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ABOUT POSITIVE DEFINITENESS OF SOURCE ENERGY LOSSES THROUGH EMISSION OF GRAVITONS WITH NONZERO REST MASS AND MASSIVE GRAVITATIONAL FIELD THEORY

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Abstract

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Gravitational radiation flux from an arbitrary spatially bounded source is positively defined in the used variant of theory of gravity with nonzero graviton mass. A link between energy losses by emission and work of sources is established. It is shown that the total work contains a part resulted from the interaction of a source with radiation field and a part resulted from self-action of the field. This is just that makes the work positively defined as a whole. A general form of radiation spectrum-angular distribution is obtained with account of spin and polarization states. For spherically symmetric sources, states with zero spin as well as zero projection on momentum of spin two make a contribution to emission.

Аннотация

Лоскутов Ю.М. О положительной определенности энергетических потерь источника на излучение гравитонов с ненулевой массой покоя и теория массивного гравитационного поля: Препринт ИФВЭ 96-18. – Протвино, 1996. – 15 с., библиогр.: 16.

В используемом варианте теории гравитации с ненулевой массой гравитона поток гравитационного излучения от произвольного пространственно ограниченного источника является положительно определенным. Установлена связь потерь энергии на излучение с работой источников. Показано, что полная работа содержит часть, обусловленную взаимодействием источника с полем излучения, и часть, обусловленную самодействием поля. В целом это и делает работу положительно определенной. Получен общий вид спектрально-углового распределения излучения с учетом спиновых и поляризационных состояний. В случае сферически-симметричных источников вклад в излучение дают состояния с нулевым спином и с нулевой проекцией спина два на импульс.

1. General Remarks

It is well known that the question about rest mass of particles like neutrino, photon, graviton is of fundamental significance. Suffice it to mention the role of the neutrino mass in various physical processes and that of the photon mass in electromagnetic phenomena. Taken alone, by virtue of its conceptual nature, theory cannot, in principle, provide an answer to whether or not a specified particle offers the rest mass. One or another agreement is simple to be adopted in it. Depending on an accepted agreement, the theory can only supply distinct consequences to be experimentally tested. On the other hand, it has been possible to recognize in the experiment (within the limits of achieved accuracy) only the upper boundary of a mass of photon and neutrino, that is, the issue always remains open below it. An incorporation of mass into the theory is not accompanied by fundamental difficulties or physically unacceptable outcomes for electroweak and electromagnetic interactions (if nature endows the corresponding particles finite, even though very small masses). A different situation arises with the theory of gravitation. Here one faces specific obstacles when introducing graviton mass to the theory. They related to the fact that the entering of mass term (after referred to as μ-term implying that μ is graviton mass) to the Lagrangian density L (and to following equations) demands, as shown for the one in [1], the introduction of supplementary metric along with the Riemannian space metric $g_{\alpha\beta}$. It is chosen as the Minkowky space metric $\gamma_{\alpha\beta}$ for a number of reasons (see [1,2,3]). This leads to two consequences.

First, the consistency of the Riemannian geometric interpretation of the theory has been lost [1,3] because space with metric $\gamma_{\alpha\beta}$ begins taking on sense as basic space, wherein gravitational (as with all other) processes occur. Metric tensor $g_{\alpha\beta}$ must now be understood as that induced by the physical gravitational field defined in space with metric $\gamma_{\alpha\beta}$. The Riemannian space has been assigned a meaning of effective, induced by physical field, space. This does not necessarily denude it of practical significance in so far as an observer together with his instruments is exposed to the action of field, i.e. "immersed" in this effective space. Hand in hand with this fact, the emergence of basic metric $\gamma_{\alpha\beta}$ allows one to transpose mental operation of field inclusion and exclusion to all physical situations. In other words, that makes possible to judge either some phenomenon caused by the action of gravitational field, or it takes place in the absence of field.

Second, the presence of two metrics involves ambiguity of μ-term entering the scalar density L, unless further requirements imposed on L are formulated.

Several versions of constructing of L with μ-term were considered in [1]. The key inference reached by authors of [1] reduces to conclusion about impossibility to develop an acceptable noncontradictory theory of gravitation with nonzero graviton mass. However, if this conclusion were true, it would be the unique case in physics, when the exact equality to zero of graviton rest mass had been proved except in a theoretical manner. Such theorem cannot be doubted. And what is the listed inference based on? It is mainly founded on the fact, that the energy of free gravitational field has a negative contribution owing to the existence of scalar component of the field, corresponding to zero spin, along with that of spin two. This, in turn must lead the authors of [1] view to the instability of field sources.

It is necessary to refine here, that the instability is caused not by negative energy density in itself, but negative investment to energy flux going from the source to infinity. For one example, the density of gravitational field (either massive or massless) energy is always negative outside a static source. But it does not tend to instability, so as energy does not bring from the source to infinity (the energy flux is equal to zero). Whenever negative contributions appear in a flux, instability should certainly arise. To elucidate its character and fix whether or not its origin is associated with some incorrectness, let us carry out proper calculations by commonly adopted methods. Running ahead, it should be said that it is these methods that the fallacy resides in.

In general case the Lagrangian density of field with μ-term can be written (see [1]) as follows[1]:

$$L_g \equiv L_g^0 + \mu^2 f(g, \gamma), \tag{1}$$

where L_g^0 is μ-independent part. Since, according to [1], the conclusion about the appearance of negative contribution to energy density (and also to energy flux density – when using standard methods of calculations) does not depend on the choise of f, take μ-term in the form, which was first proposed in [3] (the substitution of this construction will be presented below in Sec.2):

$$\mu^2 f(g, \gamma) \equiv -\frac{\mu^2}{16\pi}\Big(\frac{1}{2}\tilde{g}^{\alpha\beta}\gamma_{\alpha\beta} - \sqrt{-g} - \sqrt{-\gamma}\Big), \tag{2}$$

where $-\tilde{g}^{\alpha\beta} \equiv \sqrt{-g \cdot g^{\alpha\beta}}$. Considering the fundamental character of metric $\gamma_{\alpha\beta}$, density L_g^0 is obtained by generalization of [3]:

$$L_g^0 \equiv \frac{1}{16\pi}\tilde{g}^{\varepsilon\lambda}(G^\alpha_{\varepsilon\lambda}G^\beta_{\alpha\beta} - G^\alpha_{\varepsilon\beta}G^\beta_{\lambda\alpha}), \tag{3}$$

with the third rank tensor

$$G^\alpha_{\varepsilon\lambda} \equiv \frac{1}{2}g^{\alpha\beta}(G_\varepsilon g_{\beta\lambda} + D_\lambda g_{\beta\varepsilon} - D_\beta g_{\varepsilon\lambda}), \tag{4}$$

[1]Throughout this paper the units system with $c = \hbar = G = 1$ is used.

and D_α is covariant derivative in the Minkowsky space[2]; in Galileian coordinates $D_\alpha = \partial_\alpha$ and $G^\alpha_{\varepsilon\lambda} = \Gamma^\alpha_{\varepsilon\lambda}$. Metric energy-momentum tensor density $\tau^{\varepsilon\lambda} \equiv -2(\delta L_g/\delta g_{\varepsilon\lambda})$ of massive gravitational field in such a situation is determined by the expression

$$\begin{aligned} \tau^{\varepsilon\lambda} \;\; &\equiv \;\; -\frac{1}{8\pi\sqrt{-g}}\Big[\sqrt{-g}\tilde{R}^{\varepsilon\lambda} - \frac{1}{2}\tilde{g}^{\varepsilon\lambda}\tilde{R} + \frac{1}{2}\mu^2\Big(\sqrt{-g}\tilde{g}^{\varepsilon\lambda} + \\ &+ \;\; \tilde{g}^{\varepsilon\alpha}\tilde{g}^{\lambda\beta}\gamma_{\alpha\beta} - \frac{1}{2}\tilde{g}^{\varepsilon\lambda}\tilde{g}^{\alpha\beta}\gamma_{\alpha\beta}\Big)\Big]. \end{aligned} \tag{5}$$

Here $\tilde{R}^{\varepsilon\lambda} \equiv \sqrt{-g}R^{\varepsilon\lambda}$, $\tilde{R} \equiv \sqrt{-g}R$, $R \equiv R_{\alpha\beta}g^{\alpha\beta}$, and $R_{\alpha\beta}$ is Ricci tensor. Therewith the metric energy-momentum tensor of the field in the Minkowsky space $t_g^{\varepsilon\lambda} \equiv -2(\delta L_g/\delta\gamma_{\varepsilon\lambda})$ is equal to

$$t_g^{\varepsilon\lambda} \equiv 2\sqrt{-\gamma}\Big(\gamma^{\varepsilon\alpha}\gamma^{\lambda\beta} - \frac{1}{2}\gamma^{\varepsilon\lambda}\gamma^{\alpha\beta}\Big)\frac{\delta L_g}{\delta\tilde{g}^{\alpha\beta}} - \frac{1}{16\pi}\Big[J^{\varepsilon\lambda} - \mu^2(\tilde{g}^{\varepsilon\lambda} - \tilde{\gamma}^{\varepsilon\lambda})\Big], \tag{6}$$

where $J^{\varepsilon\lambda} \equiv D_\alpha D_\beta(\gamma^{\alpha\varepsilon}\tilde{g}^{\beta\lambda} + \gamma^{\alpha\lambda}\tilde{g}^{\beta\varepsilon} - \gamma^{\varepsilon\lambda}\tilde{g}^{\alpha\beta} - \gamma^{\alpha\beta}\tilde{g}^{\varepsilon\lambda})$, $\tilde{\gamma}^{\varepsilon\lambda} \equiv \sqrt{-\gamma}\gamma^{\varepsilon\lambda}$. Density $\tau^{\varepsilon\lambda}$ vanishes outside a source or for a free filed, since variational derivative $\delta L_g/\delta g_{\varepsilon\lambda}$ dictates dynamic equations for the field at once ($\delta L_g/\delta g_{\varepsilon\lambda} = 0$). With a source at hand field equations appear as

$$\tau^{\varepsilon\lambda} + T^{\varepsilon\lambda} = 0, \tag{7}$$

where $T^{\varepsilon\lambda} \equiv -2(\delta L_M/\delta g_{\varepsilon\lambda})$ is the energy-momentum tensor density of a substance[3]. It is up to the point to note that in the absence of μ-term equation (7) comes to the ordinary Hilbert-Einstein equation for massless field. Define (just as in [1,3]) the gravitational potentials in the Minkowsky space as deviations

$$\tilde{\Phi}^{\varepsilon\lambda} \equiv \sqrt{-\gamma}\Phi^{\varepsilon\lambda} \equiv \tilde{g}^{\varepsilon\lambda} - \tilde{\gamma}^{\varepsilon\lambda}. \tag{8}$$

Then by virtue of dynamic equation of a substance

$$\nabla_\lambda T^{\varepsilon\lambda} = 0, \tag{9}$$

where ∇_λ is the covariant derivative in the Riemannian space, from (7) follows the field condition:

$$D_\lambda\tilde{g}^{\varepsilon\lambda} = D_\lambda\tilde{\Phi}^{\varepsilon\lambda} = 0. \tag{10}$$

[2]Depicted structure of L_g^0 is generally covariant. Authors of [3] did not find that generally covariant formulation of theory with μ-term, allowing for the choice of coordinates together with the Minkowsky metric by an arbitrary way, was possible; because of this in [3] the theory was developed on the base of diagonal metric $\gamma_{\alpha\beta}$ (in Galileian coordinates). Moreover, the absence of general covariance of the theory was outlined by them as a necessity afforded by the introduction (on the strength of μ-term entering) of the Minkowky metric. About generally covariant approach to the construction of the theory with μ-term see in Sec.2.

[3]We will name all the forms of matter except for the gravitational field as "substance" for the sake of convenience.

When it is considered that for field equations $\delta L_g/\delta\tilde{g}^{\alpha\beta}=0$, i.e. outside a source identity (6) becomes faithful to (7) — without $T^{\varepsilon\lambda}$ equation —

$$J^{\varepsilon\lambda}+\mu^2\tilde{\Phi}^{\varepsilon\lambda}=16\pi t_g^{\varepsilon\lambda}, \tag{11}$$

and with account of equality $D_\lambda t_g^{\varepsilon\lambda}=0$, thus (10) can be also easily derived, because $D_\lambda J_g^{\varepsilon\lambda}\equiv=0$. By reference [4,5] one can verify that field condition (10) excludes the states corresponding to spin 1 and $0'$ from gravitational field and transforms it to scalar-tensor mixture with spin states 2 and 0.

Now let us make recourse to the universally adopted procedure and expand (5) in powers of field $\Phi^{\varepsilon\lambda}$ correct to the second order inclusive. Then we obtain

$$\begin{aligned}
\tau^{\varepsilon\lambda} &\backsimeq \frac{\sqrt{-\gamma}}{16\pi}\Big\{-\gamma^{\alpha\beta}D_\alpha D_\beta\Phi^{\varepsilon\lambda}-\mu^2\Phi^{\varepsilon\lambda}+\\
&+\frac{1}{2}(\gamma^{\varepsilon\alpha}\gamma^{\lambda\beta}-\frac{1}{2}\gamma^{\varepsilon\lambda}\gamma^{\alpha\beta})(D_\alpha\Phi^\nu_\tau D_\beta\Phi^\tau_\nu-\frac{1}{2}D_\alpha\Phi D_\beta\Phi)-\\
&-\mu^2[\Phi^\varepsilon_\alpha\Phi^{\lambda\alpha}-\frac{1}{4}\gamma^{\varepsilon\lambda}(\Phi^\alpha_\beta\Phi^\beta_\alpha-\frac{1}{2}\Phi\Phi)]-\\
&-\gamma^{\varepsilon\beta}D_\alpha\Phi^{\lambda\nu}D_\beta\Phi^\alpha_\nu-\gamma^{\lambda\alpha}D_\alpha\Phi^{\beta\nu}D_\beta\Phi^\varepsilon_\nu+\gamma^{\alpha\beta}D_\alpha\Phi^\varepsilon_\nu D_\beta\Phi^{\lambda\nu}+\\
&+\frac{1}{2}\gamma^{\varepsilon\lambda}D_\alpha\Phi^\beta_\nu D_\beta\Phi^{\alpha\nu}+D_\alpha\Phi^{\varepsilon\beta}D_\beta\Phi^{\lambda\alpha}-\Phi^{\alpha\beta}D_\alpha D_\beta\Phi^{\varepsilon\lambda}\Big\},
\end{aligned} \tag{12}$$

where $\Phi\equiv\Phi^\alpha_\alpha\equiv\Phi^{\alpha\beta}\gamma_{\alpha\beta}$, and indexes of $\Phi^{\varepsilon\lambda}$ are raised and lowered by metric tensor $\gamma_{\alpha\beta}$. If one write tensor density $t_g^{\varepsilon\lambda}$ in Minkovsky space, so it will turn out to be identical to quadratic function in (12). This causes us to consider that, in the proper approximation, the energy-momentum tensor density of massive gravitational *field as a whole* is determined in the Minkowsky space by quadratic function from (12), which will be symbolized as $t_2^{\varepsilon\lambda}$; hence

$$\tau^{\varepsilon\lambda}\equiv\frac{\sqrt{-\gamma}}{16\pi}(-\gamma^{\alpha\beta}D_\alpha D_\beta\Phi^{\varepsilon\lambda}-\mu^2\Phi^{\varepsilon\lambda})+t_2^{\varepsilon\lambda}. \tag{13}$$

Now one can easily see that, when (10) is allowed for equation (11) is faithful to $\tau^{\varepsilon\lambda}=0$, that is, further analysis may start from any of them.

We shall use this fact when covering commonly employed method for obtaining energy-momentum tensor density of radiation field. The first and the basic step on the road to its construction is the representation of potentials $\Phi^{\varepsilon\lambda}$ as the sum $\Phi^{\varepsilon\lambda}=\overset{(0)}{\Phi^{\varepsilon\lambda}}+\overset{(1)}{\Phi^{\varepsilon\lambda}}$ in which the major term is governed by zeroth-order equation

$$\Box\overset{(0)}{\Phi^{\varepsilon\lambda}}+\mu^2\overset{(0)}{\Phi^{\varepsilon\lambda}}=16\pi\overset{(0)}{T^{\varepsilon\lambda}}, \tag{14}$$

and the correction one $(\overset{(1)}{\Phi^{\varepsilon\lambda}})$ — by the next order equation, whose right side contains quadratic in field terms replacing $\Phi^{\varepsilon\lambda}$ by $\overset{(0)}{\Phi^{\varepsilon\lambda}}$ in them. Thus, outside a source linear part of $\tau^{\varepsilon\lambda}$ connected with radiation field becomes zero and the density $t^{\varepsilon\lambda}_{rad}$ identify with the density $t^{\varepsilon\lambda}_2$, constructed from solutions on null approximation equation.

The source will be considered to be quasistationary. Then instead of $t^{\varepsilon\lambda}_{rad}$ one can treat its time-average value $\bar{t}^{\varepsilon\lambda}_{rad}$ (in practically such a manner it was always done). When doing so, terms from $t^{\varepsilon\lambda}_{rad}$, which can be reduced (with the taken accuracy) to the form of 4-divergence with the help of field condition (10) or by some other way (for instance, $D_\alpha \overset{(0)}{\Phi}{}^{\varepsilon\beta} D_\beta \overset{(0)}{\Phi}{}^{\lambda\alpha} \simeq D_\alpha(\overset{(0)}{\Phi}{}^{\varepsilon\beta} D_\beta \overset{(0)}{\Phi}{}^{\lambda\alpha})$ and the like), do not contribute to $\bar{t}^{\varepsilon\lambda}_{rad}$. Therefore they can be dropped out in $t^{\varepsilon\lambda}_{rad}$. Besides, one can remove from $t^{\varepsilon\lambda}_{rad}$ a number of other terms applying (14). Cite an example

$$\gamma^{\alpha\beta} D_\alpha \overset{(0)}{\Phi}{}^{\varepsilon}_{\nu} D_\beta \overset{(0)}{\Phi}{}^{\lambda\nu} - \mu^2 \overset{(0)}{\Phi}{}^{\varepsilon}_{\nu} \overset{(0)}{\Phi}{}^{\lambda\nu} = D_\alpha(\gamma^{\alpha\beta} \overset{(0)}{\Phi}{}^{\varepsilon}{}_{\nu} D_\beta \overset{(0)}{\Phi}{}^{\lambda\nu}) - \overset{(0)}{\Phi}{}^{\varepsilon}_{\nu} (\gamma^{\alpha\beta} D_\alpha D_\beta \overset{(0)}{\Phi}{}^{\lambda\nu} + \mu^2 \overset{(0)}{\Phi}{}^{\lambda\nu}).$$

In the end we have arrived at

$$t^{\varepsilon\lambda}_{rad} = \frac{\sqrt{-\gamma}}{32\pi} \gamma^{\varepsilon\alpha}\gamma^{\lambda\beta} \Big(D_\alpha \overset{(0)}{\Phi}{}^{\nu}_{\tau} D_\beta \overset{(0)}{\Phi}{}^{\tau}_{\nu} - \frac{1}{2} D_\alpha \overset{(0)}{\Phi} D_\beta \overset{(0)}{\Phi} \Big). \tag{15}$$

By this means standard methods tend to the next expression for radiation energy flux (in Galileian coordinates)

$$\overset{(0)}{I} = -\frac{1}{32\pi} \oint_{s\to\infty} \Big[\partial_o \overset{(0)}{\Phi}{}^{\alpha}_{\beta} \partial_k \overset{(0)}{\Phi}{}^{\beta}_{\alpha} - \frac{1}{2} \partial_o \overset{(0)}{\Phi} \partial_k \overset{(0)}{\Phi} \Big] d\sigma^k, \tag{16}$$

which is identical in form to that of massless gravitational field.[4]

Represent solution $\overset{(0)}{\Phi}{}^{\varepsilon\lambda}$ away from source as Fourier series

$$\overset{(0)}{\Phi}{}^{\varepsilon\lambda} \simeq \frac{1}{r} \sum_{\omega} a^{\varepsilon\lambda}_{\omega}(-i\omega t + i\vec{k}\vec{r}) \equiv \sum_{\omega} \overset{(0)}{\Phi}{}^{\varepsilon\lambda}_{\omega}, \tag{17}$$

in which $\vec{k} \equiv \vec{n}\eta\omega$, $\eta \equiv [1 - (\mu/\omega)^2]^{1/2}$, $\vec{n} \equiv \vec{r}/r$, $\omega \equiv \nu\omega_o$ and $\omega_o \geqslant \mu$. Then we can turn our attention to the spectral-angular distibution of radiation

$$\frac{d\overset{(0)}{I}}{d\Omega} = \frac{1}{16\pi} \sum_{\nu=1}^{\infty} \omega^2 \eta \Big\{ \overset{*}{a}{}^{\varepsilon}_{\lambda}(\vec{k}) a^{\lambda}_{\varepsilon}(\vec{k}) - \frac{1}{2} \overset{*}{a}(\vec{k}) a(\vec{k}) \Big\}. \tag{18}$$

Taking into consideration (with a proper accuracy) equalities

$$a^0_0 = -\eta^2 a^3_3, \quad a^0_k = -a^k_0 = \eta a^3_k, \tag{19}$$

resulting from (10), with index 3 is associated to momentum $\vec{k}$ direction, we come to the expression:

[4] Formula (15) and thus (16) may be derived from time-averaged expression for pseudotensor of radiation field appearing in [6].

$$
\begin{aligned}
\frac{d\overset{(0)}{I}}{d\Omega} &= \frac{1}{8\pi}\sum_{\nu=1}^{\infty}\omega^2\eta\Big\{|a_2^1|^2+\frac{1}{4}|a_1^1-a_2^2|^2+\frac{\mu^2}{\omega^2}(|a_3^1|^2+|a_3^2|^2)+ \\
&+ \frac{\mu^4}{4\omega^4}|a_3^3|^2-\frac{\mu^2}{4\omega^2}\Big[(\overset{*}{a_1^1}+\overset{*}{a_2^2})a_3^3+(a_1^1+a_2^2)\,\overset{*}{a_3^3}\Big]\Big\}. \qquad (20)
\end{aligned}
$$

Negative contribution to the energy flux obtained by generally accepted methods is thus seen truly to appear and it is to graviton mass (spectral-angular distribution rearranges to the familiar form when $\mu = 0$ (see [6], for instance) and becames positively defined). Specifically, for spherically symmetric source only $a_1^1 = a_2^2 = a_3^3$ differs from zero and

$$
\frac{d\overset{(0)}{I}}{d\Omega} = -\frac{\mu^2}{8\pi}\sum_{\nu=1}^{\infty}\eta\Big(1-\frac{\mu^2}{4\omega^2}\Big)|a_3^3|^2.
$$

To clear up what contribution to $d\overset{(0)}{I}/d\Omega$ various spin states of the field make, let us fall back on the results of [4,5] and expand $a^{\varepsilon\lambda}$ in irreducible representation corresponsing to spin 2 and 0. Having regard the form of projection operators in momentum space

$$
(P_2)^{\varepsilon\lambda}_{\alpha\beta} \equiv \frac{3}{2}(Q^{\varepsilon}_{\alpha}Q^{\lambda}_{\beta}+Q^{\varepsilon}_{\beta}Q^{\lambda}_{\alpha})-Q_{\alpha\beta}Q^{\varepsilon\lambda},
$$

$$
(P_o)^{\varepsilon\lambda}_{\alpha\beta} \equiv Q_{\alpha\beta}Q^{\varepsilon\lambda},
$$

$$
Q^{\varepsilon\lambda} \equiv \frac{1}{\sqrt{3}}\Big(\gamma^{\varepsilon\lambda}-\frac{k^\varepsilon k^\lambda}{k^2}\Big),
$$

we arrive at

$$
c^{\varepsilon\lambda} \equiv \frac{1}{3}\Big(\gamma^{\varepsilon\lambda}-\frac{k^\varepsilon k^\lambda}{k^2}\Big)a, \quad c\equiv a, \quad b^{\varepsilon\lambda}\equiv a^{\varepsilon\lambda}-c^{\varepsilon\lambda}, \qquad (21)
$$

wherein $b^{\varepsilon\lambda}$ refers to spin 2 states, and $c^{\varepsilon\lambda}$ — to zero spin states. Independent amplitudes $b_2^1 = a_2^1$, $(b_1^1 - b_2^2) = (a_1^1 - a_2^2)$, $b_3^1 = a_3^1$, $b_3^2 = a_3^2$ and $b_3^3 = a_3^3 - (\omega^2/3k^2)a$ accord with two transverse-transverse (related to projection $s_3 = \pm 2$ of spin 2), two transverse-longitudinal (related to $s_3 = \pm 1$) and one longitudinal-longitudinal (related to $s_3 = 0$) states, and amplitude $c = a$ is due to a scalar component ($s = 0$). Thus we deduce instead of (20)

$$
\frac{d\overset{0}{I}}{d\Omega} = \frac{1}{8\pi}\sum_{\nu=1}^{\infty}\omega^2\eta\Big\{|b_2^1|^2+\frac{1}{4}|b_1^1-b_2^2|^2+\frac{\mu^2}{\omega^2}(|b_3^1|^2+|b_3^2|^2)+\frac{3\mu^4}{4\omega^4}|b_3^3|^2-\frac{1}{12}|c|^2\Big\}. \qquad (22)
$$

One can see that the negative contribution to radiation energy flux is given (as it was intimated in [1]) by the scalar component only.

Now that we gave a full picture of commonly used calculations, we focus upon some incorrectness of ones together with the introduction of the concept of energy-momentum tensor density of gravitational radiation field. Obviously it can appear at the outset only, when developing (16), since all other things are merely outcomes of result (16). In turn this result is by itself a consequence of the successive approximations method application. It is just what contains incorrectness. Actually, the obedience of potentials $\Phi^{\varepsilon\lambda}$ in zeroth approximation to linear equation (14) implies from the physical viewpoint that space, in which associated with nonstatic field gtavitons move, is identified with the Minkowsky space. This is clearly demonstrated by the fact that by virtue of (16) each Fourier component of the field outside the source must obey linear equation

$$\Box \overset{(0)}{\Phi^{\varepsilon\lambda}} + \mu^2 \overset{(0)}{\Phi^{\varepsilon\lambda}} = 0, \tag{23}$$

leads to connection $\gamma_{\alpha\beta}k^\alpha k^\beta = \mu^2$. However, this is completely unacceptable, because in reality gravitons move in the Riemann space induced by both field $\overset{(1)}{\Phi^{\varepsilon\lambda}}$ and their own field, i.e. momentum k^α must be actually governed by connection $g_{\alpha\beta}k^\alpha k^\beta = \mu^2$. The method applied above ignores this fact and so tend to incorrect results. In Sec.3 the other approach, taking into account that gravitons propagate in space with the Riemann metric, will be developed. This will turn out to eliminate the negative investment to flux and will make it positively defined. Thereby objections [1] against a possibility to of construct a consistent theory of gravitation with nonzero graviton mass will be removed. The fundamental tenet of it are present below.

2. Basic Equations of Theory of Gravitation with Nonzero Graviton Mass

As it was already noted above, the introduction of graviton mass, if it exists in nature, into the theory of gravitation implies (see [1,3]) the necessity of entering some supplementary metric along with the Riemannian one. For a number of reasons (a high degree of agreement of experimental data with theoretical predictions in electrodynamics, based on the concept of Minkowsky space; numerous observations testifying to the Euclidean character of three-dimensional space, etc) it is best seemed to identify this metric with Minkowsky space metric. It that case it is taken to be of fundamental significance, and metric $g_{\alpha\beta}$ was turned out to be secondary, induced by physical gravitational field. Postulating that the Minkowsky metric is basic, one assures an existence of independent laws of conservation for energy, momentum, and angular momentum of isolated system, because the Minkowsky space admits the 10-parametrical Poincare group.

Although, dynamical processes are now more naturally viewed as the ones proceeding in the Minkowsky space under the influence of gravitational fields, motion of a substance (see remark[3]) by the action of the field can be considered as its free motion in the Riemannian space induced by this field. This means that if the Lagrangian density of substance L_M taking in Minkowsky space (when gravitational fields are ignored) contains metric $\gamma_{\alpha\beta}$, then L_M must contain a metric coefficient of the Riemannian space $g_{\alpha\beta}$ instead of $\gamma_{\alpha\beta}$ when gravitational field is accounted for, i.e. [7-10]

$$L_M(\tilde{\gamma}^{\alpha\beta}(x),\ A^{(\alpha)}_{(\beta)}(x),\ D_\lambda A^{(\alpha)}_{(\beta)}(x)) \rightarrow L_M(\tilde{g}^{\alpha\beta}(x),\ A^{(\alpha)}_{(\beta)}(x),\ \nabla_\lambda A^{(\alpha)}_{(\beta)}(x)). \tag{24}$$

Here $A^{(\alpha)}_{(\beta)}$ is tensor (type of $\binom{p}{q}$) of substantial fields with $(\alpha) \equiv \alpha_1, \alpha_2, ..., \alpha_p$, $(\beta) \equiv \beta_1, \beta_2, ..., \beta_q$, and density $\tilde{\Phi}^{\alpha\beta}(x)$ of gravitational field in the Minkowsky space equates (by analogy with [1,3]) to difference $\tilde{g}^{\alpha\beta} - \tilde{\gamma}^{\alpha\beta}$ — see (8). L_M is seen to be fully geometrized (in the space with metric $g_{\alpha\beta}$) density, in that it does not explicity incorporate metric $\gamma_{\alpha\beta}$. In [7-10] accordance (24) has been called geometrization principle.

As was done in [7-10], insert the group of infinitesimal gauge fields transformations

$$\delta_\varepsilon \tilde{g}^{\alpha\beta} \equiv \delta_\varepsilon \tilde{\Phi}^{\alpha\beta} \equiv \tilde{g}^{\alpha\lambda} D_\lambda \varepsilon^\beta + \tilde{g}^{\beta\lambda} D_\lambda \varepsilon^\alpha - D_\lambda(\varepsilon^\lambda \tilde{g}^{\alpha\beta}),$$

$$\delta_\varepsilon A^{(\alpha)}_{(\beta)} \equiv F^{(\alpha)(\sigma)\nu}_{(\beta)(\tau)\mu} A^{(\tau)}_{(\sigma)} D_\nu \varepsilon^\mu - \varepsilon^\lambda D_\lambda A^{(\alpha)}_{(\beta)},$$

where $\varepsilon^\alpha(x)$ is group parameter, structural constants

$$F^{(\alpha)(\sigma)\nu}_{(\beta)(\tau)\mu} \equiv \sum_{m=1}^{p} \delta^{\sigma_1}_{\beta_1} \cdot \ \cdots \ \cdot \delta^{\sigma_q}_{\beta_q} \cdot \delta^{\alpha_1}_{\tau_1} \cdot \ \cdots \ \cdot \delta^{\alpha_m=\nu}_{\tau_m} \cdot \ \cdots \ \cdot \delta^{\alpha_p}_{\tau_p} \cdot \delta^{\alpha_m}_{\mu} -$$

$$- \sum_{m=1}^{q} \delta^{\alpha_1}_{\tau_1} \cdot \ \cdots \ \cdot \delta^{\alpha_p}_{\tau_p} \cdot \delta^{\sigma_1}_{\beta_1} \cdot \ \cdots \ \cdot \delta^{\sigma_m}_{\beta_m=\mu} \cdot \ \cdots \ \cdot \delta^{\sigma_q}_{\beta_q} \cdot \delta^{\nu}_{\beta_m},$$

and operators δ_ε form the Lie algebra and satisfy Jacobi identities. Then one can ascertain that scalar density L_M varies over a divergent term only under this transformation. It would be reasonable to demand that the Lagrangian density of gravitational field must also change over nothing but divergence. This requirement has been formulated in [7-10] and called a gauge principle. The only (the prove of uniqueness see in [7]) scalar under general mapping transormation density L^0_g complying with the minimality principle and the gauge principle is found in structure (3). Hence, the form of L^0_g suggested in [3] got not only covariant extension, but the justification for the solely possible structure as well. Both of them are of principal importance in theory. If physical gravitational field identify with mixture[5] corresponding to spin states 2 and 0, then the solely possible structure of mass term (also scalar under general mapping transformation) having entered L^0_g will be (2), since only it will permit field condition (10), excluding states with spin 1 and $0'$, to arise. Besides, it also assures the identical vanishing of full density L_g in the absence of field, which is physically essential. In this way form (2) suggested for the first time in [3] also gets a justification.

The introduction of mass term into L_g breaks the gauge symmetry: as (10) is obeyed mass term over divergence on but subset of parameters $\varepsilon^\lambda(x)$ determined by the equation

$$g^{\alpha\beta} D_\alpha D_\beta \varepsilon^\lambda = 0.$$

Analogous breaking of gauge symmetry takes place in electrodynamics with nonzero photon mass.

[5] Incorporation of spin states 1 and $0'$ as well as one of them tend to physically unacceptable consequences, like the absence of passage to the limit in solutions of equations for field potentials, when $\mu \to 0$. In particular, one can see it in [1].

In sum we arrive to unambiguosly defined expression for Lagrangian density of both substance and gravitational field [7-10]:

$$L \equiv L_M + \frac{1}{16\pi}\tilde{g}^{\varepsilon\lambda}(G^{\alpha}_{\varepsilon\lambda}G^{\beta}_{\alpha\beta} - G^{\alpha}_{\varepsilon\beta}G^{\beta}_{\lambda\alpha}) - \frac{\mu^2}{16\pi}\Big(\frac{1}{2}\tilde{g}^{\alpha\beta}\gamma_{\alpha\beta} - \sqrt{-g} - \sqrt{-\gamma}\Big). \quad (25)$$

Thus equations system for substance and gravitational field assumes the form

$$\frac{\delta L}{\delta A^{(\varepsilon)}_{(\lambda)}} = 0, \quad \frac{\delta L}{\delta \tilde{g}^{\varepsilon\lambda}} = 0. \quad (26)$$

Along with the equations for substance relation (9) must hold true. If the number of independent equations for substance in (26) is found to be equal to four, they can be totally substitued for (9) or field condition (10), as (9) and (10) are interchangeable. Such a situation is realized when a substance can be described by velocities of its elements u^k, density ρ and pressure p in full. Exclusively in such an event, system of equation (26) can be recast into the form (see also [3,7-10])

$$R^{\varepsilon\lambda} - \frac{1}{2}g^{\varepsilon\lambda}R + \frac{1}{2}\mu^2\Big[g^{\varepsilon\lambda} + (g^{\varepsilon\alpha}g^{\lambda\beta} - \frac{1}{2}g^{\varepsilon\lambda}g^{\alpha\beta})\gamma_{\alpha\beta}\Big] = \frac{8\pi}{\sqrt{-g}}T^{\varepsilon\lambda},$$

$$D_\lambda \tilde{g}^{\varepsilon\lambda} = 0. \quad (27)$$

Being supplemented by the state equation of substance $p = p(\rho)$, (27) gives us the complete system of equations enabling the determination of all fifteenth matter characteristics (u^k, ρ, p and $\tilde{g}^{\varepsilon\lambda}$).

Notice that dynamic (upper in (27)) equations of field become degenerate in the Minkowsky space metric $\gamma_{\alpha\beta}$ as $\mu \to 0$. It is common knowledge, that the removal of degeneration can lead to qualitatively new physical effects, which are latent in degenerate equations. Because of this, even if graviton mass is exactly equal to zero, it would be well to amplify equations by μ-term, lifting their degeneration in $\gamma_{\alpha\beta}$, and to make calculation just after that with μ being direct to zero at the last stage only (see [11]). In particular in [12] it has been shown that such degeneration removal separates singularities in metric coefficients g^{-1}_{00} and g_{11} (at dt^2 and dr^2) in static spherically symmetric problem, resulting in impossibility of falling particles to penetrate under the Schwarzschild sphere.

Now let us make sure that the obtained system of equations does not bring to contradiction, i.e. that flux density of massive graviton radiation is positively defined.

3. Flux Density of Massive Graviton Radiation and its Positive Definiteness

By way of identical rearrangement dynamic equations (27) can be put in the form

$$\tilde{\gamma}^{\alpha\beta}D_\alpha D_\beta\tilde{\Phi}^{\varepsilon\lambda}+\mu^2\sqrt{-\gamma}\tilde{\Phi}^{\varepsilon\lambda}=16\pi\sqrt{-g}(T^{\varepsilon\lambda}+t^{\varepsilon\lambda}), \tag{28}$$

with

$$\begin{gathered}16\pi\sqrt{-g}t^{\varepsilon\lambda}\equiv\frac{1}{2}\Big(\tilde{g}^{\varepsilon\alpha}\tilde{g}^{\lambda\beta}-\frac{1}{2}\tilde{g}^{\varepsilon\lambda}\tilde{g}^{\alpha\beta}\Big)\Big(\tilde{g}_{\nu\sigma}\tilde{g}_{\tau k}-\frac{1}{2}\tilde{g}_{\tau\sigma}\tilde{g}_{\nu k}\Big)D_\alpha\tilde{\Phi}^{\tau\sigma}D_\beta\tilde{\Phi}^{\nu k}-\\ -\mu^2\Big[\sqrt{-g}\tilde{g}^{\varepsilon\lambda}-\sqrt{-\gamma}\tilde{\Phi}^{\varepsilon\lambda}+\Big(\tilde{g}^{\varepsilon\alpha}\tilde{g}^{\lambda\beta}-\frac{1}{2}\tilde{g}^{\varepsilon\lambda}\tilde{g}^{\alpha\beta}\Big)\gamma_{\alpha\beta}\Big]-\tilde{g}^{\varepsilon\beta}\tilde{g}_{\tau\sigma}D_\alpha\tilde{\Phi}^{\lambda\sigma}D_\beta\tilde{\Phi}^{\alpha\tau}-\\ -\tilde{g}^{\lambda\alpha}\tilde{g}_{\tau\sigma}D_\alpha\tilde{\Phi}^{\beta\sigma}D_\beta\tilde{\Phi}^{\varepsilon\tau}+\tilde{g}^{\alpha\beta}\tilde{g}_{\tau\sigma}D_\alpha\tilde{\Phi}^{\varepsilon\tau}D_\beta\tilde{\Phi}^{\lambda\sigma}+\frac{1}{2}\tilde{g}^{\varepsilon\lambda}\tilde{g}_{\tau\sigma}D_\alpha\tilde{\Phi}^{\sigma\beta}D_\beta\tilde{\Phi}^{\alpha\tau}+\\ +D_\alpha\tilde{\Phi}^{\varepsilon\beta}D_\beta\tilde{\Phi}^{\lambda\alpha}-\tilde{\Phi}^{\alpha\beta}D_\alpha D_\beta\tilde{\Phi}^{\varepsilon\lambda},\end{gathered}$$

and $\tilde{g}_{\varepsilon\lambda}\equiv g_{\varepsilon\lambda}/\sqrt{-g}$. If source $T^{\varepsilon\lambda}$ in (28) loses energy by graviton emission, then fields $\Phi^{\varepsilon\lambda}$ in this equations are the sum of radiation potentials $\psi^{\varepsilon\lambda}$ and potentials $\chi^{\varepsilon\lambda}$ of rest part of fields, not contributing to radiation flux, but making up some background with a "ripple". To the second order in field, which is quite sufficient here, $\sqrt{-g}t^{\varepsilon\lambda}$ in (28) reduces to $\sqrt{-\gamma}t_2^{\varepsilon\lambda}$ determined by quadratic function from (12), and equations for $\chi^{\varepsilon\lambda}$ (see also [13,14]) become

$$\tilde{\gamma}^{\alpha\beta}D_\alpha D_\beta\tilde{\chi}^{\varepsilon\lambda}+\mu^2\sqrt{-\gamma}\tilde{\chi}^{\varepsilon\lambda}=16\pi\Big[\sqrt{-g}T^{\varepsilon\lambda}+\sqrt{-\gamma}(\tau_1^{\varepsilon\lambda}+\tau_2^{\varepsilon\lambda})\Big]. \tag{29}$$

Here

$$\begin{aligned}\tau_1^{\varepsilon\lambda}\ \equiv\ &\frac{\sqrt{-\gamma}}{16\pi}\Big\{-\gamma^{\alpha\beta}D_\alpha D_\beta\psi^{\varepsilon\lambda}-\mu^2\psi^{\varepsilon\lambda}+\frac{1}{2}\Big(\gamma^{\varepsilon\alpha}\gamma^{\lambda\beta}-\frac{1}{2}\gamma^{\varepsilon\lambda}\gamma^{\alpha\beta}\Big)\times\\ \times\ &\Big(D_\alpha\psi^\nu_\tau D_\beta\psi^\tau_\nu-\frac{1}{2}D_\alpha\psi D_\beta\psi\Big)-\mu^2\Big[\psi^\varepsilon_\alpha\psi^{\lambda\alpha}-\frac{1}{4}\gamma^{\varepsilon\lambda}(\psi^\alpha_\beta\psi^\beta_\alpha-\\ -\ &\frac{1}{2}\psi\psi)\Big]-\gamma^{\varepsilon\beta}D_\alpha\psi^{\lambda\nu}D_\beta\psi^\alpha_\nu-\gamma^{\lambda\alpha}D_\alpha\psi^{\beta\nu}D_\beta\psi^\varepsilon_\nu+\gamma^{\alpha\beta}D_\alpha\psi^\varepsilon_\nu D_\beta\psi^{\lambda\nu}+\\ +\ &\frac{1}{2}\gamma^{\varepsilon\lambda}D_\alpha\psi^\beta_\nu D_\beta\psi^{\alpha\nu}+D_\alpha\psi^{\varepsilon\beta}D_\beta\psi^{\lambda\alpha}-\psi^{\alpha\beta}D_\alpha D_\beta\psi^{\varepsilon\lambda}\Big\},\end{aligned} \tag{30}$$

and all the terms of $\tau_2^{\varepsilon\lambda}$ are certain to include $\chi^{\alpha\beta}$.

To find way of account for the fact that gravitons move in space with the Riemannian metric rather than the Minkowsky one, we consider initially asymptotics of radiation potentials away from a source. With the existence of basic metric $\gamma_{\alpha\beta}$ all gravitational fields are treated as ones defined in space with this metric. Hence asymptotics of $\psi^{\varepsilon\lambda}$ must be written as follows:

$$\psi^{\varepsilon\lambda}_{as}\simeq\frac{1}{r}\sum_\omega a^{\varepsilon\lambda}_\omega exp\Big\{-i\omega(t-\frac{r}{v})\Big\}\equiv\frac{1}{r}\sum_\omega a^{\varepsilon\lambda}_\omega exp(-i\gamma_{\alpha\beta}k^\alpha x^\beta)\equiv\sum_\omega\psi^{\varepsilon\lambda}_\omega, \tag{31}$$

where $a^{\varepsilon\lambda}_\omega$ are amplitudes of partial waves $\psi^{\varepsilon\lambda}_\omega$ and k^α are 4-momentums of gravitons. Considering (31) and keeping only major terms when taking derivatives D_α we result from linear part of (30):

$$-\gamma^{\alpha\beta}D_\alpha D_\beta\psi^{\varepsilon\lambda}_{as}-\mu^2\psi^{\varepsilon\lambda}_{as}\simeq\sum_\omega(\gamma_{\alpha\beta}k^\alpha k^\beta-\mu^2)\psi^{\varepsilon\lambda}_\omega. \tag{32}$$

Since gravitons momenta k^α obey the equation

$$g_{\alpha\beta}k^\alpha k^\beta = \mu^2, \tag{33}$$

(32) must be equal to

$$\sum_\omega(\gamma_{\alpha\beta}k^\alpha k^\beta - \mu^2)\psi_\omega^{\varepsilon\lambda} = \sum_\omega(\gamma_{\alpha\beta} - g_{\alpha\beta})k^\alpha k^\beta\psi_\omega^{\varepsilon\lambda} \simeq$$

$$\simeq \sum_\omega(\Phi_{\alpha\beta} - \frac{1}{2}\Phi\gamma_{\alpha\beta})k^\alpha k^\beta\psi_\omega^{\varepsilon\lambda} \simeq -\frac{\mu^2}{2}(\psi_{as} + \chi)\psi_{as}^{\varepsilon\lambda} - (\psi_{as}^{\alpha\beta} + \chi^{\alpha\beta})D_\alpha D_\beta\psi_{as}^{\varepsilon\lambda}.$$

All these facts indicate that the equation for radiation potentials $\psi^{\varepsilon\lambda}$, taken in Minkowsky space and determined by nonstatic part $T_1^{\varepsilon\lambda}$ of source $T^{\varepsilon\lambda}$, must be nonlinear and have the following form

$$\sqrt{-\gamma}(g_{\alpha\beta}D^\alpha D^\beta\tilde{\psi}^{\varepsilon\lambda} + \mu^2\tilde{\psi}^{\varepsilon\lambda}) = 16\pi\sqrt{-g}T_1^{\varepsilon\lambda}, \tag{34}$$

where indexes of D_α and D^ε are raised and lowered with the help of metric $\gamma_{\alpha\beta}$. One can easily make sure that asymptotics (31) satisfies corresponding to (34) homogeneous equation, which do accord with the law (33) of graviton motion in Riemannian space. Taking into consideration (34) in (29), we arrive at equation

$$\tilde{\gamma}^{\alpha\beta}D_\alpha D_\beta\tilde{\chi}^{\varepsilon\lambda} + \mu^2\sqrt{-\gamma}\tilde{\chi}^{\varepsilon\lambda} = 16\pi[\sqrt{-g}T_o^{\varepsilon\lambda} + \sqrt{-\gamma}(\tau_{rad}^{\varepsilon\lambda} + \tau_3^{\varepsilon\lambda})], \tag{35}$$

wherein $T_o^{\varepsilon\lambda}$ is static part of $T^{\varepsilon\lambda}$, $\tau_3^{\varepsilon\lambda}$ is certain to contain potentials $\chi^{\alpha\beta}$, and

$$\begin{aligned}
\tau_{rad}^{\varepsilon\lambda} \equiv\ & \frac{\sqrt{-\gamma}}{16\pi}\Big\{\frac{1}{2}\Big[(\gamma^{\varepsilon\alpha}\gamma^{\lambda\beta} - \frac{1}{2}\gamma^{\varepsilon\lambda}\gamma^{\alpha\beta})(D_\alpha\psi_\tau^\nu D_\beta\psi_\nu^\tau - \\
-\ & \frac{1}{2}D_\alpha\psi D_\beta\psi) + \gamma^{\alpha\beta}\psi D_\alpha D_\beta\psi^{\varepsilon\lambda}\Big] - \mu^2\Big[\psi_\alpha^\varepsilon\psi^{\lambda\alpha} - \frac{1}{4}\gamma^{\varepsilon\lambda}\Big(\psi_\beta^\alpha\psi_\alpha^\beta - \\
-\ & \frac{1}{2}\psi\psi\Big)\Big] - \gamma^{\varepsilon\beta}D_\alpha\psi^{\lambda\nu}D_\beta\psi_\nu^\alpha - \gamma^{\lambda\alpha}D_\alpha\psi^{\beta\nu}D_\beta\psi_\nu^\varepsilon + \\
+\ & \gamma^{\alpha\beta}D_\alpha\psi_\nu^\varepsilon D_\beta\psi^{\lambda\nu} + \frac{1}{2}\gamma^{\varepsilon\lambda}D_\alpha\psi_\nu^\beta D_\beta\psi^{\alpha\nu} + D_\alpha\psi^{\varepsilon\beta}D_\beta\psi^{\lambda\alpha} - \\
-\ & 2\psi^{\alpha\beta}D_\alpha D_\beta\psi^{\varepsilon\lambda}\Big\}.
\end{aligned} \tag{36}$$

Expression (36) is just what must be recognized as energy-momentum tensor density of radiation field with nonzero graviton mass. A correction for space distortion is seen to lead to the fact that energy-momentum tensor density of one part of field does not depend on full field tensor, but contains the contribution, conditioned by distortion (or self-action of gravitons) from linear (in the Minkowsky space) combination of this part.

Show that radiation flux density given by (36) (it is precisely what is associated with energy losses of a source) is positively defined. To do this, the case where energy losses by radiation are small and source might be considered as quasistationary will be looked at. In this approximation one can take instead of $\tau_{rad}^{\varepsilon\lambda}$ its time-averaged value, which is further represented just by $\tau^{\varepsilon\lambda}_{rad}$. Taking into account that, within the used accuracy, potentials $\psi^{\varepsilon\lambda}$ must obey field condition (10), some terms in (36) can be reduced to 4-divergences. Such terms do not contribute to averaged $\tau^{\varepsilon\lambda}_{rad}$ and thus may be left out. It is necessary to use linear (with $g_{\alpha\beta} \to \gamma_{\alpha\beta}$ and $\sqrt{-g} \to \sqrt{-\gamma}$) approximation of equation (34) in a quadratic in $\psi^{\varepsilon\lambda}$ expressions, lest a conventional accuracy in field be exceeded. Outside the one source it allows to reject some other terms from (36). In the end beyond a source, $\tau^{\varepsilon\lambda}_{rad}$ takes (see in addition [13,14]) the form:

$$\tau^{\varepsilon\lambda}_{rad} = \frac{\sqrt{-\gamma}}{32\pi}\Big[\gamma^{\varepsilon\alpha}\gamma^{\lambda\beta}\Big(D_\alpha\psi^\nu_\tau D_\beta\psi^\tau_\nu - \frac{1}{2}D_\alpha\psi D_\beta\psi\Big) - \mu^2\psi\psi^{\varepsilon\lambda}\Big]. \tag{37}$$

Hence the intensity of massive graviton emission is described (in Galileian coordinates) by the formula

$$I = -\frac{1}{32\pi}\oint_{s\to\infty}\Big[\partial_0\psi^\alpha_\beta\partial_k\psi^\beta_\alpha - \frac{1}{2}\partial_0\psi\partial_k\psi - \mu^2\psi\psi_{0k}\Big]d\sigma^k, \tag{38}$$

in which $\psi^{\varepsilon\lambda}$ are governed by equation (34) linearized in field. As $r \to \infty$, its retarded solution is

$$\psi^{\varepsilon\lambda} \simeq \frac{4}{r}\sum_\omega T^{\varepsilon\lambda}_1(\vec{k})exp(-i\omega t + i\vec{k}\vec{r}), \tag{39}$$

with $\vec{k} \equiv \vec{n}\eta\omega$, $\eta \equiv [1-(\mu/\omega)^2]^{1/2}$, $\vec{n} \equiv \vec{r}/r$, $\omega \equiv \nu\omega_0$, $\omega_0 \equiv \omega_{min} \geqslant \mu$, and

$$T^{\varepsilon\lambda}_1(\vec{k}) = \int\frac{d\omega_o t}{2\pi}d^3x T^{\varepsilon\lambda}_1(\vec{r},t)exp(i\omega t - i\vec{k}\vec{r}).$$

We mention in passing that for free gravitational field (without sources) expression (37) characterizes energy-momentum tensor density of traveling waves, which must be accompanied by background $\chi^{\varepsilon\lambda}$ determined by equation (29) with $T^{\varepsilon\lambda} = 0$. By using (39) we obtain from (37) (compare with (18))

$$\begin{aligned}\frac{dI}{d\Omega} &= \frac{1}{16\pi}\sum_{\nu=1}^{\infty}\omega^2\eta\Big\{\overset{*}{a}{}^\varepsilon_\lambda(\vec{k})a^\lambda_\varepsilon(\vec{k}) - \frac{1}{2}\overset{*}{a}(\vec{k})a(\vec{k}) - \\ &- \frac{\mu^2}{2\eta\omega^2}[\overset{*}{a}(\vec{k})a^3_0(\vec{k}) + \overset{*}{a}{}^3_0(\vec{k})a(\vec{k})]\Big\},\end{aligned} \tag{40}$$

where in accordance with (39) $a^{\varepsilon\lambda}(\vec{k}) = 4T^{\varepsilon\lambda}_1(\vec{k})$, and index 3, as in Sec.1, is related to the momentum $\vec{k}$ direction. By further applying connections (11), we recast (40) in the form [13,14]

$$\begin{aligned}\frac{dI}{d\Omega} &= \frac{1}{8\pi}\sum_{\nu=1}^{\infty}\omega^2\eta\Big\{|a^1_2|^2 + \frac{1}{4}|a^1_1 - a^2_2|^2 + \frac{\mu^2}{\omega^2}(|a^1_3|^2 + |a^2_3|^2) + \\ &+ \frac{3\mu^4}{4\omega^4}|a^3_3|^3\Big\} = \frac{2}{\pi}\sum_{\nu=1}^{\infty}\omega^2\eta\Big\{|T^{12}_1|^2 + \frac{1}{4}|T^{11}_1 - T^{22}_1|^2 + \\ &+ \frac{\mu^2}{\omega^2}(|T^{13}_1|^2 + |T^{23}_1|^2) + \frac{3\mu^4}{4\omega^4}|T^{33}_1|^2\Big\}.\end{aligned} \tag{41}$$

Contrary to (20), this relation is strictly positively defined. Above discussion demonstrates that it is due to the proper account of gravitons self-action, pertaining, in its turn, to the presence of scalar component, which provides $\psi \neq 0$, in a field.

Attention is drawn to the fact that although scalar admixture connected with trace ψ (and thus a) takes part in flux creation, trace a and consequently that of the Fourier transform of nonstatic part in energy-momentum tensor density of a substance $T_1 \equiv T_1^{\alpha\beta}\gamma_{\alpha\beta}$ does not appear in the expression for full flux in an explicit form. However, if it is subdivided into independent spin states, partial investment of a scalar component will be found[6]. In fact, considering decomposition (21) in (41), we obtain

$$\begin{aligned} \frac{dI}{d\Omega} &= \frac{1}{8\pi}\sum_{\nu=1}^{\infty}\omega^2\eta\Big\{|b_2^1|^2 + \frac{1}{4}|b_1^1 - b_2^2|^2 + \frac{\mu^2}{\omega^2}(|b_3^1|^2 + |b_3^2|^2) + \\ &+ \frac{3\mu^4}{4\omega^4}|b_3^3|^2 + \frac{1}{12}|c|^2 + \frac{\mu^2}{4\omega^2}\Big(\overset{*}{b_3^3}\,c + b_3^3\,\overset{*}{c}\Big)\Big\}. \end{aligned} \tag{42}$$

It shows that proportional to $|a_3^3|^2$ term in (41) is combined from contributions of spin states with $s_3 = 0$ and $s = 0$. If phase difference between Fourier amplitudes of radiation potentials associated with them, is chaotic, the last term in (42) can be dropped.

As the definition of radiation potentials $\psi^{\varepsilon\lambda}$ is not restricted to the exterior of a source when deducing (36), one can use it to link intensity of emission and work of sources. For this purpose let us calculate 4-divergence of $\tau_{rad}^{\varepsilon\lambda}$ applying equation (34) in linear approximation. As a result we give

$$\begin{aligned} \partial_\lambda\tau_{rad}^{0\lambda} = (\sqrt{-\gamma}/2)\Big\{T_1^{\alpha\beta}\partial_0\psi_{\alpha\beta} - \frac{1}{2}T_1\partial_0\psi + \frac{1}{16\pi}\partial_\lambda(\psi\Box\psi^{0\lambda}) - \\ -2T_1^{\alpha\beta}\partial_\alpha\psi_\beta^0 - \partial_\lambda\partial_\alpha\partial_\beta(\psi^{\alpha\beta}\psi^{0\lambda})\Big\}. \end{aligned}$$

Upon integrating it over a space area we obtain a balance equation

$$\begin{aligned} \frac{\partial W}{\partial t} + I = \frac{1}{2}\int_V d^3x\Big\{T_1^{\alpha\beta}\partial_0\psi_{\alpha\beta} - \frac{1}{2}T_1\partial_0\psi + \frac{1}{16\pi}\partial_\lambda(\psi\Box\psi^{0\lambda}) - \\ -2T_1^{\alpha\beta}\partial_\alpha\psi_\beta^0 - \partial_\lambda\partial_\alpha\partial_\beta(\psi^{\alpha\beta}\psi^{0\lambda})\Big\}, \end{aligned}$$

in which

$$W \equiv \int_V d^3x\tau_{rad}^{00}, \qquad I \equiv \oint_s \tau^{0k}d\sigma^k.$$

In a case of quasistationary source, time variation of the integrals taken over a volume might be neglected. After time-averaging we arrive to

$$I = \frac{1}{2}\int_V d^3x(T_1^{\alpha\beta}\partial_0\psi_{\alpha\beta} - \frac{1}{2}T_1\partial_0\psi) + \frac{\mu^2}{32\pi}\oint_S \psi\psi_{ok}d\sigma^k.$$

Here the first integral is the work of sources, and the second one accounts for the result of gravitons self-action. One can easily reduce this formula for I to (41) by the substitution of half-difference of retarded and advanced potentials for $\psi^{\varepsilon\lambda}$ and ψ in the solid integral.

[6] In (41) it is hidden in amplitudes a_3^3 related to that of the Fourier transform of potentials pertaining to spin states with zero projection on momentum of spin two and zero spin.

Conclusion

The main inference, which can be made from the foregoing is that, contrary to authors of [1] contention, energy losses of a sources through massive graviton emission are positively defined (see (41), (42)). What this means is theory of gravitation with nonzero graviton mass, if gravitons are endowed by nature with it, hold validity no less than theory of massless field. Basic equations of this theory are presented in Sec.2 (see (26), (27)). The previously obtained negative contributions to intensity of emission (see (20), (21)) arose from identification of space, wherein born gravitons move, with the Minkowsky space (see Sec.1). This mistake still remains uncorrected for more than 50 years. In commonly applied methods it was overlooked that, actually gravitons move in space with the Riemannian metric induced by the fields, among which is the own field of gravitons. Account of space distortion (or self-action of field and hence gravitons) is shown in Sec.3 to lead to positive definiteness of energy losses (see (41), (42)). As it follows from (42), all five states with spin two and zero spin state contribute to radiation. For a spherically symmetric source emission of massive gravitons persists, but only gravitons with zero projection on momentum of spin two as well as with zero spin are emitted. Connection between energy flux of radiation and sources work have been established (see (43)).

Difference of graviton mass from zero gives rise to some other important consequences as well. For one example, in a case of static spherically symmetric problem the presence of mass term in dynamic field equation brings about noncoincidence of metric coefficient g_{00} (at dt^2) with inverse of metric coefficient g_{11} (at dr^2) and therefore about impossibility of penetrating of falling to a center particles under the Schwarzschild sphere. Another one is that, on the strength of completeness equations system (27) together with state equation of a substance, a picture of the homogeneous isotropic Universe evolution appears to be uniquely defined (see [15]) in such theory. In doing this graviton mass is related to magnitudes of deceleration parameter and Habble function. It is estimated at 10^{-67} g. An existence of the mass tend to (see [15]) regular pulsations (with a period of the order of $7.5 \cdot 10^{10}$ years) of the Universe between states with minimal ($\sim 10^{-30}$ g/cm^3) and maximal ($\sim 10^{67}$ g/cm^3) substance density. Three-dimensional space always remains Euclidean in the process. The presence of latent mass, being about 25 times as large as the visible one, is predicted. In summary rehabilitation of theory of gravitation with nonzero graviton mass has nontrivial results. True enough, owing to its extremely small value it does not practically show up in numerous occasions. Among other things, all results of [16] connected with hypotheses for the existence of C- and P-violating gravitational interactions still stand, since areas investigated there are far apart from the ones, wherein the mass term can manifest itself.

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References

[1] Boulware D.G., Deser S.//Phys. Rev. D. 1972. V. 6. No 12. P. 3368-3382.

[2] Pauli W., Fierz M.//Proc. Roy. Soc. 1939. A73. P.211; Thirring W.//Ann. Phys. 1961. V. 16. P. 96.

[3] Freund P.G.O., Maheshwari A., Schonberg E.//Astroph. Journ. 1969. V. 157. P. 857-867.

[4] Fronsdal C.//Suppl. Nuovo Cimento. 1958. V. 9. P. 416-443.

[5] Barnes K.J.//J. Math. Phys.1965. V. 6. P. 788-794.

[6] Landau L.D., Lifshits E.M. Teoriya polya (Field theory). — Moscow: Nauka, 1988.

[7] Logunov A.A., Mestvirishvili M.A. Relyativistskaya teoriya gravitatsii (Relativistic theory of gravitation). — Moscow: Nauka, 1989.

[8] Logunov A.A., Loskutov Yu.M., Mestvirishvili M.A.//Uspekhi Fizicheskikh Nauk. 1988. V. 155. No 3. P. 369-396.

[9] Logunov A.A., Loskutov Yu.M., Mestvirishvili M.A.//Progr. of Theor. Phys. 1988. V. 80. No 6. P. 1005-1023; Intern. Journ. of Mod. Phys. A. 1988. V. 3. No 9. P. 2067-2099.

[10] Logunov A.A.//Teoreticheskaya i Matematicheskaya Fizika. 1994. V. 101. No 1. P. 3-27; Uspekhi Fizicheskikh Nauk. 1995. V. 165. P. 187-203.

[11] Logunov A.A., Loskutov Yu.M.//Doklady Academii Nauk SSSR. 1989. V. 305. No 4. P. 848-851.

[12] Loskutov Yu.M.//Teoreticheskaya i Matematicheskaya Fizika. 1990. V. 82. No 2. P. 304-312.

[13] Loskutov Yu.M.//Vestnik of Moscow Univ., Phys.-Astron. 1991. No 4. P. 49-54.

[14] Loskutov Yu.M.//Proc. of the 6th Marcel Grossman Meeting on Gen. Rel. Part B. Kyoto, Japan, 1991, p. 1658-1660.

[15] Loskutov Yu.M.//Teoreticheskaya i Matematicheskaya Fizika. 1993. V. 94. No 3. P. 515-528.

[16] Loskutov Yu.M.//Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki. 1995. V. 107. No 2. P. 283-298.

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