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$N = 2$ SUPERGRAVITY MODELS
BASED ON THE NONSYMMETRIC
QUATERNIONIC MANIFOLDS
II. GAUGE INTERACTIONS

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Abstract

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In this paper we continue our investigation of the $N = 2$ supergravity models, where scalar fields of hypermultiplets parameterize the nonsymmetric quaternionic manifolds. Using the results of our previous paper, where we have given an explicit construction for the Lagrangians and the supertransformations and, in-particular, the known global symmetries of the Lagrangians, we consider here the switching on the gauge interaction. We show that in this type of models there appears to be possible to have spontaneous supersymmetry breaking with two different scales and without a cosmological term. Moreover, such a breaking could lead to the generation of the Yukawa interactions of the scalar and spinor fields from the hypermultiplets which are absent in other known models.

Аннотация

Зиновьев Ю.М., Цокур В.А. Модели $N = 2$ супергравитации, основанные на несимметрических кватернионных многообразиях. II. Калибровочные взаимодействия: Препринт ИФВЭ 96-23. – Протвино, 1996. – 14 с., библиогр.: 8.

В этой работе мы продолжаем наше исследование моделей $N = 2$ супергравитации, в которых скалярные поля гипермультиплетов параметризуют несимметрические кватернионные многообразия. Используя результаты предыдущей работы, в которой мы дали явное построение лагранжианов и суперпреобразований, и известные глобальные симметрии лагранжианов, мы рассматриваем здесь включение калибровочного взаимодействия. Мы показываем, что в этом типе моделей оказывается возможным иметь спонтанное нарушение суперсимметрии с двумя произвольными масштабами и без космологического члена. Более того, такое нарушение может приводить к генерации юкавских взаимодействий скалярных и спинорных полей гипермультиплетов, которые отсутствуют в других известных моделях.

Introduction

In our previous paper [1] we have considered the $N = 2$ supergravity models where scalar fields of hypermultiplets parameterize one of two types [2,3] of the non-symmetric quaternionic manifolds. We have managed to give an explicit construction of the appropriate Lagrangians and supertransformations in terms of the usual hypermultiplets. One of the important general features of these models is the fact that all of them contain as a common component one of the three possible hidden sectors [4], admitting spontaneous supersymmetry breaking with two arbitrary scales and without a cosmological term. So, in this paper we consider the possibility to switch on the gauge interactions in such models, paying the main attention to the ones which lead to the spontaneous supersymmetry breaking. For the vector multiplets we choose the well known model with the scalar field geometry $O(2, m)/O(2) \otimes O(m)$. The first reason is that this model appears to be the natural generalization of the $N = 2$ hidden sector [5,6] to the case of the arbitrary number of vector multiplets as it was shown in [4], where the usual symmetric quaternionic manifolds were investigated. The second one is that such a model arises in the low-energy limit of four-dimensional superstrings with $N = 2$ supersymmetry. For completeness we reproduce all the necessary formulas below. As it was expected, both types of models do admit the spontaneous supersymmetry breaking with two different scales and without a cosmological term, in this a number of soft breaking terms arises as a result of this breaking. As we will show, apart from the mass terms for different scalar and spinor fields of vector and hypermultiplets, in one type of models the supersymmetry breaking leads to the appearance of the Yukawa interactions between scalar and spinor fields of hypermultiplets.

1. Vector multiplets

To describe the interaction of vector multiplets with N=2 supergravity, let us introduce the following fields: graviton $e_{\mu r}$, gravitini $\Psi_{\mu i}$, $i = 1, 2$, Majorana spinors ρ_i, scalar fields $\hat{\varphi}$, $\hat{\pi}$, and $(m + 2)$ vector multiplets $\{A_\mu^M, \Theta_i^M, \mathcal{Z}^M = \mathcal{X}^M + \gamma_5 \mathcal{Y}^M\}$, $M = 1, 2, ...m + 2,$

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$g^{MN} = (--,+...+)$. It is not difficult to see that the set of spinor and scalar fields is superfluous (which is necessary for symmetrical description of graviphotons and matter vector fields). The following set of constraints corresponds to the model with the geometry $O(2,m)/O(2) \otimes O(m)$:

$$\bar{\mathcal{Z}} \cdot \mathcal{Z} = -2, \qquad \mathcal{Z} \cdot \mathcal{Z} = 0, \qquad \mathcal{Z} \cdot \Theta_i = \bar{\mathcal{Z}} \cdot \Theta_i = 0. \tag{1}$$

The number of the physical degrees of freedom is correct only when the theory is invariant under the local $O(2) \approx U(1)$ transformations, the combination $(\bar{\mathcal{Z}} \partial_\mu \mathcal{Z})$ playing the role of a gauge field. Covariant derivatives for scalar fields $\mathcal{Z}$ and $\bar{\mathcal{Z}}$ look like

$$D_\mu = \partial_\mu \pm \frac{1}{2}(\bar{\mathcal{Z}} \partial_\mu \mathcal{Z}), \tag{2}$$

where covariant derivative $D_\mu \mathcal{Z}$ has the sign "+" and $D_\mu \bar{\mathcal{Z}}$ has the sign "-".

In the given notations the Lagrangian of interaction looks as follows:

$$\begin{aligned}
\mathcal{L}^F &= \frac{i}{2} \varepsilon^{\mu\nu\rho\sigma} \bar{\Psi}_{\mu i} \gamma_5 \gamma_\nu D_\rho \Psi_{\sigma i} + \frac{i}{2} \bar{\rho}^i \hat{D} \rho_i + \frac{i}{2} \bar{\Theta}^i \hat{D} \Theta_i - \\
&\quad + e^{\hat{\varphi}/\sqrt{2}} \left\{ \frac{1}{4} \varepsilon^{ij} \bar{\Psi}_{\mu i} (\mathcal{Z}(A^{\mu\nu} - \gamma_5 \tilde{A}^{\mu\nu})) \Psi_{\nu j} + \frac{1}{4} \bar{\Theta}^i \gamma^\mu (\sigma A) \Psi_{\mu i} + \right. \\
&\quad \left. + \frac{i}{4\sqrt{2}} \bar{\rho}^i \gamma^\mu (\mathcal{Z}(\sigma A)) \Psi_{\mu i} + \frac{\varepsilon^{ij}}{8} \left[2\sqrt{2} \bar{\rho}_i (\sigma A) \Theta_j + \bar{\Theta}_i^{\ M} (\mathcal{Z}(\sigma A)) \Theta_j^{\ M} \right] \right\} \\
&\quad - \frac{1}{2} \varepsilon^{ij} \bar{\Theta}_i^{\ M} \gamma^\mu \gamma^\nu D_\nu \mathcal{Z}^M \Psi_{\mu j} - \frac{1}{2} \varepsilon^{ij} \bar{\rho}_i \gamma^\mu \gamma^\nu (\partial_\nu \hat{\varphi} + \gamma_5 e^{-\sqrt{2}\hat{\varphi}} \partial_\nu \hat{\pi}) \Psi_{\mu j} \qquad (3) \\
\mathcal{L}^B &= -\frac{1}{2} R - \frac{1}{4} e^{\sqrt{2}\hat{\varphi}} \left[A_{\mu\nu}^{\ 2} + 2(\mathcal{Z} \cdot A_{\mu\nu})(\bar{\mathcal{Z}} \cdot A_{\mu\nu}) \right] - \frac{\hat{\pi}}{2\sqrt{2}} (A \cdot \tilde{A}) + \\
&\quad + \frac{1}{2} (\partial_\mu \hat{\varphi})^2 + \frac{1}{2} e^{-2\sqrt{2}\hat{\varphi}} (\partial_\mu \hat{\pi})^2 + \frac{1}{2} D_\mu \mathcal{Z}^A D_\mu \bar{\mathcal{Z}}^A. \qquad (4)
\end{aligned}$$

Derivatives of the spinor fields have the following form:

$$\begin{aligned}
D_\mu \eta_i &= D_\mu^G \eta_i - \frac{1}{4}(\bar{\mathcal{Z}} \partial_\mu \mathcal{Z}) \eta_i + \frac{1}{2\sqrt{2}} e^{-\sqrt{2}\hat{\varphi}} \gamma_5 \partial_\mu \hat{\pi} \eta_i, \\
D_\mu \rho_i &= D_\mu^G \rho_i + \frac{1}{4}(\bar{\mathcal{Z}} \partial_\mu \mathcal{Z}) \rho_i + \frac{3}{2\sqrt{2}} e^{-\sqrt{2}\hat{\varphi}} \gamma_5 \partial_\mu \hat{\pi} \rho_i, \qquad (5) \\
D_\mu \Theta_i &= D_\mu^G \Theta_i - \frac{1}{4}(\bar{\mathcal{Z}} \partial_\mu \mathcal{Z}) \Theta_i - \frac{1}{2\sqrt{2}} e^{-\sqrt{2}\hat{\varphi}} \gamma_5 \partial_\mu \hat{\pi} \Theta_i,
\end{aligned}$$

and the derivative of the field $\Psi_{\mu i}$ is the same as for η_i.

Supertransformation laws look like

$$\begin{aligned}
\delta \Theta_i^M &= -\frac{1}{2} e^{\hat{\varphi}/\sqrt{2}} \left\{ (\sigma A)^M + \frac{1}{2} \bar{\mathcal{Z}}^M (\mathcal{Z}(\sigma A)) + \frac{1}{2} \mathcal{Z}^M (\bar{\mathcal{Z}}(\sigma A)) \right\} \eta_i - i\varepsilon_{ij} \hat{D} \mathcal{Z}^M \eta_i, \\
\delta \rho_i &= -\frac{1}{2\sqrt{2}} e^{\hat{\varphi}/\sqrt{2}} \mathcal{Z}(\sigma A) \eta_i - i\varepsilon_{ij} \gamma^\mu (\partial_\mu \hat{\varphi} + \gamma_5 e^{-\sqrt{2}\hat{\varphi}} \partial_\mu \hat{\pi}) \eta_i,
\end{aligned}$$

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$$\delta\Psi_{\mu i} = 2D_\mu\eta_i + \frac{i}{4}\varepsilon_{ij}e^{\hat\varphi/\sqrt{2}}\bar{\mathcal{Z}}(\sigma A)\eta_i \qquad \delta\hat\pi = e^{\sqrt{2}\hat\varphi}\varepsilon^{ij}(\bar\rho_i\gamma_5\eta_j),$$

$$\delta\mathcal{X}^A = \varepsilon^{ij}(\bar\Theta_i{}^A\eta_j) \qquad \delta\mathcal{Y}^A = \varepsilon^{ij}(\bar\Theta_i{}^A\gamma_5\eta_j) \qquad \delta\hat\varphi = \varepsilon^{ij}(\bar\rho_i\eta_j),$$

$$\delta A_\mu^A = e^{-\hat\varphi/\sqrt{2}}\left\{\varepsilon^{ij}(\bar\Psi_{\mu i}\mathcal{Z}^A\eta_j) + i(\bar\Theta_i{}^A\gamma_\mu\eta^i) - \frac{i}{\sqrt{2}}(\bar\rho^i\gamma_\mu\mathcal{Z}^A\eta_i)\right\}. \tag{6}$$

2. $W(p, q)$-model

The W(p,q)-model has been constructed in [1] and here we shall give only a brief description of it. The main attention will be paid to the switching on a gauge interaction in this model and to the investigation of the spontaneous supersymmetry breaking and its consequences.

2.1. Description of the model

To describe W(p,q)-model let us introduce two kinds of the hypermultiplets $(\Lambda_\alpha, Y_m)^{\dot A}$ and $(\Sigma_\alpha, Z^m)^{\ddot A}$, $\dot A = 1, ..., p$, $\ddot A = 1, ..., q$, $\alpha = 1, 2$ and $m = 1, 2, 3, 4$, which we call correspondingly Y- and Z-multiplets. These multiplets interact with a hidden sector that has been constructed in [4] and contains the following fields: graviton $e_{\mu r}$, gravitini $\Psi_{\mu i}$, $i = 1, 2$, fermionic fields χ_α^a, $a = 1, 2, 3, 4$, and bosonic fields y_{ma} and $\pi^{[mn]}$. The fields π^{mn} will enter the Lagrangian through a derivative only, whereas y_{ma} will realize the nonlinear σ-model $GL(4, R)/O(4)$.

Let us denote y_{ma}^{-1} as y^{ma}, so that $y_{ma}y^{na} = \delta_m^n$.

We shall need four matrices $(\tau^a)_{i\alpha}$ and their conjugate ones $(\bar\tau^a)^{\alpha i}$ satisfying the condition:

$$\tau^a\bar\tau^b + \tau^b\bar\tau^a = 2\delta^{ab}I, \tag{7}$$

for which we shall use the following explicit representation $\tau = \{I, \vec\sigma\}$, $\bar\tau = \{I, -\vec\sigma\}$, where $\vec\sigma$ are the Pauli matrices. Let us introduce also six matrices:

$$\Sigma^{ab} = \frac{1}{2}(\tau^a\bar\tau^b - \tau^b\bar\tau^a), \qquad \Sigma^{ab} = \frac{1}{2}\varepsilon^{abcd}\Sigma^{cd}. \tag{8}$$

Recall, since in our representation for spinors the matrix γ_5 plays the role of an imaginary unit, then, e.g.,

$$\gamma_\mu(\tau^a)_{i\alpha} = (\bar\tau^a)^{\alpha i}\gamma_\mu, \qquad \gamma_\mu(\Sigma^{ab})_i{}^j = -(\Sigma^{ab})^i{}_j\gamma_\mu. \tag{9}$$

The Lagrangian of the interaction of the hidden sector with Y- and Z-multiplets have the following form:

$$\begin{aligned}
\mathcal{L}^{\mathcal{F}} &= \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}\bar\Psi_{\mu i}\gamma_5\gamma_\nu D_\rho\Psi_{\sigma i} + \frac{i}{2}\bar\chi_\alpha^a\hat D\chi_\alpha^a + \frac{i}{2}\bar\Lambda_\alpha\hat D\Lambda_\alpha + \frac{i}{2}\bar\Sigma_\alpha\hat D\Sigma_\alpha - \\
&\quad -\frac{1}{2}\bar\chi_\alpha^a\gamma^\mu\gamma^\nu(S_\nu^+ + P_\nu)_{ab}(\bar\tau^b)^{\alpha i}\Psi_{\mu i} - \frac{1}{2}\bar\Sigma_\alpha\gamma^\mu\gamma^\nu\partial_\nu Z^m y_{ma}(\bar\tau^a)^{\alpha i}\Psi_{\mu i} - \\
&\quad -\frac{1}{2}\bar\Lambda_\alpha\gamma^\mu\gamma^\nu\sqrt{\Delta}\partial_\nu Y_m y^{ma}(\bar\tau^a)^{\alpha i}\Psi_{\mu i} - \frac{i}{2}\bar\chi_\alpha^a\gamma^\mu(S_\mu^- - P_\mu)_{ab}\chi_\alpha^b +
\end{aligned}$$

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$$+i\bar{\Sigma}_\alpha\gamma^\mu\partial_\mu Z^m y_{ma}\chi^a_\alpha - \frac{i}{2}\bar{\Lambda}_\alpha\gamma^\mu\sqrt{\Delta}\partial_\mu Y_m y^{ma}\chi^a_\alpha +$$

$$+\frac{i}{2}\bar{\Lambda}_\alpha\gamma^\mu\partial_\mu Y_m y^{mb}(\Sigma^{ab})_\alpha{}^\beta\chi^a_\beta, \tag{10}$$

$$\mathcal{L}^{\mathcal{B}} = -\frac{1}{2}R + \frac{1}{2}(S^+_{\mu ab})^2 + \frac{1}{2}(P_{\mu ab})^2 + \frac{1}{2}g_{mn}\partial_\mu Z^m\partial_\mu Z^n + \frac{\Delta}{2}g^{mn}\partial_\mu Y_m\partial_\mu Y_n, \tag{11}$$

where we have denoted:

$$P_{\mu ab} = y_{ma}(\partial_\mu\pi^{mn} + \frac{1}{2}Z^m\overset{\leftrightarrow}{\partial}_\mu Z^n + \frac{1}{4}\varepsilon^{mnpq}Y_p\overset{\leftrightarrow}{\partial}_\mu Y_q)y_{nb},$$

$$S^\pm_{\mu ab} = \frac{1}{2}(\partial_\mu y_{ma}y^{mb} \pm y_{mb}\partial_\mu y^{ma}) \tag{12}$$

and the D-derivatives of the fermionic fields look like:

$$D_\mu = D^G_\mu \pm \frac{1}{4}(S^-_\mu + P_\mu)_{ab}\Sigma^{ab} \tag{13}$$

with the sign "-" for the derivatives of the parameter η and gravitino Ψ_μ and the sign "+" for all other fermion fields derivatives.

Corresponding supertransformation laws have the following form:

$$\delta\Psi_{\mu i} = 2D_\mu\eta_i \qquad \delta\chi^a_\alpha = -i\gamma^\mu(S^+_\mu + P_\mu)_{ab}(\bar{\tau}^b)^{\alpha i}\eta_i,$$

$$\delta y_{ma} = y_{mb}(\bar{\chi}^b_\alpha(\bar{\tau}^a)^{\alpha i}\eta_i), \quad \delta\pi^{mn} = \frac{1}{2}\{y^{ma}y^{nb} - y^{mb}y^{na}\}(\bar{\chi}^a_\alpha(\bar{\tau}^b)^{\alpha i}\eta_i),$$

$$\delta\Lambda_\alpha = -i\gamma^\mu\sqrt{\Delta}\partial_\mu Y_m y^{ma}(\bar{\tau}^a)^{\alpha i}\eta_i, \quad \delta Y_m = \frac{1}{\sqrt{\Delta}}y_{ma}(\bar{\Lambda}_\alpha(\bar{\tau}^a)^{\alpha i}\eta_i),$$

$$\delta\Sigma_\alpha = -i\gamma^\mu\partial_\mu Z^m y_{ma}(\bar{\tau}^a)^{\alpha i}\eta_i, \quad \delta Z^m = y^{ma}(\bar{\Sigma}_\alpha(\bar{\tau}^a)^{\alpha i}\eta_i) \tag{14}$$

with the notations defined above.

Now let us consider interaction of W(p,q)-model with the vector multiplets described in the previous section. It is easy to check that the only additional terms, which appear in the Lagrangian, are the following:

$$\Delta\mathcal{L}^{\mathcal{F}} = \frac{e^{\hat{\varphi}/\sqrt{2}}}{8}\varepsilon^{\alpha\beta}\left\{(\bar{\chi}^a_\alpha\bar{\mathcal{Z}}^M(\sigma A)^M\chi^b_\beta)+\right.$$

$$\left.+ (\bar{\Lambda}_\alpha\bar{\mathcal{Z}}^M(\sigma A)^M\Lambda_\beta) + (\bar{\Sigma}_\alpha\bar{\mathcal{Z}}^M(\sigma A)^M\Sigma_\beta)\right\}. \tag{15}$$

Besides, D-derivatives of the fermionic fields change their form. Derivatives (13) of all the fermions of W(p,q)-model acquire the following additional terms (analogously to the ones in (5)):

$$\frac{1}{4}(\bar{\mathcal{Z}}\partial_\mu\mathcal{Z}) - \frac{1}{2\sqrt{2}}e^{-\sqrt{2}\hat{\varphi}}\gamma_5\partial_\mu\hat{\pi} \tag{16}$$

and derivatives (5) of the fields ρ_i and Θ^M_i acquire the following additional terms (analogously to the ones in (13)):

$$\frac{1}{4}(S^-_\mu + P_\mu)_{ab}(\Sigma^{ab})_i{}^j. \tag{17}$$

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2.2. Gauge interaction and symmetry breaking

Our next step will be to switch on the gauge interaction and investigate a possibility to have a spontaneous supersymmetry breaking and its consequences.

Among the global symmetries of the bosonic Lagrangian there are the translations of the field π^{mn}: $\pi \to \pi + \Lambda$. It has been shown in [4], that for the three out of these six translations their gauging leads to the spontaneous supersymmetry breaking with a vanishing cosmological constant. Also, a bosonic Lagrangian of the model is invariant under the global transformations of the group $O(p) \otimes O(q)$, which touches the Y- and Z-sectors, and that allows one to switch on the gauge interaction corresponding to some subgroup of this group. For that let us make the following substitution in the Lagrangian and the supertransformation laws:

$$
\begin{aligned}
\partial_\mu Y_m^{\dot A} &\to \partial_\mu Y_m^{\dot A} - A_\mu^M (T^M)^{\dot A \dot B} Y_m^{\dot B}, &\quad \partial_\mu Z^{m\ddot A} &\to \partial_\mu Z^{m\ddot A} - A_\mu^M (T^M)^{\ddot A \ddot B} Z^{m\ddot B}, \\
\partial_\mu \mathcal{Z}^M &\to \partial_\mu \mathcal{Z}^M - f^{MNK} A_\mu^{\ N} \mathcal{Z}^K, &\quad \partial_\mu \Theta_i^{\ M} &\to \partial_\mu \Theta_i^{\ M} - f^{MNK} A_\mu^{\ N} \Theta_i^{\ K}, \\
\partial_\mu \pi^{mn} &\to \partial_\mu \pi^{mn} - A_\mu^M (M^M)^{mn}.
\end{aligned}
\tag{18}
$$

In order to restore the invariance of the Lagrangian under the supertransformations, one has to add the following terms to the Lagrangian

$$
\begin{aligned}
\mathcal{L}'^{\mathcal{F}} = e^{-\hat\varphi/\sqrt 2} \Bigg\{ &-\frac14 \bar\Psi_{\mu i} \sigma^{\mu\nu} \varepsilon^{ij} (\mathcal{Z}R)_{ab} (\Sigma^{ab})_j^{\ k} \Psi_{\nu k} + \frac{i}{2\sqrt 2} \bar\Psi_{\mu i} \gamma^\mu (\bar{\mathcal{Z}}R)_{ab} (\Sigma^{ab})_i^{\ j} \rho_j - \\
&-\frac{i}{2} \bar\Psi_{\mu i} \gamma^\mu R_{ab} (\Sigma^{ab})_i^{\ j} \Theta_j - i \bar\Psi_{\mu i} \gamma^\mu \varepsilon_{ij} (\bar\tau^a)^{\alpha j} (\bar{\mathcal{Z}}R)_{ab} \chi_\alpha^b - \\
&-\frac{i}{2} \bar\Psi_{\mu i} \gamma^\mu \sqrt\Delta Y_m^{\dot A} \bar{\mathcal{Z}}^{\dot A \dot B} y^{ma} (\tau^a)_{i\alpha} \varepsilon^{\alpha\beta} \Lambda_\beta^{\dot B} - \frac{i}{2} \bar\Psi_{\mu i} \gamma^\mu Z^{m\ddot A} \bar{\mathcal{Z}}^{\ddot A \ddot B} y_{ma} (\tau^a)_{i\alpha} \varepsilon^{\alpha\beta} \Sigma_\beta^{\ddot B} \\
&+\frac{1}{\sqrt 2} \bar\rho_i \varepsilon^{ij} R_{ab} (\Sigma^{ab})_j^{\ k} \Theta_k - \sqrt 2 \bar\rho_i (\bar{\mathcal{Z}}R)_{ab} (\bar\tau^a)^{\alpha i} \chi_\alpha^b - \frac14 \bar\Theta_i^{\ M} \varepsilon^{ij} (\mathcal{Z}R) \Sigma_j^{\ k} \Theta_k^{\ M} - \\
&-\frac{1}{\sqrt 2} \bar\rho_i \sqrt\Delta Y_m^{\dot A} \bar{\mathcal{Z}}^{\dot A \dot B} y^{ma} (\bar\tau^a)^{\alpha i} \Lambda_\alpha^{\dot B} - \frac{1}{\sqrt 2} \bar\rho_i Z^{m\ddot A} \bar{\mathcal{Z}}^{\ddot A \ddot B} y_{ma} (\bar\tau^a)^{\alpha i} \Sigma_\alpha^{\ddot B} + \\
&+2 \bar\Theta_i R_{ab} (\bar\tau^a)^{\alpha i} \chi_\alpha^b - \bar\chi_\alpha^a \varepsilon^{\alpha\beta} (\bar{\mathcal{Z}})_{ab} \chi_\beta^b - \frac12 \bar\chi_\alpha^a \varepsilon^{\alpha\beta} (\bar{\mathcal{Z}}R)_{bc} (\Sigma^{bc})_\beta^{\ \gamma} \chi_\gamma^a + \\
&+\bar\Theta_i^{\ M} \sqrt\Delta Y_m^{\dot A} (T^M)^{\dot A \dot B} y^{ma} (\bar\tau^a)^{\alpha i} \Lambda_\alpha^{\dot B} + \bar\Theta_i^{\ M} Z^{m\ddot A} (T^M)^{\ddot A \ddot B} y_{ma} (\bar\tau^a)^{\alpha i} \Sigma_\alpha^{\ddot B} + \\
&+\frac14 \bar\Sigma_\alpha (\bar{\mathcal{Z}}R)_{ab} (\Sigma^{ab})_\beta^{\ \alpha} \varepsilon^{\beta\gamma} \Sigma_\gamma - \frac14 \bar\Lambda_\alpha (\bar{\mathcal{Z}}R)_{ab} (\Sigma^{ab})_\beta^{\ \alpha} \varepsilon^{\beta\gamma} \Lambda_\gamma - \\
&-\frac12 f^{MNK} \left(\frac{i}{2} \bar\Psi_{\mu i} \gamma^\mu \mathcal{Z}^N \bar{\mathcal{Z}}^K \Theta_i^M - \varepsilon^{ij} \bar\Theta_i^M \mathcal{Z}^N \Theta_j^K + \frac{1}{\sqrt 2} \varepsilon^{ij} \bar\Theta_i^M \mathcal{Z}^N \bar{\mathcal{Z}}^K \rho_j \right) \Bigg\}, \tag{19}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}'^{\mathcal{B}} = e^{-\hat\varphi\sqrt 2} \Bigg\{ &-2(\bar{\mathcal{Z}}R)_{ab} (\mathcal{Z}R)_{ab} - R_{ab}(R_{ab} + \frac12 \varepsilon_{abcd} R_{cd}) + \\
&+\frac{\Delta}{2} g^{mn} Y_m \bar{\mathcal{Z}} \mathcal{Z} Y_n + \frac12 g_{mn} Z^m \bar{\mathcal{Z}} \mathcal{Z} Z^n + \frac18 e^{-\hat\varphi\sqrt 2} (f^{MNK} \mathcal{Z}^N \bar{\mathcal{Z}}^K)^2 \Bigg\} \tag{20}
\end{aligned}
$$

and to the supertransformation laws, respectively,

$$
\delta'\Psi_{\mu i} = \frac{i}{2} e^{-\hat\varphi/\sqrt 2} \gamma_\mu \varepsilon^{ij} (\mathcal{Z}R)_{ab} (\Sigma^{ab})_j^{\ k} \eta_k,
$$

$$\delta'\chi^a_\alpha = -2e^{-\hat{\varphi}/\sqrt{2}}(\mathcal{Z}R)_{ab}(\tau^b)_{i\alpha}\varepsilon^{ij}\eta_j,$$

$$\delta'\rho_i = -\frac{1}{\sqrt{2}}e^{-\hat{\varphi}/\sqrt{2}}(\mathcal{Z}R)_{ab}(\Sigma^{ab})_i{}^j\eta_j,$$

$$\delta'\Lambda^{\dot{A}}_\alpha = -e^{-\hat{\varphi}/\sqrt{2}}\varepsilon_{\alpha\beta}\sqrt{\Delta}\mathcal{Z}^{\dot{A}\dot{B}}Y^{\dot{B}}_m y^{ma}(\bar{\tau}^a)^{\beta i}\eta_i,$$

$$\delta'\Sigma^{\ddot{A}}_\alpha = -e^{-\hat{\varphi}/\sqrt{2}}\varepsilon_{\alpha\beta}\mathcal{Z}^{\ddot{A}\ddot{B}}Z^{m\ddot{B}}y_{ma}(\bar{\tau}^a)^{\beta i}\eta_i,$$

$$\delta'\Theta_i = e^{-\hat{\varphi}/\sqrt{2}}\{R + \frac{1}{2}\mathcal{Z}(\bar{\mathcal{Z}}R)\frac{1}{2}\bar{\mathcal{Z}}(\mathcal{Z}R)\}_{ab}(\Sigma^{ab})_i{}^j\eta_j +$$

$$+\frac{1}{2}e^{-\hat{\varphi}/\sqrt{2}}f^{MNK}\mathcal{Z}^N\bar{\mathcal{Z}}^K\eta_i, \tag{21}$$

where

$$R^M_{ab} = y_{ma}\{\frac{1}{2}(M^M)^{mn} + \frac{1}{2}Z^mT^MZ^n + \frac{1}{4}\varepsilon^{mnpq}Y_pT^MY_q\}y_{nb}. \tag{22}$$

Now one can investigate minimum of the potential $V = -\mathcal{L}'^{\mathcal{B}}$ defined above. Without losing the generality one can always choose:

$$<y_{ma}> = \delta_{ma}, \qquad <\mathcal{Z}^M> = (1, i, 0, ..., 0). \tag{23}$$

In this case the potential has the minimum at $<Y_m> = <Z^m> = 0$ and one can easily calculate the value of the potential at the minimum:

$$V_0 = \frac{1}{4}M^M_{ab}\{M^M_{ab} - \frac{1}{2}\varepsilon_{abcd}M^M_{cd}\}, \tag{24}$$

where $M = 1, 2$. If the matrices M^M_{ab} are self-dual, then we have the spontaneous supersymmetry breaking and the cosmological constant vanishes as a result of the gauge group choice (i. e., which global translations were made to be local ones) and not of a fine tuning of the parameters. Let us choose $M^1_{12} = M^1_{34} = m_1$, $M^2_{14} = M^2_{23} = m_2$ and the other parameters equal zero. In this case diagonalized gravitino mass matrix has the form:

$$M^{ik} = \varepsilon^{ij} <(\mathcal{Z}R)_{ab}> (\Sigma^{ab})_j{}^k \sim \begin{pmatrix} m_1 + m_2 & 0 \\ 0 & m_1 - m_2 \end{pmatrix}. \tag{25}$$

Unfortunately, the spontaneous symmetry breaking does not generate the masses for the Y- and Z-sectors. It is easy to check, that matrices $(\Sigma^{ab})_\alpha{}^\beta$ are anti-self-dual and spinors Λ_α and Σ_α do not acquire masses because of a vanishing of the expression $<(\bar{\mathcal{Z}}R)_{ab}> (\Sigma^{ab})_\alpha{}^\beta$. Scalar fields Y_m and Z^m also remain massless, which can be seen taking into account that the generators $(M^M)^{mn}$, $(T^M)^{\dot{A}\dot{B}}$ and $(T^M)^{\ddot{A}\ddot{B}}$ are nontrivial only when $M = 1, 2$, $M = 3, ..., 3 + p$ and $M = 4 + p, ..., 4 + p + q$, correspondingly.

3. V(p,q)-model

The N=2 supergravity model with the second general type of nonsymmetric quaternionic geometry, a so called V(p,q)-model [2,3], has also been constructed in [1]. Here we again refrain from a detailed description of it, paying the main attention to the symmetry breaking in this model.

3.1. Description of the model

The hidden sector of this model is essentially the same as in the previous one, but in different parameterization. It contains the following fields: graviton $e_{\mu r}$, gravitini $\Psi_{\mu i}$, fermions λ_a^i and χ^i, where $i = 1, 2$ and $a = 1, 2, 3$, and bosonic fields y_{ma}, φ, $\pi^{[mn]}$, l^m and π_m, where $m = 1, 2, 3$. There are three sets of hypermultiplets interacting with the hidden sector: $(\Omega^i, X^m, Z)^A$, $(\Lambda^i, Y_m, Y)^{\dot{A}}$ and $(\Sigma^i, Z^m, Z)^{\ddot{A}}$, which we will call X-, Y- and Z-multiplets, correspondingly, and it turns out to be necessary to introduce γ-matrices $\Gamma^{A\dot{A}\ddot{A}}$ in order to connect fields from different kinds of the multiplets in the Lagrangian. The X-multiplet carries vector index A of the $O(p)$ group and Y- and Z-multiplets carry the corresponding spinor indices in full correspondence with [2,3].

The fermionic Lagrangian of the V(p,q)-model has the following form:

$$
\begin{aligned}
\mathcal{L}^{\mathcal{F}} &= \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\Psi}_{\mu i}\gamma_5\gamma_\nu D_\rho\Psi_{\sigma i} + \frac{i}{2}\bar{\lambda}_a^i\hat{D}\lambda_a^i + \frac{i}{2}\bar{\chi}^i\hat{D}\chi^i + \\
&+ \frac{i}{2}\bar{\Omega}^i\hat{D}\Omega^i + \frac{i}{2}\bar{\Lambda}^i\hat{D}\Lambda^i + \frac{i}{2}\bar{\Sigma}^i\hat{D}\Sigma^i - \\
&- \frac{1}{2}\bar{\chi}^i\gamma^\mu\gamma^\nu\left\{\partial_\nu\varphi\delta_i{}^j - 2e^\varphi Q_\nu^{a+}\tau^a{}_i{}^j\right\}\Psi_{\mu j} - \\
&- \frac{1}{2}\bar{\lambda}_a^i\gamma^\mu\gamma^\nu\left\{(S_\nu^+ + P_\nu)_{ab}\tau^b{}_i{}^j + 2e^\varphi Q_\nu^{a-}\delta_i{}^j\right\}\Psi_{\mu j} - \\
&- \frac{1}{2}\bar{\Psi}_{\mu i}\gamma^\nu\gamma^\mu\left\{e^\varphi D_\nu X\delta_i{}^j + y_{ma}\partial_\nu X^m\tau^a{}_j{}^i\right\}\Omega^j - \\
&- \frac{1}{2}e^{\varphi/2}\bar{\Psi}_{\mu i}\gamma^\nu\gamma^\mu(V_\nu)_i{}^j\Lambda^j - \frac{1}{2}e^{\varphi/2}\bar{\Psi}_{\mu i}\gamma^\nu\gamma^\mu(W_\nu)_i{}^j\Sigma^j + \\
&+ \frac{i}{2}(S_\mu^- + P_\mu)_{ab}(\bar{\lambda}_a^i\gamma_\mu\lambda_b^i) + 2ie^\varphi Q_\mu^{a-}(\bar{\lambda}_a^i\gamma_\mu\chi^i) - \\
&- (\bar{\lambda}_a^i\gamma^\mu\Omega^i)y_{ma}\partial_\mu X^m - i(\bar{\chi}^i\gamma^\mu\Omega^i)e^\varphi D_\mu X - \\
&- \frac{i}{2}e^{\varphi/2}\bar{\chi}^i\gamma^\mu(V_\mu)_i{}^j\Lambda^j - \frac{i}{2}e^{\varphi/2}\bar{\chi}^i\gamma^\mu(W_\mu)_i{}^j\Sigma^j - \\
&- \frac{i}{2}e^{\varphi/2}\bar{\lambda}_a^i\gamma^\mu\tau^a{}_j{}^i(V_\mu)_k{}^j\Lambda^k + \frac{i}{2}e^{\varphi/2}\bar{\lambda}_a^i\gamma^\mu\tau^a{}_j{}^i(W_\mu)_k{}^j\Sigma^k - \\
&- \frac{i}{2}e^{\varphi/2}\Gamma^{A\dot{A}\ddot{A}}\bar{\Omega}^{iA}\gamma^\mu(V_\mu)_j{}^i\Sigma^{j\ddot{A}} + \frac{i}{2}e^{\varphi/2}\Gamma^{A\dot{A}\ddot{A}}\bar{\Omega}^{iA}\gamma^\mu(W_\mu)_j{}^i\Lambda^{j\dot{A}} - \\
&- \frac{i}{2}\Gamma^{A\dot{A}\ddot{A}}\bar{\Lambda}^{i\dot{A}}\gamma^\mu\left\{e^\varphi D_\mu X^A\delta_j{}^i - \partial_\mu X^{mA}y_{ma}\tau^a{}_j{}^i\right\}\Sigma^{j\ddot{A}}.
\end{aligned}
\tag{26}
$$

The conventions in this Lagrangian are the following:

$$
\begin{aligned}
P_{\mu ab} &= y_{ma}\left\{\partial_\mu\pi^{mn} + \frac{1}{2}(X^m\overset{\leftrightarrow}{\partial}_\mu X^n)\right\}y_{nb}, \\
S_{\mu ab}^\pm &= \frac{1}{2}[y^{ma}\partial_\mu y_{mb} \pm y^{mb}\partial_\mu y_{ma}], \quad Q_\mu^{a\pm} = y_{ma}L_\mu^m \pm \frac{1}{4}y^{ma}U_{\mu m}, \\
U_{\mu m} &= \partial_\mu\pi_m + (Y_m^{\dot{A}}\overset{\leftrightarrow}{\partial}_\mu Y^{\dot{A}}) - \frac{1}{2}\varepsilon_{mnk}(Z^{n\ddot{A}}\overset{\leftrightarrow}{\partial}_\mu Z^{k\ddot{A}}),
\end{aligned}
$$

$$
\begin{aligned}
D_\mu X^A &= \partial_\mu X^A + X^{mA}U_{\mu m} + \frac{1}{2}\Gamma^{A\dot{A}\ddot{A}}[(Y^{\dot{A}}\overset{\leftrightarrow}{\partial_\mu}Z^{\ddot{A}}) + (Y_m^{\dot{A}}\overset{\leftrightarrow}{\partial_\mu}Z^{m\ddot{A}})], \\
L_\mu^m &= \partial_\mu l^m + \frac{1}{2}\pi^{mn}U_{\mu n} + \frac{1}{2}X^{mA}D_\mu X^A - \frac{1}{4}X^{mA}X^{nA}U_{\mu n} - \\
&\quad -\frac{1}{8}\varepsilon^{mnk}(Y_n^{\dot{A}}\overset{\leftrightarrow}{\partial_\mu}Y_k^{\dot{A}}) + \frac{1}{4}(Z^{m\ddot{A}}\overset{\leftrightarrow}{\partial_\mu}Z^{\ddot{A}}), \\
(V_\mu)_i{}^j &= \frac{1}{\sqrt{\Delta}}\partial_\nu Y \delta_i{}^j + \sqrt{\Delta}D_\nu Y_m y^{ma}\tau^a{}_i{}^j, \\
(W_\mu)_i{}^j &= \sqrt{\Delta}D_\mu Z\delta_i{}^j + \frac{1}{\sqrt{\Delta}}D_\mu Z^m y_{ma}\tau^a{}_i{}^j, \\
D_\mu Z^{m\ddot{A}} &= \partial_\mu Z^{m\ddot{A}} + \Gamma^{A\dot{A}\ddot{A}}X^{mA}\partial_\mu Y^{\dot{A}}, \\
D_\mu Y_m^{\dot{A}} &= \partial_\mu Y_m^{\dot{A}} - \varepsilon_{mnk}[\pi^{nk}\delta^{\dot{A}\dot{B}} + \frac{1}{2}X^{nA}X^{kB}(\Sigma^{AB})^{\dot{A}\dot{B}}]\partial_\mu Y^{\dot{B}} - \\
&\quad -\varepsilon_{mnk}X^{nA}\Gamma^{A\dot{A}\ddot{A}}\partial_\mu Z^{k\ddot{A}}, \\
D_\mu Z^{\ddot{A}} &= \partial_\mu Z^{\ddot{A}} + \varepsilon_{mnk}[\pi^{mn}\delta^{\ddot{A}\ddot{B}} + \frac{1}{2}X^{mA}X^{nB}(\Sigma^{AB})^{\ddot{A}\ddot{B}}]\partial_\mu Z^{k\ddot{B}} - \\
&\quad -X^{mA}\Gamma^{A\dot{A}\ddot{A}}(\partial_\mu Y_m^{\dot{A}} - \varepsilon_{mnk}\pi^{nk}\partial_\mu Y^{\dot{A}}) - \\
&\quad -\frac{1}{6}\varepsilon_{mnk}X^{mA}X^{nB}X^{kC}(\Gamma^{ABC})^{\dot{A}\ddot{A}}\partial_\mu Y^{\dot{A}} \qquad (27)
\end{aligned}
$$

and D-derivatives for the fermions have the following form:

$$
(D_\mu)_i{}^j = D_\mu^G\delta_i^j \pm \frac{1}{4}\varepsilon^{abc}(S_\mu^- - P_\mu)_{ab}\tau^c{}_i{}^j + e^\varphi Q_\mu^{a+}\tau^a{}_i{}^j. \qquad (28)
$$

Here derivatives of $\Psi_{\mu i}$ and η_i have the sign "-" and derivatives of all the other fermion fields – the sign "+".

The corresponding supertransformations of the fermionic fields are the following:

$$
\begin{aligned}
\delta\Psi_{\mu i} &= 2D_\mu\eta_i \qquad \delta\chi^i = -i\gamma_\mu\left\{\partial_\mu\varphi\delta_i{}^j - 2e^\varphi Q_\mu^{a+}\tau^a{}_i{}^j\right\}\eta_j, \\
\delta\lambda_a^i &= -i\gamma^\mu\left\{(S_\mu^+ + P_\mu)_{ab}\tau^b{}_i{}^j + 2e^\varphi Q_\mu^{a-}\delta_i{}^j\right\}\eta_j, \\
\delta\Omega^i &= -i\gamma^\mu\left\{e^\varphi D_\mu X\delta_i{}^j + y_{ma}\partial_\mu X^m\tau^a{}_i{}^j\right\}\eta_j, \\
\delta\Lambda^i &= -i\gamma^\mu(V_\mu)_i{}^j\eta_j, \qquad \delta\Sigma^i = -i\gamma^\mu(W_\mu)_i{}^j\eta_j. \qquad (29)
\end{aligned}
$$

Here we use the same conventions as in (27) and (28).

The bosonic Lagrangian of the V(p,q)-model has the following form:

$$
\begin{aligned}
\mathcal{L}^\mathcal{B} &= \frac{1}{2}(\partial_\mu\varphi)^2 + \frac{1}{2}(S_{\mu ab}^+)^2 + \frac{1}{2}(P_{\mu ab})^2 + 4e^{2\varphi}g_{mn}L_\mu^n L_\nu^n + \\
&\quad +\frac{1}{4}e^{2\varphi}g^{mn}U_{\mu m}U_{\mu n} + \frac{1}{2}e^{2\varphi}(D_\mu X)^2 + \frac{1}{2}g_{mn}\partial_\mu X^m\partial_\mu X^n + \frac{e^\varphi}{2\Delta}(\partial_\mu Y)^2 + \\
&\quad +e^\varphi\frac{\Delta}{2}(D_\mu Y_m)(D_\mu Y_n)g^{mn} + e^\varphi\frac{\Delta}{2}(D_\mu Z)^2 + \frac{e^\varphi}{2\Delta}(D_\mu Z^m)(D_\mu Z^n)g_{mn}, \qquad (30)
\end{aligned}
$$

where $g_{mn} = y_{ma}y_{na}$ and $g^{mn} = y^{ma}y^{na}$. We do not give here the corresponding super-transformations of the bosonic fields, because they are awkward and nonessential for our considerations.

Now let us consider interaction of the V(p,q)-model with the vector multiplets, described in Section I. It is easy to check, that the only additional terms, which appear in the fermionic Lagrangian, are the following:

$$
\begin{aligned}
\Delta\mathcal{L}^{\mathcal{F}} = \ & \frac{e^{\hat{\varphi}/\sqrt{2}}}{8}\varepsilon_{ij}\left\{(\bar{\chi}^i\bar{\mathcal{Z}}^M(\sigma A)^M\chi^j) + (\bar{\lambda}^i_a\bar{\mathcal{Z}}^M(\sigma A)^M\lambda^j_a) + \right.\\
& \left. + (\bar{\Omega}^i\bar{\mathcal{Z}}^M(\sigma A)^M\Omega^j) + (\bar{\Lambda}^i\bar{\mathcal{Z}}^M(\sigma A)^M\Lambda^j) + (\bar{\Sigma}^i\bar{\mathcal{Z}}^M(\sigma A)^M\Sigma^j)\right\}.
\end{aligned}
\tag{31}
$$

Besides, D-derivatives of the fermionic fields change their form. Derivatives (28) of all the fermions of the V(p,q)-model acquire the following additional terms (analogously to the ones in (5)):

$$
\frac{1}{4}(\bar{\mathcal{Z}}\partial_\mu\mathcal{Z}) - \frac{1}{2\sqrt{2}}e^{-\sqrt{2}\hat{\varphi}}\gamma_5\partial_\mu\hat{\pi}
\tag{32}
$$

and derivatives (5) of the fields ρ_i and Θ^M_i acquire the following additional terms (analogously to the ones in (28)):

$$
\frac{1}{4}\varepsilon^{abc}(S^-_\mu - P_\mu)_{ab}\tau^c{}_i{}^j + e^{\varphi}Q^{a+}_\mu\tau^a{}_i{}^j.
\tag{33}
$$

3.2. Gauge interaction and symmetry breaking

To investigate the possibilities of the symmetry breaking, we have to switch on a gauge interaction in the hidden sector of the model. The hidden sector is the same as for $W(p,q)$-model, it has just been rewritten in other variables. And it has translations $\pi_m \to \pi_m + \Lambda_m$ and $l^m \to l^m + \Lambda^m$ as part of its global symmetry group. These translations correspond to the translation of the field π^{mn} in $W(p,q)$-model. And, as it has been shown in the previous section, by making part of these translations local one can obtain the spontaneous symmetry breaking with two arbitrary mass scales and vanishing cosmological constant.

In order to learn, if the mass splitting in the X-, Y- and Z-multiplets appear in our model, we also have to switch on the gauge interaction, which touches the corresponding sectors. The question, we are interested to answer as well, is: if the Yukawa couplings, mixing the fields from the different multiplets, appear in the Lagrangian after the symmetry breaking.

In general case there are two global symmetries of the sector, including X-,Y- and Z-multiplets, which do not touch the hidden sector. The first one is the following:

$$
\delta X^A = \alpha^{M_1}(T^{M_1}_1)^{AB}X^B, \quad \delta Y^{\dot{A}} = \alpha^{M_1}(\dot{T}^{M_1}_1)^{\dot{A}\dot{B}}Y^{\dot{B}}, \quad \delta Z^{\ddot{A}} = \alpha^{M_1}(\ddot{T}^{M_1}_1)^{\ddot{A}\ddot{B}}Z^{\ddot{B}},
\tag{34}
$$

where generators $\dot{T}_1$ and $\ddot{T}_1$ are connected to generator T_1:

$$
(\dot{T}^{M_1}_1)^{\dot{A}\dot{B}} = \frac{1}{4}(T^{M_1}_1)^{AB}(\Sigma^{AB})^{\dot{A}\dot{B}}, \qquad (\ddot{T}^{M_1}_1)^{\ddot{A}\ddot{B}} = \frac{1}{4}(T^{M_1}_1)^{AB}(\Sigma^{AB})^{\ddot{A}\ddot{B}}.
\tag{35}
$$

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The generators T_1 are chosen to be real, antisymmetric and correspond to some subgroup of the orthogonal group $O(p)$, where p is the number of the X-multiplets. The fields of the X-multiplets transform under the vector representation of this group and the fields of the Y- and Z-multiplets transform under the spinor representation.

As it has been shown in [7,8], depending of the values of p, q, there exists an additional global symmetry group:

$$\delta Y^{\dot{A}} = \alpha^{M_2}(\dot{T}_2^{M_2})^{\dot{A}\dot{B}}Y^{\dot{B}}, \qquad \delta Z^{\ddot{A}} = \alpha^{M_2}(\ddot{T}_2^{M_2})^{\ddot{A}\ddot{B}}Z^{\ddot{B}}, \qquad \delta X^A = 0, \qquad (36)$$

where generators $\dot{T}_2$ and $\ddot{T}_2$ commute with Γ-matrices:

$$\dot{T}_2^{\dot{A}\dot{B}}(\Gamma^A)^{\dot{B}\ddot{A}} - (\Gamma^A)^{\dot{A}\ddot{B}}\ddot{T}_2^{\ddot{B}\ddot{A}} = 0. \qquad (37)$$

The generators T_i have to obey the following commutation relations:

$$T_i^{M_i} \cdot T_i^{N_i} - (M_i \leftrightarrow N_i) = f_i^{M_i N_i K_i}T_i^{K_i}, \qquad (38)$$

where $f_i^{M_i N_i K_i}$ are the structure constants of the corresponding symmetry groups and the indices M_i are the indices of the adjoint representations.

Let us denote these two sets of indices by a general index M: $M = \{(M_1), (M_2)\}$, structure constants of the direct product of these symmetry groups by f^{MNK} and

$$
\begin{aligned}
(\dot{T}^M)^{\dot{A}\dot{B}} &= \{(\dot{T}_1^{M_1})^{\dot{A}\dot{B}}, (\dot{T}_2^{M_2})^{\dot{A}\dot{B}}\}, \qquad (\ddot{T}^M)^{\ddot{A}\ddot{B}} = \{(\ddot{T}_1^{M_1})^{\ddot{A}\ddot{B}}, (\ddot{T}_2^{M_2})^{\ddot{A}\ddot{B}}\}, \\
(T^M)^{AB} &= \{(T_1^{M_1})^{AB}, 0\}.
\end{aligned}
\qquad (39)
$$

Then the commutation relations (38) can be rewritten in the general form:

$$T^M \cdot T^N - (M \leftrightarrow N) = f^{MNK}T^K \qquad (40)$$

and relationship (37), taking into account that $\Gamma^A \Sigma^{BC} - \Sigma^{BC}\Gamma^A = 2(\delta^{AB}\Gamma^C - \delta^{AC}\Gamma^B)$, can be rewritten in the following form:

$$(\dot{T}^M)^{\dot{A}\dot{B}}(\Gamma^A)^{\dot{B}\ddot{A}} - (\Gamma^A)^{\dot{A}\ddot{B}}(\ddot{T}^M)^{\ddot{B}\ddot{A}} = (T^M)^{AB}(\Gamma^B)^{\dot{A}\ddot{A}}. \qquad (41)$$

Now we can switch on the gauge interaction. For that let us make the following substitution in the Lagrangian and the supertransformation laws:

$$
\begin{aligned}
\partial_\mu \mathcal{Z}^M &\rightarrow \partial_\mu \mathcal{Z}^M - f^{MNK}A_\mu{}^N \mathcal{Z}^K, \qquad \partial_\mu \Theta_i{}^M \rightarrow \partial_\mu \Theta_i{}^M - f^{MNK}A_\mu{}^N \Theta_i{}^K, \\
\partial_\mu l^m &\rightarrow \partial_\mu l^m - A_\mu^M M^{Mm}, \qquad \partial_\mu \pi_m \rightarrow \partial_\mu \pi_m - A_\mu^M N_m^M, \\
\partial_\mu X^A &\rightarrow \partial_\mu X^A - A_\mu^M (T^M)^{AB}X^B, \qquad \partial_\mu \Omega^A \rightarrow \partial_\mu \Omega^A - A_\mu^A (T^M)^{AB}\Omega^B
\end{aligned}
\qquad (42)
$$

and analogous expressions for the fields from Y- and Z-multiplets.

In order to restore the invariance of the Lagrangian under the supertransformations, one has to add to the Lagrangian the following terms:

$$
\begin{aligned}
\mathcal{L}'^{\mathcal{F}} &= e^{-\hat{\varphi}/\sqrt{2}} \left\{ -\frac{1}{4}\bar{\Psi}_{\mu i}\sigma^{\mu\nu}\varepsilon^{ij}(R_+^a + G^a)\tau^a{}_j{}^k\Psi_{\nu k} + \right. \\
&\quad + \frac{i}{2\sqrt{2}}\bar{\Psi}_{\mu i}\gamma^\mu(\bar{R}_+^a + \bar{G}^a)\tau^a{}_i{}^j\rho_j - \frac{i}{2}\bar{\Psi}_{\mu i}\gamma^\mu(R_+^{Ma} + G^{Ma})\tau^a{}_i{}^j\Theta_j^M - \\
&\quad - \frac{i}{2}\bar{\Psi}_{\mu i}\gamma^\mu\bar{R}_+^a\tau^a{}_i{}^j\varepsilon_{jk}\chi^k - \frac{i}{2}\bar{\Psi}_{\mu i}\gamma^\mu\varepsilon_{ij}(\bar{R}_-^a\delta_j{}^k - \varepsilon^{abc}\bar{G}^b\tau^c{}_j{}^k)\lambda_a^k + \\
&\quad + \frac{1}{\sqrt{2}}\bar{\rho}_i\bar{R}_+^a\tau^a{}_j{}^i\chi^j - \frac{1}{\sqrt{2}}\bar{\rho}_i(\bar{R}_-^a\delta_i{}^j - \varepsilon^{abc}\bar{G}^b\tau^c{}_i{}^j)\lambda_a^j - \bar{\chi}^i\varepsilon_{ij}\bar{R}_-^a\lambda_a^j + \\
&\quad + \frac{1}{\sqrt{2}}\bar{\rho}_i\varepsilon^{ij}(R_+^{Ma} + G^{Ma})\tau^a{}_j{}^k\Theta_k^M + \frac{1}{4}\bar{\lambda}_a^i\varepsilon_{ij}(\bar{R}_+^b - \bar{G}^b)\tau^b{}_k{}^j\lambda_a^k + \\
&\quad + \bar{\lambda}_a^i(R_-^{Ma}\delta_i{}^j - \varepsilon^{abc}G^{Mb}\tau^c{}_i{}^j)\Theta_j^M - \frac{1}{4}\bar{\Theta}_i^M\varepsilon_{ij}(R_+^a - G^a)\tau^a{}_j{}^k\Theta_k^M + \\
&\quad + \frac{1}{4}\bar{\chi}^i\varepsilon_{ij}(\bar{R}_+^a - \bar{G}^a)\tau^a{}_k{}^j\chi^k - \bar{\chi}^i R_+^{Ma}\tau^a{}_i{}^j\Theta_j^M - \frac{1}{2}\varepsilon^{abc}\bar{\lambda}_a^i\varepsilon_{ij}\bar{G}^c\lambda_b^j + \\
&\quad + \frac{i}{2}\bar{\Omega}^{iA}\gamma^\mu\varepsilon^{ij}(\mathcal{F}_X{}^A)_j{}^k\Psi_{\mu k} + \frac{i}{2}\bar{\Lambda}^{i\dot{A}}\gamma^\mu\varepsilon^{ij}(\mathcal{F}_Y{}^{\dot{A}})_j{}^k\Psi_{\mu k} + \frac{i}{2}\bar{\Sigma}^{i\ddot{A}}\gamma^\mu\varepsilon^{ij}(\mathcal{F}_Z{}^{\ddot{A}})_j{}^k\Psi_{\mu k} \\
&\quad + \frac{1}{\sqrt{2}}\bar{\Omega}^{iA}(\bar{\mathcal{F}}_X{}^A)_i{}^j\rho_j + \frac{1}{\sqrt{2}}\bar{\Lambda}^{i\dot{A}}(\bar{\mathcal{F}}_Y{}^{\dot{A}})_i{}^j\rho_j + \frac{1}{\sqrt{2}}\bar{\Sigma}^{i\ddot{A}}(\bar{\mathcal{F}}_Z{}^{\ddot{A}})_i{}^j\rho_j - \\
&\quad - \bar{\Omega}^{iA}(\mathcal{F}_X^{MA})_i{}^j\Theta_j^M - \bar{\Lambda}^{i\dot{A}}(\mathcal{F}_Y^{M\dot{A}})_i{}^j\Theta_j^M - \bar{\Sigma}^{i\ddot{A}}(\mathcal{F}_Z^{M\ddot{A}})_i{}^j\Theta_j^M - \\
&\quad + \frac{1}{2}\bar{\Lambda}^{i\dot{A}}(\bar{\mathcal{F}}_Y{}^{\dot{A}})_i{}^j\varepsilon_{jk}\chi^k - \frac{1}{2}\bar{\Lambda}^{i\dot{A}}\tau^a{}_i{}^j(\bar{\mathcal{F}}_Y{}^{\dot{A}})_j{}^k\varepsilon_{kp}\lambda_a^p - e^\varphi\bar{\Omega}^{iA}\varepsilon_{ij}\bar{F}_X{}^A\chi^j + \\
&\quad + \frac{1}{2}\bar{\Sigma}^{i\ddot{A}}(\bar{\mathcal{F}}_Z{}^{\ddot{A}})_i{}^j\varepsilon_{jk}\chi^k + \frac{1}{2}\bar{\Sigma}^{i\ddot{A}}\tau^a{}_i{}^j(\bar{\mathcal{F}}_Z{}^{\ddot{A}})_j{}^k\varepsilon_{kp}\lambda_a^p - \bar{\Omega}^{iA}\varepsilon_{ij}\bar{\mathcal{Z}}^{AB}X^{mB}y_{ma}\lambda_a^j - \\
&\quad - \frac{1}{2}\Gamma^{A\dot{A}\ddot{A}}\{\bar{\Sigma}^{i\ddot{A}}\varepsilon_{ij}(\bar{\mathcal{F}}_X{}^A)_k{}^j\Lambda^{k\dot{A}} + \bar{\Lambda}^{i\dot{A}}(\bar{\mathcal{F}}_Z{}^{\ddot{A}})_i{}^j\varepsilon_{jk}\Omega^{kA} - \bar{\Sigma}^{j\ddot{A}}(\bar{\mathcal{F}}_Y{}^{\dot{A}})_i{}^j\varepsilon_{jk}\Omega^{kA}\} - \\
&\quad - \frac{1}{2}\bar{\Omega}^{iA}\varepsilon_{ij}\bar{\mathcal{Z}}^{AB}\Omega^{jB} - \frac{1}{2}\bar{\Lambda}^{i\dot{A}}\varepsilon_{ij}\bar{\mathcal{Z}}^{\dot{A}\dot{B}}\Lambda^{j\dot{B}} - \frac{1}{2}\bar{\Sigma}^{i\ddot{A}}\varepsilon_{ij}\bar{\mathcal{Z}}^{\ddot{A}\ddot{B}}\Sigma^{j\ddot{B}} + \\
&\quad + \frac{1}{4}(R_+^{Ma} - G^{Ma})(\bar{\Omega}^i\varepsilon_{ij}\bar{\mathcal{Z}}^M\tau^a{}_k{}^j\Omega^k) + \\
&\quad + \frac{1}{4}(R_-^{Ma} - G^{Ma})\{\bar{\Lambda}^i\varepsilon_{ij}\bar{\mathcal{Z}}^M\tau^a{}_k{}^j\Lambda^k + \bar{\Sigma}^i\varepsilon_{ij}\bar{\mathcal{Z}}^M\tau^a{}_k{}^j\Sigma^k\} - \\
&\quad \left. - \frac{1}{2}f^{MNK}\left(\frac{i}{2}\bar{\Psi}_{\mu i}\gamma^\mu\mathcal{Z}^N\bar{\mathcal{Z}}^K\Theta_i^M - \varepsilon^{ij}\bar{\Theta}_i^M\mathcal{Z}^N\Theta_j^K + \frac{1}{\sqrt{2}}\varepsilon^{ij}\bar{\Theta}_i^M\mathcal{Z}^N\bar{\mathcal{Z}}^K\rho_j\right)\right\}, \quad (43)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}'^{\mathcal{B}} &= -\frac{1}{2}e^{-\sqrt{2}\hat{\varphi}}\left\{ (R_+^{Ma} + G^{Ma})^2 + |R_+^a|^2 + |R_-^a|^2 + 2|G^a|^2 + e^{2\varphi}|F_X{}^A|^2 - \right. \\
&\quad - g_{mn}(X^m\mathcal{Z}\bar{\mathcal{Z}}X^n) - \frac{e^\varphi}{\Delta}(Y\mathcal{Z}\bar{\mathcal{Z}}Y) + \Delta e^\varphi g^{mn}F_{Y_m}{}^{\dot{A}}\bar{F}_{Y_n}{}^{\dot{A}} + \Delta e^\varphi|F_Z{}^{\ddot{A}}|^2 + \\
&\quad \left. + \frac{e^\varphi}{\Delta}g_{mn}F_Z{}^{m\ddot{A}}\bar{F}_Z{}^{n\ddot{A}} + \frac{1}{8}(f^{MNK}\mathcal{Z}^N\bar{\mathcal{Z}}^K)^2\right\}, \quad (44)
\end{aligned}
$$

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where the following notations are used:

$$R_{\pm}^{Ma} = e^{\varphi}\{y_{ma}(A^{mM} + 2\pi^{mn}B_n^M + X^m X^n B_n^M) \pm y^{ma}B_m^M\},$$

$$A^{mM} = -2M^{mM} + XT^M X^m - \frac{1}{2}X^{mA}\{Y\Gamma^A\ddot{T}^M Z + Y\dot{T}^M\Gamma^A Z +$$

$$+ Y_m\Gamma^A\ddot{T}^M Z^m + Y_m\dot{T}^M\Gamma^A Z^m\} + \frac{1}{2}\varepsilon^{mnk}Y_n\dot{T}^M Y_k + Z\ddot{T}^M Z^m,$$

$$B_m^M = -\frac{1}{2}N_m^M + Y\dot{T}^M Y_m + \frac{1}{2}\varepsilon_{mnk}Z^n\ddot{T}^M Z^k,$$

$$G_a^M = \frac{1}{2}\varepsilon^{abc}y_{mb}(X^m T^M X^n)y_{nc},$$

$$(\mathcal{F}_X^{MA})_i{}^j = e^{\varphi}F_X^{MA}\delta_i{}^j + (T^M)^{AB}X^{mB}y_{ma}\tau^a{}_i{}^j,$$

$$(\mathcal{F}_Y^{M\dot{A}})_i{}^j = e^{\varphi/2}\{\frac{1}{\sqrt{\Delta}}(T^M)^{\dot{A}\dot{B}}Y^{\dot{B}}\delta_i{}^j + \sqrt{\Delta}F_{Ym}^{M\dot{A}}y^{ma}\tau^a{}_i{}^j\},$$

$$(\mathcal{F}_Z^{M\ddot{A}})_i{}^j = e^{\varphi/2}\{\sqrt{\Delta}F_Z^{M\ddot{A}}\delta_i{}^j + \frac{1}{\sqrt{\Delta}}F_Z^{Mm\ddot{A}}y_{ma}\tau^a{}_i{}^j\},$$

$$F_X^{MA} = (T^M)^{AB}X^B - 2X^{mA}B_m^M + \frac{1}{2}X^{mA}\{Y\Gamma^A\ddot{T}^M Z + Y\dot{T}^M\Gamma^A Z +$$

$$+ Y_m\Gamma^A\ddot{T}^M Z^m + Y_m\dot{T}^M\Gamma^A Z^m\},$$

$$F_Z^{Mm\ddot{A}} = (T^M)^{\ddot{A}\dot{B}}Z^{m\dot{B}} + X^{mA}(\Gamma^A)^{\dot{A}\ddot{A}}(T^M)^{\dot{A}\dot{B}}Y^{\dot{B}},$$

$$F_{Ym}^{M\dot{A}} = (T^M)^{\dot{A}\dot{B}}Y_m^{\dot{B}} - \varepsilon_{mnk}\{\pi^{nk}\delta^{\dot{A}\dot{B}} + \frac{1}{2}X^{nA}X^{kB}(\Sigma^{AB})^{\dot{A}\dot{B}}\}(T^M)^{\dot{B}\dot{C}}Y^{\dot{C}} -$$

$$- \varepsilon_{mnk}X^{nA}(\Gamma^A)^{\dot{A}\ddot{A}}(T^M)^{\ddot{A}\dot{B}}Z^{k\dot{B}},$$

$$F_Z^{M\ddot{A}} = (T^M)^{\ddot{A}\ddot{B}}Z^{\ddot{B}} + \varepsilon_{mnk}\{\pi^{mn}\delta^{\ddot{A}\ddot{B}} + \frac{1}{2}X^{mA}X^{nB}(\Sigma^{AB})^{\ddot{A}\ddot{B}}\}(T^M)^{\ddot{B}\ddot{C}}Z^{k\ddot{C}} -$$

$$- X^{mA}(\Gamma^A)^{\dot{A}\ddot{A}}(T^M)^{\dot{A}\dot{B}}Y_m^{\dot{B}} + \varepsilon_{mnk}X^{mA}(\Gamma^A)^{\dot{A}\ddot{B}}\{\pi^{nk}\delta^{\ddot{A}\ddot{B}} -$$

$$- \frac{1}{6}X^{nB}X^{kC}(\Sigma^{BC})^{\ddot{B}\ddot{A}}\}(T^M)^{\dot{A}\dot{B}}Y^{\dot{B}}. \tag{45}$$

Additional terms to the supertransformation laws are the following:

$$\delta'\Psi_{\mu i} = \frac{i}{2}e^{-\hat{\varphi}/\sqrt{2}}\gamma_\mu\varepsilon^{ij}(R_+^a + G^a)\tau^a{}_j{}^k\eta_k,$$

$$\delta'\chi^i = -e^{-\hat{\varphi}/\sqrt{2}}\varepsilon^{ij}(R_+^a + G^a)\tau^a{}_j{}^k\eta_k,$$

$$\delta'\lambda_a^i = e^{-\hat{\varphi}/\sqrt{2}}\varepsilon^{ij}(R_-^a\delta_j{}^k - \varepsilon^{abc}G^b\tau^c{}_j{}^k)\eta_k,$$

$$\delta'\rho_i = -\frac{1}{\sqrt{2}}e^{-\hat{\varphi}/\sqrt{2}}(R_+^a + G^a)\tau^a{}_i{}^j\eta_j,$$

$$\delta'\Theta_i^M = e^{-\hat{\varphi}/\sqrt{2}}\{(R_+^{Ma} + G^{Ma}) + \frac{1}{2}\bar{\mathcal{Z}}^M(R_+^a + G^a) +$$

$$+ \frac{1}{2}\mathcal{Z}^M(\bar{R}_+^a + \bar{G}^a)\}\tau^a{}_i{}^j\eta_j + \frac{1}{2}e^{-\hat{\varphi}/\sqrt{2}}f^{MNK}\mathcal{Z}^N\bar{\mathcal{Z}}^K\eta_i,$$

$$\delta'\Omega^{iA} = -e^{-\hat{\varphi}/\sqrt{2}}\varepsilon^{ij}(\mathcal{F}_X{}^A)_j{}^k\eta_k,$$

$$\delta'\Lambda^{i\dot{A}} = -e^{-\hat{\varphi}/\sqrt{2}}\varepsilon^{ij}(\mathcal{F}_Y{}^{\dot{A}})_j{}^k\eta_k, \quad \delta'\Sigma^{i\ddot{A}} = -e^{-\hat{\varphi}/\sqrt{2}}\varepsilon^{ij}(\mathcal{F}_Z{}^{\ddot{A}})_j{}^k\eta_k. \tag{46}$$

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Let us again choose vacuum expectation values of the scalar fields in the following form: $< y_{ma} > \sim \delta_{ma}$ and $< \mathcal{Z}^M > = (1, i, 0, ..., 0)$. In this, the potential has the minimum at vanishing vacuum expectation values for the fields X, Y and Z, its value at the minimum being: $V_0 = 2(M^{Ma} - \frac{1}{4}N_a^M)^2$, where indices $M = 1, 2$, and $a = 1, 2, 3$. One can see from this formula, that if one makes the following choice of the gauge group:

$$M^{11} = \frac{1}{4}N_1^1 = m_1 \qquad M^{22} = \frac{1}{4}N_2^2 = m_2,$$

(47)

all the other parameters being equal to zero, then it is easy to check, that a cosmological term vanishes.

The gravitino mass matrix takes the form:

$$M^{ij} = -\frac{1}{2}\varepsilon^{ij} < R_+^a > \tau^a{}_j{}^k \sim \begin{pmatrix} m_1 + m_2 & 0 \\ 0 & -m_1 + m_2 \end{pmatrix}$$

(48)

in a full correspondence with (25). Scalar fields X^{1A} and X^{2A} acquire masses $2m_1$ and $2m_2$ correspondingly, the spinors Ω_i^A — the same masses as the gravitini, while the other scalar and fermionic fields of X-, Y- and Z-multiplets remain massless. Also, as one should have expected, all fields of the vector multiplet except the vector ones acquire masses.

Moreover, it is interesting that for such a vacuum expectation values of the scalar fields there exist non-trival Yukawa couplings, mixing fields from different hypermultiplets:

$$-\frac{1}{2}\bar{\Sigma}^{i\ddot{A}}\Gamma^{A\dot{A}\ddot{A}}\varepsilon_{ij}(X^{mA}\bar{\mathcal{Z}}^M N_m^M)\Lambda^{j\dot{A}}.$$

(49)

Note, that we work in a system where gravitational coupling constant $k = 1$. The value of Yukawa couplings above, which is determined by the vacuum expectation value of $\bar{\mathcal{Z}}^M N_m^M$, is $m_{1,2}/m_{pl}$, so for such a coupling to be essential one has to have the scale of $N = 2 \to N = 1$ supersymmetry breaking not much below the Plank scale, e.g. of the order of the grand unification scale.

Conclusion

Thus, in this paper we have considered the possibility to switching on the gauge interaction for both types of $N = 2$ supergravity models, where the scalar fields of the hypermultiplets parameterize the nonsymmetric quaternionic manifolds. First of all, we were interested in the possibility to have the spontaneous supersymmetry breaking without a cosmological term in these models. As we have shown, for the $W(p, q)$ models the pattern of supersymmetry breaking resembles very much the one for the usual $O(4, p)/O(4) \otimes O(p)$ quaternionic model. In turn, the spontaneous supersymmetry breaking in the $V(p, q)$ models leads to a more interesting picture. First, the mass terms are generated for some of the fields from the hypermultiplets and not for the vector multiplet ones only. Second, we have shown that as one of the byproducts of supersymmetry breaking one obtains the Yukawa couplings for the scalar and spinor fields of the hypermultiplets. Such couplings,

which are absent in the globally supersymmetric $N = 2$ gauge theories as well as in the $N = 2$ supergravity models with the symmetric quaternionic manifolds, could lead to interesting phenomenological consequences. We have not considered here a possibility to introduce the nonzero vacuum expectation values for the matter scalar fields along the flat directions of the potentials, which could give the gauge symmetry breaking, because we concentrated here on the general properties of these models and have not considered any specific models. But the results already obtained make these models quite promising and deserve further study.

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References

[1] V. A. Tsokur and Yu. M. Zinoviev, $N = 2$ supergravity models, based on the nonsymmetric quaternionic manifolds. I. Symmetries and Lagrangians, preprint IHEP 96-22, Protvino, 1996.

[2] D. V. Alekseevskii, *Math. USSR Izvestija* **9** (1975) 297.

[3] S. Cecotti, *Com. Math. Phys.* **124** (1989) 23.

[4] Yu. M. Zinoviev, *Int. J. of Mod. Phys.* **A7** (1992) 7515.

[5] S. Cecotti, L. Girardello and M. Porrati, *Phys. Lett.* **168B** (1986) 83.

[6] Yu. M. Zinoviev, *Yad. Fiz.* **46** (1987) 943.

[7] B. de Wit, F. Vanderseypen and A. van Proeyen, *Nucl. Phys.* **B400** (1993) 463.

[8] B. de Wit and A. van Proeyen, *Int. J. Mod. Phys.* **D3** (1994) 31.

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Модели $N = 2$ супергравитации, основанные на несимметрических кватернионных многообразиях. II. Калибровочные взаимодействия.

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