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**HADRONIC PRODUCTION
OF S - AND P -WAVE STATES OF $\bar{b}c$ -QUARKONIUM**

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Abstract

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In the leading $O(\alpha_s^4)$ order of the perturbative QCD, the hadronic production cross-sections of S - and P -wave states of B_c meson are calculated. The results for the S -wave levels are compared with the values given elsewhere as well as in the model of the b quark fragmentation into B_c . In the given order, the cross-sections of the hadronic production of the P -wave states are calculated for the first time. Their contribution into the B_c meson production is less than 10%. There is a strong difference between the predictions of the fragmentation model and the exact perturbative calculations. These differences are discussed in details for the differential distributions over various kinematical quantities.

Аннотация

Бережной А.В., Киселев В.В., Лиходед А.К. Адронное образование S - и P -волновых состояний $\bar{b}c$ -кваркония: Препринт ИФВЭ 96-2. – Протвино, 1996. – 15 с., 9 рис., 1 табл., библиогр.: 18.

В ведущем $O(\alpha_s^4)$ -порядке пертурбативной КХД вычислены сечения адронного рождения S - и P -волновых состояний B_c -мезона. Для S -волновых уровней результаты сравниваются с величинами, полученными в других работах и в модели фрагментации b -кварка в B_c . В данном порядке сечения адронного образования P -волновых состояний вычислены впервые. Их вклад в рождение B_c -мезонов меньше 10%. Существует сильное расхождение между предсказаниями модели фрагментации и точными пертурбативными вычислениями, которое детально обсуждается для дифференциальных распределений по различным кинематическим переменным.

Introduction

During recent two years an essential progress has been reached in the understanding of the production mechanisms for the heavy quarkonium, composed of two heavy quarks with different flavours.

The $\bar{b}c$ -quarkonium production process has the simplest consideration in e^+e^- -annihilation, where in the limit of high energies ($M^2/s \ll 1$), the differential cross-section has the evident factorizable form

$$\frac{d\sigma_H}{dz} = \sigma_{b\bar{b}}(s) \cdot D_{\bar{b} \rightarrow H}(z), \quad (1)$$

where $z = 2E_{B_c}/\sqrt{s}$, and $D_{\bar{b} \rightarrow H}(z)$ is interpreted as the $\bar{b} \rightarrow H+X$ fragmentation function with H being $\bar{b}c$ -quarkonium [1-3].

Recently, the functions of fragmentation into the S -, P -, and D -wave states for the $\bar{b}c$ -quarkonium production in e^+e^- collisions have been found [1-4]. Everywhere below mentioning the fragmentation model we will imply expressions like (1).

The description is more complicated for $\gamma\gamma$ and hadron-hadron collisions. As for these interactions, in addition to the mechanism of the \bar{b} quark production with the following fragmentation into the $\bar{b}c$ -quarkonium, there is a new type of diagrams, corresponding to the photon (gluon) dissociation into the pair of heavy quarks with the following recombination of the quarks into the $\bar{b}c$ -quarkonium.

In the photon-photon collisions, one can separate three gauge invariant subgroups (6+6+8) within the complete set of 20 diagrams in the leading Born approximation, so that the subgroups allow one to have the self-consistent interpretations: the \bar{b} quark fragmentation, c quark one and recombination. The contribution of the first mentioned type is quite reliably described by expression (1). For the c quark fragmentation diagrams one observes a strong deviation from the picture of the fragmentation model. The recombination contribution dominates in the complete region of kinematical variables. As was shown in [5], the given conclusion is valid for the S -wave state production as well as the P -wave one [6].

The process of hadronic production of the $\bar{b}c$ -quarkonium is more difficult for the analysis. In the leading order of the perturbation theory it is described by 36 diagrams in the fourth order over α_s . At present, the results of calculations by several groups giving controversial numerical values and conclusions for the $\bar{b}c$ -quarkonium production cross-sections are available [7-10]. Moreover, it was offered to simplify the consideration of the heavy quarkonium production by its reduction to the straightforward fragmentation of \bar{b} quark with the usage of fragmentation functions, derived for e^+e^- -annihilation case [11].

In this paper we give the comparative analysis of the results from different papers devoted to the production of the S -wave states. We prove that the straightforward fragmentation model does not properly describe the $\bar{b}c$ -quarkonium production. Next, we present the results of calculations for the P -wave level production over the complete set of the leading order diagrams.

1. Calculation technique

The A^{SJj_z} amplitude of the B_c meson production can be expressed through the amplitude of four free quarks production $T^{Ss_z}(p_i, k(\mathbf{q}))$ and the orbital wave function of the B_c meson, $\Psi^{Ll_z}(\mathbf{q})$, in the meson rest frame as

$$A^{SJj_z} = \int T^{Ss_z}(p_i, k(\mathbf{q})) \cdot (\Psi^{Ll_z}(\mathbf{q}))^* \cdot C_{s_z l_z}^{J j_z} \frac{d^3 \mathbf{q}}{(2\pi)^3}, \quad (2)$$

where J and j_z are the total spin of the meson and its projection on z axis in the B_c rest frame, correspondingly; L and l_z are the orbital momentum and its projection; S and s_z are the sum of quark spins and its projection; $C_{s_z l_z}^{J j_z}$ are the Clebsh-Gordan coefficients; p_i are four-momenta of B_c , b and \bar{c} , \mathbf{q} is the three-momentum of \bar{b} quark in the B_c meson rest frame; $k(\mathbf{q})$ is the four-momentum, obtained from the four-momentum $(0, \mathbf{q})$ by the Lorentz transformation from the B_c rest frame to the system, where the calculation of $T^{Ss_z}(p_i, k(\mathbf{q}))$ is performed. Then, the four-momenta of \bar{b} and c quarks, composing the B_c meson, will be determined by the following formulae with the accuracy up to $|\mathbf{q}|^2$ terms

$$\begin{aligned} p_{\bar{b}} &= \frac{m_b}{M} P_{B_c} + k(\mathbf{q}), \\ p_c &= \frac{m_c}{M} P_{B_c} - k(\mathbf{q}), \end{aligned} \quad (3)$$

where m_b and m_c are the quark masses, $M = m_b + m_c$, and P_{B_c} is the B_c momentum. Let us note that for the P -wave states it is enough to take into account only terms, linear over \mathbf{q} in eq.(2), and $\mathbf{q} = 0$ in the S -wave production.

The product of spinors $v_{\bar{b}} \bar{u}_c$, corresponding to the \bar{b} and c quarks in the $T^{Ss_z}(p_i, k(\mathbf{q}))$ amplitude of eq.(2), should be substituted by the projection operator

$$\mathcal{P}(\Gamma) = \sqrt{M} \left(\frac{\frac{m_b}{M} \hat{P}_{B_c} + \hat{k} - m_b}{2m_b} \right) \Gamma \left(\frac{\frac{m_c}{M} \hat{P}_{B_c} - \hat{k} + m_c}{2m_c} \right), \quad (4)$$

where $\Gamma = \gamma^5$ for $S = 0$, or $\Gamma = \hat{\varepsilon}^*(P_{B_c}, s_z)$ for $S = 1$, where $\varepsilon(P_{B_c}, s_z)$ is the polarization vector for the spin-triplet state.

For the sake of convenience, one can express the $\mathcal{P}(\Gamma)$ operator through the spinors of the following form

$$\begin{aligned} v'_b(p_b + k, \pm) &= \left(1 - \frac{\hat{k}}{2m_b}\right) v_b(p_b, \pm), \\ u'_c(p_c - k, \pm) &= \left(1 - \frac{\hat{k}}{2m_c}\right) u_c(p_c, \pm), \end{aligned} \quad (5)$$

where $v_b(p_b, \pm)$ and $u_c(p_c, \pm)$ are the spinors with the given projection of quark spin on z axis in the B_c meson rest frame. Note, that the spinors in eq.(5) satisfy the Dirac equation for the antiquark with the momentum $p_b + k$ and mass m_b or for the quark with the momentum $p_c - k$ and mass m_c up to the linear order over k (i.e. over \mathbf{q} too), correspondingly.

One can easily show that the following equalities take place

$$\begin{aligned} \sqrt{\frac{2M}{2m_b 2m_c}} \frac{1}{\sqrt{2}} \{v'_b(p_b + k, +) \bar{u}'_c(p_c - k, +) - v'_b(p_b + k, -) \bar{u}'_c(p_c - k, -)\} &= \\ &= \mathcal{P}(\gamma^5) + O(k^2), \\ \sqrt{\frac{2M}{2m_b 2m_c}} v'_b(p_b + k, +) \bar{u}'_c(p_c - k, -) &= \\ &= \mathcal{P}(\hat{\varepsilon}^*(P, -1)) + O(k^2), \\ \sqrt{\frac{2M}{2m_b 2m_c}} \frac{1}{\sqrt{2}} \{v'_b(p_b + k, +) \bar{u}'_c(p_c - k, +) + v'_b(p_b + k, -) \bar{u}'_c(p_c - k, -)\} &= \\ &= \mathcal{P}(\hat{\varepsilon}^*(P, 0)) + O(k^2), \\ \sqrt{\frac{2M}{2m_b 2m_c}} v'_b(p_b + k, -) \bar{u}'_c(p_c - k, +) &= \\ &= \mathcal{P}(\hat{\varepsilon}^*(P, +1)) + O(k^2). \end{aligned} \quad (6)$$

In the B_c rest frame the polarization vectors of the spin-triplet state have the form

$$\begin{aligned} \varepsilon^{rf}(-1) &= \frac{1}{\sqrt{2}}(0, 1, -i, 0), \\ \varepsilon^{rf}(0) &= (0, 0, 0, 1), \\ \varepsilon^{rf}(+1) &= -\frac{1}{\sqrt{2}}(0, 1, i, 0). \end{aligned} \quad (7)$$

In calculations the Dirac representation of γ -matrices is used and the following explicit form of the spinors is applied

$$\begin{aligned} u(p, +) &= \frac{1}{\sqrt{E+m}} \begin{pmatrix} E+m \\ 0 \\ p_z \\ p_x + ip_y \end{pmatrix}, \quad u(p, -) = \frac{1}{\sqrt{E+m}} \begin{pmatrix} 0 \\ E+m \\ p_x - ip_y \\ -p_z \end{pmatrix}, \\ v(p, +) &= -\frac{1}{\sqrt{E+m}} \begin{pmatrix} p_z \\ p_x + ip_y \\ 0 \\ E+m \end{pmatrix}, \quad v(p, -) = \frac{1}{\sqrt{E+m}} \begin{pmatrix} p_x - ip_y \\ p_z \\ 0 \\ E+m \end{pmatrix}. \end{aligned} \quad (8)$$

The S -wave production amplitude can be written down as

$$A^{Ss_z} = iR_S(0) \sqrt{\frac{2M}{2m_b 2m_c}} \sqrt{\frac{1}{4\pi}} \left(T^{Ss_z}(p_i, k(\mathbf{q}=0)) \right), \quad (9)$$

where $R_S(0)$ is the radial wave function at the origin, so that

$$R_S(0) = \sqrt{\frac{\pi}{3}} \tilde{f}_{B_c},$$

and the \tilde{f}_{B_c} value is related to the leptonic constants of pseudoscalar and vector B_c states

$$\begin{aligned} \langle 0 | J_\mu(0) | V \rangle &= i f_V M_V \epsilon_\mu, \\ \langle 0 | J_{5\mu}(0) | P \rangle &= i f_P p_\mu, \end{aligned}$$

where $J_\mu(x)$ and $J_{5\mu}(x)$ are the vector and axial-vector currents of the constituent quarks. Then the account for hard gluon corrections in the first order over α_s [12] results in

$$\tilde{f} = f_V \left[1 - \frac{\alpha_s^H}{\pi} \left(\frac{m_2 - m_1}{m_2 + m_1} \ln \frac{m_2}{m_1} - \frac{8}{3} \right) \right], \quad (10)$$

$$\tilde{f} = f_P \left[1 - \frac{\alpha_s^H}{\pi} \left(\frac{m_2 - m_1}{m_2 + m_1} \ln \frac{m_2}{m_1} - 2 \right) \right], \quad (11)$$

where $m_{1,2}$ are the masses of quarks composing the quarkonium. For the vector currents of quarks with equal masses, the BLM procedure of the scale fixing in the "running" coupling constant of QCD [13] gives (see paper by M.B.Voloshin in ref.[12])

$$\alpha_s^H = \alpha_s^{\overline{\text{MS}}} (e^{-11/12} m_Q^2).$$

The estimates of the \tilde{f}_{B_c} value within the potential models have the essential uncertainty, $\tilde{f}_{B_c} = 500 \pm 100$ MeV [14]. The QCD sum rule estimate of the f_{B_c} value for the pseudoscalar state gives $f_{B_c} = 385 \pm 25$ MeV [15], which is in a good agreement with the evaluation in the framework of recent lattice computations [16], where $f_{B_c} = 395(2)$ MeV with the error bar, giving the statistical uncertainty only. The \tilde{f}_{B_c} estimate strongly depends on the α_s^H scale choice, which is not yet calculated in the BLM procedure. So, we use $\tilde{f}_{B_c} = 570$ MeV.

For the P -wave states in eq.(2), the $T^{Ss_z}(p_i, k(\mathbf{q}))$ amplitude can be expanded into the Taylor series up to the terms linear over \mathbf{q} . Then one gets

$$A^{SJj_z} = i R'_P(0) \sqrt{\frac{2M}{2m_b 2m_c}} \sqrt{\frac{3}{4\pi}} C_{s_z l_z}^{Jj_z} \mathcal{L}^{l_z} (T^{Ss_z}(p_i, k(\mathbf{q}))), \quad (12)$$

where $R'_P(0)$ is the first derivative of the radial wave function at the origin, and \mathcal{L}^{l_z} has the following form

$$\begin{aligned} \mathcal{L}^{-1} &= \frac{1}{\sqrt{2}} \left(\frac{\partial}{\partial q_x} + i \frac{\partial}{\partial q_y} \right), \\ \mathcal{L}^0 &= \frac{\partial}{\partial q_z}, \\ \mathcal{L}^{+1} &= -\frac{1}{\sqrt{2}} \left(\frac{\partial}{\partial q_x} - i \frac{\partial}{\partial q_y} \right), \end{aligned} \quad (13)$$

where $\frac{\partial}{\partial q_x}$, $\frac{\partial}{\partial q_y}$, $\frac{\partial}{\partial q_z}$ are the differential operators acting on $T^{Ss_z}(p_i, k(\mathbf{q}))$ as the function of $\mathbf{q} = (q_x, q_y, q_z)$ at $\mathbf{q} = 0$.

As all considered matrix elements are calculated in the system distinct from the B_c rest frame, the four-momentum $k(\mathbf{q})$ has been calculated by the following formulae

$$\begin{aligned} k^0 &= \frac{\mathbf{v} \cdot \mathbf{q}}{\sqrt{1-\mathbf{v}^2}}, \\ \mathbf{k} &= \mathbf{q} + \left(\frac{1}{\sqrt{1-\mathbf{v}^2}} - 1 \right) \frac{\mathbf{v} \cdot \mathbf{q}}{\mathbf{v}^2} \mathbf{v}, \end{aligned} \quad (14)$$

where \mathbf{v} is the B_c velocity in the system, where the calculations have been performed. The matrix element $T^{Ssz}(p_i, k(\mathbf{q}))$ is computed, so that the four-momenta of \bar{b} and c quarks are determined by eq.(3), taking into account eq.(14).

The first derivatives in eq.(13) are substituted by the following approximations

$$\left. \frac{\partial T^{Ssz}(p_i, k(\mathbf{q}))}{\partial q_j} \right|_{\mathbf{q}=\mathbf{0}} \approx \frac{T^{Ssz}(p_i, k(\mathbf{q}^j)) - T^{Ssz}(p_i, 0)}{\Delta}, \quad (15)$$

where Δ is some small value, and \mathbf{q}^j have the following form

$$\begin{aligned} \mathbf{q}^x &= (\Delta, 0, 0), \\ \mathbf{q}^y &= (0, \Delta, 0), \\ \mathbf{q}^z &= (0, 0, \Delta). \end{aligned} \quad (16)$$

With the chosen values of quark masses and interaction energies, the increment value $\Delta = 10^{-5}$ GeV has provided a stability of 4-5 meaning digits in the squared matrix elements summed over j_z for all P -wave states with the given value of J and S , when performing the Lorentz transformations along the beam axis or the rotation around the same axis.

It is noteworthy that because of such transformations, the new vectors $k(\mathbf{q}^j)$ do not correspond to the transformed old vectors. Therefore, the applied test is not only a check of correct input of the $T^{Ssz}(p_i, k(\mathbf{q}))$ amplitude, but it is also the check of correct choice of the phases in eq.(12).

The matrix element A^{SJj_z} squared, which has been calculated by the method described above, must be summed over j_z as well as the spin states of free b and \bar{c} quarks. It must be also averaged over spin projections of initial particles.

The phase space integration has been made by the Monte Carlo method of RAMBO program [11].

2. S -wave states

Let us consider the calculation results for the gg -production of the S -wave $\bar{b}c$ -quarkonium levels (B_c and B_c^*) in comparison with the values given elsewhere. The cross-section of the process under consideration is proportional to the \tilde{f}_{B_c} squared as well as to the fourth power of α_s . To compare the results of different papers, we rescale all the numbers to the values, determined by the same set of $\alpha_s = 0.2$ and $\tilde{f}_{B_c} = 570$ MeV, which are used in the given calculations. We also fix the mass values of $m_b = 4.8$ GeV and $m_c = 1.5$ GeV.

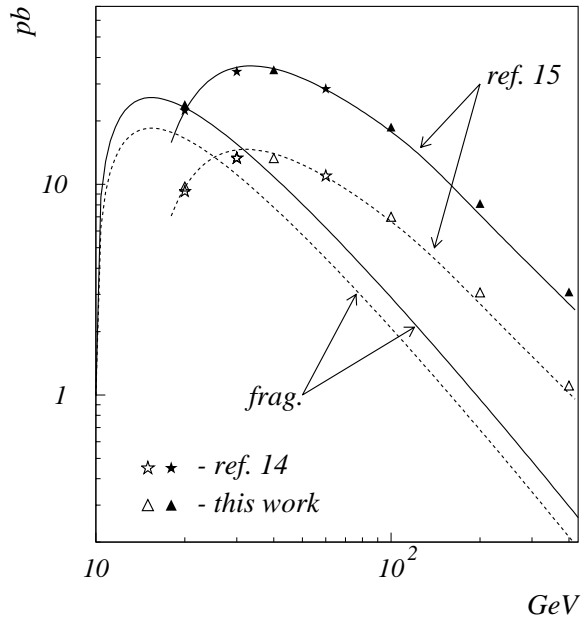


Fig. 1. The total cross-section of the gluon-gluon production of B_c (empty triangle) and B_c^* (solid triangle) in comparison with the predictions of the fragmentation model (frag.) and the results of ref.[8] (curves) and ref.[10] (stars).

In the studied region of energies, the ratio of the B_c and B_c^* yields ($R = \sigma_{B_c^*}/\sigma_{B_c} \simeq 3$) strongly deviates from $R \simeq 1.4$ predicted by the fragmentation model. One can also see in Fig. 1, that in the region of applicability determined by the condition $M^2/s \ll 1$, the fragmentation model gives the total cross-section, which is essentially smaller than the exact result evaluated over the complete set of diagrams.

As one can see in Fig. 2, the total cross-section of the $B_c^{(*)}$ production is basically accumulated in the region of the transverse momenta close to $p_T \sim M_{B_c}$. One could expect, that the fragmentation mechanism begins to dominate at large p_T . Indeed, the p_T -distribution shown in Fig. 2 for B_c and B_c^* at $\sqrt{s} = 100$ GeV points to the fact, that there is quite a good coincidence of the distribution tails obtained from exact perturbative calculations and in the fragmentation model. Numerically, this agreement takes place at $p_t > 40$ GeV for B_c . Thus, one can draw a certain conclusion, that the fragmentation contribution is not dominant in the gg -collisions. It does not also describe the distribution over the variable defined as $z = 2|\vec{P}_{B_c}|/\sqrt{\hat{s}}$. This fact is evident in Fig. 3. At z values close to unit, the fragmentation model overestimates the exact perturbative result.

To get the cross-section and p_T spectra in hadron-hadron collisions, one should convert the partonic gg cross-section with the distribution functions of gluons in the initial hadrons. We use the parametrization of ref.[17] for the parton distributions at the fixed virtuality scale $Q = 2m_b \sim 10$ GeV. In the framework of the given approximation, we also

The cross-sections of the gluonic production of $B_c^{(*)}$ versus the total energy of the gg collisions are presented in Fig. 1. In addition to the results of the complete numerical calculations of the $O(\alpha_s^4)$ contribution, we also give the values, obtained in the fragmentation model and, hence, calculated as the product of the gg -production cross-section for the $b\bar{b}$ -pair and the probability of the $\bar{b} \rightarrow B_c^{(*)}$ fragmentation, calculated at the same set of parameters.

As one can see in this figure, there is a good agreement of our previous calculations [7] with the results of refs.[8,10]. The values given in two other papers of ref.[9] are approximately three times greater than our results. In this paper we have recalculated the cross-sections in the axial gauge, used in ref.[9], in contrast to the covariant Feynman gauge applied in our previous consideration [7]. The corresponding numerical results have not changed after the replacement of the gauge. This fact points to errors in the computations, performed in ref.[9].

fix also the coupling constant value $\alpha_s = 0.2$. Note, that the use of the parton distributions given in ref.[18] shows a small variation of numerical results, so that the differences are not essential in the consideration of the production mechanism.

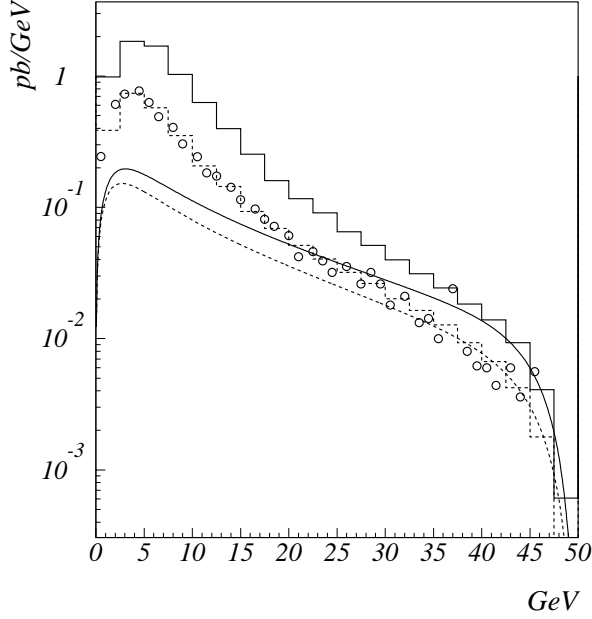


Fig. 2. $d\sigma/dp_T$ at the gluon interaction energy of 100 GeV for B_c^* (solid histogram) and B_c (dashed histogram) in comparison with the predictions of the fragmentation model (solid and dashed curves for B_c^* and B_c , respectively), and the result of ref.[8] for B_c (dots).

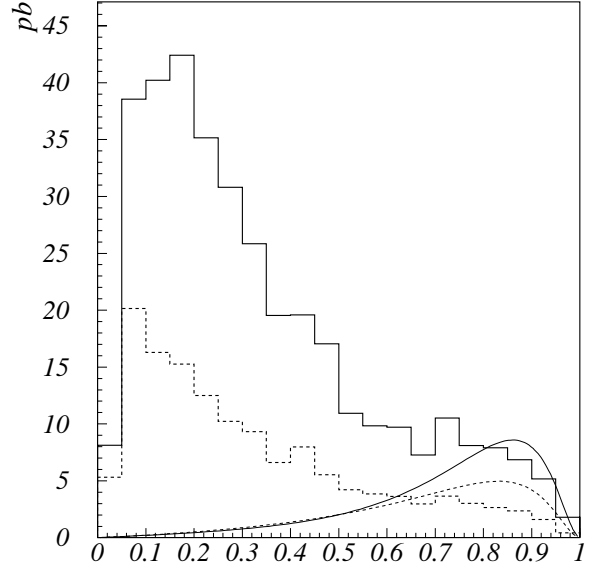


Fig. 3. $d\sigma/dz$ at the gluon interaction energy of 100 GeV for B_c^* (solid histogram) and B_c (dashed histogram) in comparison with the predictions of the fragmentation model (curves).

The convolution result is presented in Fig. 4 for the energy of the FNAL Tevatron ($\sqrt{s} = 1.8$ TeV). The histograms correspond to the exact perturbative results, whereas the curves are calculated according to the following formula

$$\begin{aligned} \frac{d\sigma}{dp_T} \quad (\bar{p}p \rightarrow H_{(p_T)}x) &= \sum_{i,j} \int dx_1 dx_2 dz f_{i/p}(x_1, \mu) f_{j/\bar{p}}(x_2, \mu) \times \\ &\times \frac{d\hat{\sigma}}{dp_T} \quad (ij \rightarrow \bar{b}(p_T/z) + x) \times D_{\bar{b} \rightarrow H}(z, \mu), \end{aligned} \quad (17)$$

where $D(z, \mu)$ is the function of the $\bar{b} \rightarrow H$ fragmentation with $H = B_c, B_c^*, \dots$, $d\hat{\sigma}/dp_T$ is the differential cross-section of the \bar{b} quark production, and $f_{i/A}(x, \mu)$ is the distribution of the i -kind parton in the A -hadron.

As one can see in the figure, the curves certainly deviate from the histograms of the exact perturbative calculations in the whole region of p_T . At small p_T , the fragmentation model gives the overestimation, whereas at large p_T , on the contrary, it underestimates

the exact result. It is significant to note the fact, that the perturbative calculations and the model give the different values for the ratio of the differential cross-sections of B_c^* and B_c . One finds the yield ratio $R \sim 2 \div 3$, whereas the model gives $R \sim 1.3 \div 1.5$.

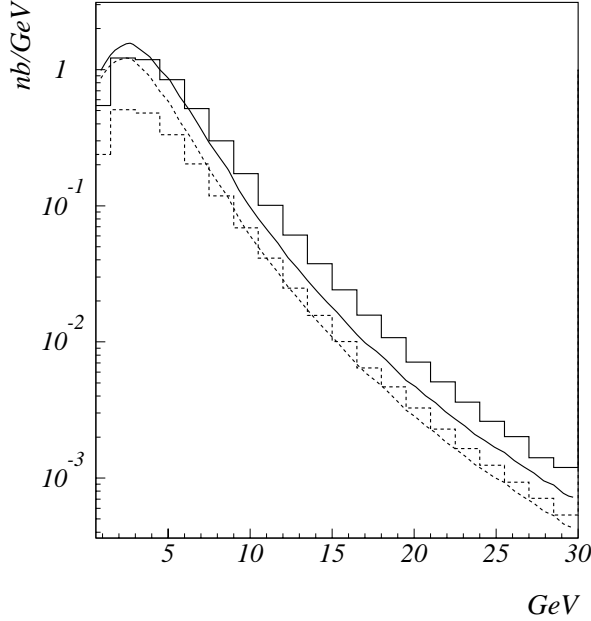


Fig. 4. $d\sigma/dp_T$ in the hadronic production of B_c^* (solid histogram) and B_c (dashed histogram) in comparison with the predictions of the fragmentation model (curves) at the $p\bar{p}$ -collision energy of 1.8 TeV.

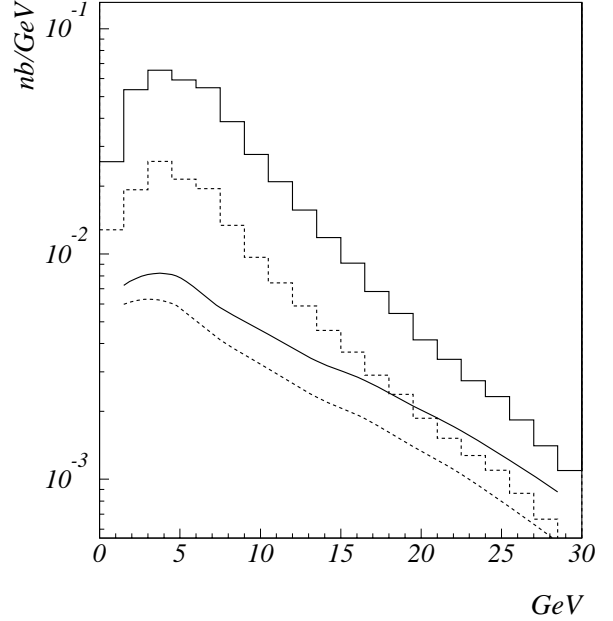


Fig. 5. The same as in Fig. 4 with the sub-process energy > 60 GeV cut off.

The qualitative agreement between the model and perturbative calculations for the $d\sigma/dp_T$ distribution of B_c meson has allowed the authors of ref.[8] to conclude on the satisfactory description of the exact perturbative $O(\alpha_s^4)$ -contribution by the fragmentation model. The misleading is related to the fact, that those values of the gg -collision energy are included in the integration region of expression (17), where the approximation of the fragmentation does not work or it is not strictly defined, i.e., where the $M^2/s \ll 1$ condition is evidently invalid. As one can see in Fig. 1 at these low energies, the cross-section in the fragmentation model and that of the perturbative result have different dependency on the energy. The latter fact is because of the use of the two-particle phase space in the fragmentation approximation of the heavy quark production instead of the real three-particle phase space (B_c , \bar{b} and c are in the final state).

To avoid the uncertainties related to the low energy of gg -collisions, we have started the integration over \hat{s} in expression (17) from $\sqrt{\hat{s}} > 60$ GeV. As one can see in Fig. 5, the given cut drastically changes the relation between the fragmentation model contribution and the exact perturbative result. It indicates that the fragmentation model cannot provide an adequate approximation for the correct description of the hadronic B_c meson production.

One has to note, that the given conclusion does not depend on the definite choice of the structure functions for the initial hadrons as well as on a special modification of the fragmentation expression.

Thus, the conclusion drawn in ref.[8] about the dominance of the fragmentation mechanism in the hadronic production of B_c mesons, is incorrect.

3. Production of P -wave states

As was mentioned above, to get the cross-section for the production of the P -wave quarkonium levels it is necessary to calculate the first derivative of the quark production matrix element over \vec{q} , i.e. the relative momentum of the quarks inside the B_c meson. This procedure of calculations has been tested by comparison of the numerical computation of the fragmentation functions for the \bar{b} quark into the P -wave states with known analytical expressions for the corresponding functions obtained in ref.[4]. We have found a good agreement between the 1P_1 , 3P_0 , 3P_1 , and 3P_2 -level cross-sections calculated numerically, and those obtained from the analytical expressions for the same set of parameters, m_b , m_c , $|R'_P(0)|^2$ and α_s . The total cross-sections of the gluonic production of P -wave levels are presented versus the total energy of collisions in Tab. 1.

Table 1. The dependence of production cross-sections for different P -wave states of B_c on the total energy of colliding gluons. (The calculation errors are shown in parenthesis).

$\sqrt{\hat{s}}$, GeV	$\sigma_{^1P_1}$, pb	$\sigma_{^3P_0}$, pb	$\sigma_{^3P_1}$, pb	$\sigma_{^3P_2}$, pb
15.	0.04677(6)	0.07076(13)	0.04355(8)	0.09018(14)
20.	0.3663(7)	0.1837(7)	0.3454(9)	0.7191(18)
30.	0.685(3)	0.2082(15)	0.585(3)	1.370(7)
40.	0.744(5)	0.206(2)	0.597(5)	1.487(13)
60.	0.645(8)	0.169(3)	0.490(8)	1.27(2)
80.	0.521(8)	0.132(3)	0.384(8)	1.02(2)
100.	0.418(9)	0.105(4)	0.304(8)	0.82(3)

The dependence of the cross-section, summed over the spin states of the P -levels, on the energy of the interacting gluons is shown in Fig. 6. This dependence can be approximately described by the following expression

$$\sigma_{B_c(L=1)} = 25. \cdot \left(1 - \left(\frac{2(m_b+m_c)}{\sqrt{s}}\right)\right)^{1.95} \times \left(\frac{2(m_b+m_c)}{\sqrt{s}}\right)^{1.2} \text{ pb.} \quad (18)$$

The contribution, determined by the fragmentation model, is also shown in the same figure, where it is found as the corresponding total $gg \rightarrow b\bar{b}$ cross-section multiplied by the integral probability of the $b \rightarrow B_c(L=1)$ fragmentation, $W = 5.34 \cdot 10^{-5}$. The c -quark fragmentation is suppressed by the order of magnitude in the hadron collisions, and we will neglect it below.

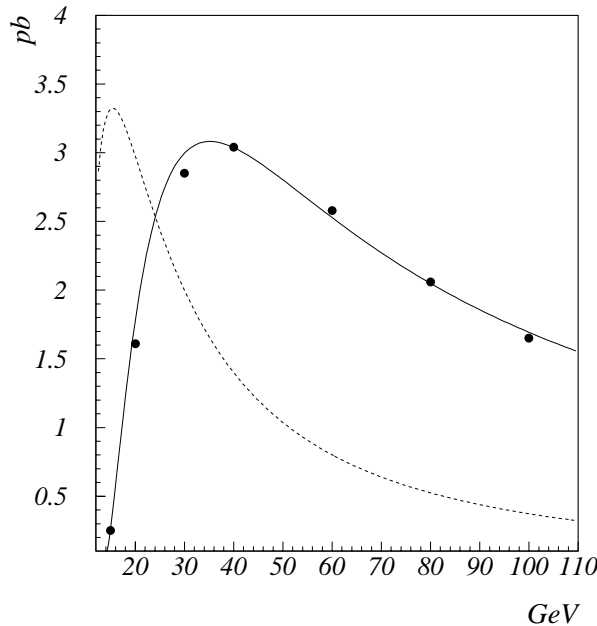


Fig. 6. The total cross-section summed over spins of the P -levels in the gluon-gluon production (dots) and its fit (solid line curve) in comparison with the prediction of the fragmentation model (dashed line curve).

the tails. This fact is quite expectable, since the same picture has been observed in the production of S -levels.

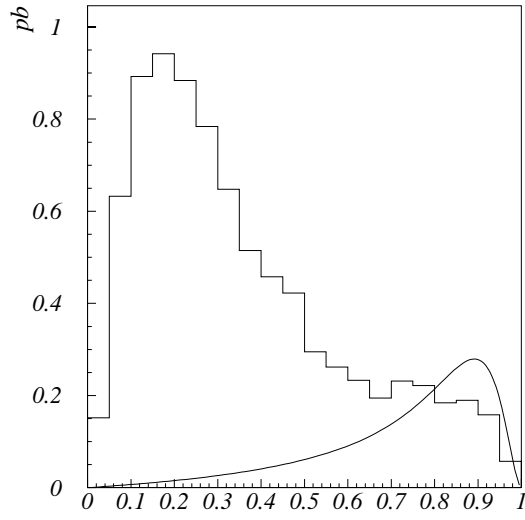
Thus, our calculations of the P -wave level production cross-sections show that the total hadronic cross-sections are about 10% of the corresponding cross-sections of the S -wave levels, and the recombination mechanism dominates for both S - and P -levels.

The differential distributions for the P -wave states calculated by convoluting the partonic cross-sections with the gluon distributions, as it was described in the previous section, are shown in Fig. 9 for the FNAL Tevatron energy, $\sqrt{s} = 1.8$ TeV.

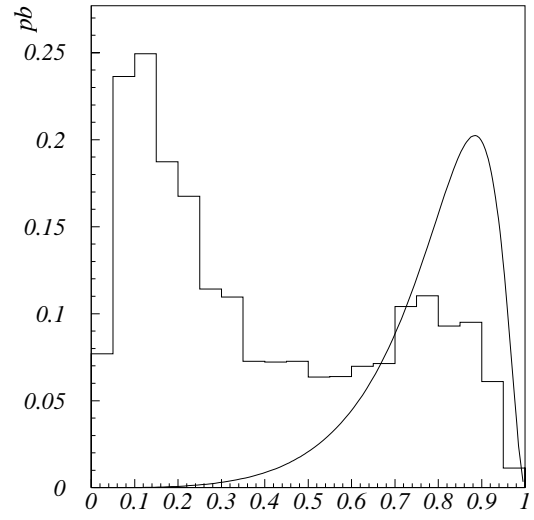
As it comes for the S -wave states, the calculation over formula (1) of the fragmentation model gives a rather approximate qualitative value for the P -level cross-section. The reason for such "description" is generally related to the uncertainty of the fragmentation approach in the region of energies close to the kinematical threshold of the reaction, where the fragmentation model overestimates the cross-section. This incorrect contribution compensates the underestimation in the region of large energies and small or moderate p_T , where the recombination dominates. This results in the approximate description of the exact perturbative distribution by the fragmentation formula at $p_T \sim 15$ GeV (see Fig. 9). As was shown in the previous section, the \hat{s} cut clarifies the problem, and one can evidently conclude, that the exact perturbative calculations and the fragmentation model strongly differ in the predictions.

As one can see in Fig. 6, the total cross-section of the P -wave level production is much greater than the value predicted by the fragmentation model, which has a strict meaning in the region of high energies, and, thereby, the contribution of the recombination diagrams dominates. The fragmentation contribution does not also describe the $d\sigma/dz$ distributions, which are the differential cross-sections integrated over the transverse momentum of P -wave levels. These distributions are shown in Fig. 7. It is interesting to notice that in contrast to the $\gamma\gamma$ -collisions, the result of the fragmentation model essentially overestimates the exact perturbative values in the gluonic collisions. This fact, related to the interference of different contributions, will be discussed elsewhere.

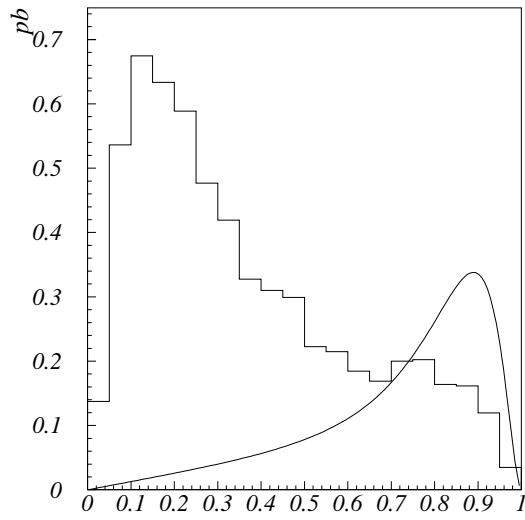
The distributions over the transverse momentum of the produced P -wave states are presented in Fig. 8 in comparison with the results of the fragmentation model. One can see, that the fragmentation is valid only on



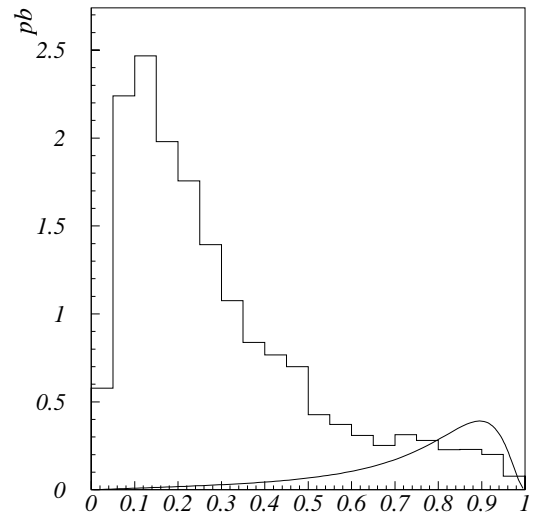
a



b



c



d

Fig. 7. $d\sigma/dz$ at the gluon interaction energy of 100 GeV for 1P_1 -, 3P_0 -, 3P_1 -, and 3P_2 -levels (a, b, c and d figures, correspondingly) in comparison with the predictions of the fragmentation model (curves).

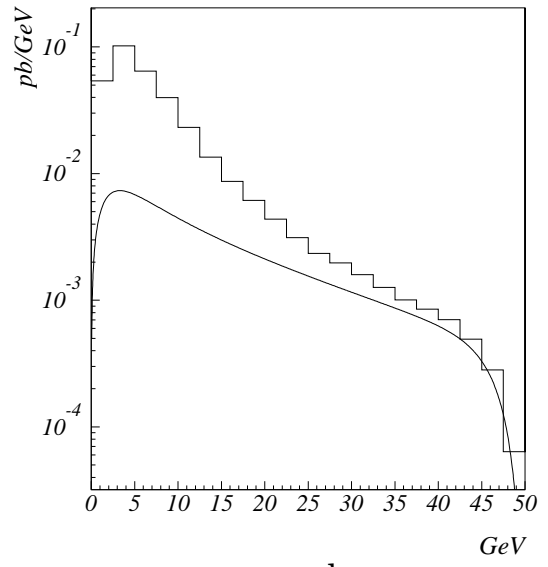
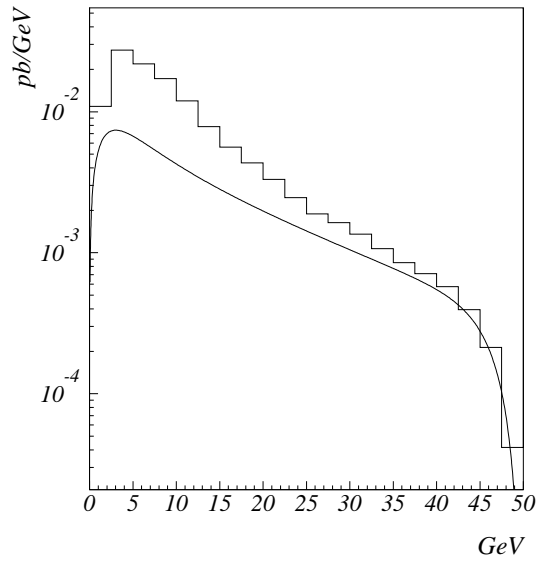
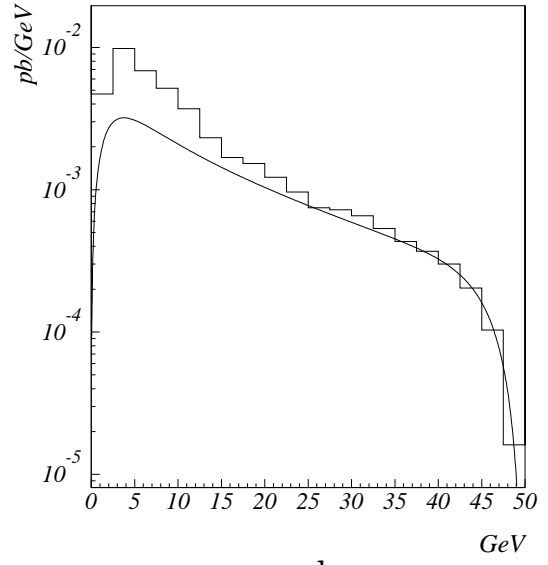
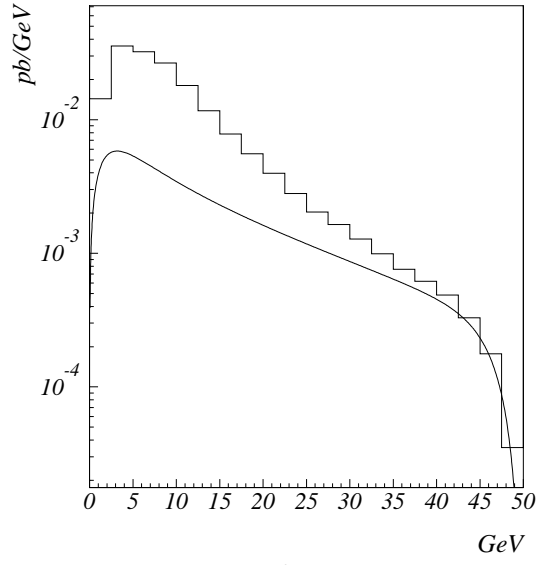


Fig. 8. $d\sigma/dp_T$ with the same notations as in Fig. 7.

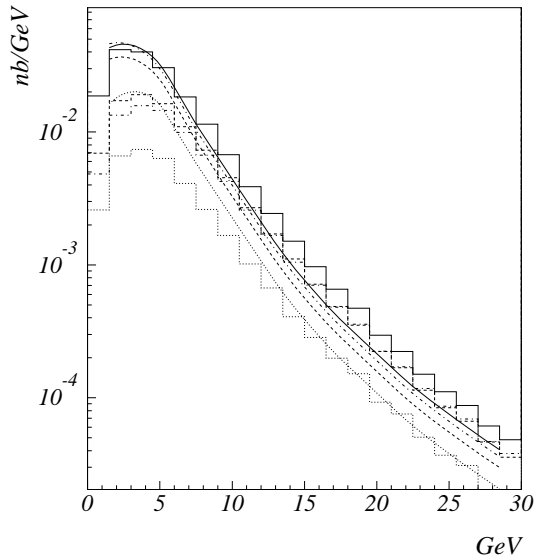


Fig. 9. The differential cross-section of the hadronic $p\bar{p}$ production of the P -level B_c mesons at the hadron interaction energy of 1.8 TeV (histograms) in comparison with the fragmentation model results: 3P_2 — solid curve, 3P_0 — dotted curve, 3P_1 — dash-dotted curve, and 1P_1 — dashed curve.

Conclusion

Let us analyse the results of the numerical perturbative calculations performed for the hadronic production cross-sections of the S - and P -wave levels of the $\bar{b}c$ -quarkonium in the leading $O(\alpha_s^4)$ order of the perturbative QCD.

There are two scales of virtualities in the problem. The first one is of the order of the heavy quark masses, and it appears in the heavy quark production. The second scale is determined by the characteristic relative momentum of the quarks inside the $\bar{b}c$ -meson. Since the latter is much less than the masses of the produced quarks, the process of the B_c production can be separated into two steps:

- 1) the heavy quark production, described in the perturbative QCD, and
- 2) the formation of the bound state, described by the quarkonium wave function at the origin.

The results, hence, linearly depend on α_s^4 , $\tilde{f}_{B_c}^2$ and $|R'_P(0)|^2$. Moreover, there are the additional parameters defining the quark masses, m_b and m_c . The bulk of the results uncertainty is connected with the α_s , \tilde{f}_{B_c} , and $|R'_P(0)|$ values. For instance, the use of the running coupling constant $\alpha_s(\hat{s})$ instead of $\alpha_s(4m_b^2)$ as well as the use of a smaller value of \tilde{f}_{B_c} can decrease the cross-section by an order of magnitude.

From our point of view, the use of $\alpha_s(\hat{s})$ is not correct, since the analysis shows that typical virtualities in the production of four heavy quarks are less than \hat{s} at large \hat{s} . This is why we have fixed the value of $\alpha_s = 0.2$. The comparison with the results of other papers shows, that we agree with the calculations of the S -level production in ref.[8], but, in contrast to [8], we compare the yields of pseudoscalar and vector states of B_c in the hadronic production to emphasize the invalidity of the fragmentation model to the problem under consideration. The results of ref.[9] disagree with the values given in our paper for both total cross-section of the pseudoscalar state and the dependence on the

total energy. The production of the vector state was not considered in ref.[9]. So, our computations, performed in covariant and axial gauges, point to errors in papers of ref.[9].

The exact perturbative cross-sections for the P -wave states are found for the first time. As well as for the hadronic production of S -levels, one observes a strong discrepancy in the values of the relative yields of 1P_1 , 3P_0 , 3P_1 and 3P_2 -levels in comparison with the production in e^+e^- -annihilation. This fact points to different dominating mechanisms. The fragmentation of the \bar{b} quark into $B_c^{(*)}$ is the basic mechanism in e^+e^- -annihilation, whereas the recombination of heavy quarks dominates in the hadronic production of $B_c^{(*)}$.

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