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RELATIVISTIC MECHANICS WITH REDUCED FIELDS

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Abstract

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A new relativistic classical mechanics of interacting particles using a concept of a reduced field (RF) is proposed. RF is a mediator of interactions the state of which is described by a finite number of two-argument functions. Ten of these functions correspond to the generators of the Poincare group. Equations of motion contain the retardation of interactions required by the causality principle and have form of a finite system of ordinary hereditary differential equations. RFs may be considered as generalizations of ordinary classical fields and may enter equations of motion together with electromagnetic field. Interaction with RF may go without the radiation of RF, so RF may mediate relativistic elastic interactions of particles.

Аннотация

Соколов С.Н. Релятивистская механика с редуцированными полями: Препринт ИФВЭ 96-15. – Протвино, 1996. – 14 с., библиогр.: 24.

Предложена новая релятивистская классическая механика взаимодействующих частиц, использующая концепцию редуцированного поля (РП). РП является медиатором взаимодействий, состояние которого описывается конечным числом двухаргументных функций. Десять из этих функций соответствуют генераторам группы Пуанкаре. Уравнения движения содержат запаздывание взаимодействий, требуемое принципом причинности, и имеют вид системы конечного числа обычных дифференциальных уравнений с запаздыванием. Редуцированные поля могут рассматриваться как обобщения обычных классических полей и могут входить в уравнения движения вместе с электромагнитным полем. Взаимодействие с РП может происходить без излучения РП, так что РП может быть посредником релятивистских упругих взаимодействий частиц.

1. Introduction

Relativistic classical mechanics with a reduced field (RF) as a mediator of interactions lies between the relativistic direct interaction theories [1,13] and the classical relativistic field theories.

In equations of motion, RF appears as a force depending on the space-time positions of interacting particles and as a set of integral values corresponding to the set of ten generators of the Poincare' group and interpreted as the energy-momentum and the spin of the field. The physical idea leading to the notion of RF is as follows.

The forces acting between particles are transmitted by some mediator and various methods to describe the interaction correspond to various physical assumptions about the properties of the mediator. In the non-relativistic case, forces usually depend only on the positions of particles and are defined in such a way that the sum of the linear momenta of particles, as well as the sum of their angular momenta, do not change with time, while the sum of their kinetic energies changes. From the physical viewpoint, it means that the mediator transmitting the force from one particle to another (a rope, or a spring, or an electrostatic field, or something else) has small inertia, so its linear and angular momenta are negligible, retardation in the transmission is unessential, and excitation of internal degrees of freedom (e.g. of waves in the spring) is small. The state of such mediator is fully fixed by one function of particle positions (for example, by the force in the Newtonian picture, or by the interaction potential in the Hamiltonian and the Lagrangian formulations).

In the relativistic case, the structure of the Poincare group requires more detailed description of the mediator of interaction but leaves a certain freedom in the choice of the mediator properties.

The field theories assume that the mediator is continuous and its excitations propagate according to some wave equation.

The direct interaction theories seek the most concise description compatible with the structure of the Poincare' group. In the Hamiltonian formulation, they usually allow for the mediator either to have zero (Lorentz) angular momentum with nonzero linear momentum (such choice is called the point form of dynamics [6,7]), or to have some zero elements of linear momentum with nonzero angular momentum (the instant form [2,4,5,11] and other forms of dynamics [13,15-18]). But the direct interaction theories still exclude causal retardation, assuming that the mediator transmits interaction instantly in some coordinate system (related with the choice of the foliation of the Minkowski space and with the form of dynamics). This formal "instanteneity" leads to difficulties with the space-time interpretation of motion and with causality. These difficulties look differently in the quantum and the classical versions of these theories.

In the quantum case, the difficulties appear as a nonlocality of operators and as an uncertainty of the coupling with the electromagnetic field. Their space-time nature is obscured by quantum effects.

In the classical case [8,13-18], the difficulties appear as an ambiguity of the relation between the canonical and spatial coordinates and as various pathologies of trajectories. However, since the Hamiltonian classical models [1,8,13-18] can be, in principle, obtained from (more technically developed) quantum ones [2-7,11] by a certain limiting procedure [12], the source of difficulties is physically the same in both cases and stems from the inherent ambiguity of the coordinate measurements.

Indeed, whatever definition of physical coordinates is chosen, the results of measurement of the trajectories of interacting particles by means of probing particles or external fields turn out to be non-unique. Therefore, for the isolated systems of interacting particles, the direct interaction theories may well predict the properties observable at the infinity (like energy levels, scattering, rotation frequencies), but cannot predict unique trajectories of particle and, thus, are utterly unable to treat particles that interact directly between themselves and, at the same time, interact with an external electromagnetic field. This excludes their application to many physically interesting cases.

Clearly, to overcome the difficulties with the causality and coordinates, one has to make more explicit account of retardation than is possible in the usual Lagrangian formalism with finite number of degrees of freedom. First attempts to generalize the Lagrangian formalism in that direction were done in [19-21]. But this leads to nonlocal Lagrangians which are not easy to connect with ordinary differential equations without additional assumptions of analyticity.

In this paper, we propose an alternative way to built a relativistic classical mechanics respecting the causality principle and having a natural space-time interpretation. Unlike preceding attempts, we do not try to generalize the Hamiltonian or Lagrangian formalisms, or use some kind of "instanteneity" which would impose complicated consistency conditions on the accelerations, but propose to generalize the equations of motion by the explicit inclusion into them of some equations for the evolution of the state of the mediator. In the additional equations for the mediator, we allow to the mediator to transmit forces with the causal retardation, but limit the ability of the mediator to excite its internal degrees of freedom, so the motion of particles and of the mediator can be governed by a finite number of ordinary hereditary differential equations.

It is often thought that retardation always leads to radiation, i.e. to the excitation of waves carrying to infinity some energy, momentum, and angular momentum, and that only quantum effects can prohibit the radiation, if the energy is insufficient for the emission of mediating quanta (for example, for the emission of pions in a nucleon-nucleon collision). However, the proposed relativistic classical mechanics with RF gives a counter-example: there the excitation of the mediator does not obligatory propagates to infinity, so this theory allows elastic interactions with no radiation.

2. Equations of Motion

Let a particle have position x, (constant) mass m, velocity h, $h^2 = 1$, momentum $p = mh$, and acceleration $dh/d\tau$, where τ is the proper time. Let $[x_a]$ denote the past part of the trajectory $x_a(\tau)$ of particle a up to the point x_a included and $[x_a)$ denote the same part with the end point x_a excluded. The expression $A \cdot B$, where A, B are tensors or vectors, will mean $A^{\cdots}_{\cdots\mu} B^{\mu\cdots}_{\cdots}$, the metric is $(1, -1, -1, -1)$, and $\wedge$ is the external product.

Let retarded position $x_a^{r(x_b)}$ of particle a with respect to point x_b be defined by conditions $(x_b - x_a^{r(x_b)})^2 = 0, x_{b0} \geq x_{a0}^{r(x_b)}$.

We shall define a RF as a mediator of interaction the state of which is described by a set of functionals of the past parts $[x_a]$ of particle trajectories and which produces the forces depending both on $[x_a]$ and on the state of the mediator. We will imply that the set of functionals fixing the state of the mediator always includes the energy-momentum Π and the spin S transferred to the mediator from the particles. The quantity Π is a 4-vector (with no limitations on the sign of Π^2), and S is an antisymmetric tensor with 6 independent elements describing all the angular momenta corresponding both to the spatial and the Lorentz rotations. The variables $\Psi = (\Pi, S)$ will be called, collectively, the fill of the mediator.

Besides the fill, the description of the state of the mediator may include other functionals of $[x]$ and forces may depend on these functionals. In the simplest case, each pair of particles interacts via its own mediator and the state of each mediator is described by its fill only.

We shall impose upon RF the causality condition: the force acting on a given particle a at point x_a may depend only on the causally accessible parts $[x_b^{r(x_a)}]$ of the trajectories of other particles b ending by the retarded positions $x_b^{r(x_a)}$ of these particles. Otherwise, the RF may be quite arbitrary.

The system of equations of motion for two particles a, b interacting via one RF is

$$\begin{aligned} dx_a &= h_a d\tau_a, & dx_b &= h_b d\tau_b, \\ dh_a &= F_a d\tau_a / m_a, & dh_b &= F_a d\tau_b / m_b, \\ d\pi_a &= -F_a d\tau_a, & d\pi_b &= -F_b d\tau_b, \\ d\lambda_a &= x_a \wedge d\pi_a, & d\lambda_b &= x_b \wedge d\pi_b, \end{aligned} \tag{1}$$

where $\pi(\tau), \lambda(\tau)$ are auxiliary (vector- and tensor-valued) one-time functions through which the fill Ψ of the mediator is defined as the set of two-time functions

$$\Pi(\tau_a, \tau_b) = \pi_a(\tau_a) + \pi_b(\tau_b),$$

$$S(\tau_a,\tau_b) = \lambda_a(\tau_a) + \lambda_b(\tau_b) - x_a \wedge (K_a \cdot \Pi(\tau_a,\tau_b)) - x_b \wedge (K_b \cdot \Pi(\tau_a,\tau_b)). \qquad (2)$$

Here K_a are any tensors of rank 2 with the property $K_a + K_b = \mathbf{1}$. The choice of tensors K_a is related with the choice of a center-of-mass variable and is a matter of convenience.

Tensor S is translationally invariant and can be alternatively written as

$$S = -\sum_n \int dx_n \wedge \pi_n + L, \qquad L = \sum_n x_n \wedge (\pi_a - K_n \cdot \Pi). \qquad (3)$$

If K_a are proportional to unit tensors, $K_a = k_a\mathbf{1}$, we obtain

$$L = \sum_n (x_a - x^c) \wedge \pi_a, \qquad x^c = k_a x_a + k_b x_b.$$

Obviously, if $k_a = m_a/(m_a + m_b)$, x^c can be interpreted as a center-of mass coordinate of two particles and L, as an orbital momentum of the mediating RF with respect to this coordinate.

Force F, due to condition $h^2 = 0$ implying $dh \cdot h = 0$, is subject to the transversality condition

$$F \cdot h = 0 \qquad (4)$$

and, according to the causality condition, may have form

$$F_a = F_a([x_a],[h_a],[x_b^r],[h_b^r],\Pi(\tau_a,\tau_b^r),S(\tau_a,\tau_b^r),\ldots),$$

where the dots stand for other possible functionals that fix the state of RF or RFs and depend on the parts of the particle trajectories causally accessible for particle a. For example, the mediating field may "remember" the partial fills π_a, $s_a = \lambda_a - x_a \wedge \pi_a$ and force may depend on π_a, s_a separately. This generalization (leading to the self-interaction of particles) will not be illustrated in the present paper.

For the system of two particles and one RF, the conservation laws

$$\sum p + \Pi = \text{const}, \qquad \sum x \wedge p + \sum \lambda = \text{const},$$

are obviously true. Hence, if before and after the collision of particles the fill Ψ is zero (and, hence, due to the definition (2), $\lambda_a + \lambda_b = 0$), the usual relativistic conservation laws for the scattering processes are true. If the fill Ψ of RF changes after the scattering, this change is interpreted as an energy-momentum and angular momentum carried away by radiation.

The case of many particles is similar, except that each RF accumulates its fill from the forces it transfers, the total force acting upon a particle is a sum of forces from all other particles, and the summation over all RFs for all pairs of particles is made in the conservation laws.

The interaction via RF is fixed by the choice of functions F. In contrast to the Hamiltonian approach, where the interaction potential is a primary object and forces are secondary objects derivable from the given potential, an RF approach considers forces as primary objects, and the potential-like quantities Π are functionals of the forces in the past.

Since the equations of motion are hereditary, corresponding initial data should contain, besides the current values of the particle variables and of RF fills, the previous values of these quantities sufficiently far in the past. Namely, the semi-open subsets $[x_a)$ of the initially given trajectories $[x_a]$ should include the retarded points $x_a^{r(x_b)}$ with respect to all the other particles b in the system. Then the forces $F_a(..., [x_b^{r(x_a)}], ...)$ can be calculated and equations of motion for each particle can be integrated independently with forces

$$F_a(\tau) = F_a([x_a(\tau)], [x_b^{r(x_a(\tau))}], ...),$$

while all the retarded points $[x_b^{r(x_a(\tau))}]$ belong to the initially given trajectories. The independent integration extends the known parts of trajectories up to some points $x_a(\tau_1)$. Then the integration of equations of motion for each particle can be repeated. This process gives the explicit analytic solution of the equations of motion in the form of multiple integrals of the initially given forces. The solution is evidently unique and may be easily calculated numerically. The only case, where the described integration process fails, is the case of the central collision of particles, where at some moment the coordinates of two particles exactly coincide. However, other methods (based on the Taylor expansion) may be used near such points, so uniqueness of solutions is generally true.

We see that the equations of motion are asynchronous and no choice of a specific synchronization relation between times of different particles is needed during the integration. However, the use of some synchronization may be convenient both in numerical and analytical calculations. The choice of such synchronization, provided it is compatible with causality, does not influence the results.

The forces F may depend on the arbitrary regular functionals of the trajectories, but should not depend on the time derivatives of h. If F were dependent on acceleration $dh/d\tau$, the equations of motion would belong to the hereditary equations of the so-called neutral type, about the properties of which little is known, except in some particular cases. The dependence of F on higher derivatives of h would lead to serious nonuniqueness, unacceptable for a self-consistent physical theory. The objects that are similar to RFs but produce forces dependent on the accelerations will be called improper RFs.

The presence of retarded arguments in the equations of motion makes the mechanics with RFs time-asymmetrical (irreversible) even in the absence of radiation and makes the particle motion more complicated than in the case of "instantaneous" interaction. In particular, the qualitative behavior of the solutions crucially depends on the magnitude of forces.

If the forces are small, retardation has small influence on the solutions and solutions are smooth and monotonous, if such are the expressions for forces. The possible roughness of initial data is quickly smoothed out (due to multiple integrations) and the solution tends to an analytic one. This phenomenon is the basis for the so-called spontaneous predictivisation [10].

If the forces are so large that the fill of the mediator changes more than by the factor of e during the retardation time, the solutions become oscillating and growing in amplitude.

(Physically it can be interpreted as a resonant excitation of the mediating field.) The field nature of RF may show itself in many other ways, for example, by the threshold effects.

In the present paper, we will consider only the examples of RFs mediating elastic interactions of particles.

3. Virtual Fields

The dependence of forces on the state of RF, and, first of all, on the fill Ψ, gives the possibilities of describing such relativistic interactions which are difficult or impossible to describe otherwise. The important example is the elastic scattering of relativistic particles.

To make the interaction elastic it is sufficient, in the RF approach, to include, into the expressions of forces, the terms that depend on Ψ and constantly diminish $|\Psi|$, if it is greater than zero. Then, after the collision, when distances become large and other forces become small, these terms will dominate and the fill will tend to zero. This process can be interpreted as a decay of RF, so the relevant terms will be called decay forces. The reduced fields that totally decay after a collision of particles are distant analogies of the virtual fields of the relativistic quantum field theories: they both appear temporally during the collision of particles and do not show up at infinity. For that reason, we will call such RFs classical virtual fields (VFs).

The construction of the decay forces needs some efforts. We consider here the simplest examples of such forces. Let us first reduce the asynchronous two-time equations (1) to a single-time evolution equation for the fill Ψ only, choosing as a common scalar evolution parameter the retardation time T and considering the proper times τ_a, τ_b as functions of T, satisfying the relations

$$T = (x_a - x_b^r) \cdot (h_a + h_b^r), \qquad T = (x_b - x_a^r) \cdot (h_a^r + h_b).$$

To simplify the analysis of the asymptotic solutions, we make an adiabatic assumption that forces F are not large, vectors h change little, and their derivatives $\dot{h} = dh/dT$ and differences $h - h^r$ may be neglected.

It will be convenient to introduce vectors

$$H_a = (ch_b - h_a)/(c^2 - 1), \qquad H_b = (ch_a - h_b)/(c^2 - 1),$$

where $c = h_a \cdot h_b$, with properties

$$H_a \cdot h_a = H_b \cdot h_b = 1, \qquad H_a \cdot h_b = H_b \cdot h_a = 0,$$

and define tensors

$$Q_a = H_a \otimes h_a, \quad Q_b \otimes h_b$$

having properties

$$Q_a \cdot Q_a = Q_a, \quad Q_a \cdot Q_b = 0.$$

Operators $Q\cdot$ will be used as projectors, their sum

$$\mathcal{P} = Q_a \cdot + Q_b$$

is a projector on the (h_a, h_b)-plane in the tangent space. We will denote $\bar{\mathcal{P}} = \mathbf{1} - \mathcal{P}$ and mark by bar the parts of vectors orthogonal to the (h_a, h_b)-plane.

In the definition of S we choose

$$K_a \cdot = Q_b \cdot + \frac{1}{2} \bar{\mathcal{P}}.$$

Then the evolution equations for the fill become

$$\dot{\Pi} = -F_a \dot{\tau}_a - F_b \dot{\tau}_b,$$

$$\dot{S} = -\frac{1}{2}(x_a - x_b) \wedge (\bar{F}_a \dot{\tau}_a - \bar{F}_b \dot{\tau}_b) - \frac{1}{2}(h_a \dot{\tau}_a + h_b \dot{\tau}_b) \wedge \bar{\Pi} - n h_a \wedge h_b \, v \cdot \Pi, \tag{5}$$

where $v = h_a \dot{\tau}_b - h_b \dot{\tau}_a, \quad n = 1/(c^2 - 1)$, and terms with $\dot{K}$ are omitted according to the adiabatic approximation.

Let forces be linear functions of the fill. Then equations (5) become a system of linear differential equations with retardation. System (5) can be shortly written as

$$\dot{\Psi} = M \Psi,$$

where M is 10×10 matrix of coefficients and operators of retardation. Clearly, if all the proper values of M have a negative real part, solutions Ψ must decrease with time. The only problem is that the structure of RHS of (5) restricts the choice of M in a complicated fashion: vectors F_a, F_b have (due to the transversality condition (4)) only 6 independent components, so in the general case only 6 constants in M can be chosen freely, while fill Ψ has 10 components.

In cases of planar motion and of motion along one line, when fewer dimensions come into play, the freedom in the choice of M is limited in a similar way. In case of planar motion, there are 4 independent components of F_a, F_b against 6 independent components of Ψ. In case of motion along one line, there are 2 independent components of forces against 3 independent components of the fill. Here we consider the case of motion along one line and postpone other more cumbersome cases for further publications.

The motion along one line physically means that in a certain coordinate frame the trajectories of both particles lie on the same straight line, so in this frame the colliding particles either turn back or pass through each other. To preserve the explicit covariance, we will not use a special frame, but single out the case of motion along one line by the condition that all (relative) vectors between particle positions are linear combinations of h_a, h_b.

The pass from the proper times τ, τ^r to variables T, T^r may be done as follows. Define two constant null-vectors

$$o_a = h_a C - h_b, \quad o_b = h_b C - h_a, \quad o_a^2 = o_b^2 = 0, \quad C = c + \sqrt{c^2 - 1},$$

proportional to $x_a - x_b^r, \quad x_b - x_a^r$ and entering equalities

$$x_a - x_b^r = o_a T q_a, \qquad x_b - x_a^r = o_b T q_b, \tag{6}$$

where q_a, q_b are unknown coefficients. Multiplying both parts of (6) by $(h_a + h_b)\cdot$ and using the adaiabatic approximation $h^r = h$, we obtain

$$q_a = q_b = q \equiv 1/(c+1)(C-1).$$

Differentiating (6) with respect to T and denoting d/dT by the dot, we obtain

$$h_a \dot{\tau}_a - h_b \dot{\tau}_b = o_a q,$$

whence, multiplying both parts by $H_a\cdot, H_b\cdot$, we get

$$\dot{\tau}_a = \dot{\tau}_b = Cq, \qquad \dot{\tau}_a^r = \dot{\tau}_b^r = q.$$

The relation between T and the retarded retardation time T^r, defined by the relation

$$x_b^r - x_a^{rr} = T^r o_b q,$$

where x_a^{rr} is the point retarded with respect to x_b^r, can be found from the equalities

$$x_a - x_a^{rr} = h_a(\tau_a - \tau_a^{rr}),$$

$$x_a - x_a^{rr} = x_a - x_b^r + x_b^r - x_a^{rr} = T o_a q + T^r o_b q.$$

Multiplying them by $o_a\cdot, o_b\cdot$, we obtain

$$(\tau_a - \tau_a^{rr}) h_a \cdot o_a = T^r o_b \cdot o_a q,$$

$$(\tau_a - \tau_a^{rr}) h_a \cdot o_b = T o_b \cdot o_a q,$$

whence it follows that

$$T = CT^r.$$

Now we come to the equations of motion. Forces compatible with motion along a line may be written as

$$F_a = H_b \phi_b / \dot{\tau}_a, \qquad F_b = H_a \phi_a / \dot{\tau}_b,$$

where ϕ_a, ϕ_b are scalar functions. Tensor of spin in this case may be written as

$$S = n h_a \wedge h_b S_{ba},$$

where scalar S_{ba} is

$$S_{ba} = h_b \cdot S \cdot h_a = \text{const} + \int^{\tau_a} \phi_b \tau d\tau - \int^{\tau_b} \phi_a \tau d\tau - (h_a \tau_b - h_b \tau_a) \cdot \Pi. \tag{7}$$

Note that $S_{ab} = h_a \cdot S \cdot h_b = -S_{ba}$.

The interpretation of the spin element S_{ab} assiciated with the Lorentz rotation deserves a comment. If the center of mass of the two particles is at rest before the collision, the value of S_{ab} is proportional to the shift of the center of mass of the two particles due to the interaction with the mediating field. If S_{ab} vanishes after the collision, the center of mass of the two particles returns to its former place. Otherwise it does not. The vanishing S_{ab} does not follow from the vanishing of the energy-momentum of RF. Generally, the amount of radiated spin is independent from the amount of radiated energy, so the decay forces of VFs must take equal care of the decay of all the components of the fill.

Using (7) (or (3)), in the adiabatic approximation we obtain

$$\dot{S}_{ba} = -v \cdot \Pi.$$

The first of the eqs.(5) reduces to two scalar equations $\dot{\Pi}_a = -h_a \cdot \Pi, \qquad \dot{\Pi}_b = -h_a \cdot \Pi.$

Combining the last expressions and the definition of $v = h_a \dot{\tau}_b - h_b \dot{\tau}_a$, we see that the evolution equations for the fill reduce to the system of three scalar equations

$$\begin{aligned} \dot{\Pi}_a &= -\phi_a, \\ \dot{\Pi}_b &= -\phi_b, \\ \dot{S}_{ab} &= (\Pi_a - \Pi_b) q, \end{aligned} \tag{8}$$

where $\Pi_a = h_a \cdot \Pi$, $\Pi_b = h_b \cdot \Pi$.

The simplest choice of functions ϕ is a linear combination of scalars Π_a, Π_b, S_{ab}. To make coefficients dimensionless, we will use, instead of Π, S, the quantities having the dimension of force:

$$\Pi_a/D,\ \Pi_b/D,\ S_{ab}/D^2,$$

where $D = T + T^0$, and $T^0 > 0$ is an arbitrary constant introduced in order to avoid singularity when the particles meet and $T = 0$.

Let forces be symmetric with respect to particles:

$$\begin{aligned} \phi_a &= a\Pi_a/D + a_1 \Pi_b/D + b S_{ab}/D^2, \\ \phi_b &= a\Pi_b/D + a_1 \Pi_a/D + b S_{ba}/D^2. \end{aligned} \tag{9}$$

Since function ϕ_a enters into the expression of the force acting on particle b and vice versa, the causality condition requires that the times $\tau_1, \ldots, \tau_4$ in the expressions

$$\phi_a = a\Pi_a(\tau_1, \tau_2)/D + \ldots, \qquad \phi_b = a\Pi_b(\tau_3, \tau_4)/D + \ldots,$$

obey the restrictions $\tau_{1,4} \leq \tau^r(T)$, $\tau_{2,3} \leq \tau(T), \ldots$. Within these restrictions, different choices of times are of interest.

We consider first the simplest choice, when all the arguments in forces are equally retarded

$$\tau_1 = \tau_2 = \ldots = \tau^r(T).$$

We choose the coefficient a_1 before the mixing term in (9) to be zero. Denoting $\Pi(T) = \Pi(\tau(T), \tau(T)), \ldots$, we obtain from (8) the system

$$\begin{aligned} \dot{\Pi}_a(T) &= -a\Pi_a(T^r)/D - b S_{ab}(T^r)/D^2, \\ \dot{\Pi}_b(T) &= -a\Pi_b(T^r)/D - b S_{ba}(T^r)/D^2, \end{aligned}$$

$$\dot{S}_{ab}(T) = (\Pi_a(T) - \Pi_b(T))q.$$

These equations contain variable deviation of argument $T - T^r = T(1 - 1/C)$. To make them closer to equations with a constant deviation of argument studied in [22-24], we perform the substitution

$$D = e^\theta, \quad S_{ab} = D\sigma, \quad \dot{S}_{ab} = \sigma' + \sigma, \tag{10}$$

where prime means $d/d\theta$. It gives

$$\begin{aligned} \Pi'_a(\theta) &= -a\Pi_a(\theta^r) - b\sigma(\theta^r), \\ \Pi'_b(\theta) &= -a\Pi_b(\theta^r) + b\sigma(\theta^r), \\ \sigma'(\theta) &= (\Pi_a(\theta) - \Pi_b(\theta))q - \sigma(\theta), \end{aligned} \tag{11}$$

where $\theta^r = \theta - \mu$, $\mu = \ln C$. The general solution $\Psi = (\Pi_a, \Pi_b, \sigma)$ of (11) is a (possibly infinite) superposition of solutions

$$\Psi_n(\theta) = e^{\lambda_n \theta}\Psi_n(0)$$

with different proper values λ_n. Substitution of $\Psi = e^{\lambda\theta}\Psi(0)$ into (11) gives for λ the characteristic equations with the matrix

$$Z = \begin{pmatrix} \lambda + ae^{-\lambda\mu} & 0 & be^{-\lambda\mu} \\ 0 & \lambda + ae^{-\lambda\mu} & -be^{-\lambda\mu} \\ -q & q & \lambda + 1 \end{pmatrix}.$$

Equation

$$\det Z = (\lambda + ae^{-\lambda\mu})[(\lambda + 1)(\lambda + ae^{-\lambda\mu}) + 2bqe^{-\lambda\mu}] = 0 \tag{12}$$

determines the spectrum of proper values λ_n. Since

$$S_{ab} = e^\theta\sigma = e^{(\lambda+1)\theta}\sigma(0),$$

the fulfillment of the condition

$$\mathrm{Re}\,\lambda_n < -1 \tag{13}$$

is sufficient to make both Π and S_{ab} decrease with time. Consider from this point of view the roots of eq. (12).

Let us look first at the real roots. The equation

$$\lambda + ae^{-\lambda\mu} = 0$$

for $a < 1/e\mu$ has two real roots. If $a > 0$, they are both negative and the root λ_0, closest to zero, is limited by $\lambda > -1/\mu$. If $\mu < 1$ condition (13) can be fulfilled.

The closest to zero real root of the second equation

$$(\lambda + 1)(\lambda + ae^{-\lambda\mu}) + 2bqe^{-\lambda\mu} = 0,$$

if it exists, lies between -1 and 0, if $b < 0$, and is smaller than -1, if $b > 0$. So, to fulfill (13), we should choose $b > 0$.

Calculating $\det Z$ for complex λ and looking at the pattern of the zeros at the complex plane, one can see that the real parts of the complex roots lie to the left of $\lambda = -1$ at the same conditions on a and b as for the real roots. Therefore, if $\mu < 1/e$, the choice $a = 1/e\mu$ and $b > 0$ gives the forces that make any state of RF vanish with time, when particles fly apart and forces (9) dominate.

The limitation $\lambda > -1/\mu$ means that the RF with considered forces decays and behaves as a VF, if μ is not large, and cannot decay if $\mu > 1$. Since μ is the function of velocities

$$\mu = \ln\left(h_a \cdot h_b + \sqrt{(h_a \cdot h_b)^2 - 1}\right),$$

there is the energy threshold (more exact, the relative velocity threshold) below which the RF is a VF and the scattering of particles is elastic, and above which some energy-momentum and spin is carried away by the radiated RF. This behaviour reminds the energy threshold for the emission of real particles in the quantum field theory.

Let the full force contain, besides the slowly decreasing parts fixed by functions ϕ_a, ϕ_b, some other forces, fixed by functions f_a, f_b

$$F_a = H_b(\phi_b + f_b)/\dot{\tau}_a, \qquad F_b = H_a(\phi_a + f_a)/\dot{\tau}_b.$$

Then the numerical methods may be used for the study of the solutions. The numerical solutions of the exact equations of motion with different (repulsive and attractive) forces f have shown that, if functions f become small at large distances, the decay process due to forces ϕ takes over as soon as $|f|$ becomes smaller than $|\phi|$.

The existence of the velocity threshold above which a RF does not fully decay, is not obligatory. Consider the modification of the above example of decay forces with another choice of time arguments:

$$\phi_a(T) = a\Pi_a(\tau^r(T), \tau(T))/D + bS_{ab}(\tau^r(T), \tau^r(T))/D^2,$$

$$\phi_b(T) = a\Pi_b(\tau(T), \tau^r(T))/D + bS_{ba}(\tau^r(T), \tau^r(T))/D^2. \tag{14}$$

Since function ϕ_a enters the force acting on particle b, this choice also is compatible with causality. The presence of mixed (retarded, unretarded) arguments generally complicates the system of equations. However, in the adiabatic approximation Π_a does not depend on the first argument:

$$\Pi_a(\tau^r(T), \tau(T)) = \Pi_a(\tau(T), \tau(T)).$$

One can see this by calculating $\partial\Pi_a(\tau_1, \tau_2)/\partial\tau_1 = -h_a \cdot F_a^r$ and using that in the adiabatic approximation $h_a = h_a^r$. Therefore, the choice (14) gives

$$\dot{\Pi}_a(T) = -\Pi_a(T)/D - bS_{ab}(T^r),$$

$$\dot{\Pi}_b(T) = -\Pi_b(T)/D - bS_{ba}(T^r),$$

$$\dot{S}_{(ab)}(T) = (\Pi_a(T) - \Pi_b(T))q.$$

Making the same substitution (10), we obtain

$$\Pi_a'(\theta) = -a\Pi_a(\theta) - b\sigma(\theta^r),$$

$$\Pi_b'(\theta) = -a\Pi_b(\theta) + b\sigma(\theta^r),$$

$$\sigma'(\theta) = (\Pi_a(\theta) - \Pi_b(\theta))q - \sigma(\theta),$$

and come to the matrix

$$Z_1 = \begin{pmatrix} \lambda + a & 0 & be^{-\lambda\mu} \\ 0 & \lambda + a & -be^{-\lambda\mu} \\ -q & q & \lambda + 1 \end{pmatrix}$$

and to the equation

$$\det Z_1 = (\lambda + a)[(\lambda + 1)(\lambda + a) + 2bqe^{-\lambda\mu}] = 0.$$

Now, if $a > 1$ and $b > 0$, the first equation $\lambda + a = 0$ gives $\lambda < -1$, and the second equation

$$(\lambda + 1)(\lambda + a) = -2be^{-\lambda\mu}q$$

has only roots with $\mathrm{Re}\lambda < -1$. So, the RF with the decay forces (14) with $a > 1$ and $b > 0$ totally decays, when particles fly apart, no matter what their energies (or velocities) are. The presence of any short-range forces f_a, f_b should not change the result. Such RF is a perfect VF, mediating purely elastic interactions of particles at all energies.

4. Concluding Remarks

The version of RFs outlined in this paper is the basic one, using the simplest description of the mediator compatible with the causal retardation. It contains large freedom of choice of particular interactions. However, more detailed descriptions of the mediator may be advantageous. In particular, the changeable masses of particles may describe the parts of the mediating field which are concentrated near the particles and move together with them. The replacement of the transversality condition by the equations for the mass evolution may simplify the expressions for decay forces.

The RF formulation contains many arbitrary functions and is suitable for the construction of various internally consistent and fully relativistic models of elastic and inelastic processes with interacting particles. Addition of external fields is no problem here: the corresponding terms are simply added to the forces, acting on particles, while forces, standing in the RHSs of equations for the fill of RFs remain unchanged. In case of the electromagnetic interaction, the time derivative of the particle acceleration, entering the standard expression for the radiation friction and spoiling the good properties of the equations of motion, may be removed by a small-distance modification of the electromagnetic forces equivalent to some smearing of the particle charge.

The numerical solution of the equations of motion for RFs is almost as easy, as the solution of the Hamiltonian equations of motion. The first numerically solved cases showed a rather nontrivial behavior of the solutions in the regions, where the forces are large and the energy-momentum of a RF is comparable with that of particles. The results of the numerical analysis will be published elsewhere.

Due to the explicit retardation of forces, the interaction via RFs is always time-asymmetrical and this should have experimentally observable consequences, especially in the processes like $pp \to pp\gamma$, where the gamma emission must be sensitive to the shape of the trajectory of the protons near the collision point. The relevant models with RFs may give more specific predictions of the retardation effects, than the dispersion relations of the quantum field theory based on the causality principle reflecting just on the mere fact of causal retardation.

There are several theoretical questions, concerning RFs, that are not studied yet. The first one is the low velocity limit. The other one is the limiting cases when the time-asymmetry effects become unessential. In such cases, the relativistic VF description should become approximately equivalent to the relativistic Hamiltonian description. Such questions, as well as the relationships of the RF theory with the formalism of nonlocal Lagrangians, need the clarification.

The most intriguing problem of the classical RF theory is the construction of its quantum counterpart. The problem here is the description of history of motion in terms of the Hilbert space vectors.

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