STATE RESEARCH CENTER OF RUSSIA

INSTITUTE FOR HIGH ENERGY PHYSICS

IHEP 96-22

V.A.Tsokur and Yu.M.Zinoviev [1]

$N = 2$ SUPERGRAVITY MODELS BASED ON THE NONSYMMETRIC QUATERNIONIC MANIFOLDS I. SYMMETRIES AND LAGRANGIANS

Submitted to *Int. J. Mod. Phys.*

[1] E-mail address: ZINOVIEV@MX.IHEP.SU

Protvino 1996

UDK 539.1.01 M–24

Abstract

Tsokur V.A., Zinoviev Yu.M. $N = 2$ supergravity models based on the nonsymmetric quaternionic manifolds. I. Symmetries and Lagrangians: IHEP Preprint 96-22. – Protvino, 1996. – p. 20, figs. 2, refs.: 13.

In this paper we consider the $N = 2$ supergravity models in which the hypermultiplets realize the nonlinear σ-models, corresponding to the nonsymmetric (but homogeneous) quaternionic manifolds. By exploiting the isometries of appropriate manifolds we give an explicit construction for the Lagrangians and supertransformation laws in terms of usual hypermultiplets in the form suitable for the investigation of general properties of such models as well as for the studying of concrete models.

Аннотация

Зиновьев Ю. М., Цокур В. А. Модели $N = 2$ супергравитации, основанные на несимметрических кватернионных многообразиях. I. Симметрии и лагранжианы: Препринт ИФВЭ 96-22. – Протвино, 1996. – 20 с., 2 рис., библиогр.: 13.

В этой работе мы рассматриваем модели $N = 2$ супергравитации, в которых гипермультиплеты реализуют нелинейные σ-модели, соответствующие несимметрическим (но однородным) кватернионным многообразиям. Используя изометрии соответствующих многообразий, мы даем явное построение лагранжианов и законов суперпреобразований в терминах обычных гипермультиплетов в форме удобной как для исследования общих свойств таких моделей, так и для изучения конкретных моделей.

© State Research Center of Russia
Institute for High Energy Physics, 1996

Introduction

Recently an interest to the theories with $N = 2$ supersymmetry has revived. In the stringy context the reason was that one could use the same Calabi-Yao manifolds for compactification of heterotic string (obtaining the theory with $N = 1$ supersymmetry) and for the type II string thus having $N = 2$ supersymmetry. The latter leads to the restrictions on the geometrical properties of such models like moduli spaces and so on. Another reason is related with quantum properties of the theories with $N = 2$ supersymmetry giving a possibility to study nonperturbative phenomena [1]. From the phenomenological point of view models with extended supersymmetries do not look promising due to severe problems arising in any attempts to construct even semirealistic theory. Generally, one faces three kinds of problems:

- Spontaneous supersymmetry breaking.
  Any realistic model should rely on the mechanism of spontaneous supersymmetry breaking which gives two essentially different scales, in this the cosmological term must be automatically equal to zero for any values of these parameters. Two scales of breaking are necessary because the breaking with only one scale (when two gravitini remain mass degenerate) leaves the theory vectorlike [2,3]. In order to be able to reproduce at low energies the "standard" $N = 1$ supersymmetric phenomenology, such a model should admit, as a particular case, the partial super-Higgs effect then the $N = 1$ supersymmetry remains unbroken and the corresponding gravitino — massless. It turns out that such a breaking is indeed possible [4,5], moreover it has been shown [6] that there exist three different hidden sectors having desirable properties. This, in turn, allowed one to consider the spontaneous supersymmetry breaking in a wide class of$N = 2$ supergravity models and calculate the soft breaking terms that arose after the breaking had taken place [7,6].
- Gauge symmetry breaking.
  Generally, the scalar field potential in $N = 2$ gauge theories as well as in $N = 2$ supergravity models has quite a lot of flat directions giving one a possibility to introduce non-zero vacuum expectation values for the appropriate scalar fields thus

breaking the gauge symmetry spontaneously. However, after the spontaneous supersymmetry breaking at least part of these flat directions is lost. For example, one of the general properties of the mechanism of spontaneous supersymmetry breaking described above is that the scalar fields from the vector multiplets unavoidably acquire masses. Let us stress, however, that analogous problem (the positive mass square for the Higgs field) appears in the $N = 1$ theories as well.

- Fermionic mass spectrum.
  In the theories with $N = 2$ supersymmetry this problem turns out to be much more difficult. First of all, as we have already mentioned, with the unbroken supersymmetry or even when there is only one breaking scale, the theory is vectorlike and one has to have spontaneous supersymmetry breaking with two different scales before trying to lift the mass degeneracy between usual and mirror fermions. But even after the problem with the supersymmetry breaking has been solved, one has to have an appropriate set of Yukawa couplings to generate correct fermionic mass spectrum. As is well known, in $N = 2$ gauge theories there exists only one type of such couplings, where the scalar fields come from the vector multiplets. In the "minimal" coupling of these theories to $N = 2$ supergravity (based on the quaternionic spaces that are symmetric ones) the situation remains to be essentially the same — there are no Yukawa couplings between scalar and spinor fields of the hypermultiplets.

However, there exist quaternionic manifolds that are not symmetric manifolds (but they are homogeneous ones, which is important for the possibility for such models to arise in superstrings). For the first time the classification of these spaces has been given by Alekseevsky [8], later on the properties of these manifolds, especially their symmetry properties, were studied in a number of papers [9,10,11,12,13]. Two properties will be very important for us in this work:

- As we will show below, all these models contain as a universal part one of the three hidden sectors [6] (corresponding to the nonlinear σ-model $O(4,4)/O(4) \otimes O(4)$) admitting a spontaneous supersymmetry breaking with two arbitrary scales and without a cosmological term.
- In the most general case (we will denote such models $V(p,q)$, see below) there exist cubic invariants (due to the fact that part of the hypermultiplets transformed under the vector representation of group $O(p)$, while others — under the spinor ones) related to the appropriate γ-matrices. This leads to a possibility for Yukawa couplings to be generated.

The Alekseevsky classification [8] for the quaternionic manifolds is reduced to the classification of admittable quaternionic algebras with the dimension (always multiple of four) equal to the dimension of the manifold. This means, that each generator in the algebra corresponds to the physical scalar field in the hypermultiplets being the coordinate of the manifold and playing the role of the Goldstone one for this transformation. In the cases when the quaternionic manifold is symmetric, the isometry algebra turns out to be larger (e.g. $Sp(2,2n)$, $SU(2,n)$ or $O(4,n)$ for the well known cases). But even in general

cases the isometry algebra of the quaternionic manifold appears to be larger than the minimal one arising in classification [10]-[13]. This allows one by using the invariance under these algebras to construct and investigate the appropriate nonlinear σ-models. In particular, we will see that in all cases it is possible to have linearly realized $SU(2)$ subgroup, corresponding to the natural symmetry of the $N = 2$ superalgebra.

All generators in the quaternionic algebra (apart from the ones in hidden sector) come in three types which following Alekseevsky we will denote as X, Y and Z. In this, there exist two general series of models.[1] In the first one, that we will denote $W(p, q)$, the X-type hypermultiplets are absent, while the number p of Y-type hypermultiplets and the number q of Z-type ones are arbitrary. In the second one denoted as $V(p, q)$ there exist p sets of X-type hypermultiplets while the number of Y and Z hypermultiplets are equal to $qd(p)$, where $d(p)$ — dimension of spinor representation of $O(p)$ group. In the four special cases ($q = 1$ and $p = 1, 2, 4, 8$) these manifolds turn out to be symmetric with the isometry algebras being F_4, E_6, E_7 and E_8. Thus, the minimal model that contains all three types of hypermultiplets is the F_4 one and we will use this model as our starting point. Namely, in the following section we will show how one can construct the linear combinations of the F_4 generators corresponding to the generators in the Alekseevsky classification. Moreover, we will explicitly construct the subalgebra of the whole F_4 algebra which admits natural generalization to the case of nonsymmetric quaternionic manifolds.

In the next section we consider the $W(p, q)$ models for arbitrary p and q. As is known [9], in the partial case $q = 0$ this model coincides with the well known $O(4, 4+p)/O(4) \otimes O(4+p)$ one. This allows one to have a very simple realization of this model which could be easily generalized to the case of arbitrary q.

Later, as a preliminary step to the construction of the general $V(p, q)$ models, we consider one more partial case — $q = 0$, i.e. the case when only X-type hypermultiplets are present. This model also corresponds to the similar $O(4, 4 + p)/O(4) \otimes O(4 + p)$ model but in different parameterization. Using this fact, we have managed to construct the realization with the correct isometry algebra (i.e. global symmetry of the bosonic Lagrangian).

Both models $W(p, q)$ and $V(p, 0)$ contain the same hidden sector, corresponding to the non-linear σ-model $O(4, 4)/O(4) \otimes O(4)$ but in the very different parameterizations. So, to join these models into the general $V(p, q)$ one we have to make a reduction for both of them in order to bring the hidden sectors to the similar form. Unfortunately, this enlarges the number of fields in terms of which the model is described and makes all the formulas rather long. Nevertheless, we have managed to construct an explicit Lagrangian invariant under the local $N = 2$ supertransformations and global bosonic transformations, corresponding to the appropriate quaternionic algebras.

[1] Really, there exist more models, as it has been shown in [11,12], but in this paper we restrict ourselves to only two general types of models

1. F_4-model

As we have already mentioned in Introduction, the minimal model that contains all necessary ingredients for the construction of the quaternionic non-linear σ-models we are interested in is the F_4-model. This model (as well as all other ones) contains a universal "hidden sector", based on the $O(4,4)$ group. The latter has the $SU(2)^4$ as its maximal compact subgroup, whose generators we will denote as ${t_i}^j$, ${t_\alpha}^\beta$, ${t_{\dot\alpha}}^{\dot\beta}$ and ${t_{\ddot\alpha}}^{\ddot\beta}$, correspondingly. Commutation relations for these generators are normalized so that

$$[{t_i}^j, {t_k}^l] = {\delta_k}^j {t_i}^l - {\delta_i}^l {t_k}^j \tag{1}$$

and analogously for other ones. Besides, we will use the notation $t_{ij} = \varepsilon_{jk}{t_i}^k = t_{ji}$ and so on. In this basis the non-compact generators of the $O(4,4)$ group form a multispinor $T_{i\alpha\dot\alpha\ddot\alpha}$, satisfying a pseudo-reality condition

$$(T_{i\alpha\dot\alpha\ddot\alpha})^* = T^{i\alpha\dot\alpha\ddot\alpha} = \varepsilon^{ij}\varepsilon^{\alpha\beta}\varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\ddot\alpha\ddot\beta}T_{j\beta\dot\beta\ddot\beta}. \tag{2}$$

Commutation relations look like

$$\begin{aligned}
[{t_i}^j, T_{k\alpha\dot\alpha\ddot\alpha}] &= {\delta_k}^j T_{i\alpha\dot\alpha\ddot\alpha} - \frac{1}{2}{\delta_i}^j T_{k\alpha\dot\alpha\ddot\alpha} \quad \text{plus similar ones for} \quad \alpha, \dot\alpha, \ddot\alpha \\
[T_{i\alpha\dot\alpha\ddot\alpha}, T^{j\beta\dot\beta\ddot\beta}] &= +\{{t_i}^j {\delta_\alpha}^\beta {\delta_{\dot\alpha}}^{\dot\beta} {\delta_{\ddot\alpha}}^{\ddot\beta} + \cdots\}.
\end{aligned} \tag{3}$$

Note that in our normalization the plus sign in the last commutator corresponds to the noncompact group $O(4,4)$, while for the minus sign one would have an $O(8)$ group.

The key elements of the whole construction [8] are the four commuting algebras of the form $[h, g] = 2g$. Let us choose

$$\begin{aligned}
h_1 &= (T_{1111} + T_{2222}), \qquad h_2 = (T_{1122} + T_{2211}), \\
h_3 &= (T_{1212} + T_{2121}), \qquad h_4 = (T_{1221} + T_{2112}).
\end{aligned} \tag{4}$$

Then one has

$$\begin{aligned}
g_1 &= \frac{1}{2}(t_{12}^{(1)} + t_{12}^{(2)} + t_{12}^{(3)} + t_{12}^{(4)}) + \frac{1}{2}(T_{1111} - T_{2222}), \\
g_2 &= \frac{1}{2}(t_{12}^{(1)} + t_{12}^{(2)} - t_{12}^{(3)} - t_{12}^{(4)}) + \frac{1}{2}(T_{1122} - T_{2211}), \\
g_3 &= \frac{1}{2}(t_{12}^{(1)} - t_{12}^{(2)} + t_{12}^{(3)} - t_{12}^{(4)}) + \frac{1}{2}(T_{1212} - T_{2121}), \\
g_4 &= \frac{1}{2}(t_{12}^{(1)} - t_{12}^{(2)} - t_{12}^{(3)} + t_{12}^{(4)}) + \frac{1}{2}(T_{1221} - T_{2112}),
\end{aligned} \tag{5}$$

where $t^{(1)}$ stands for t_{ij}, $t^{(2)}$ for $t_{\alpha\beta}$ and so on. Besides the generators given above, algebra $O(4,4)$ (as well as all algebras, corresponding to symmetric quaternionic spaces) contains

also four generators $\hat{g}$, such that $[h, \hat{g}] = -2\hat{g}$, $[g, \hat{g}] = h$. They look like

$$\begin{aligned}
\hat{g}_1 &= \frac{1}{2}(t_{12}^{(1)} + t_{12}^{(2)} + t_{12}^{(3)} + t_{12}^{(4)}) - \frac{1}{2}(T_{1111} - T_{2222}), \\
\hat{g}_2 &= \frac{1}{2}(t_{12}^{(1)} + t_{12}^{(2)} - t_{12}^{(3)} - t_{12}^{(4)}) - \frac{1}{2}(T_{1122} - T_{2211}), \\
\hat{g}_3 &= \frac{1}{2}(t_{12}^{(1)} - t_{12}^{(2)} + t_{12}^{(3)} - t_{12}^{(4)}) - \frac{1}{2}(T_{1212} - T_{2121}), \\
\hat{g}_4 &= \frac{1}{2}(t_{12}^{(1)} - t_{12}^{(2)} - t_{12}^{(3)} + t_{12}^{(4)}) - \frac{1}{2}(T_{1221} - T_{2112}).
\end{aligned} \tag{6}$$

Let us stress that it is the presence of all or some of the generators $\hat{g}$ in the algebra that determines a possibility to "restore" all or some of the four initial $SU(2)$ subgroups.

Now one can combine all other generators of $O(4,4)$ algebra (as well as all other generators of F_4) into the linear combinations that will be the eihgenvectors for all four generators h. For example, the 16 remaining generators of $O(4,4)$ form two octets with h_1 eihgenvalues ± 1:

$$\begin{aligned}
\Omega_1^{\pm} &= t_{22}^{(1)} \pm T_{2111} \qquad \Omega_2^{\pm} = t_{22}^{(2)} \pm T_{1211}, \\
\Omega_3^{\pm} &= t_{22}^{(3)} \pm T_{1121} \qquad \Omega_4^{\pm} = t_{22}^{(4)} \pm T_{1112}, \\
\Omega_5^{\pm} &= t_{11}^{(1)} \mp T_{1222} \qquad \Omega_6^{\pm} = t_{11}^{(2)} \mp T_{2122}, \\
\Omega_7^{\pm} &= t_{11}^{(3)} \mp T_{2212} \qquad \Omega_8^{\pm} = t_{11}^{(4)} \mp T_{2221}.
\end{aligned} \tag{7}$$

By combining them further into the linear combinations that are eighenvectors for h_2, h_3 and h_4 one ends up with sixteen combinations with the eighenvalues ± 1 (see Appendix). Figure 1 shows the two-dimensional projection of this four-dimensional diagram.

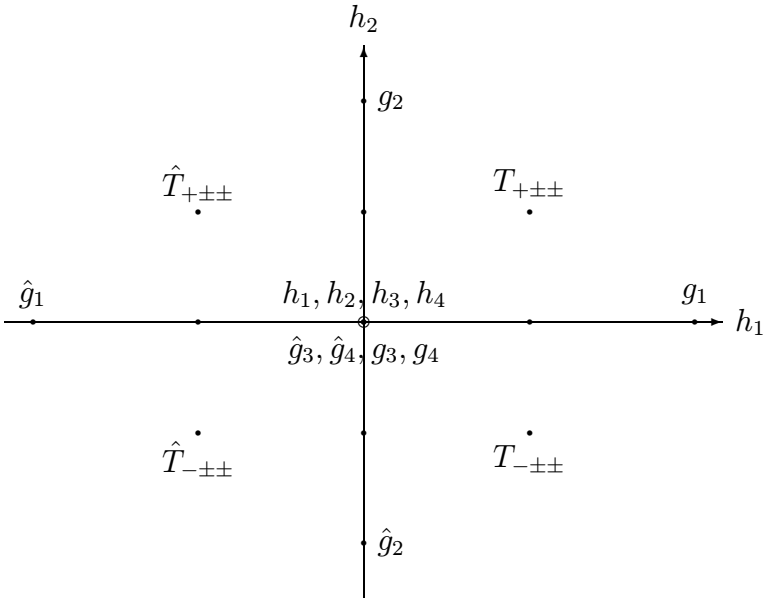

Fig. 1. Generators of $O(4,4)$.

.

One can easily get convinced that sixteen generators h, g and $T_{\pm\pm\pm}$ form some closed algebra. It is this algebra that determines the corresponding quaternionic manifold. Namely, for each its generator one has scalar field in the appropriate nonlinear σ-model playing the role of a Goldstone one. As for the remaining generators all of them are present for the case of symmetric quaternionic manifolds only. As we know [11,12] the generator $\hat{g}_1$ is present then and only then the manifold is symmetric. But if we exclude this generator, one immediately see that among eight generators $\hat{T}_{\pm\pm\pm}$ at most four commuting generators $\hat{T}_{+\pm\pm}$ could present because their commutators with $\hat{T}_{-\pm\pm}$ give $\hat{g}_1$. Moreover, taking into account that $[\hat{g}_2, \hat{T}_{+\pm\pm}] \sim \hat{T}_{-\pm\pm}$ one must exclude the $\hat{g}_2$ as well. All this means, in particular, that from the initial four $SU(2)$ subgroups not more than two could survive in the cases of interest. Their generators are formed by the linear combinations of $g_{3,4}$, $\hat{g}_{3,4}$, $T_{+\pm\pm}$ and $\hat{T}_{+\pm\pm}$:

$$t_a = \begin{pmatrix} t^1_{22} + t^2_{11} \\ t^1_{12} - t^2_{12} \\ t^1_{11} + t^2_{22} \end{pmatrix}, \qquad t_{\dot{a}} = \begin{pmatrix} t^3_{22} + t^4_{11} \\ t^3_{12} - t^4_{12} \\ t^3_{11} + t^4_{22} \end{pmatrix}, \tag{8}$$

where $a, \dot{a} = 1, 2, 3$. The remaining six linear combinations together with $h_{1,2,3,4}$ form a singlet $T = h_1 + h_2$ and $(3, 3)$ representation:

$$T_{a\dot{a}} = \begin{pmatrix} T_{2121} & T_{2111} - T_{2122} & T_{2112} \\ T_{1121} - T_{2221} & h_1 - h_2 & T_{1112} - T_{2212} \\ T_{1221} & T_{1211} - T_{1222} & T_{1212} \end{pmatrix}. \tag{9}$$

In turn, the generators $g_{1,2}$ and $T_{+\pm\pm}$ form two triplets $(3, 1)$ and $(1, 3)$:

$$T_a = \begin{pmatrix} \Omega^+_1 - \Omega^+_6 \\ g_1 + g_2 \\ \Omega^+_2 - \Omega^+_5 \end{pmatrix}, \qquad T_{\dot{a}} = \begin{pmatrix} \Omega^+_3 - \Omega^+_8 \\ g_1 - g_2 \\ \Omega^+_4 - \Omega^+_7 \end{pmatrix}. \tag{10}$$

In this covariant under two $SU(2)$ groups notations the commutation relations have the form:

$$\begin{aligned} [T_a, T_b] &= 0, \qquad [T_a, T_{\dot{b}}] = 0, \qquad [T_{\dot{a}}, T_{\dot{b}}] = 0, \\ [T, T_a] &= T_a, \qquad [T, T_{\dot{a}}] = T_{\dot{a}}, \qquad [T, T_{a\dot{a}}] = 0, \\ [T_{a\dot{a}}, T_b] &= \delta_{ab} T_{\dot{a}}, \qquad [T_{a\dot{a}}, T_{\dot{b}}] = \delta_{\dot{a}\dot{b}} T_a, \\ [T_{a\dot{a}}, T_{b\dot{b}}] &= \varepsilon_{abc} t^c \delta_{\dot{a}\dot{b}} + \delta_{ab} \varepsilon_{\dot{a}\dot{b}\dot{c}} t^{\dot{c}}. \end{aligned} \tag{11}$$

From these relations we see that it is just the $O(3, 3) \otimes D \otimes T_{3,3}$, where $T_{3,3}$ — six translations. Let us stress that it is this subalgebra of the whole $O(4, 4)$ algebra that "survives" for all the quaternionic manifilds we are considering. For what follows it will be useful to note that as a result of $O(3, 3) \simeq SL(4)$ this algebra is equivalent to $GL(4) \otimes T_6$, where the generators t_a, $t_{\dot{a}}$, $T_{a\dot{a}}$ and T form the $GL(4)$ algebra, while six translations are transformed as skew-symmetric tensor Π^{mn}, $m, n = 1, 2, 3, 4$:

$$\begin{aligned} [T_m{}^n, T_k{}^l] &= \delta_m{}^l T_k{}^n - \delta_k{}^n T_m{}^l, \\ [T_m{}^n, \Pi^{kl}] &= \delta_m{}^k \Pi^{nl} - \delta_m{}^l \Pi^{nk}, \\ [\Pi^{mn}, \Pi^{kl}] &= 0. \end{aligned} \tag{12}$$

Now let us turn to the whole group F_4. The noncompact version of this group that we need contains as a maximal regular (noncompact) subgroup an $O(4,5)$ group formed by the $O(4,4)$ group described above and by the eight generators $(\Lambda_{i\alpha}, \Lambda_{\dot\alpha\ddot\alpha})$. The remaining sixteen generators are transformed as a spinor representation of this $O(4,5)$ group and in our basis take the form: $(\Lambda_{i\dot\alpha}, \Lambda_{i\ddot\alpha}, \Lambda_{\alpha\dot\alpha}, \Lambda_{\alpha\ddot\alpha})$. Note, that all these Λ-generators are complex ones, satisfying the pseudoreality condition $(\Lambda_{i\alpha})^* = \Lambda^{i\alpha} = \varepsilon^{ij}\varepsilon^{\alpha\beta}\Lambda_{j\beta}$ and so on. All commutation relations for these generators are given in Appendix. In the same way as for the $O(4,4)$ generators we can construct linear combinations which are eighenvectors for the $h_{1,2,3,4}$. The explicit form of such combinations also given in the Appendix. From 24 generators we get 12 combinations with $h_1 = 0$ and 6 combinations with $h_1 = 1$ (see Figure 2) as well as 6 combinations with $h_1 = -1$ which we denote as $\hat{\tilde X}_\pm$, $\hat{\tilde Y}_\pm$ and $\hat{\tilde Z}_\pm$.

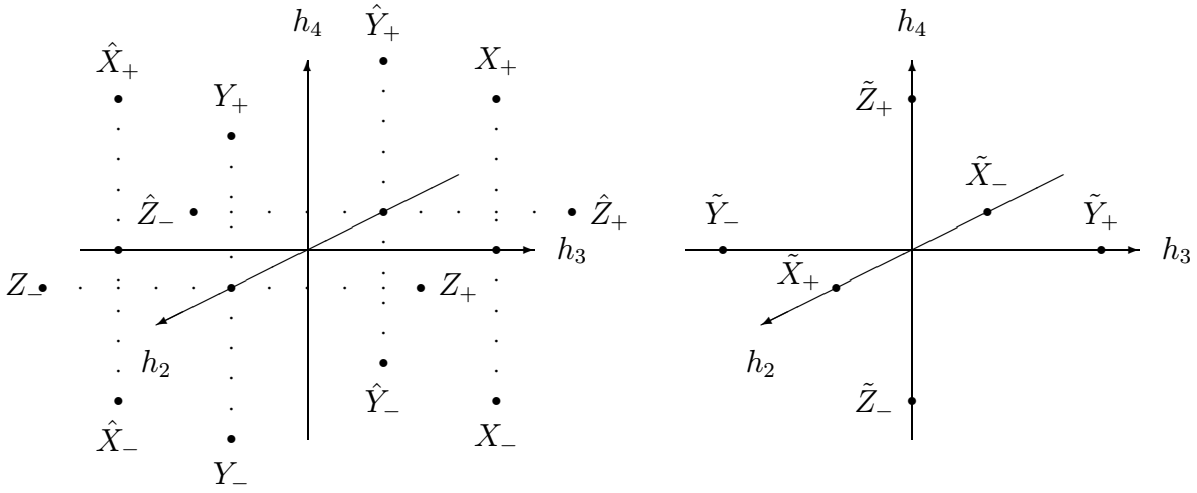

Fig. 2. Generators of $F_4/O(4,4)$ with $h_1 = 0$ (left) and $h_1 = 1$ (right).

.

Note, the letters X, Y, Z are chosen so that they match the Alekseevsky notations in [8]. As we have seen, the F_4 algebra has just one set of each type of generators (that is why we choose it as our starting point). In this, together with the generators $h_{1,2,3,4}$, $g_{1,2,3,4}$ and $T_{\pm\pm\pm}$, described before, the generators $\{X_\pm, \tilde X_\pm, Y_\pm, \tilde Y_\pm, Z_\pm, \tilde Z_\pm\}$ form the closed algebra with dimension 28, corresponding to this quaternionic manifold.

In general, the quaternionic algebra can contain different numbers of X, Y, Z generators, the whole dimensions of the algebras being, of course, multiples of four. As it has been shown [8,9] there exist two general types of quaternionic manifolds which are not symmetric ones. In the first one the X type generators are absent, while one can have the arbitrary number p of the sets $(Y_\pm, \tilde Y_\pm)$ and the arbitrary number q of the $(Z_\pm, \tilde Z_\pm)$. In the second case we have q sets of X-type generators, while the quaternionic dimensions of Y's and Z's are both equal to the $p \times d(q)$, where p is arbitrary and $d(q)$ is the dimension of the spinor representation of $O(q)$. In the following sections we will give explicit realizations for such algebras and construct the corresponding $N = 2$ supersymmetric nonlinear σ-models.

2. $W(p,q)$–model

Let us first consider the quaternionic algebras, which arise in the absence of the X-type generators. Under the two "surviving" $SU(2)$ subgroups the generators $(Y_\pm, \tilde{Y}_\pm)$ and $(Z_\pm, \tilde{Z}_\pm)$, defined previously are transformed as the bispinors $Y_{i\alpha}$ and $Z_{i\alpha}$ (note that in this section indices i, α correspond to the two new $SU(2)$ groups and do not coincide with the ones used before). The commutation relations look like:

$$\begin{aligned}
[T_{a\dot{a}}, Y_{i\alpha}] &= (\sigma^a)_i{}^j(\sigma^{\dot{a}})_\alpha{}^\beta Y_{j\beta}, \qquad [T_{a\dot{a}}, Z_{i\alpha}] = -(\sigma^a)_i{}^j(\sigma^{\dot{a}})_\alpha{}^\beta Z_{j\beta}, \\
[Y_{i\alpha}, Y_{j\beta}] &= (\sigma^a)_{ij}\varepsilon_{\alpha\beta}T_a + \varepsilon_{ij}(\sigma^{\dot{a}})_{\alpha\beta}T_{\dot{a}}, \\
[Z_{i\alpha}, Z_{j\beta}] &= -(\sigma^a)_{ij}\varepsilon_{\alpha\beta}T_a + \varepsilon_{ij}(\sigma^{\dot{a}})_{\alpha\beta}T_{\dot{a}}, \\
[T_a, Y_{i\alpha}] &= [T_{\dot{a}}, Y_{i\alpha}] = [T_a, Z_{i\alpha}] = [T_{\dot{a}}, Z_{i\alpha}] = 0, \\
[T, Y_{i\alpha}] &= Y_{i\alpha} \qquad [T, Z_{i\alpha}] = Z_{i\alpha},
\end{aligned} \tag{13}$$

where σ^a, $\sigma^{\dot{a}}$ are Pauli matrices. Under the $GL(4)$ group mentioned at the end of the previous section we have covariant and contravariant vectors Y^m and Z_m, correspondingly. Then the algebra takes a very simple form:

$$\begin{aligned}
[T_m{}^n, Y^k] &= -\delta_m{}^k Y^n, \qquad [T_m{}^n, Z_k] = \delta_k{}^n Z_m, \\
[Y^m, Y^n] &= \Pi^{mn}, \qquad [Z_m, Z_n] = \frac{1}{2}\varepsilon_{mnkl}\Pi^{kl}, \\
[\Pi^{mn}, Y^k] &= 0, \qquad [\Pi^{mn}, Z_k] = 0, \qquad [Y^m, Z_n] = 0.
\end{aligned} \tag{14}$$

Now, it is easy to see that there is nothing to prevent us from considering an arbitrary number of generators $Y^{n\dot{A}}$, $\dot{A} = 1, 2\cdots p$ and $Z_N{}^{\ddot{A}}$, $\ddot{A} = 1, 2\cdots q$. The commutation relations remain the same except that one now can introduce new generators $t^{[\dot{A}\dot{B}]}$ and $t^{[\ddot{A}\ddot{B}]}$, corresponding to $O(p)$ and $O(q)$ groups with evident commutation relations. In this, the generators that we have excluded are transformed as $\hat{Y}_n$ and $\hat{Z}^n$, correspondingly. It appears that in the case when, say, $q = 0$, i.e. the generators of Z-type are absent, we may include the generators $\hat{Y}_n$ into the algebra (together with the previously excluded $\hat{g}_{1,2}$ and $T_{-\pm\pm}$) extending the algebra up to the full $O(4, 4+p)$ one. Thus, in the absence of Z-type generators we have just the well known $O(4, 4+p)/O(4) \otimes O(4+p)$ nonlinear σ-model. It allows one to get a very simple description of such a model which turns out to be convenient for the generalization on the $q > 0$ case.

The simplest way to describe the required σ-model is to introduce the $p+8$ hypermultiplets $(\Phi_a{}^{\hat{A}}, \Lambda_\alpha{}^{\hat{A}})$, $a = 1, 2, 3, 4$, $\hat{A} = 1, 2\cdots p+8$, $\alpha = 1, 2$, where Φ are the real scalar fields and Λ — Majorana spinors, satisfying the following constraints:

$$\Phi_a{}^{\hat{A}} \cdot \Phi_{b\hat{A}} = -\delta_{ab}, \qquad \Phi_a{}^{\hat{A}} \cdot \Lambda_{\alpha\dot{A}} = 0. \tag{15}$$

In this, the theory has the local $O(4)$ invariance, the corresponding covariant derivatives being, for example:

$$\begin{aligned}
D_\mu \Phi_a{}^{\hat{A}} &= \partial_\mu \Phi_a{}^{\hat{A}} - (\Phi_a \partial_\mu \Phi_b)\Phi_b{}^{\hat{A}}, \qquad \Phi_a D_\mu \Phi_b = 0, \\
D_\mu \Lambda^{\hat{A}} &= \partial_\mu \Lambda^{\hat{A}} + \frac{1}{2}(\Phi_a \partial_\mu \Phi_b)\bar{\sigma}^{ab}\Lambda^{\hat{A}}.
\end{aligned} \tag{16}$$

In this notations the Lagrangian of the interaction of such hypermultiplets with $N = 2$ supergravity has the form:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{N=2sugra} + \mathcal{L}_{hyper},\\ \mathcal{L}_{hyper} &= \frac{i}{2}\bar{\Lambda}\gamma^\mu D_\mu\Lambda + \frac{1}{2}D_\mu\Phi_a D^\mu\Phi_a - \frac{1}{2}\bar{\Lambda}\gamma^\mu\gamma^\nu D_\nu\Phi_a\bar{\tau}^a\Psi_\mu.\end{aligned} \tag{17}$$

Here we introduced four matrices $(\tau_a)_{i\alpha}$ and $(\bar{\tau}_a)^{\alpha i}$ such that:

$$(\tau^a)_{i\alpha}(\bar{\tau}^b)^{\alpha j} + (a \leftrightarrow b) = 2\delta^{ab}\delta_i{}^j \tag{18}$$

in this,

$$(\sigma^{ab})_i{}^j = \frac{1}{2}((\tau^a)_{i\alpha}(\bar{\tau}^b)^{\alpha j} - (a \leftrightarrow b)), \qquad (\bar{\sigma}^{ab})_\alpha{}^\beta = \frac{1}{2}((\tau^a)_{i\alpha}(\bar{\tau}^b)^{\beta i} - (a \leftrightarrow b)). \tag{19}$$

Now let us rewrite this Lagrangian and the supertransformations in terms of independent scalar and spinor fields. For that purpose we introduce a kind of light cone variables:

$$\Phi_a{}^{\hat{A}} = (x_a{}^m + E_{am}, x_a{}^m - E_{am}, E_{am}Y^{m\dot{A}}), \quad m = 1,2,3,4, \quad \dot{A} = 1,2\cdots p. \tag{20}$$

Now, by introducing a new field

$$\Pi^{mn} = (E^{-1})^{am}x_a{}^n - (E^{-1})^{an}x_a{}^m \tag{21}$$

we can solve the constraint for $x_a{}^m$:

$$x_a{}^m = \frac{1}{2}(E^{-1})^{am} + \frac{1}{4}E_{an}Y^{n\dot{A}}Y^{m\dot{A}} + \frac{1}{2}E_{an}\Pi^{nm} \tag{22}$$

and rewrite all the bosonic expressions in terms of the E_{am}, Π^{mn} and $Y^{m\dot{A}}$. In this, the fields E_{am} realise the nonlinear σ-model $GL(4)/O(4)$, while Π^{mn} enter the Lagrangian through the derivatives $\partial_\mu\Pi^{mn}$ only, the translations $\Pi^{mn} \to \Pi^{mn} + \Lambda^{mn}$ being the global symmetry.

Analogously, we solve the constraint for the spinor fields introducing new fields $\Lambda^{\hat{A}} = (\xi^m + \chi_m, \xi^m - \chi_m, \Lambda^{\dot{A}})$. This gives

$$\xi^m = -(E^{-1})^{am}x_a{}^n\chi_n + \frac{1}{2}Y^{m\dot{A}}\Lambda_{\dot{A}}. \tag{23}$$

Here two changes of variables $\chi_m \to \frac{1}{\sqrt{2}}E_{am}\chi^a$ and $\Lambda \to \Lambda + Y^m\chi_m$ are necessary to have canonical kinetic terms for spinors.

In terms of these new variables the Lagrangian could be written as:

$$\begin{aligned}\mathcal{L}_{hyper} &= \frac{i}{2}\bar{\chi}\gamma^\mu D_\mu\chi + \frac{1}{2}(S_\mu^+)^2 + \frac{1}{2}(P_\mu)^2 + \frac{i}{2}\bar{\Lambda}\gamma^\mu D_\mu\Lambda + \frac{1}{2}E_{ma}E_{na}\partial_\mu Y^m\partial_\mu Y^n\\ &-\frac{1}{2}\bar{\chi}^a\gamma^\mu\gamma^\nu(S_\nu{}^+ + P_\nu)_{ab}\bar{\tau}^b\Psi_\mu - \frac{1}{2}\bar{\Lambda}\gamma^\mu\gamma^\nu E_{ma}\partial_\nu Y^m\bar{\tau}^a\Psi_\mu\\ &-\frac{i}{8}\varepsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu\gamma_5\gamma_\nu(S_\rho{}^- + P_\rho)_{ab}\sigma^{ab}\Psi_\sigma + \frac{i}{8}\bar{\chi}^c\gamma^\mu(S_\mu^- + P_\mu)_{ab}\bar{\sigma}^{ab}\chi^c\\ &-\frac{i}{2}\bar{\chi}^a\gamma^\mu(S_\mu^- - P_\mu)_{ab}\chi^b + \frac{i}{8}\bar{\Lambda}\gamma^\mu(S_\mu^- - P_\mu)_{ab}\bar{\sigma}^{ab}\Lambda + \frac{i}{2}\bar{\chi}^a\gamma^\mu E_{ma}\partial_\mu Y^m\Lambda.\end{aligned} \tag{24}$$

Here we use the following notations:

$$(S^{\pm}_{\mu})_{ab} = \frac{1}{2}(E^{-1}\partial_{\mu}E \pm \partial_{\mu}EE^{-1})_{ab}, \qquad (P_{\mu})_{ab} = E_{ma}E_{nb}(\partial_{\mu}\Pi^{mn} - \frac{1}{2}(Y^m \overset{\leftrightarrow}{\partial}_{\mu} Y^n)). \quad (25)$$

In turn, the supertransformations look like:

$$\begin{aligned}
\delta\Psi_{\mu} &= 2D_{\mu}\eta - \frac{1}{2}(S^{-}_{\mu} + P_{\mu})_{ab}\sigma^{ab}\eta, \\
\delta\chi^{a} &= -i\gamma^{\mu}(S^{+}_{\mu} + P_{\mu})_{ab}\bar{\tau}^{b}\eta, \qquad \delta\Lambda = -i\gamma^{\mu}E_{ma}\partial_{\mu}Y^{m}\bar{\tau}^{a}\eta, \\
\delta E_{ma} &= (\bar{\chi}^{b}E_{bm}\bar{\tau}^{a}\eta), \qquad\qquad \delta Y^{m} = (\bar{\Lambda}(E^{-1})^{ma}\bar{\tau}^{a}\eta), \\
\delta\Pi^{mn} &= (\bar{\chi}^{a}(E^{-1})_{a}{}^{[m}(E^{-1})_{b}{}^{n]}\bar{\tau}^{b}\eta) + (\bar{\Lambda}Y^{[m}(E^{-1})^{n]a}\bar{\tau}^{a}\eta).
\end{aligned} \quad (26)$$

This Lagrangian, besides the $GL(4)$ transformations (acting on the "world" indices m, n only), is invariant under the six translations $\Pi^{mn} \to \Pi^{mn} + \Lambda^{mn}$ as well as under the following transformations:

$$\delta Y^{m} = \xi^{m}, \qquad \delta\Pi^{mn} = Y^{[m}\xi^{n]}. \quad (27)$$

Now it is an easy task to add to this model additional hypermultiplets whose scalar fields are transformed as $Z_m{}^{\ddot{A}}$, $\ddot{A} = 1, 2 \cdots q$, under $GL(4)$. All one needs for that is to complete the Lagrangian with:

$$\begin{aligned}
\Delta L_{hyper} &= \frac{i}{2}\bar{\Sigma}\gamma^{\mu}D_{\mu}\Sigma + \frac{\Delta}{2}(E^{-1})^{am}(E^{-1})^{an}\partial_{\mu}Z_{m}\partial_{\mu}Z_{n} \\
&-\frac{1}{2}\bar{\Sigma}\gamma^{\mu}\gamma^{\nu}\sqrt{\Delta}(E^{-1})^{am}\partial_{\nu}Z_{m}\bar{\tau}^{a}\Psi_{\mu} + \frac{i}{8}\bar{\Sigma}\gamma^{\mu}(S^{-}_{\mu} - P_{\mu})_{ab}\bar{\sigma}^{ab}\Sigma \\
&+\frac{i}{2}\bar{\chi}^{a}\gamma^{\mu}\sqrt{\Delta}(E^{-1})^{am}\partial_{\mu}Z_{m}\Sigma + \frac{i}{2}\bar{\chi}^{a}\gamma^{\mu}\sqrt{\Delta}(E^{-1})^{bm}\partial_{\mu}Z_{n}\bar{\sigma}^{ab}\Sigma,
\end{aligned} \quad (28)$$

where $\Delta = det(E_{ma})$, and the supertransformations with

$$\begin{aligned}
\delta\Sigma &= -i\gamma^{\mu}\sqrt{\Delta}(E^{-1})^{am}\partial_{\mu}Z_{m}\bar{\tau}^{a}\eta, \\
\delta Z_{m} &= (\bar{\Sigma}\sqrt{\Delta}(E^{-1})^{am}\bar{\tau}^{a}\eta), \\
\delta'\Pi^{mn} &= \frac{1}{2\sqrt{\Delta}}(\bar{\Sigma}\varepsilon^{mnpq}E_{pa}Z_{q}\bar{\tau}^{a}\eta).
\end{aligned} \quad (29)$$

Besides, one has to change the definition for the P_{μ} by

$$(P_{\mu})_{ab} = E_{am}E_{bn}(\partial_{\mu}\Pi^{mn} - \frac{1}{2}(Y^m \overset{\leftrightarrow}{\partial}_{\mu} Y^n) + \frac{1}{4}\varepsilon^{mnpq}(Z_p\partial_{\mu}Z_q)). \quad (30)$$

Now, besides the bosonic transformations given above, the whole Lagrangian is also invariant under:

$$\delta Z_{m} = \eta_{m}, \qquad \delta\Pi^{mn} = \frac{1}{2}\varepsilon^{mnpq}Z_{p}\eta_{q}. \quad (31)$$

It is an easy task to check that these bosonic transformations have the commutation relations which coincide with the ones given at the beginnig of this section. Thus the bosonic part of the Lagrangian constructed is indeed the nonlinear σ-model, corresponding to the quaternionic manifold of the desired form.

3. $V(p,0)$–model

As a preliminary step to the full $V(p,q)$–model, let us consider first one more simple case — the one without Y and Z multiplets. Under the two "surviving" $SU(2)$ subgroups the X generators form two triplets X_a, $X_{\dot a}$ and two singlets X and $\tilde X$. In this, the minimal quaternionic algebra could be extended so that to include the X_a, $X_{\dot a}$ and X generators together with the $O(3,3)\otimes D\otimes T_{3,3}$ ones, defined earlier. The commutation relations have the form:

$$\begin{array}{rcllll} [T_{a\dot a},X_b] &=& \delta_{ab}X_{\dot a}, & [T_{a\dot a},X_{\dot b}]=\delta_{\dot a\dot b}X_a, & & \\ [X_a,X_b] &=& -\varepsilon_{abc}t^c & [X_{\dot a},X_{\dot b}]=\varepsilon_{\dot a\dot b\dot c}t^{\dot c}, & [X_a,X_{\dot a}]=-T_{a\dot a}, & \\ [X_a,T_b] &=& \delta_{ab}X, & [X_{\dot a},T_{\dot b}]=-\delta_{\dot a\dot b}X, & [X_a,X]=T_a, & [X_{\dot a},X]=T_{\dot a}. \end{array} \tag{32}$$

From these relations one can see that the generators X_a and $X_{\dot a}$ together with the t_a, $t_{\dot a}$ and $T_{a\dot a}$ form the $O(3,4)$ algebra, while the X generators together with the T_a and $T_{\dot a}$ form seven translations $T_{3,4}$. Moreover, there is nothing to prevent us from considering the generalization on the case of arbitrary number p of generators X_a, $X_{\dot a}$ and X. In this case the algebra consists of the $O(3,p+3)$ group, scale transformations D, and $p+6$ translations. In the absence of the Y, Z multiplets this algebra can be completed up to the whole $O(4,p+4)$ one. So, we start once more with the usual non-linear σ-model $O(4,p+4)/O(4)\otimes O(p+4)$ described by the real scalar fields $\Phi_{\hat a}{}^{\hat A}$, $\hat a=1,2,3,4$, $\hat A=0,1,2\cdots p+7$, but this time (in order to preserve linearly realized $O(3,3+p)$ symmetry) we will solve the constraint $\Phi_{\hat a}\Phi_{\hat b}=-\delta_{\hat a\hat b}$ only partially introducing the following parameterization:

$$\Phi_{\hat a}{}^{\hat A}=\left(\begin{array}{c|c|c} \phi_- & X^A & \phi_+ \\ \hline L_a^- & L_a{}^A & L_a^+ \end{array}\right), \tag{33}$$

where now $a=1,2,3$ and $A=1,2,\cdots p+6$. In this notations the constraint takes the form:

$$\begin{array}{rcl} -L_a^-L_b^- + (L_aL_b) + L_a^+L_b^+ &=& -\delta_{ab}, \\ -(\phi_-)^2+(X)^2+(\phi_+)^2 &=& -1, \\ -\phi_-L_a^- + (L_aX) + \phi_+L_a^+ &=& 0. \end{array} \tag{34}$$

As usual, we have an $O(4)\simeq O(3)\otimes O(3)$ local invariance, so one can use one of these $O(3)$ groups in order to set $L_a^+=L_a^-$. Then the first equation becomes $(L_aL_b)=-\delta_{ab}$, i.e. just the usual constraint for the $O(3,p+3)/O(3)\otimes O(p+3)$ non-linear σ-model! Two other equations allow one to rewrite all the formulas in terms of $L_a{}^A$, X^A and, say, $\Phi=(\phi_--\phi_+)$. The bosonic part of the corresponding quaternionic model looks like:

$$\mathcal{L}_B=\frac{1}{2}\frac{(\partial_\mu\Phi)^2}{\Phi^2}+\frac{1}{2}\Phi^2((\partial_\mu X)^2+2(\vec L\partial_\mu X)^2)+\frac{1}{2}D_\mu\vec L D_\mu\vec L, \tag{35}$$

where D_μ is $O(3)$ covariant derivative. As we see the X scalar fields enter the Lagrangian through the derivatives only, so the Lagrangian is trivially invariant under the translations

$X^A \to X^A + \Lambda^A$, as well as $O(3,p+3)$ rotations and scale transformations. By rather long but strightforward calculations one can check that this Lagrangian is invariant also under the "special conformal" transformations of the form:

$$\begin{aligned}\delta\Phi &= \Phi(X\Lambda), \qquad \delta\vec{L}^A = (\vec{L}X)\Lambda^A - (\vec{L}\Lambda)X^A,\\ \delta X^A &= \frac{1}{2}\Phi^{-2}\Lambda^A - (X\Lambda)X^A + \frac{1}{2}(XX)\Lambda^A + \Phi^{-2}(\vec{L}\Lambda)\vec{L}^A,\end{aligned} \tag{36}$$

which, together with the ones mentioned above, form the full $O(4,p+4)$ algebra.

Analogously, by solving partially the constraint for the spinor fields and making field redefinitions to bring the fermionic kinetic terms to the canonical forms, one can express the fermionic part of the corresponding quaternionic model in terms of the spinors χ^i and Ω^{iA}, satisfying $\vec{L}^A\Omega^{iA} = 0$. The results are:

$$\begin{aligned}\mathcal{L}_F &= -\frac{1}{2}\bar{\Omega}^i\gamma^\mu\gamma^\nu[\Phi(\partial_\nu X + \vec{L}(\vec{L}\partial_\nu X))\delta_i{}^j + D_\nu L_i{}^j]\Psi_{\mu j}\\ &\quad -\frac{1}{2}\bar{\chi}^i\gamma^\mu\gamma^\nu(\Phi^{-1}\partial_\nu\Phi\delta_i{}^j - \Phi(L_i{}^j\partial_\nu X))\Psi_{\mu j} + \frac{i}{4}\Phi\varepsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu{}^i\gamma_5\gamma_\nu(L_i{}^j\partial_\rho X)\Psi_{\sigma j}\\ &\quad +\frac{i}{4}\Phi\bar{\chi}_i\gamma^\mu(L_i{}^j\partial_\mu X)\chi^j + \frac{i}{4}\Phi\bar{\Omega}_i\gamma^\mu(L_i{}^j\partial_\mu X)\Omega^j - i\Phi\bar{\chi}_i\gamma^\mu\partial_\mu X\Omega^i\end{aligned} \tag{37}$$

and

$$\begin{aligned}\delta\Psi_{\mu i} &= 2D_\mu\eta_i + \Phi(L_i{}^j\partial_\mu X)\eta_j,\\ \delta\chi^i &= -i\gamma^\mu[\Phi^{-1}\partial_\mu\Phi\delta_i{}^j - \Phi(L_i{}^j\partial_\mu X)]\eta_j,\\ \delta\Omega^i &= -i\gamma^\mu[\Phi(\partial_\mu X + \vec{L}(\vec{L}\partial_\mu X))\delta_i{}^j + D_\mu L_i{}^j]\eta_j,\\ \delta\Phi &= \Phi(\bar{\chi}^i\eta_i) \qquad \delta\vec{L} = (\bar{\Omega}^i(\vec{\tau})_i{}^j\eta_j),\\ \delta X &= \Phi^{-1}[(\bar{\Omega}^i\eta_i) + (\bar{\chi}^i L_i{}^j\eta_j)].\end{aligned} \tag{38}$$

4. $V(p,q)$–model

Thus, both for the YZ-sector and for the X-sector we have managed to construct rather simple realizations having essentially the same hidden sector (the $O(4,4)/O(4)\otimes O(4)$ model) but in the very different parameterizations. So, to join these models together we have to reduce them to the forms having identical parameterization. Let us start with the X-sector. In this case all that we need is to solve the remaining constraints $L^aL^b = -\delta^{ab}$ and $\vec{L}\Omega = 0$ exactly in the same way as we have done it for the initial $O(4,m)$ model above. In this, the fields $\vec{L}^A$ give y_{ma}, $\pi^{[mn]}$ and X^{mA}, where now $A = 7,8,\dots p+3$, $a,m = 1,2,3$, while X^A give l^m, π_m and X^A, correspondingly. The bosonic Lagrangian looks like

$$\begin{aligned}\mathcal{L}^B &= \frac{1}{2}(\partial_\mu\varphi)^2 + \frac{1}{2}(S^+_{\mu ab})^2 + \frac{1}{2}(P_{\mu ab})^2 + 4e^{2\varphi}g_{mn}L^n_\mu L^n_\nu +\\ &\quad +\frac{1}{4}e^{2\varphi}g^{mn}U_{\mu m}U_{\mu n} + \frac{1}{2}e^{2\varphi}(D_\mu X)^2 + \frac{1}{2}g_{mn}\partial_\mu X^m\partial_\mu X^n,\end{aligned} \tag{39}$$

where $g_{mn} = y_{ma}y_{na}$, $g^{mn} = y^{ma}y^{na}$, $\Phi = e^{\varphi}$ and we have introduced the following notations:

$$\begin{aligned}
S^{\pm}_{\mu ab} &= \frac{1}{2}[y^{ma}\partial_\mu y_{mb} \pm y^{mb}\partial_\mu y_{ma}], \qquad U_{\mu m} = \partial_\mu \pi_m, \\
L^m_\mu &= \partial_\mu l^m + \frac{1}{2}\pi^{mn}U_{\mu n} + \frac{1}{2}X^n D_\mu X - \frac{1}{4}X^m X^n U_{\mu n}, \\
P_{\mu ab} &= y_{ma}\left\{\partial_\mu \pi^{mn} + \frac{1}{2}(X^m \overset{\leftrightarrow}{\partial_\mu} X^n)\right\} y_{nb}, \\
Q^{a\pm}_\mu &= y_{ma}L^m_\mu \pm \frac{1}{4}y^{ma}U_{\mu m}, \qquad D_\mu X = \partial_\mu X + X^m U_{\mu m}.
\end{aligned} \tag{40}$$

This Lagrangian, besides the $GL(3)$ group acting on the "world" indices m, n, scale transformations and trivial translations for the fields l^m, π_m and X^A, is invariant under the following global transformations:

$$\begin{aligned}
\delta X^m &= \zeta^m, \quad \delta X = -\zeta^m \pi_m, \quad \delta \pi^{mn} = X^{[m}\zeta^{n]}, \quad \delta l^m = \frac{1}{2}\zeta^m X, \\
\delta \pi^{mn} &= -2\Lambda^{mn}, \qquad \delta l^m = \Lambda^{mn}\pi_n.
\end{aligned} \tag{41}$$

Analogously, by solving the constraint for the spinor field in terms of $(\lambda^{ia}, \Omega^{iA})$ one can find the fermionic part of the Lagrangian:

$$\begin{aligned}
\mathcal{L}_F &= \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}\bar\Psi_{\mu i}\gamma_5\gamma_\nu D_\rho \Psi_{\sigma i} + \frac{i}{2}\bar\lambda^i_a \hat D \lambda^i_a + \frac{i}{2}\bar\chi^i \hat D \chi^i + \frac{i}{2}\bar\Omega^i \hat D \Omega_i - \\
&-\frac{1}{2}\bar\chi^i\gamma^\mu\gamma^\nu\left\{\partial_\nu\varphi\delta_i{}^j - 2e^\varphi Q^{a+}_\nu \tau^a{}_i{}^j\right\}\Psi_{\mu j} - \\
&-\frac{1}{2}\bar\lambda^i_a\gamma^\mu\gamma^\nu\left\{(S^+_\nu + P_\nu)_{ab}\tau^b{}_i{}^j + 2e^\varphi Q^{a-}_\nu \delta_i{}^j\right\}\Psi_{\mu j} + \\
&+\frac{i}{2}(S^-_\mu + P_\mu)_{ab}(\bar\lambda^i_a\gamma_\mu\lambda^i_b) + 2ie^\varphi Q^{a-}_\mu(\bar\lambda^i_a\gamma_\mu\chi^i) \\
&-(\bar\lambda^i_a\gamma^\mu\Omega^i)y_{ma}\partial_\mu X^m - i(\bar\chi^i\gamma^\mu\Omega^i)e^\varphi D_\mu X - \\
&-\frac{1}{2}\bar\Psi_{\mu i}\gamma^\nu\gamma^\mu\left\{e^\varphi D_\nu X\delta_i{}^j + y_{ma}\partial_\nu X^m \tau^a{}_i{}^j\right\}\Omega^j,
\end{aligned} \tag{42}$$

where D-derivatives for the fermions have the following form:

$$(D_\mu)_i{}^j = D^G_\mu \delta_i{}^j \pm \frac{1}{4}\varepsilon^{abc}(S^-_\mu - P_\mu)_{ab}(\tau^c)_i{}^j + e^\varphi Q^{a+}_\mu(\tau^a)_i{}^j \tag{43}$$

Here the derivatives of $\Psi_{\mu i}$ and η_i have the sign "-" and derivatives of χ^i, λ^i_a and Ω^{iA} – the sign "+".

In this, the total Lagrangian is invariant under the following local $N = 2$ supertransformations:

$$\begin{aligned}
\delta\Psi_{\mu i} &= 2D_\mu\eta_i, \qquad \delta\chi^i = -i\gamma_\mu\left\{\partial_\mu\varphi\delta_i{}^j - 2e^\varphi Q^{a+}_\mu(\tau^a)_i{}^j\right\}\eta_j, \\
\delta\lambda^i_a &= -i\gamma^\mu\left\{(S^+_\mu + P_\mu)_{ab}(\tau^b)_i{}^j + 2e^\varphi Q^{a-}_\mu\delta_i{}^j\right\}\eta_j,
\end{aligned}$$

$$
\begin{aligned}
\delta\Omega^i &= -i\gamma^\mu\left\{e^\varphi D_\mu X\delta^j_i + y_{ma}\partial_\mu X^m(\tau^a)_i{}^j\right\}\eta_j,\\
\delta\varphi &= (\bar\chi^i\eta_i), \qquad \delta y_{ma} = y_{mb}(\bar\lambda^i_b(\tau^a)_i{}^j\eta_j),\\
\delta\pi^{mn} &= \frac{1}{2}(\bar\lambda^i_a(\tau^a)_i{}^j\eta_j)[y^{ma}y^{nb} - (m\leftrightarrow n)] - \frac{1}{2}\left\{X^m\delta X^n - (m\leftrightarrow n)\right\},\\
\delta\pi_m &= -e^{-\varphi}y_{ma}[(\bar\lambda^i_a\eta_i) + (\bar\chi^i(\tau^a)_i{}^j\eta_j)],\\
\delta l^m &= \frac{1}{4}e^{-\varphi}y^{ma}[(\bar\lambda^i_a\eta_i) - (\bar\chi^i(\tau^a)_i{}^j\eta_j)] - \frac{1}{2}\pi^{mn}\delta\pi_n - \frac{1}{2}e^{-\varphi}X^m(\bar\Omega^i\eta_i) - \frac{1}{4}X^mX^n\delta\pi_n,\\
\delta X^m &= y^{ma}(\bar\Omega^i(\tau^a)_i{}^j\eta_j), \qquad \delta X = e^{-\varphi}(\Omega^i\eta_i) - X^m\delta\pi_m.
\end{aligned}
\tag{44}
$$

Note, that in what follows by the hidden sector we will mean the part of this model with the bosonic fields $(\varphi, Y_{ma}, \pi^{mn}, l^m, \pi_n)$ and the fermionic ones (χ^i, λ^{ia}) and the formulae given above where the fields $(X^A, X^{mA}, \Omega^{iA})$ are set to zero.

Now, let us turn to the YZ-sector. The bosonic part of the hidden sector consists of the field E_{ma}, corresponding to the non-linear σ-model $GL(4,R)/O(4)$ and antisymmetric tensor Π^{mn}. In this formulation the theory has local $O(4)$ invariance, so one can use one of its $O(3)$ subgroup to bring matrix E to block-triangle form. We will use the following concrete parameterization:

$$
E = \begin{pmatrix} \sqrt{\Delta}e^{\varphi/2} & 0 \\ -\frac{1}{2}\sqrt{\Delta}e^{\varphi/2}\varepsilon_{mnk}\pi^{nk} & \frac{1}{\sqrt{\Delta}}e^{\varphi/2}y_{ma} \end{pmatrix}, \qquad \Pi = \begin{pmatrix} 0 & -2l^m \\ 2l^n & \frac{1}{2}\varepsilon^{mnk}\pi_k \end{pmatrix}, \tag{45}
$$

where now $m, a = 1, 2, 3$, $\Delta = det(y_{ma})$. In this, the hidden sector takes exactly the same form as in the formulas given above (of course, in the absence of the fields X, X^m and Ω^i). Scalar fields of Y and Z multiplets, which are now $(Y, Y_m)^{\dot A}$ and $(Z, Z^m)^{\ddot A}$), give the following contribution to the bosonic Lagrangian:

$$
\begin{aligned}
\mathcal{L}^B &= \frac{e^\varphi}{2\Delta}(\partial_\mu Y)^2 + e^\varphi\frac{\Delta}{2}(D_\mu Y_m)(D_\mu Y_n)g^{mn} + \\
&\quad + e^\varphi\frac{\Delta}{2}(D_\mu Z)^2 + \frac{e^\varphi}{2\Delta}(D_\mu Z^m)(D_\mu Z^n)g_{mn},
\end{aligned}
\tag{46}
$$

where

$$
\begin{aligned}
D_\mu Y_m &= \partial_\mu Y_m - \varepsilon_{mnk}\pi^{nk}\partial_\mu Y, \qquad D_\mu Z^m = \partial_\mu Z^m,\\
D_\mu Z &= \partial_\mu Z + \varepsilon_{mnk}\pi^{mn}\partial_\mu Z^k.
\end{aligned}
\tag{47}
$$

At the same time our definitions for $U_{\mu m}$ and L^m_μ change to:

$$
U_{\mu m} = \partial_\mu\pi_m + (Y_m \overset{\leftrightarrow}{\partial_\mu} Y) - \frac{1}{2}\varepsilon_{mnk}(Z^n \overset{\leftrightarrow}{\partial_\mu} Z^k), \tag{48}
$$

$$
L^m_\mu = \partial_\mu l^m + \frac{1}{2}\pi^{mn}U_{\mu n} - \frac{1}{8}\varepsilon^{mnk}(Y_n \overset{\leftrightarrow}{\partial_\mu} Y_k) + \frac{1}{4}(Z^m \overset{\leftrightarrow}{\partial_\mu} Z). \tag{49}
$$

The resulting bosonic Lagrangian, besides the usual $GL(3,R)$, scale transformations and trivial translations for the fields l^m and π_m, is invariant under the following global

transformations:

$$\begin{aligned}
\delta\pi^{mn} &= -2\Lambda^{mn}, \quad \delta l^m = \Lambda^{mn}\pi_n, \quad \delta Y_m = -\varepsilon_{mnk}\Lambda^{nk}Y, \quad \delta Z = \varepsilon_{mnk}\Lambda^m Z^k, \\
\delta Y &= \xi, \quad \delta\pi_m = \xi Y_m, \qquad \delta Y_m = \xi_m, \quad \delta l^m = \frac{1}{4}\varepsilon^{mnk}\xi_n Y_k, \quad \delta\pi_m = -\xi_m Y, \\
\delta Z &= \eta^m, \quad \delta l^m = \frac{1}{4}\eta Z^m, \qquad \delta Z^m = \eta^m, \quad \delta l^m = -\frac{1}{4}\eta^m Z, \quad \delta\pi_m = \varepsilon_{mnk}\eta^n Z^k.
\end{aligned} \tag{50}$$

The fermionic Lagrangian, containing the fields of the Y- and Z-multiplets, has the following form:

$$\begin{aligned}
\mathcal{L}^F &= \frac{i}{2}\bar{\Lambda}^i\hat{D}\Lambda^i - \frac{1}{2}e^{\varphi/2}\bar{\Psi}_{\mu i}\gamma^\nu\gamma^\mu(V_\nu)_i{}^j\Lambda^j + \\
&+\frac{i}{2}\bar{\Omega}^i\hat{D}\Omega^i - \frac{1}{2}e^{\varphi/2}\bar{\Psi}_{\mu i}\gamma^\nu\gamma^\mu(W_\nu)_i{}^j\Sigma^j - \\
&-\frac{i}{2}e^{\varphi/2}\bar{\chi}^i\gamma^\mu(V_\mu)_i{}^j\Lambda^j - \frac{i}{2}e^{\varphi/2}\bar{\chi}^i\gamma^\mu(W_\mu)_i{}^j\Sigma^j - \\
&-\frac{i}{2}e^{\varphi/2}\bar{\lambda}^i_a\gamma^\mu(V_\mu)_j{}^i(\tau^a)_k{}^j\Lambda^k + \frac{i}{2}e^{\varphi/2}\bar{\lambda}^i_a\gamma^\mu(W_\mu)_j{}^i(\tau^a)_k{}^j\Sigma^k,
\end{aligned} \tag{51}$$

where D_μ for the Λ^i and Σ^i are the same as one for χ^i and

$$\begin{aligned}
(V_\mu)_i{}^j &= \frac{1}{\sqrt{\Delta}}\partial_\nu Y\delta_i{}^j + \sqrt{\Delta}D_\nu Y_m y^{ma}(\tau^a)_i{}^j, \\
(W_\mu)_i{}^j &= \sqrt{\Delta}D_\mu Z\delta_i{}^j + \frac{1}{\sqrt{\Delta}}D_\mu Z^m y_{ma}(\tau^a)_i{}^j,
\end{aligned} \tag{52}$$

while the supertransformations for the fields of Y and Z hypermultiplets are the following:

$$\begin{aligned}
\delta Y &= \sqrt{\Delta}e^{-\varphi/2}(\bar{\Lambda}^i\eta_i), \qquad \delta Z^m = \sqrt{\Delta}e^{-\varphi/2}y^{ma}(\bar{\Sigma}^i(\tau^a)_i{}^j\eta_j), \\
\delta Z &= \frac{1}{\sqrt{\Delta}}e^{-\varphi/2}(\bar{\Sigma}^i\eta_i) - \varepsilon_{mnk}\pi^{mn}\delta Z^k, \\
\delta Y_m &= \frac{1}{\sqrt{\Delta}}e^{-\varphi/2}y_{ma}(\bar{\Lambda}^i(\tau^a)_i{}^j\eta_j) + \varepsilon_{mnk}\pi^{mn}\delta Y, \\
\delta\Lambda^i &= -i\gamma^\mu(V_\mu)_i{}^j\eta_j, \qquad \delta\Sigma^i = -i\gamma^\mu(W_\mu)_i{}^j\eta_j.
\end{aligned} \tag{53}$$

Besides, some new terms in the supertransformation laws of the fields π_m and l^m appear:

$$\begin{aligned}
\delta' l^m &= \frac{1}{4}\varepsilon^{mnk}Y_n\delta Y_k + \frac{1}{4}Z\delta Z^m - \frac{1}{4}Z^m\delta Z, \\
\delta'\pi_m &= Y\delta Y_m - Y_m\delta Y + \varepsilon_{mnk}Z^n\delta Z^k.
\end{aligned} \tag{54}$$

Now we are ready to construct a model, containing both the X-sector and the YZ-sector, described above. The method is, starting from the X-sector, to add Y and Z-multiplets by the use of the usual Noether procedure, extending $U_{\mu m}$ and L^m_μ as in formulae

(48), (49). As a result, all the terms, which are present in the pure YZ-model, appear as well as some new, "crossing" terms, containing fermions from both X- and Y,Z-multiplets. In this, we have to introduce constant matrices $\Gamma^{A\dot{A}\ddot{A}}$, carrying all three kinds of the indices in order to connect the fields of different multiplets. The bosonic symmetries and the supersymmetry impose certain constraints on these matrices — they turn out to be γ-matrices for the $O(p)$ group. The calculations are tedious and formidable, but, at least partly, interesting results can be obtained by means of the symmetry considerations.

All the changes in the bosonic sector of unified XYZ-model as compared with pure X and YZ-sectors can be rather easily seen by exploring the symmetries of the bosonic Lagrangian. For example, consider one of the bosonic symmetries of the X-model that we denoted ζ^m (first line in (41)). The combination $D_\mu X^A$ (40) is invariant under this transformation. But, as it has been shown above, adding Y and Z-multiplets, we have to "extend" $\partial_\mu \pi_m$ to $U_{\mu m}$ according to (48). It is easy to check, that the combination $D_\mu X^A$ with such $U_{\mu m}$ is already noninvariant under the ζ^m transformations and in order to restore this symmetry we have to "extend" "covariant derivative" $D_\mu X$:

$$D_\mu X^A \to D_\mu X^A + \frac{1}{2}\Gamma^{A\dot{A}\ddot{A}}[(Y^{\dot{A}} \overset{\leftrightarrow}{\partial_\mu} Z^{\ddot{A}}) + (Y_m^{\dot{A}} \overset{\leftrightarrow}{\partial_\mu} Z^{m\ddot{A}})]. \tag{55}$$

In this, the fields Y and Z are transformed under ζ^m-transformations according to the following formulas:

$$\begin{aligned} \delta Y^{\dot{A}} &= 0, \qquad \delta Y_m^{\dot{A}} = \Gamma^{A\dot{A}\ddot{A}}\varepsilon_{mnk}\zeta^{nA}Z^{k\ddot{A}}, \\ \delta Z^{\ddot{A}} &= \Gamma^{A\dot{A}\ddot{A}}\zeta^{mA}Y_m^{\dot{A}}, \qquad \delta Z^{m\ddot{A}} = -\Gamma^{A\dot{A}\ddot{A}}\zeta^{mA}Y^{\dot{A}} \end{aligned} \tag{56}$$

that leaves "extended" $U_{\mu m}$ (48) invariant.

The requirement of the invariance of $D_\mu X$ under ζ^m-transformations as well as the requirement of the closure of the corresponding algebra leads to the constraint on the Γ-matrices:

$$\Gamma^{A\dot{A}\ddot{A}}\Gamma^{B\dot{B}\ddot{A}} + (A \leftrightarrow B) = 2\delta^{AB}\delta^{\dot{A}\dot{B}}, \tag{57}$$

i. e., just as it should be, they are the γ-matrices for the $O(p)$ group. The commutator of the two ζ^m-transformations gives a transformation, which is a part of the O(4,4) group (second line in (41)):

$$\begin{aligned} \delta\pi^{mn} &= -2\Lambda^{mn}, \qquad \delta l^m = \Lambda^{mn}\pi_n, \\ \delta Y_m &= -2\Lambda_m Y, \qquad \delta Z = 2\Lambda_m Z^m, \end{aligned} \tag{58}$$

where $\Lambda^{mn} \sim [\zeta_1^{mA}\zeta_2^{nA} - (m \leftrightarrow n)]$, $\Lambda_m = \varepsilon^{mnk}\Lambda^{nk}$ and all other fields are inert under this transformation.

Due to the fact, that, for example, field Z^m transforms nontrivially under the ζ^m-transformation, the derivative $\partial_\mu Z^m$ is noninvariant under it. In order to restore the invariance we have to "extend" this derivative to

$$D_\mu Z^{m\ddot{A}} = \partial_\mu Z^{m\ddot{A}} + \Gamma^{A\dot{A}\ddot{A}}X^{mA}\partial_\mu Y^{\dot{A}}. \tag{59}$$

A bit more formidably, but by means of the same considerations, one can obtain the corresponding expressions, invariant under the ζ^m-transformations, for the "extended" derivatives of the $Z^{\ddot{A}}$ and $Y_m^{\dot{A}}$ fields:

$$\begin{aligned} D_\mu Y_m^{\dot{A}} &= \partial_\mu Y_m^{\dot{A}} - \varepsilon_{mnk}[\pi^{nk}\delta^{\dot{A}\dot{B}} + \frac{1}{2}X^{nA}X^{kB}(\Sigma^{AB})^{\dot{A}\dot{B}}]\partial_\mu Y^{\dot{B}} - \\ &\quad -\varepsilon_{mnk}X^{nA}\Gamma^{A\dot{A}\ddot{A}}\partial_\mu Z^{k\ddot{A}}, \\ D_\mu Z^{\ddot{A}} &= \partial_\mu Z^{\ddot{A}} + \varepsilon_{mnk}[\pi^{mn}\delta^{\ddot{A}\ddot{B}} + \frac{1}{2}X^{mA}X^{nB}(\Sigma^{AB})^{\ddot{A}\ddot{B}}]\partial_\mu Z^{k\ddot{B}} - \\ &\quad -X^{mA}\Gamma^{A\dot{A}\ddot{A}}(\partial_\mu Y_m^{\dot{A}} - \varepsilon_{mnk}\pi^{nk}\partial_\mu Y^{\dot{A}}) - \\ &\quad -\frac{1}{6}\varepsilon_{mnk}X^{mA}X^{nB}X^{kC}(\Gamma^{ABC})^{\dot{A}\ddot{A}}\partial_\mu Y^{\dot{A}}, \end{aligned} \tag{60}$$

where $\Sigma^{AB} = \frac{1}{2}(\Gamma^A\Gamma^B - \Gamma^B\Gamma^A)$ and $(\Gamma^{ABC})^{\dot{A}\ddot{A}} = \frac{1}{6}[\Gamma^{A\dot{A}\ddot{B}}\Gamma^{B\dot{B}\ddot{B}}\Gamma^{C\dot{B}\ddot{A}} + (ABC - cycle)]$. The derivative $\partial_\mu Y^{\dot{A}}$ does not change its form because the field $Y^{\dot{A}}$ is inert under ζ^m-transformation.

The total bosonic Lagrangian of the $V(p,q)$-model is just the sum of the bosonic Lagrangians (39) and (46), where now

$$\begin{aligned} L_\mu^m &= \partial_\mu l^m + \frac{1}{2}\pi^{mn}U_{\mu n} + \frac{1}{2}X^{mA}D_\mu X^A - \frac{1}{4}X^{mA}X^{nA}U_{\mu n} - \\ &\quad -\frac{1}{8}\varepsilon^{mnk}(Y_n^{\dot{A}} \overset{\leftrightarrow}{\partial_\mu} Y_k^{\dot{A}}) + \frac{1}{4}(Z^{m\ddot{A}} \overset{\leftrightarrow}{\partial_\mu} Z^{\ddot{A}}) \end{aligned} \tag{61}$$

and all other covariant objects are defined in (40), (48), (55) and (60).

The corresponding fermionic Lagrangian is the sum of the fermionic Lagrangians (42) and (51) with some additional "crossing" terms, which have the following form:

$$\begin{aligned} \Delta\mathcal{L}^F = &- \frac{i}{2}e^{\varphi/2}\Gamma^{A\dot{A}\ddot{A}}\bar{\Omega}^{iA}\gamma^\mu(V_\mu)_j{}^i\Sigma^{j\ddot{A}} + \frac{i}{2}e^{\varphi/2}\Gamma^{A\dot{A}\ddot{A}}\bar{\Omega}^{iA}\gamma^\mu(W_\mu)_j{}^i\Lambda^{j\dot{A}} - \\ &- \frac{i}{2}\Gamma^{A\dot{A}\ddot{A}}\bar{\Lambda}^{i\dot{A}}\gamma^\mu\left\{e^\varphi D_\mu X^A\delta_j{}^i - \partial_\mu X^{mA}y_{ma}(\tau^a)_j{}^i\right\}\Sigma^{j\ddot{A}}. \end{aligned} \tag{62}$$

In this, the supertransformations for the fermionic fields are the same as in formulas (44) and (53) taking into account the changes in the expressions such as $D_\mu X$, $U_{\mu m}$ and so on.

The supertransformations of the bosonic fields are the following:

$$\begin{aligned} \delta\pi^{mn} &= \frac{1}{2}(\bar{\lambda}_a^i(\tau^a)_i{}^j\eta_j)[y^{ma}y^{nb} - (m \leftrightarrow n)] - \frac{1}{2}[X^{mA}\delta X^{nA} - (m \leftrightarrow n)], \\ \delta Y^{\dot{A}} &= \sqrt{\Delta}e^{-\varphi/2}(\bar{\Lambda}^{i\dot{A}}\eta_i) \qquad \delta X^{mA} = y^{mA}(\bar{\Omega}^{iA}(\tau^a)_i{}^j\eta_j), \\ \delta Z^{m\ddot{A}} &= \sqrt{\Delta}e^{-\varphi/2}y^{ma}(\bar{\Sigma}^{i\ddot{A}}(\tau^a)_i{}^j\eta_j) - X^{mA}\Gamma^{A\dot{A}\ddot{A}}\delta Y^{\dot{A}}, \\ \delta Y_{m\dot{A}} &= \frac{1}{\sqrt{\Delta}}e^{-\varphi/2}y_{ma}(\bar{\Lambda}^{i\dot{A}}(\tau^a)_i{}^j\eta_j) + \varepsilon_{mnk}X^{nA}\Gamma^{A\dot{A}\ddot{A}}\delta Z^{k\ddot{A}} + \\ &\quad +\varepsilon_{mnk}[\pi^{mn}\delta^{\dot{A}\dot{B}} + \frac{1}{2}X^{nA}X^{kB}(\Sigma^{AB})^{\dot{A}\dot{B}}]\delta Y^{\dot{B}}, \end{aligned}$$

$$
\begin{aligned}
\delta Z^{\ddot{A}} &= \frac{1}{\sqrt{\Delta}} e^{-\varphi/2} (\bar{\Sigma}^{i\ddot{A}} \eta_i) + X^{mA} \Gamma^{A\dot{A}\ddot{A}} \delta Y_m^{\dot{A}} - \\
&\quad - \varepsilon_{mnk} [\pi^{mn} \delta^{\ddot{A}\ddot{B}} + \frac{1}{2} X^{mA} X^{nB} (\Sigma^{AB})^{\ddot{A}\ddot{B}}] \delta Z^{k\ddot{B}} - \\
&\quad - \varepsilon_{mnk} X^{mA} \Gamma^{A\dot{A}\ddot{B}} [\pi^{nk} \delta^{\ddot{A}\ddot{B}} - \frac{1}{6} X^{nB} X^{kC} (\Sigma^{BC})^{\ddot{B}\ddot{A}}] \delta Y^{\dot{A}}, \\
\delta \pi_m &= -e^{-\varphi} y_{ma} [(\bar{\lambda}_a^i \eta_i) + (\bar{\chi}^i (\tau^a)_i{}^j \eta_j)] + Y^{\dot{A}} \delta Y_m^{\dot{A}} - Y_m^{\dot{A}} \delta Y^{\dot{A}} + \varepsilon_{mnk} Z^{n\ddot{A}} \delta Z^{k\ddot{A}}, \\
\delta X^A &= e^{-\varphi} (\Omega^{iA} \eta_i) - X^{mA} \delta_0 \pi_m - \\
&\quad - \frac{1}{2} \Gamma^{A\dot{A}\ddot{A}} \left\{ Y^{\dot{A}} \delta Z^{\ddot{A}} - Z^{\ddot{A}} \delta Y^{\dot{A}} Y_m^{\dot{A}} \delta Z^{m\ddot{A}} - Z^{m\ddot{A}} \delta Y_m^{\dot{A}} \right\}, \\
\delta l^m &= \frac{1}{4} e^{-\varphi} y^{ma} [(\bar{\lambda}_a^i \eta_i) - (\bar{\chi}^i (\tau^a)_i{}^j \eta_j)] - \frac{1}{2} \pi^{mn} \delta_0 \pi_n - \frac{1}{2} e^{-\varphi} X^{mA} (\bar{\Omega}^{iA} \eta_i) \\
&\quad - \frac{1}{4} X^m X^n \delta_0 \pi_n + \frac{1}{4} \varepsilon^{mnk} Y_n^{\dot{A}} \delta Y_k^{\dot{A}} + \frac{1}{4} Z^{\ddot{A}} \delta Z^{m\ddot{A}} - \frac{1}{4} Z^{m\ddot{A}} \delta Z^{\ddot{A}}, \qquad (63)
\end{aligned}
$$

where $\delta_0 \pi_m = -e^{-\varphi} y_{ma} [(\bar{\lambda}_a^i \eta_i) + (\bar{\chi}^i (\tau^a)_i{}^j \eta_j)]$. As it has already been said, all these formulas can be obtained by the use of the straightforward Noether procedure.

Conclusion

So, we have constructed the Lagrangian and supertransformations for the two general types of $N = 2$ supergravity models, based on the nonsymmetric quaternionic manifolds. In the following paper we will consider the gauge interactions which are possible in such models, in-particular, the ones that lead to the spontaneous supersymmetry breaking.

Acknowledgements

The work was supported by the International Science Foundation and Russian Government grant RMP300 and by the Russian Foundation for Fundamental Research grant 95-02-06312.

References

[1] N. Seiberg and E. Witten, *Nucl. Phys.* **B426** (1994) 19.

[2] E. Cremmer, C. Kounnas, A. van Proeyen, J. P. Derendinger, S. Ferrara, B. de Wit, and L. Girardello, *Nucl. Phys.* **B250** (1985) 385.

[3] H. Itoyama, L. McLerran, T. R. Taylor, and J. J. van der Bij, *Nucl. Phys.* **B279** (1987) 380.

[4] S. Cecotti, L. Girardello, and M. Porrati, *Phys. Lett.* **168B** (1986) 83.

[5] Yu. M. Zinoviev, *Yad. Fiz.* **46** (1987) 943.

[6] Yu. M. Zinoviev, *Int. J. of Mod. Phys.* **A7** (1992) 7515.

[7] Yu. M. Zinoviev, *Yad. Fiz.* **46** (1987) 1240.

[8] D. V. Alekseevskii, *Math. USSR Izvestija* **9** (1975) 297.

[9] S. Cecotti, *Com. Math. Phys.* **124** (1989) 23.

[10] B. de Wit and A. van Proeyen, *Phys. Lett.* **252B** (1990) 221.

[11] B. de Wit, F. Vanderseypen, and A. van Proeyen, *Nucl. Phys.* **B400** (1993) 463.

[12] B. de Wit and A. van Proeyen, *Int. J. Mod. Phys.* **D3** (1994) 31.

[13] B. de Wit and A. Van Proeyen, Isometries of special manifolds, hep-th/9505097 (1995).

Received March 14, 1996

Appendix 1

Let us first give the explicit expressions for the $T_{\pm\pm\pm}$ generators in terms of $\Lambda^{\pm}$ combinations determined in Section 1:

$$\begin{aligned}
T_{+++} &= \Omega_1+\Omega_2+\Omega_3+\Omega_4-\Omega_5-\Omega_6-\Omega_7-\Omega_8,\\
T_{-++} &= \Omega_1+\Omega_2-\Omega_3-\Omega_4+\Omega_5+\Omega_6-\Omega_7-\Omega_8,\\
T_{+-+} &= \Omega_1-\Omega_2+\Omega_3-\Omega_4+\Omega_5-\Omega_6+\Omega_7-\Omega_8,\\
T_{--+} &= \Omega_1-\Omega_2-\Omega_3+\omega_4-\Omega_5+\Omega_6+\Omega_7-\Omega_8,\\
T_{++-} &= \Omega_1-\Omega_2-\Omega_3+\Omega_4+\Omega_5-\Omega_6-\Omega_7+\Omega_8,\\
T_{-+-} &= \Omega_1-\Omega_2+\Omega_3-\Omega_4-\Omega_5+\Omega_6-\Omega_7+\Omega_8,\\
T_{+--} &= \Omega_1+\Omega_2-\Omega_3-\Omega_4-\Omega_5-\Omega_6+\Omega_7+\Omega_8,\\
T_{---} &= \Omega_1+\Omega_2+\Omega_3+\Omega_4+\Omega_5+\Omega_6+\Omega_7+\Omega_8,
\end{aligned} \tag{64}$$

where all Ω stand for Ω^+. The expressions for the $\hat{T}_{\pm\pm\pm}$ are similar to the ones given above.

Now we give the commutation relations for the F_4 generators in our multispinor basis. They look like:

$$[\Lambda_{i\alpha},\Lambda^{j\beta}]=(t_i{}^j\delta_\alpha{}^\beta+\delta_i{}^j t_\alpha{}^\beta),\qquad [\Lambda_{\alpha\dot\alpha},\Lambda^{\beta\dot\beta}]=-(t_\alpha{}^\beta\delta_{\dot\alpha}{}^{\dot\beta}+\delta_\alpha{}^\beta t_{\dot\alpha}{}^{\dot\beta}), \tag{65}$$

$$[\Lambda_{i\alpha},\Lambda_{j\dot\alpha}]=\varepsilon_{ij}\Lambda_{\alpha\dot\alpha},\qquad [\Lambda_{i\alpha},\Lambda_{\beta\dot\alpha}]=-\varepsilon_{\alpha\beta}\Lambda_{i\dot\alpha}, \tag{66}$$

$$[\Lambda_{i\alpha},\Lambda_{\dot\alpha\ddot\alpha}]=[\Lambda_{i\dot\alpha},\Lambda_{\alpha\ddot\alpha}]=[\Lambda_{i\ddot\alpha},\Lambda_{\alpha\dot\alpha}]=T_{i\alpha\dot\alpha\ddot\alpha}, \tag{67}$$

$$[T_{i\alpha\dot\alpha\ddot\alpha}, \Lambda^{j\beta}] = -\frac{1}{\sqrt{2}}\delta_i{}^j\delta_\alpha{}^\beta\Lambda_{\dot\alpha\ddot\alpha}, \tag{68}$$

and a lot of similar ones for other positions of indices.

The following combinations of these generators are the eighenvectors for the $h_{1,2,3,4}$:

$$\begin{aligned}
X_\pm &= \Lambda_{0012} \pm \Lambda_{0021} \pm \Lambda_{1200} + \Lambda_{2100}, \\
\hat{X}_\pm &= \Lambda_{0012} \mp \Lambda_{0021} \pm \Lambda_{1200} - \Lambda_{2100}, \\
Y_\pm &= \Lambda_{0102} \pm \Lambda_{0201} \pm \Lambda_{1020} + \Lambda_{2010}, \\
\hat{Y}_\pm &= \Lambda_{0102} \mp \Lambda_{0201} \pm \Lambda_{1020} - \Lambda_{2010}, \\
Z_\pm &= \Lambda_{0120} \pm \Lambda_{0210} \pm \Lambda_{1002} + \Lambda_{2001}, \\
\hat{Z}_\pm &= \Lambda_{0120} \mp \Lambda_{0210} \pm \Lambda_{1002} - \Lambda_{2001},
\end{aligned} \tag{69}$$

where, for example, Λ_{1200} stands for $\Lambda_{i\alpha}$, $i = 1, \alpha = 2$ as well as another twelve combinations which we denote as $\tilde{X}_\pm$, $\hat{\tilde{X}}_\pm$, $\tilde{Y}_\pm$, $\hat{\tilde{Y}}_\pm$, $\tilde{Z}_\pm$ and $\hat{\tilde{Z}}_\pm$.

Зиновьев Ю. М., Цокур В. А.
Модели $N = 2$ супергравитации, основанные на несимметрических кватернионных многообразиях. I. Симметрии и лагранжианы.

Оригинал-макет подготовлен с помощью системы LaTeX.
Редактор Е.Н.Горина. Технический редактор Н.В.Орлова.

Подписано к печати 14.03.96. Формат 60 × 84/8. Офсетная печать.
Печ.л. 2,5. Уч.-изд.л. 1,92. Тираж 250. Заказ 645. Индекс 3649.
ЛР №020498 17.04.97.

ГНЦ РФ Институт физики высоких энергий
142284, Протвино Московской обл.

Индекс 3649

П Р Е П Р И Н Т 96-22, И Ф В Э, 1996