$$R_{1s}^{KG} = \frac{\int |\chi|^2 d\vec{x}}{\int |\varphi|^2 d\vec{x}} = 1 - 4\left[2 + \left(y + \frac{1}{2}\right)^{\frac{1}{2}} + \frac{(y + \frac{1}{2})^{\frac{3}{2}}}{2y}\right]^{-1}, \qquad (45)$$

which increases from 0 (when $Z = 0$) to the upper limit 1 (when $Z \longrightarrow \frac{1}{2\alpha}$).

5.2. Dirac particle

The WF of Dirac Eq. is a spinor with four components:

$$\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}. \tag{46}$$

Here φ and χ (each with two components), usually called as the "positive" and "negative" energy components in the literature, are just the counterparts of φ and χ for KG particle.

However, instead of Eq. (35), now we have

$$\rho_{Dirac} = \psi^\dagger \psi = \varphi^\dagger \varphi + \chi^\dagger \chi. \tag{47}$$

On the other hands, the invariant quantity during the boosting process is

$$\bar{\psi}\psi = \varphi^\dagger \varphi - \chi^\dagger \chi > 0. \tag{48}$$

Now both $\varphi^\dagger \varphi$ and $\chi^\dagger \chi$ are constrained under the ρ. In the limiting case, $\varphi^\dagger \varphi \geq \chi^\dagger \chi \longrightarrow \frac{1}{2}\rho$. The relevant ratio for freely moving wave packet read:

$$R_{free}^{Dirac} = \frac{1 - \sqrt{1 - \frac{v^2}{c^2}}}{1 + \sqrt{1 - \frac{v^2}{c^2}}}. \tag{49}$$

And the $1S$ state energy of Hydrogenlike atom is

$$E_{1s}^{Dirac} = mc^2 \sqrt{1 - z^2\alpha^2} \overset{z \to \frac{1}{\alpha}}{\longrightarrow} 0 \tag{50}$$

with relevant ratio being

$$R_{1s}^{Dirac} = \frac{1 - \sqrt{1 - z^2\alpha^2}}{1 + \sqrt{1 - z^2\alpha^2}}. \tag{51}$$

5.3. Comparison between nonrelativistic QM and relativistic QM

Let us look at the coupling form of KG Eq. (23) and ignore all the χ field terms. Then we go back to the Schrodinger Eq. i. e., to the nonrelativistic QM (NRQM).

Now we are able to define the NRQM being the QM for particle with all the hidden antiparticle ingredient ignored. In this case, neither upper bound for particle velocity v nor lower bound for its energy E in an external field exists. Meanwhile, the mass of particle, m, is an invariant and there is no any relationship between m and E.

The situation changes radically in relativistic QM(RQM), as we just see from either KG Eq. or Dirac Eq. Once the hidden antiparticle ingredient is taken into account, there must be an upper bound for the velocity of particle, $v < c$, also a lower bound for its energy in an external field, $E_{min} \geq 0$. Both these bounds are determined by the ratio (R) of hidden antiparticle ingredient to that of particle ingredient: $R \longrightarrow 1$. Meanwhile, the particle mass m becomes a variable and is precisely proportional to the energy E of particle: $E = mc^2$.

Now we begin to understand the essence of mass generation. It is also the Essence of SR and could be ascribed to the equal existence of particle with its antiparticle and the underlying symmetry shown as Eq. (25).

6.   Derivation of special relativity

We are now in a position to derive the special relativity (SR) from new point of view. Actually, what we shall do is nothing but looking at the problem upside down [7].

We will work at one inertial frame, i. e., the laboratory coordinate system and begin with the NRQM, i. e., the Schrodinger equation. Then comes the crucial step. We regard the basic symmetry Eq. (25) as the only "relativistic postulate" and inject it into the Schrodinger Eq. for establishing Eq. (23), i. e., the KG Eq. Let us stress the main points as follows:

(a) A particle is always not pure. It always contains two contradictory fields, $\varphi(x,t)$ and $\chi(x,t)$, which are coupled eachother.

(b) Under the (newly defined) space-time inversion $(x \longrightarrow -x, t \longrightarrow -t)$,

$$\varphi(-\vec{x}, -t) \longrightarrow \chi(\vec{x}, t), \chi(-\vec{x}, -t) \longrightarrow \varphi(\vec{x}, t) \tag{52}$$

together with the transformation property of external field:

$$V(-\vec{x}, -t) \longrightarrow -V(\vec{x}, t) \tag{53}$$

the theory(equation) must be invariant.

(c) The new symmetry, i. e., the "invariance under space-time inversion" exhibits itself as the only "relativistic postulate". It should be assuned as the general feature of all relativistic theory.

(d) Of course, for dealing with the specific property of particle, we have to add another postulate of "nonrelativistic" nature. For example, starting from Schrodinger Eq., we already assume the kinetic energy of spinless particle being in the form of $\frac{p^2}{2m}$ and add a "rest energy" term $m_0 c_1^2$. On the other hand, we have to assume that the kinetic energy of spin $1/2$ particle is in the form of $c_2, \vec{\sigma} \cdot \vec{p}$ while rest energy being $m_0 c_2^2$. Based on the ansatz $\psi = \binom{\varphi}{\chi}$ with φ or χ being two component spinor, Dirac equation can be derived similarly by the invariance postulate under the space-time inversion.

(e) After establishing the KG Eq. or Dirac Eq., we obtain from the plane wave solution the following relation easily:

$$E^2 = p^2 c_i^2 + m_0^2 c_i^4, (i = 1, 2). \tag{54}$$

Then it is easy to see that the group velocity v_g of wave equals to the particle velocity v:

$$v_g = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{pc_i^2}{E} = v \overset{E \to \infty}{\longrightarrow} c_i. \tag{55}$$

The experiments show that the limiting velocity of particle, c_i, is equal to the speed of light c. (Otherwise we would have no idea about space and time). Hence we have found relativistic dynamical law including Eqs. (53), (54) and

$$p = mv, E = mc^2, m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{56}$$

(f) Next, we turn to relativistic kinematics. The main task is to find the relationship between the coordinates of two inertial frames, i. e. the Lorentz transformation. For this purpose, we need some invariance. Instead of the invariance of speed of light, this time we will resort to an invariance of nonrelativistic nature, i. e., the invariance of phase with respect to the coordinate transformation, (which was introduced by de'Broglie as a "law of phase harmony"). A particle is moving in the laboratory system with velocity v, moment p along x axis and energy E. We take a motion system resting on the particle and compare the phase of plane wave described in two systems:

$$\exp[\frac{i}{\hbar}(px - Et)] = \exp[\frac{i}{\hbar}(p'x' - E't')] = \exp(-\frac{i}{\hbar}m_0 c^2 t'), \tag{57}$$

$(p' = 0, E' = E_0 = m_0 c^2)$. Substituting Eq. (53) into (56), we find

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{58}$$

which is precisely the key relation in Lorentz transformation.

(g) In summary, since we adopt the approach from dynamics to kinematics, we can say more than Einstein in 1905. For instance, the two fields $\varphi(x,t)$ and $\chi(x,t)$ have opposite tendency of space-time evolution in their phases essentially:

$$\varphi(\vec{x}, t) \quad \sim \quad \exp[\frac{i}{\hbar}(\vec{p} \cdot \vec{x} - Et)], \tag{59}$$

$$\chi(\vec{x}, t) \quad \sim \quad \exp[-\frac{i}{\hbar}(\vec{p} \cdot \vec{x} - Et)]. \tag{60}$$

So in a concrete particle state with $|\varphi| > |\chi|$, though χ has to obey φ and follow the evolution as (58), it does exhibit itself as a drag and enhance the inertial mass. It is interesting to explain the time dilation effect in SR. According to Eqs. (58) and (59), in some sense, the time reading of the "clock" for φ field is "clockwise" whereas that for χ field is "anticlockwise" essentially. Thus in accompanying with the increase of particle velocity, though the time reading remains "clockwise", it runs slower and slower due to the enhancement of hidden χ field.

7.   Relativistic two-body stationary Schrodinger equation and quarkonium

In particle physics, the heavy quarkonium $Q\bar{Q}(c\bar{c}$ or $b\bar{b})$ is often treated by Schrodinger equation with potential $V(r) = \sigma r$, where r is the distance between Q and $\bar{Q}$ while σ is called "string tension constant". Why can this "nonrelativistic" potential model be so successful? Could it be further improved at the level of QM in a simple manner? [20]

We assume that the two-particle system should also be described by two fields $\varphi(\vec{r}_1, \vec{r}_2, t)$ and $\chi(\vec{r}_1, \vec{r}_2, t)$ in coupling($M = m_1 + m_2, \vec{r} = \vec{r}_2 - \vec{r}_1$):

$$\begin{cases} i\hbar\frac{\partial}{\partial t}\varphi = M\varphi - (\frac{\hbar^2}{2m_1}\nabla_1^2 + \frac{\hbar^2}{2m_2}\nabla_2^2)(\varphi + \chi) + V(r)(\varphi + \chi) \\ i\hbar\frac{\partial}{\partial t}\chi = -M\chi + (\frac{\hbar^2}{2m_1}\nabla_1^2 + \frac{\hbar^2}{2m_2}\nabla_2^2)(\chi + \varphi) - V(r)(\chi + \varphi) \end{cases} . \tag{61}$$

Eq. (60) is invariant under the space-time inversion with

$$\varphi(-\vec{r}_1, -\vec{r}_2, -t) \longrightarrow \chi(\vec{r}_1, \vec{r}_2, t), \chi(-\vec{r}_1, -\vec{r}_2, -t) \longrightarrow \varphi(\vec{r}, \vec{r}, t) \tag{62}$$

$$V(-\vec{r}_1, -\vec{r}_2, -t) \longrightarrow V(\vec{r}_1, \vec{r}_2, t) \tag{63}$$

Note that, however, Eq. (62) is different from Eq. (26) where there is a sign change in $V(\vec{x}, t)$ under space-time inversion. This is because in one-body equation V is treated as an external field with the neucleus as an inert core whereas here both two particles are turning into their antiparticles under space-time inversion.

Introducing the center-of-mass coordinate $\vec{R} = (m_1\vec{r}_1 + m_2\vec{r}_2)/M$, reduced mass $\mu = m_1 m_2/M$ and setting

$$\varphi = \Phi + \frac{i\hbar}{Mc^2}\dot{\Phi}, \chi = \Phi - \frac{i\hbar}{Mc^2}\dot{\Phi}, \tag{64}$$

$$\Phi(\vec{R}, \vec{r}, t) = \psi(\vec{r})\exp[\frac{i}{\hbar}(\vec{P} \cdot \vec{R} - Et)] \tag{65}$$

($\vec{P}$ is the momentum of center-of-mass and E is the total energy of system) we arrive at

$$\left[-\frac{\hbar^2}{2\mu}\nabla_r^2 + V(\vec{r})\right]\psi(\vec{r}) = \epsilon\, psi(\vec{r}) \tag{66}$$

$$\epsilon = \frac{1}{2Mc^2}(E^2 - M^2c^4 - P^2c^2). \tag{67}$$

We will set $P = 0$ and denote the binding energy of system as $B = Mc^2 - E$. Hence

$$B = Mc^2\left[1 - (1 + \frac{2\epsilon}{Mc^2})^{\frac{1}{2}}\right]. \tag{68}$$

To our surprise, after taking the antiparticle effect into account, the form of stationary Schrodinger Eq. undergoes no change but the eigenvalue ϵ in the right side should not be directly identified with $(-B)$ in the bound state. Rather, we should evaluate the B fom ϵ via Eq. (67). Note that there is a lower bound for ϵ : $\epsilon_{min} = -Mc^2/2$, i. e., $E_{min} = 0$.

Let us use Eqs. (65), (66) for heavy quarkonium $Q\bar{Q}$ system. The eigenvalues for S states reads:

$$\epsilon_n = \lambda_n\left(\frac{\sigma^2}{2\mu}\right)^{\frac{1}{3}}, (n = 1, 2...) \tag{69}$$

with λ_n being the zero point of Airy function. So the total energy E of $Q\bar{Q}$ system should be calculated from (66), yielding

$$E_n = 4\mu\left[1 + \frac{1}{2}\lambda_n\left(\frac{\sigma^2}{2\mu^4}\right)^{\frac{1}{3}}\right]^{\frac{1}{2}} \tag{70}$$

($\mu = m/2 = M/4$, m being the quark mass). As a comparison, if one treated $\epsilon = E' - Mc$ directly, then the energy of $Q\bar{Q}$ sysyem would be

$$E'_n = 4\mu' + \lambda_n\left(\frac{\sigma'^2}{2\mu'}\right)^{\frac{1}{3}} \tag{71}$$

The following Table 1 gives the comparison between the experimental values E_n^{exp} for six S states in Upsilon($b\bar{b}$) system and the theoretical fitted values either from Eq. (69) (E_n) or from (70) (E'_n).

Table 1. The energy levels of S states in Upsilon($b\bar{b}$) system

n	1	2	3	4	5	6
E (GeV)	9. 46037	10. 023	10. 355	10. 580	10. 865	11. 019
E (GeV)	9. 46037	10. 023	10. 461	10. 834	11. 163	11. 462
E' (GeV)	9. 46037	10. 023	10. 483	10. 890	11. 262	11. 609
λ_n	2. 338	4. 088	5. 521	6. 787	7. 944	9. 023

In fitting procedure, we have fixed two parameters σ and μ by two experimental values for $n = 1$ and 2. Thus we find from Eq. (69) that

$$\begin{cases} m_b &= 2\mu = 4.326\,GeV \\ \sigma &= 0.4530 GeV^2 \end{cases} \tag{72}$$

while from Eq. (70) that

$$\begin{cases} m'_b &= 2\mu' = 4.354\,GeV \\ \sigma' &= 0.3804\,GeV^2. \end{cases} \tag{73}$$

The similar fit for Charmonium $J/\psi(c\bar{c})$ system yields:

$$\begin{cases} m_c &=& 1.031\,GeV \\ \sigma &=& 0.4183\,GeV^2. \end{cases}, \tag{74}$$

or

$$\begin{cases} m'_c &=& 1.155\,GeV \\ \sigma' &=& 0.2099\,GeV^2 \end{cases}. \tag{75}$$

We can see that the value for σ' given by (72) and (74) are in discrepancy too big while that for σ given by (71) and (73) seems much better.

In summary, now we understand two points:

(a) The reason why the potential model in "nonrelativistic" stationary Schrodinger Eq. was so successful for describing the $Q\bar{Q}$ system lies in the fact that the potential $V(r)$ is actually a "four-dimensional scalar potential" satistying Eq. (62) rather than a "vector potential" satisfying Eq. (26). In recent years, this kind of confining scalar potential $V(r) \sim \sigma r$ can be derived from quantum chromodynamics(QCD) as discussed by Brambilla and Faustov [21, 22].

(b) We are able to make "relativistic correction" on "nonrelativistic" model at QM level by using Eq. (67), as shown at the improvement of Eq. (69) vs (70), in a very simple way.

8.   Field operators in quantum field theory

The vector potential of classical electromagnetic field is real:

$$\vec{A}(\vec{x},t) = \frac{1}{\sqrt{V}} \sum_{\vec{k},\lambda} \vec{\varepsilon}_{\vec{k}}^{(\lambda)}[C_{\vec{k}\lambda}(t)e^{i\vec{k}\cdot\vec{x}} + C_{\vec{k}\lambda}^*(t)e^{-i\vec{k}\cdot\vec{x}}], \tag{76}$$

which can be quantized into a Hermitian field operator in quantum field theory(QFT) as follows:

$$\hat{\vec{A}}(\vec{x},t) = c\sqrt{\frac{\hbar}{2\omega V}} \sum_{\vec{k},\lambda} \vec{\varepsilon}_{\vec{k}}^{(\lambda)}[a\hat{}_{\vec{k}\lambda}(t)e^{i\vec{k}\cdot\vec{x}} + \hat{a}_{\vec{k}\lambda}^\dagger(t)e^{-i\vec{k}\cdot\vec{x}}]. \tag{77}$$

Here the amplitudes $C_{\vec{k}\lambda}(t)$ in (75) had been promoted into operators $\hat{a}_{\vec{k}\lambda}(t)$ in Fock space on photon with commutation relations:

$$[\hat{a}_{\vec{k}\lambda}(t), \hat{a}_{\vec{k}'\lambda'}^\dagger(t)] = \delta_{\vec{k}\vec{k}'}\delta_{\lambda\lambda'}. \tag{78}$$

Next, for "classical" Dirac field, one has

$$\psi(\vec{k},t) = \frac{1}{\sqrt{V}} \sum_{\vec{p}} \sqrt{\frac{mc^2}{|E|}}\{ \sum_{r=1,2(E>0)} b_{\vec{p}}^{(r)}u^{(r)}(\vec{p})e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)} + \sum_{r=3,4(E<0)} b_{\vec{p}}^{(r)}u^{(r)}(\vec{p})e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)}\}. \tag{79}$$

Then, depending on the "hole" concept, one defined the operators for antiparticle as:

$$\hat{d}_{\vec{p}}^{(s)\dagger} = \mp\hat{b}_{-\vec{p}}^{(r)}(s=1, r=4; s=2, r=3), \qquad v^{(s)}(\vec{p}) = \mp u^{(r)}(-\vec{p}) \tag{80}$$

to get the field operator for Dirac particles:

$$\hat{\psi}(\vec{x},t) = \frac{1}{\sqrt{V}} \sum_{\vec{p}} \sum_{s=1,2(E>0)} \sqrt{\frac{mc^2}{E}}\{\hat{b}_{\vec{p}}^{(s)}u^{(s)}(\vec{p})e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)} + \hat{d}_{\vec{p}}^{(s)\dagger}v^{(s)}(\vec{p})e^{-\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)}\} \tag{81}$$

and anticommutation relations:

$$[\hat{b}_{\vec{p}}^{(s)}, \hat{b}_{\vec{p}'}^{\dagger(s')}]_+ = [\hat{d}_{\vec{p}}^{(s)}, \hat{d}_{\vec{p}'}^{\dagger(s')}]_+ = \delta_{\vec{p}\vec{p}'}\delta_{\vec{s}\vec{s}'}. \tag{82}$$

However, for complex KG field, though there are infinite solutions with negative energy, the "hole" concept can't work. One had to quantize it similar to that in Eq. (80) by defining

$$\hat{\phi}(\vec{x}, t) = \sum_{\vec{p}} \frac{1}{\sqrt{2\omega_{\vec{p}}V}} [\hat{a}_{\vec{p}} e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)} + \hat{b}_{\vec{p}}^{\dagger} e^{-\frac{i}{\hbar}(\vec{p}\cdot\vec{x}-Et)}] \tag{83}$$

with

$$[\hat{a}_{\vec{p}}, \hat{a}_{\vec{p}'}^{\dagger}] = [\hat{b}_{\vec{p}}, \hat{b}_{\vec{p}'}^{\dagger}] = \delta_{\vec{p}\vec{p}'} \tag{84}$$

to describe the annihilation and creation of KG particle and its antiparticle.

The question remains as "what is the reason for doing so?" Or more generally, what is the unified basis for the definition of above three kind of field operators $(\hat{\vec{A}}, \hat{\psi}$ and $\hat{\phi})$?

We propose the following answer [7]. It is just the (newly defined) "invariance of field operator under space-time inversion" that ensures its correctness. Say, for Eq. (82), it reads:

$$\hat{\phi}(-\vec{x}, -t) = \hat{\phi}(\vec{x}, t), \tag{85}$$

which contains the transformation

$$\hat{a}_{\vec{p}} \rightleftharpoons \hat{b}_{\vec{p}}^{\dagger}, (\vec{x} \longrightarrow -\vec{x}, t \longrightarrow -t), \tag{86}$$

as an intuitive complement to Eq. (13).

9.   Connection between spin and statistics

We should address a further question in the previous section: "Why one must quantize the KG field by commutation relation whereas the Dirac field by anticommutation relation?"

As a preparation, we first note that Dirac equation has two forms [7]. In Pauli metric, it is usually written as

$$(\gamma_{\mu}\frac{\partial}{\partial x_{\mu}} + m)\psi(x) = 0 \tag{87}$$

with $\psi \sim \begin{pmatrix} \varphi \\ \chi \end{pmatrix} e^{-\frac{iEt}{\hbar}}$. As discussed in previous section, if$|\frac{\varphi}{\chi}| > 1, E > 0, \psi$ describes the electron. If$|\frac{\chi}{\varphi}| > 1, E < 0, \psi$describes the positron. Let us perform a space-time inversion: $\psi(x) \longrightarrow \psi(-x) = \psi_c(x)$, then Eq. (86) changes to

$$(\gamma_{\mu}\frac{\partial}{\partial x_{\mu}} - m)\psi_c(x) = 0, \tag{88}$$

which remains effective with$\psi_c \sim \begin{pmatrix} \chi_c \\ \varphi_c \end{pmatrix} e^{\frac{iEt}{\hbar}}$. If $|\frac{\chi_c}{\varphi_c}| > 1, E > 0, \psi_c$describes the positron. If $|\frac{\varphi_c}{\chi_c}| > 1, E < 0, \psi_c$ describes the electron. The difference between Eqs. (86) and (87), or between ψ and ψ_c, is merely a representation transformation: $\psi \longrightarrow \psi_c = \gamma_5\psi, (\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4)$.

We are now in a position to prove the connection between spin and statistics in QFT. Beginning from the "Principle of microscopic causality", one can first arrive at the general commutation relation for KG field as

$$[\hat{\phi}(x), \hat{\phi}^{\dagger}(y)]_{\omega} = i\hbar c \triangle (x - y) \tag{89}$$

with

$$\triangle (x) = 0, x^2 > 0, \tag{90}$$

which means that there is no connection between two points with space- like distance. However, the subscript ω of the bracket in the left side is not fixed yet either to be $\omega = -$ (commutation relation) or $\omega = +$ (anticommutation relation).

Now we use the "invariance under the space-time inversion" to fix the $\omega = -$. Actually, note that $\triangle(-x) = -\triangle(x)$ and

$$\hat{\phi}(x)\hat{\phi}^\dagger(y) \xrightarrow{x\to-x} \hat{\phi}^\dagger(-y)\hat{\phi}(-x) = \hat{\phi}^\dagger(y)\hat{\phi}(x). \tag{91}$$

(The order of operator product has to be reversed.) Then $\omega = -$ follows immediately.

Similarly, for Dirac field, one first arrives at

$$[\hat{\psi}(x), \hat{\bar{\psi}}(y)]_\omega = -i(\gamma_\mu\frac{\partial}{\partial x_\mu} - \frac{mc}{\hbar})\triangle(x-y). \tag{92}$$

Next, under the space-time inversion, we have

$$[\hat{\psi}_c(x), \hat{\bar{\psi}}_c(y)]_\omega = \begin{cases} -i(\gamma_\mu\frac{\partial}{\partial x_\mu} + \frac{mc}{\hbar})\triangle(x-y), & \text{if} \quad \omega = + \\ i(\gamma_\mu\frac{\partial}{\partial x_\mu} + \frac{mc}{\hbar})\triangle(x-y), & \text{if} \quad \omega = - \end{cases}. \tag{93}$$

Hence the invariance demands $\omega = +$ up to a representation transformation $(\psi \longrightarrow \psi_c)$. the other choice $\omega = -$ is certainly excluded.

In summary, the combination of the principle of microscopic causality with the invariance under the (newly defined)space-time inversion leads to the correct connection between spin and statistics unambiguously.

10. The Feynman propagator for electron

In the nonrelativistic QM (NRQM), the evolution of WF for a particle is described by Feynman as

$$\psi(x_b, t_b) = \int K(b, a)\psi(x_a, t_a)dx_a, \tag{94}$$

$$K(b, a) = \sum_{a\to b,(all\ paths)} const \cdot \exp(\frac{i}{\hbar}S[x(t)]) \tag{95}$$

with classical action $S[x(t)]$ calculated along a path connecting points a and b. However, all paths, in spite of their arbitrariness, must go forward in time.

This can also be seen from the Green function of Schrodinger Eq.:

$$(i\hbar\frac{\partial}{\partial t} - \hat{H})G(x, t|x', t') = \delta(x - x')\delta(t - t'), \tag{96}$$

$$G(x, t|x', t') = -\frac{i}{\hbar}K(x, t|x', t')\theta(t - t'), \tag{97}$$

$$\theta(t - t') = \begin{cases} 1, & t > t' \\ 0, & t < t' \end{cases}. \tag{98}$$

Hence the Green function G given by (96) is precisely the same as the Feynman kernel function K given by (94), except the existence of θ function showing explicitly that the reversal of evolution in time is not allowed. We remind our readers onceagain of the misnomer of so-called "time-reversal" in NRQM as discussed in section 2.

The situation is radically different in relativistic QM (RQM) where the Feynman propagator for electron, $K_F(x, x')$, is defined as

$$(\gamma_\mu\frac{\partial}{\partial x_\mu} + m)K_F(x, x') = -i\delta^{(4)}(x - x'), \tag{99}$$

$$K_F(x, x') = \sum_{\vec{p},s} \frac{m}{EV} \{ u^{(s)}(\vec{p}) \bar{u}^{(s)}(\vec{p}) e^{ip \cdot (x-x')} \theta(t - t') - v^{(s)}(\vec{p}) \bar{v}^{(s)}(\vec{p}) e^{-ip \cdot (x-x')} \theta(t' - t) \}. \quad (100)$$

Let us perform a space-time inversion [7]:

$$\begin{aligned}
K_F(x, x') \quad &\longrightarrow \quad K_F(-x, -x') = K_F^c(x, x'), \\
&= \quad \sum_{\vec{p},s} \frac{m}{EV} \{ u^{(s)}(\vec{p}) \bar{u}^{(s)}(\vec{p}) e^{-ip \cdot (x-x')} \theta(t' - t) - v^{(s)}(\vec{p}) \bar{v}^{(s)}(\vec{p}) e^{ip \cdot (x-x')} \theta(t - t') \}, \\
&= \quad \gamma_5 K_F(x, x') \gamma_5 \sim K_F(x, x'), \quad (101)
\end{aligned}$$

which means that $K_F(x, x')$ is invariant under the space-time inversion up to a representation transformation.

11. Superluminal theory for neutrino

Einstein's theory of SR and the principle of causality imply that the speed of any moving object cannot exceed that of light, c. However, there were many attempts in literature discussing the particle moving with speed $u > c$, called as superluminal particle or tachyon. In recent years, the experimental data show that the measured neutrino mass-square is negative. It was reported in Ref. [23] that

$$m^2(\nu_e) = -2.5 + 3.3 eV^2, m^2(\nu_\mu) = -0.016 + 0.023 MeV^2, \quad (102)$$

which strongly hint that neutrino might be a superluminal particle having energy and momentum relation as

$$E^2 = c^2 p^2 - m_s^2 c^4 \quad (103)$$

with the "proper mass" m_s being real and positive [24]. Then it is easy to prove that

$$p = m_s u \left(\frac{u^2}{c^2} - 1 \right)^{-\frac{1}{2}}, E = m_s c^2 \left(\frac{u^2}{c^2} - 1 \right)^{-\frac{1}{2}} \quad (104)$$

with u being the velocity of tachyon. (see, e. g., [24]).

For deriving the relation (102) from a quantum theory and following Dirac's idea, Chang proposed a Dirac-type equation as follows [25]:

$$i\hbar \frac{\partial}{\partial t} \psi_s = [-c\vec{\alpha} \cdot \hat{\vec{p}} + \beta_s m_s c^2] \psi_s, \quad (105)$$

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i, \\ \sigma_i & 0 \end{pmatrix}, \qquad \beta_s = \begin{pmatrix} 0 & I, \\ -I & 0 \end{pmatrix},$$

which gives the relation (102) straightforwardly.

At first sight, the nonhermitian property of β_s would obstruct Eq. (104) from being accepted. However, after careful examination [25, 26], it is shown that Eq. (104) precisely satisfies the basic symmetry discussed in previous sections. Indeed, setting $\psi_s = \binom{\varphi}{\chi}$, we can write Eq. (104) as

$$\begin{cases} i\hbar \frac{\partial}{\partial t} \varphi = ic\hbar \vec{\sigma} \cdot \nabla \chi + m_s c^2 \chi \\ i\hbar \frac{\partial}{\partial t} \chi = ic\hbar \vec{\sigma} \cdot \nabla \varphi - m_s c^2 \varphi \end{cases} . \quad (106)$$

Evidently, it is invariant under the space-time inversion with transformation (25), just like Dirac Eq., which reads:

$$\begin{cases} i\hbar \frac{\partial}{\partial t} \varphi_D = ic\hbar \vec{\sigma} \cdot \nabla \chi_D + m_0 c^2 \varphi_D \\ i\hbar \frac{\partial}{\partial t} \chi_D = ic\hbar \vec{\sigma} \cdot \nabla \varphi_D - m_0 c^2 \chi_D \end{cases} . \quad (107)$$

But what is the difference between them? After introducing $\xi = \frac{1}{\sqrt{2}}(\varphi + \chi)$ and $\eta = \frac{1}{\sqrt{2}}(\varphi - \chi)$ to recast (105) and (106) into

$$\begin{cases} i\hbar\frac{\partial}{\partial t}\xi = ic\hbar\vec{\sigma}\cdot\triangledown\xi - m_s c^2\eta \\ i\hbar\frac{\partial}{\partial t}\eta = -ic\hbar\vec{\sigma}\cdot\triangledown\eta + m_s c^2\xi \end{cases} \tag{108}$$

and

$$\begin{cases} i\hbar\frac{\partial}{\partial t}\xi_D = ic\hbar\vec{\sigma}\cdot\triangledown\xi_D + m_0 c^2\eta_D \\ i\hbar\frac{\partial}{\partial t}\eta_D = -ic\hbar\vec{\sigma}\cdot\triangledown\eta_D + m_0 c^2\xi_D \end{cases} \tag{109}$$

respectively, we see that while Eq. (108) is invariant under the space inversion ($\vec{x} \longrightarrow -\vec{x}, t \longrightarrow t$) and transformation

$$\xi_D(-\vec{x}, t) \longrightarrow \eta_D(\vec{x}, t), \eta_D(-\vec{x}, t) \longrightarrow \xi_D(\vec{x}, t). \tag{110}$$

Eq. (107) fails to do so because of the mass terms with opposite signs before them. Hence we realize that the violation of hermitian property is due to the violation of parity which was discovered in 1956 [3, 4]. The new observation is that the maximum parity violation is triggered by nonzero mass (m_s) which in turn implies that neutrino must be a superluminal particle with permanent helicity.

In Ref. [26], after introducing two parameters $R = \frac{\chi^\dagger\chi}{\varphi^\dagger\varphi}$ and $W = \frac{\eta^\dagger\eta}{\xi^\dagger\xi}$, and analizing the anticorrelation between them in the whole range of particle speed ($0 < u < \infty$), we are able to understand the intrinsic (dynamical) reason responsible for the strange kinematic behavior of particle with $u < c$(i. e. , the SR effect as discussed in section 6) as well as that with $u > c$ as shown in Eq. (103).

12.   Summary and discussion

(a) The special relativity (SR) and quantum mechanics (QM) are in conformity in essence, so their combination can lead to the vigorous relativistic QM (RQM), quantum field theory (QFT) and particle physics (PP). The genes(DNA) in their inheritance are, respectively:

$$QM: \quad \hat{\vec{p}} = -i\hbar\triangledown \quad SR: \quad \hat{\vec{p}}_c = i\hbar\triangledown$$
$$\hat{E} = i\hbar\frac{\partial}{\partial t} \qquad \hat{E}_c = -i\hbar\frac{\partial}{\partial t}.$$

(b) A particle is always not pure. Its wavefunction(WF) always contain two contradictory fields, $\varphi(x, t)$ and $\chi(x, t)$. Essentially,

$$\varphi \sim \exp[\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - Et)], \qquad (E > 0)$$

$$\chi \sim \exp[-\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - Et)], \qquad (E > 0).$$

If $|\varphi| > |\chi|$, the particle exibits itself as a "particle" with

$$\psi \sim \varphi \sim \chi \sim \exp[\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - Et)], \qquad (E > 0),$$

If $|\chi_c| > |\varphi_c|$, the particle exibits itself as an "antiparticle" with

$$\psi_c \sim \chi_c \sim \varphi_c \sim \exp[-\frac{i}{\hbar}(\vec{p}\cdot\vec{x} - Et)], \qquad (E > 0).$$

(c) There is no any "negative energy state", no "sea" or "hole" in the vacuum at all. The historical mission of "hole" theory for electron is coming to an end.

(d) We should not regard the $\vec{x}$ in the WF $\psi(x, t)$ as the coordinate of "point particle" before the measurement. Rather, $\vec{x}$ and t are the flowing coordinates of fields [27, 28].

(e) The CPT theorem already exhibits itself as a basic postulate:

$$\mathcal{PT} = \mathcal{C}.$$

The operator at the left side ($\mathcal{PT}$) just means $x \longrightarrow -x, t \longrightarrow -t$, the newly defined space-time inversion. The operator at the right side ($\mathcal{C}$) means the physical particle-antiparticle transformation, a definition being contained right here (not coming from elsewhere).

(f) The above symmetry should be pushed forward to

$$\varphi(-\vec{x}, -t) \quad \longrightarrow \quad \chi(\vec{x}, t),$$
$$\chi(-\vec{x}, -t) \quad \longrightarrow \quad \varphi(\vec{x}, t),$$

forming a starting point to construct the theory of SR, the RQM, the QFT and the PP. The superluminal theory for neutrino is just the new and interesting manifestation of the subtlety of QM and the basic symmetry.

(g) Actually, the basic symmetry lies in the essential equivalence of "i" and "$-i$", which is relevant to the fundamental interpretation of QM. Einstein had pointed out that "space and time are closely related and inseparable." We could add that "space-time and matter are also closely related and inseparable". It is time for the revival of "Ether" [8, 15, 27, 28].

Acknowledgments

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1. Author's note: In 21-23 June 2000, I attended the 23rd International Workshop on the Fundamental Problems of High Energy Physics and Field Theory held at Protvino, Russia and presented a lecture titled "Nine arguments for the essence of special relativity".Fortunately, in July 2000, Dr.T.Chang brought a equation for neutrino as a superluminal particle to my attention, which results in collaboration of us and two papers[25, 26]. So I add section 11 to this paper and change the title.

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