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**POSSIBLE ORIGIN OF EXTRA STATES
IN PARTICLE PHYSICS**

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Abstract

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In the framework of the Friedrichs model the transforming of discrete state into resonance is considered. The number of resonances depends on the formfactor which describes the interaction of discrete state and continuum and for its reasonable form this number exceeds the usual one to one correspondence. The physical implication of the phenomenon is discussed.

Аннотация

Лиходед А.К., Пронько Г.П. Возможное происхождение дополнительных состояний в физике частиц: Препринт ИФВЭ 96-102. – Протвино, 1996. – 12 с., библиогр.: 9.

В рамках модели Фридрихса мы рассматриваем превращение стабильных состояний в резонансы. Число резонансов определяется форм-фактором, описывающим взаимодействие дискретного состояния с континуумом и, как правило, ожидаемое однозначное соответствие между резонансами и дискретными состояниями не имеет места. Обсуждаются физические следствия этого явления.

Hadron's spectroscopy is one of the most intricate parts of modern particle physics. The theoretician and experimentalists are competing with each other in interpretation of already discovered resonances and invention of the new kinds of it. The pragmatic approach to the problem of classification of resonances consists in identification of standard, nonexotic set of the particles which admits conventional quark model interpretation and after this being done, the superfluous states could be considered as candidates into exotic ones — glueballs, hybrids, molecules etc. Indeed the direct observation of exotic quantum numbers forbidden in the usual quark model extremely simplifies the problem of discovery of these new kinds of matter, but until now very few examples of open exotics exist, but not confirmed. Also, it is generally believed that the new kinds of mesons should have specific decay modes and specific creation processes, e.g. the glueballs should easily be created in decays of J/ψ and have large couplings with $\eta\eta$, $\eta\eta'$ and $\eta'\eta'$ channels and suppressed electromagnetic ones [1]. Unfortunately these statements have only qualitative character because in the case of hidden exotics, its mixing with quark states may drastically change branching ratios into different channels[2].

So, finally we return to the usual procedure of separating superfluous states with nonexotic quantum numbers as the main tool for discovery of new kinds of mesons (the similar problem for baryons also exists and is waiting for its discussion). In this situation we must have the firm statement that the number of states which we predict in the framework of some model (e.g. potential, bag, string et cet.) is not influenced by interaction. In the present paper we try to analyze the validity of the hypothesis that there is always the one to one correspondence between the number of predicted states and the number of resonances. It is generally believed that the interaction with decay channel provides the width of resonance, it may shifts the mass, but never changes the number of states. We show that this common point of view (we also were the believers until recently) is generally wrong. In the simple but rather universal model we demonstrate that, as a rule, the number of states, which is defined by interaction is greater, when the expected one and only weak coupling regime gives the desired one to one correspondence.

We begin our discussion with the motivation of the model, which will be used in this paper for the description of the unstable particles. Switching on the interaction of one particle with two (or few) others, whose total mass permits the transition on the mass

shell, makes this particle unstable. This statement is trivial for any particle physicist. On the other hand, from the point of view of the field theory the question is not that simple, as it seems. As an example, let us consider the theory of two scalar fields $\phi(x)$ and $\psi(x)$ with masses M and m , respectively, though

$$M > 2m. \tag{1}$$

Now, if we will switch on among others the interaction which is described by the vertex

$$S_{int} = \lambda \int d^4x \phi(x) \psi^2(x), \tag{2}$$

the theory becomes unstable. The latter means that the usual asymptotic conditions fail in this case. Of course, we can still calculate the Green functions in this theory, investigate the complex singularity which corresponds to the initially stable ϕ -particle, but it is only part of the story. The most important questions from the physical point of view are:

- What are the asymptotic states in this field theory?
- How can we calculate the S -matrix for these asymptotic states?

A general discussion of these questions in the relativistic case is rather complicated and we do not dwell on them here, addressing the readers to paper [3]. Now we will give a schematic consideration, sufficient for qualitative description of many physical situations, where the nonrelativistic approximation is valid.

In terms of the field theory, the direct consequence of instability condition (1) is the nonvanishing interaction due to (2) for $t \rightarrow \pm\infty$, what could be established, e.g. in the interaction picture. The nonvanishing part of interaction requires the redefinition of asymptotic hamiltonian (which is usually taken equal to the free one) and the true asymptotic states should be defined as eigenstates of this modified hamiltonian. The correct scattering theory should be considered now for these true asymptotic states due to residue (well behaved at $t \rightarrow \pm\infty$) interaction. At first sight it seems that the whole problem becomes technically very difficult (new Feynmann diagrams with new nonlocal propagators, vertexes, etc.), but it is not really true. To see it the path integral approach is very useful. In this approach it is obvious that the basic object is the total action and that the perturbation theory with respect to free hamiltonian makes it (the scattering theory) ill-defined. So, if we will use as asymptotic states the true ones, we can use usual Feynmann diagrams for internal parts of the processes, the modification concerns only external lines.

Now, after this very short and schematic general introduction we shall start considering the subject of the present paper — a possible picture of asymptotic states, their properties and the correspondence with the common point of view on the resonances in particle physics. The appropriate framework for the description of resonances is provided by well-known Friedrichs model [4] the touchstone of general theory of perturbation of unbounded operators, which goes back to the late forties. This model is rather popular even nowadays but unfortunately not among the particle physicists. To simplify formulas we will not consider the second-quantized version of the Friedrichs model, which is directly connected

with asymptotic hamiltonian of the field theory with interaction in the form of (2), limiting ourselves only with the lowest level of it.

Let us denote $|1\rangle$ – the discrete state with energy ω_1 and $|\omega\rangle$ – the state with continuous spectrum for 0 to ∞ . These states span the space of states of our system \mathcal{H} . The scalar products of the basis states are

$$\begin{aligned} \langle 1|1\rangle &= 1 \\ \langle \omega|\omega'\rangle &= \delta(\omega - \omega'). \end{aligned} \quad (3)$$

The unperturbed hamiltonian in \mathcal{H} could be written as follows:

$$H_0 = \omega_1|1\rangle\langle 1| + \int_0^\infty d\omega \omega|\omega\rangle\langle \omega| \quad (4)$$

Now let us add to H_0 perturbation which describes the transitions between $|1\rangle$ and $|\omega\rangle$:

$$H_{int} = \lambda \int_0^\infty d\omega [f(\omega)|\omega\rangle\langle 1| + f^*(\omega)|1\rangle\langle \omega|], \quad (5)$$

where λ is coupling constant and $f(\omega)$ is smooth, square integrable function, which satisfies the condition:

$$\omega_1 > \lambda^2 \int_0^\infty d\omega \frac{|f|^2(\omega)}{\omega}. \quad (6)$$

The physical sense of (6) will be clear later. As is seen from (4) and (5) our model describes exactly the situation which we have discussed in Introduction: the discrete state $|1\rangle$ is the state predicted in some model (potential, bag, string, et cet). The hamiltonian

$$H = H_0 + H_{int} \quad (7)$$

describes the interaction of this state with continuum.

Apparently $\omega_1 > 0$, to make the decay possible on the mass shell. The position of the threshold in the origin is unessential, we placed it there for simplicity.

The eigenvalue problem for hamiltonian (7)

$$(H - E)\Psi(E) = 0, \quad (8)$$

we have to solve in the space \mathcal{H} , i.e. we shall seek the eigenvector $\Psi(E)$ in the following form:

$$\Psi(E) = \psi(E)|1\rangle + \int_0^\infty d\omega \psi(E, \omega)|\omega\rangle, \quad (9)$$

where $\psi(E)$ and $\psi(E, \omega)$ - the unknown amplitudes for which, making use of the (3) and (8) we obtain the system of equations:

$$\begin{aligned} (\omega_1 - E)\psi(E) + \lambda \int_0^\infty d\omega \psi(E, \omega)f^*(\omega) &= 0 \\ (\omega - E)\psi(E, \omega) + \lambda\psi(E)f(\omega) &= 0 \end{aligned} \quad (10)$$

To solve this system, let us begin with the second equation and express $\psi(E, \omega)$ via $\psi(E)$:

$$\psi(E, \omega) = A\delta(\omega - E) - \frac{\lambda f(\omega)}{\omega - E}\psi(E), \quad (11)$$

where A is an arbitrary constant. Note that the first term in r.h.s. of (11) arises because the factor $\omega - E$ in the equation, as a function of E has a real zero at $E = \omega$. This expression for $\psi(E, \omega)$ can be used in the first equation (10) and the equation for amplitude $\psi(E)$ is finally derived:

$$\left[\omega_1 - \lambda^2 \int_0^\infty d\omega \frac{|f|^2(\omega)}{\omega - E} \right] \psi(E) = -\lambda A f^*(E). \quad (12)$$

It's worth saying that in this form equation (12) is only symbolic. The matter is that initially we have considered system (10) for real values of E . The factor in l.h.s. of (12), in square brackets could be defined as a boundary value of the analytic function

$$\eta^{-1}(E) = \omega_1 - E - \lambda^2 \int_0^\infty d\omega \frac{|f|^2(\omega)}{\omega - E}. \quad (13)$$

This function, as is seen from its representation has a cut $[0, \infty)$ and for real energy we can define its value from above and from below of the cut:

$$\eta_{\pm}^{-1}(E) = \omega_1 - E - \lambda^2 \int_0^\infty d\omega \frac{|f|^2(\omega)}{\omega - (E \pm i\epsilon)}. \quad (14)$$

Apparently these two functions $\eta_{\pm}(E)$ correspond to two different solutions of our eigenvalue problem (8) — in-going and out-going waves. So the proper form of equation (12) for real energy is the following:

$$\eta_{\pm}^{-1}(E)\psi_{\pm} = -\lambda A f^*(E), \quad (15)$$

We see that the $\psi(E)$ (as well, as $\psi(E, \omega)$) also acquires subscript \pm .

In mathematical literature the function $\eta(E)$ on the whole complex plane E is called the partial (or one particle) resolvent of H . For a particle physicist more familiar term is the Green function or the propagator. The solution of (15) can be written in the following form:

$$\psi_{\pm} = \psi^0 - A\eta_{\pm}(E)\lambda f^*(E), \quad (16)$$

where $\psi^0(E)$ is the solution of (15) with vanishing r.h.s. The latter depends upon the properties of the resolvent $\eta(E)$: if it has a pole on the first sheet on the real axis, then

$$\psi^0 = B\delta(E - E_0), \quad (17)$$

where E_0 is the position of the pole. Close inspection of equation (14) shows that this pole may exist only below a threshold. From the physical point of view it seems rather

pathological and to prevent creation of this unwanted pole it is sufficient to impose condition (6) on the formfactor $f(\omega)$. If it had been done, then the first term in (16) would be absent and gathering together (8), (11) and (16) we obtain the final form of eigenvector $\Psi(E)$:

$$\Psi_{\pm}(E) = \left\{ |E\rangle + \lambda f^*(E) \eta_{\pm}(E) \left[|1\rangle + \lambda \int_0^{\infty} d\omega \frac{f(\omega)}{\omega - (E \pm i\epsilon)} |\omega\rangle \right] \right\}. \quad (18)$$

This formula is the key point of our present discussion and therefore we must carefully investigate it and its consequences.

First of all, the most important fact that follows from (18) is that the hamiltonian of our system (7) has only continuous spectrum — the discrete state $|1\rangle$ has been dissolved in the continuum.¹ The comparison of eigenvectors of H_0 and H leads us to the conclusion that in an unstable case there is no analyticity in coupling constant λ . To understand the fate of discrete level with $E = \omega_1$ we must investigate the resolvent $\eta(E)$ on the complex plane E .

The common point of view on this question is the following: the pole at the point $E = \omega_1$ moves to the second sheet acquiring negative imaginary part transforms into the Breit-Wigner resonance. This point of view is supported by calculations in the limit $\lambda \rightarrow 0$. Indeed, the inverse resolvent $\eta_+^{-1}(E)$ could be represented in the following form:

$$\eta_+^{-1}(E) = \omega_1 - E - \lambda^2 \left(r(E) + i\pi |f|^2(E) \right), \quad (19)$$

where $r(E)$ is the real part of integral in r.h.s. of (13). If we assume that $r(E)$ and $|f|^2(E)$ is smooth function in the vicinity of ω_1 , then from (19) it follows that a new pole of the resolvent will be at the point

$$\begin{aligned} E_c &= \omega_1 - \lambda^2 r(\omega_1) - i\pi \lambda^2 |f|^2(\omega_1) = \\ &= \tilde{\omega}_1 - i\Gamma. \end{aligned} \quad (20)$$

Note, that representation (19) is valid if we start from the upper rim of the cut and continue to the second sheet from above. We also can start from the lower rim of the cut and continue to the second sheet from below. There we of course will find the complex conjugated partner of (20).

This consideration is valid only for infinitesimal values of coupling constant and can't be applied even qualitatively for the case of hadronic resonances, where typical coupling with decay products is large. In this case we have to consider the equation for complex poles of the resolvent without approximation and the whole formfactor $f(\omega)$ becomes important. To illustrate it let us consider several examples which should get us convinced that the result of switching off the interaction may lead to qualitatively unexpected consequences.

¹This phenomenon has to be compared to the case when the ω_1 lays below threshold. In this case equation (15) has nonpathological homogeneous solution (17) and finally we obtain two eigenvectors of H — the perturbed discrete state and perturbed continuous one.

Example 1. Let us take the formfactor $f(\omega)$ in the following form:

$$|f|^2(\omega) = \frac{\omega^{1/2}}{\omega + \rho^2}, \quad (21)$$

where ρ is real. The inverse resolvent according to (13) is given by

$$\eta^{-1}(E) = \omega_1 - E - \lambda^2 \int_0^\infty d\omega \frac{\omega^{1/2}}{\omega + \rho^2} \frac{1}{\omega - E}, \quad (22)$$

and after integration we arrive at

$$\eta^{-1}(E) = \omega_1 - z^2 - \frac{i\pi\lambda^2}{z - i\rho}, \quad (23)$$

where we have defined the variable $z = \sqrt{E}$ in such a way that the first sheet of E -plane corresponds to the upper half-plane of z and the second sheet of E — to the lower half-plane of z . Condition (6) means in this case that

$$\omega_1 > \frac{\pi\lambda^2}{\rho}, \quad (24)$$

what in turn implies that equation $\eta^{-1}(E) = 0$ has the following roots:

$$\begin{aligned} z_{1,2} &= \pm \left[\omega_1 - 2\gamma d - \gamma^2 \right]^{1/2} - i\gamma, \\ z_3 &= -id, \end{aligned} \quad (25)$$

where d and γ is given by

$$\rho = d + 2\gamma, \pi\lambda^2 = 2\gamma(\omega_1 + d^2). \quad (26)$$

For new parameters γ and d , inequality (24) reads as

$$\omega_1 > 2\gamma d. \quad (27)$$

Recall, that (27) prevents penetration of z_i to the upper half-plane of z (or to the first sheet of E). As is seen from equations (25) and (27), here may be two different situations:

- Two complex conjugated poles on the second sheet of E -plane and one antibound state, below the threshold, is also on the second sheet. ($\omega_1 - 2\gamma d > \gamma^2$).
- Three antibound states and no resonances. ($\gamma^2 > \omega_1 - 2\gamma d > 0$)

The whole resolvent looks like

$$\eta(E) = \frac{z + i(d + 2\gamma)}{(z + id) [(z + i\gamma)^2 - (\omega_1 - 2\gamma d - \gamma^2)]}, \quad (28)$$

and apparently that for $\lambda \rightarrow 0$ ($\gamma \rightarrow 0$) it has only two complex conjugated poles

$$E_c = \omega_1 - 2\gamma d \pm 2i\gamma\omega_1^{1/2} + O(\lambda^4) \quad (29)$$

and no antibound states.

Example 2. In this case the formfactor is given by

$$|f|^2(\omega) = \frac{\lambda^2\omega^{1/2}}{(\omega - \rho^2)(\omega - \rho^{*2})}, \quad (30)$$

now ρ is a complex number. Proceeding as in the previous example we obtain the inverse resolvent:

$$\eta^{-1}(E) = \omega_1 - z^2 + \frac{i\pi\lambda^2}{\rho - \rho^*} \frac{1}{(z + \rho)(z - \rho^*)}, \quad (31)$$

where z is the square root of energy, defined as above. The condition on the parameters now looks like

$$\omega_1 - \frac{i\pi\lambda^2}{(\rho - \rho^*)|\rho|^2} > 0. \quad (32)$$

In this example the equation $\eta^{-1}(E) = 0$ has four solutions which correspond to the following situations:

- Two pairs of complex conjugated poles (resonances).
- Pair of complex conjugated double poles. (Here a fine tuning of parameters is needed [5]: $Re\rho = \sqrt{\omega_1}$, $Im\rho = \left[\frac{\pi\lambda^2}{16\omega_1}\right]^{1/3}$).
- One pair of complex poles and two antibound states.
- Four antibound states.

All these cases in the limit $\lambda \rightarrow 0$ fuse together at E_c

$$E_c = \omega_1 + \frac{\pi\lambda^2(\omega_1 - |\rho|^2)}{2\rho_2 [(\omega_1 - |\rho|^2)^2 + 4\rho_2\omega_1]} - \frac{i\pi\lambda^2\sqrt{\omega_1}}{[(\omega_1 - |\rho|^2)^2 + 4\rho_2\omega_1]} + O(\lambda^4), \quad (33)$$

where $\rho_2 = Im\rho$. We may continue this set of examples, but the general idea is now clear: as far as the consideration of singularities is fulfilled nonperturbatively, the number of resonances, created out of stable states exceeds the expected one to one correspondence.

The question, which immediately arises during the discussions with colleagues from particle physics establishment is: "How can the number of state be changed? It should contradict the completeness relation!" To answer these questions and may be prevent it in more elaborated form we first should prove the completeness of solutions which we have obtained and after that will be able to treat the problem of discrete states in the case of resonances.

First of all let us fix the arbitrary constant $A = 1$ in the final expression for eigenvector $\Psi(E)$ – eq. (18). Making use of normalization conditions (4) and the Sokhotsti-Plemel relation one can prove that

$$(\Psi_{\pm}(E))^{\dagger} \Psi_{\pm}(E') = \delta(E - E'). \quad (34)$$

Further the Sokhotski-Plemel relation provides us with the following equation for real E :

$$\eta_+(E) - \eta_-(E) = 2\pi i \lambda^2 |f|^2(E) \eta_+(E) \eta_-(E). \quad (35)$$

Using (35) one can get convinced that the following remarkable relation is valid:

$$\int_0^\infty dE \Psi_+(E) (\Psi_+(E))^+ = |1 \rangle \langle 1| + \int_0^\infty d\omega |\omega \rangle \langle \omega|. \quad (36)$$

The same is true also for out-going solution $\Psi_-(E)$. Equation (34) and (36) tell us that the set of solutions $\Psi_+(E)$ (or $\Psi_-(E)$) forms the complete system in our case of unstable particle.²The other question which is now rather difficult to be formulated precisely concerns the status of resonances as a "particle" or "discrete state". This question has been intensively discussed in the series of papers of Brussels-Austin group[6] and in the textbook "Quantum Mechanics" by A.Bohm [7]. Unfortunately the comprehensive discussion of this subject will lead us to the functional analysis, very far from particle physics and therefore we again will schematically present the general ideas.

Let us return to solution (18) of eigenvalue problem (for definiteness we shall speak about $\Psi_+(E)$ solution) and consider it as a function of complex energy. As we already know the resolvent $\eta_{+(E)}$ which enters into the r.h.s. of (18) has a pole (poles) on the second sheet in the point (points) say E_c . The residue in this pole is proportional to the expression in the square brackets, taken at $E = E_c$. Let us denote it via $\Psi_+^G(E_c)$:

$$\Psi_+^G(E_c) = |1 \rangle + \lambda \int_0^\infty d\omega \frac{f(\omega)}{\omega - E} |\omega \rangle \Big|_{E \rightarrow E_c}, \quad (37)$$

where the continuation to the point E_c should be performed from the above of the real axis. The superscript G stands for Gamov. It is this state, being properly continued to the second sheet is the eigenvector of H with complex eigenvalue E_c and there exists the generalized spectral decomposition of H , where $\Psi_+^G(E_c)$ enter as a discrete state. It goes without saying that the last sentence is a heresy from the point of view of the Hilbert space formulation of quantum theory, but we worked not in the Hilbert space from the very beginning, when we considered the hamiltonians with continuous spectrum. Usually we do not pay too much attention to the difference between the Hilbert space and the rigged Hilbert space (it is the space where the operators with continuous spectrum are defined). The reason for that is probably the Dirac bright invention of bra and cat vectors, which enter into the formulas in a very symmetric way as far as the real spectrum is concerned. The general situation is nevertheless the following: if we consider the operators with continuous spectrum, we may use as states the wave packets, which are good, square integrable elements of some Hilbert space \mathcal{H} . But among these vectors we can't find the eigenvectors of our operators and we must extend our space including into it also nonnormalizable vectors if we want to construct the spectral decomposition of operators.

²It is useful to compare this case with the situation when the discrete level lies below continuum.

This extended space Φ^+ is really the space of the functionals, not the functions (recall the most popular example of δ -function). The space of functionals Φ^+ should be supplied with the space of test functions Φ , where these functionals may be defined. In such a way there arises the rigged Hilbert space or Gelfand triplet of spaces (though more appropriate to say the trinity of spaces)

$$\Phi \subset \mathcal{H} \subset \Phi^+. \quad (38)$$

Now we can return to the Gamov vector and explain its place in the present construction. First of all we want to emphasize that the continuation to the complex point E_c in (37) should be performed starting from the above of real axis (if we simply put $E = E_c$ in the integrand, the answer will be wrong). The obstacle for direct continuation is the contour of integration: to move E below real axis we have to deform the path of integration to the complex plane, what is impossible because the state $|\omega\rangle$ is defined for real ω only. In this point let us recall that the state $\Psi_+(E)$ belongs to the space Φ^+ . If we will be able to find the appropriate space of test functions Φ , such that the $\langle \phi|\omega\rangle$ (where $\langle \phi|$ belongs to Φ) could be analytically continued to the lower half plane, we will be able to make the analytic continuation of (37). The space Φ , which we need for this purpose, does exist and its elements have the following form:

$$\Phi \supset \langle \phi| = \int_0^\infty d\omega \langle \omega|\phi(\omega), \quad (39)$$

where function $\phi(\omega)$ belongs to the space of Hardy class functions from above i.e. the functions which could be analytically continued to the lower half plane.

The above discussion shows that in spite of the absence of resonances in unity decomposition (36) we can construct the corresponding states in the rigged Hilbert space. Moreover, there exists a generalized spectral of hamiltonian which could be analytically continued in the rigged space in such a way that the resonances will explicitly enter it. We will not present here this construction and will devote the end of the paper to the discussion of physical consequences of our approach.

As is seen from the general expression for eigenstates and our examples, the function $f(\omega)$ plays very important role in the formation of resonances, their masses, widths and number. Certainly, this important object should be derived in the framework of fundamental theory — QCD, but in the present situation it is hardly possible and we have to introduce it as a phenomenological one. Therefore we must investigate if there are some general requirements on these functions which follow from quantum theory. One of it we have already used in our approach forbidding the appearance of the stable state below the threshold by condition (6). Particle physicists have recognized in the formfactors, which we had used in our examples, the factor $\sqrt{\omega}$ — the two particles phase volume, which defines the vanishing of transition amplitude of one scalar into a pair of scalars. In the general case the power of relative momentum — $\sqrt{\omega}$, will be $l + 1$, where l is the relative orbital momentum. Apart from that we imposed the requirement of square integrability of the formfactor. Actually we can relax this condition — all our arguments hold true even for formfactors which vanish at infinity. As we see these conditions leave

too much room for different parameterization of the formfactor and there may arise the impression that among these different possibilities there also exists the case when the number of states coincides with the initial one. Unfortunately, this very case does not fit into aforementioned conditions. Indeed, let us take the following formfactor:

$$|f|^2(\omega) = \sqrt{\omega}, \quad (40)$$

which does not vanish at infinity. To define the inverse resolvent we need to make one subtraction in the dispersion integral (13) in some point $E = -E_0$, where $E_0 = \rho^2 > 0$. This subtraction of infinite constant from the integral term may be absorbed into the infinite renormalization of ω_1 , in (13). After this renormalization we arrive at the following expression for $\eta^{-1}(E)$:

$$\eta^{-1}(E) = \omega_1^r - z^2 - i\pi\lambda^2(z - i\rho), \quad (41)$$

where we have used the notations from our first example and superscript r means renormalized. Apparently the equation $\eta^{-1}(E) = 0$ now is quadratic and has exactly one pair of complex conjugated solutions. So, principally we may have the desired one to one correspondence, but the price for it is the infinite renormalization in the model which is considered as a phenomenological one. Apart from that, this subtraction of integral may be absorbed into renormalization of physical quantity only in the case of S-wave decay, for higher waves there is nothing to renormalize, therefore we consider this possibility as unsatisfactory.

The model which we have considered may be generalized for many channels and several discrete states to describe more realistic situation in particle physics — mixing of states via interaction with mutual continuum. The most important features of this generalization are the following: all discrete states dissolve in continuum, the number of eigenstates of hamiltonian is equal to the number of different continuums, the equation which defines the positions of the poles are mutual for all states — it becomes the equation for the poles of determinant of partial resolvent, but the intensities of different poles depend on the specific channel. If again we will consider the meromorphic class of functions, the number of resonances exceeds one to one correspondence.

Apparently, the model we have considered is rather general and universal and, certainly, QCD, as the fundamental theory of strong interaction should provide us with some prescription for the key object of our approach to the formfactor $f(\omega)$. As we have already mentioned the relativistic generalization of the Friedrichs model is also possible [3] and the role of the square of formfactor in this case is played by spectral density of propagator of bound discrete state, therefore, in the realistic situation the usage of exponential functions is hardly possible, sooner it should be the function with the usual threshold singularity and meromorphic character of the appropriate complex plane and therefore also should lead to qualitatively the same picture. On the other hand, one can argue, that among the lowest multiplets we do not observe any doubling of states, all of them are very nicely described by single Breit-Wigner poles. That is true, but at the same time, when we consider the excited states, the situation changes rather drastically. Sometimes the hypothesis about extra states fits better than the single state. The example of most

advanced analysis of resonance picture in the singlet channel 0^{++} , in the framework of $K\bar{K}$ - matrix formalism with channels $K\bar{K}, \eta\eta, \eta\eta'$ and $\pi\pi$ [8] shows that the number of states exceeds the quark model predictions. The most favorable interpretation of these extra states, of course, is the glueball one, but appearance of poles splitting cannot be rejected. Certainly more clear situation is in the isospin 1 states, because here we have no admixture of glueballs and here we find the region of masses 1450-1700 Gev with quantum numbers 1^{--} where different fittings give several states [9]. Also, if the shape of resonance differs from the usual Breit-Wigner one, it may be the reflection of several poles which is not separated well and more accurate measurement of phase may clarify its interpretation. The last point we want to mention in conclusion is the dependence of shape of resonances on the channel even for the well established ones. This phenomenon is very well known, but usually it is interpreted as experimental errors or influence of different interaction of different decay products. Multi-channel generalization of our approach, which we have not considered here, clearly shows that this dependence is the other manifestation of formfactors $f_i(\omega)$, which is different for different channels and can be used for its investigation.

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