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**WILL UNITARITY SLOW DOWN THE RISE OF $F_2(x, Q^2)$
AT $x \rightarrow 0$?**

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Abstract

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On the grounds of the U -matrix form of s -channel unitarization, we consider constraints the unitarity provides for the total cross-section of the virtual photon-induced reactions and discuss the preasymptotic nature of hadron and photon scatterings at $\sqrt{s} \leq 0.5$ TeV.

Аннотация

Трошин С.М., Тюрин Н.Е. Замедлит ли унитарность рост $F_2(x, Q^2)$ при $x \rightarrow 0$?: Препринт ИФВЭ 96-29. – Протвино, 1996. – 8 с., библиогр.: 16.

На основе U -матричной формы s -канальной унитарности мы рассматриваем ограничения, которые унитарность накладывает на поведение полного сечения реакций, инициированных виртуальным фотоном, и обсуждаем предасимптотическую природу адронного и фотонного рассеяния при $\sqrt{s} \leq 0.5$ ТэВ.

Introduction

Recent HERA data [1,2] have clearly demonstrated a rising behavior of the structure function $F_2(x, Q^2)$ at small x and led to many interesting discussions [3] on the Pomeron structure in soft and hard interactions. These data can be interpreted as the dependence of the virtual photon–proton total cross–section $\sigma_{tot}^{\gamma^*p}(W)$ on the center of mass energy W . The observed rise of $\sigma_{tot}^{\gamma^*p}(W)$ is consistent with a linear dependence on W and has been treated somewhere as a manifestation of hard BFKL Pomeron [4]. This linear rise of $\sigma_{tot}^{\gamma^*p}(W)$ has been considered, to some extent, as a surprising fact based on our knowledge of the energy dependence of total cross–sections in hadronic interactions.

Indeed, the above comparison between photon–induced and hadron–induced interactions is quite legitimate since the photon is demonstrating its hadronlike nature for a long time. It has been accounted by the Vector Dominance Model (VDM) [5]. The apparent contradiction between the hadron and virtual photon total cross–section behaviors has no fundamental meaning in the preasymptotic energy region where the Froissart–Martin bound does not restrict the particular form of hadronic cross–sections. Indeed, it has been shown that the total hadronic cross–sections rise is, in fact, consistent with the linear dependence on \sqrt{s} [6] up to $\sqrt{s} \leq 0.5$ TeV and thus there is no qualitative disagreement with the trends observed in the $\sigma_{tot}^{\gamma^*p}(W)$ at HERA energies ($W \leq 0.3$ TeV).

At higher energies, the unitarity leads to deviation from the linear rise with \sqrt{s} in hadronic total cross–sections. It converts a powerlike preasymptotic increase into the asymptotic $\log^2 s$ rise.

However, it could not be the case for $\sigma_{tot}^{\gamma^*p}(W)$ since the asymptotic theorems cannot be directly applied to the virtual photon scattering. In this context, a practice to treat the virtual photon similar to the on–shell hadron cannot be considered as generally valid because of the off–shell effects.

The problem was addressed to in [7] on the basis of unitarity for off–shell scattering starting from the eikonal representation for the scattering amplitude. It was argued that the observed rise of $F_2(x, Q^2)$ at small x or $\sigma_{tot}^{\gamma^*p}(W)$ rise at large W values could be

considered as a true asymptotic behavior and the extension of the eikonal representation for off-shell particles did not provide limitations for the structure function F_2 at $x \rightarrow 0$.

In this paper we treat similar problems on the basis of the U -matrix approach to the scattering amplitude unitarization. We obtain the unitary representation for the case of off-shell particles and discuss assumptions when the unitarity does provide limitations for the amplitude of off-shell particle scattering. We also consider a specific model parameterization for the U -matrix obtained on the basis of chiral quark model in [8] and discuss the corresponding results for the structure function $F_2(x, Q^2)$ at small x values.

1. Off-shell scattering amplitude in the U -matrix approach

In the process of photon-hadron interactions there is a significant probability for photon coupling directly to vector mesons, i.e. a photon fluctuates into intermediate vector meson states which afterwards interact with a hadron. It can be expressed as the field current identity [9]:

$$J_\mu = \sum_V \frac{e}{f_V} m_V^2 V_\mu, \quad (1)$$

where J_μ is the electromagnetic current and V_μ — vector meson fields ($V = \rho, \omega, \varphi, J/\psi$), f_V^{-1} is the γ - V coupling constant and $e = \sqrt{4\pi\alpha_{em}}$. Owing to this identity the amplitude of virtual-photon Compton scattering can be represented in the form:

$$F_{\gamma\gamma} = \sum_{V,V'} \left(\frac{e}{f_V} \right) F_{VV'} \left(\frac{e}{f_{V'}} \right), \quad (2)$$

where $F_{VV'}$ is the scattering amplitude of the process $VN \rightarrow V'N$. We use diagonal approximation

$$F_{VV'} = F_{VV} \delta_{VV'}$$

since we expect that diffraction dissociation amplitudes can be neglected as compared to elastic scattering amplitudes [10]. The amplitudes $F_{VV'}$ in Eq. (2) describe the scattering of off-shell vector mesons. We consider for a while a single vector meson field and denote $F^{**}(s, t, Q^2)$, $F^*(s, t, Q^2)$ and $F(s, t)$ the amplitudes when both mesons (initial and final) are off mass shell, only an initial meson is off mass shell and both mesons are on mass shell, correspondingly. The virtualities of initial and final vector mesons were chosen equal to Q^2 since further we will need the amplitude of the forward virtual Compton scattering.

Our aim now is to consider unitarity constraints for the amplitudes F^* and F^{**} . We use the U -matrix form of unitary representation for the scattering amplitude. It is grounded on the relativistic generalization of the Heitler equation in the theory of radiation damping [11]. The U -matrix equation has been derived in the relativistic theory in [12] in the framework of the single-time formalism in QFT. It provides a simple algebraic form for the scattering amplitude of scalar on-shell particles in impact parameter representation:

$$F(s, b) = \frac{U(s, b)}{1 - iU(s, b)}, \quad (3)$$

where $U(s, b)$ is the generalized reaction matrix. It is considered as an input dynamical quantity similar, e.g. to the eikonal function. The inelastic overlap function is connected with $U(s, b)$ by the relation

$$\eta(s, b) = \text{Im}U(s, b)|1 - iU(s, b)|^{-2}. \quad (4)$$

Eq. (3) ensures the s -channel unitarity provided that $\text{Im}U(s, b) \geq 0$.

Eq. (3) has completely different analytical structure as compared to the eikonal form, in particular, it does not generate essential singularity in the complex s -plane at infinity while the eikonal representation does.

It is to be noted that the solution of unitarity in potential scattering for the case of off-shell particles in the K -matrix form (the U -matrix is its relativistic analog) was given for the first time in [13].

Eq. (3) was obtained on the basis of relation between the matrix element of the radiation operator

$$R(x_1, x_2; y_1, y_2) = -\langle 0 | \frac{\delta^4 S}{\delta \varphi_1^*(x_1) \delta \varphi_2^*(x_2) \delta \varphi_1(y_1) \delta \varphi_2(y_2)} S^+ | 0 \rangle$$

and the function $U(x_1, x_2; y_1, y_2)$ which parameterizes the evolution operator [12]. In the momentum space the relation has the form:

$$R(p_1, p_2; q_1, q_2) = U(p_1, p_2; q_1, q_2) + \frac{1}{2(2\pi)^3} \int dk_1 dk_2 U(p_1, p_2; k_1, k_2) D^-(k_1) D^-(k_2) R(k_1, k_2; q_1, q_2), \quad (5)$$

where

$$D^-(k) = 2\pi i \theta(k^0) \delta(k^2 - m^2).$$

In Eq. (5) only momenta of intermediate particles k_1 and k_2 always lie on the mass shell while the momenta of external particles can be shifted from the mass shell. Following Ref. [12] one can easily obtain equations for the amplitudes F^{**} and F^* . These equations have the same structure as the equation for the on-shell amplitude F but relate the different amplitudes. In the impact parameter representation ($s \gg 4m^2$) they can be written as follows

$$\begin{aligned} F^{**}(s, b, Q^2) &= U^{**}(s, b, Q^2) + iU^*(s, b, Q^2)F^*(s, b, Q^2), \\ F^*(s, b, Q^2) &= U^*(s, b, Q^2) + iU^*(s, b, Q^2)F(s, b). \end{aligned} \quad (6)$$

The solutions are

$$F^*(s, b, Q^2) = \frac{U^*(s, b, Q^2)}{1 - iU(s, b)}, \quad (7)$$

$$F^{**}(s, b, Q^2) = \frac{U^{**}(s, b, Q^2)}{1 - iU(s, b)} + i \frac{[U^*(s, b, Q^2)]^2 - U^{**}(s, b, Q^2)U(s, b)}{1 - iU(s, b)}. \quad (8)$$

Eq. (7) is quite similar to Eq. (3). The appearance of the second term in Eq. (8) reflects the role of off-shell effects. If this term were different from zero then we would arrive to the conclusions made in [7] on the basis of the generalization of eikonal representation for the off-shell particles, i.e. the unitarity does not lead to the constraint $|F^{**}(s, b, Q^2)| \leq 1$ and consequently a power-like asymptotic rise of the cross-sections does not contradict the unitarity.

However, there is another possibility, when

$$[U^*(s, b, Q^2)]^2 - U^{**}(s, b, Q^2)U(s, b) = 0. \quad (9)$$

We consider the latter in some details. Eq. (9) will be fulfilled identically if the following factorization occurs:

$$\begin{aligned} U^{**}(s, b, Q^2) &= \omega(s, b, Q^2)U(s, b)\omega(s, b, Q^2), \\ U^*(s, b, Q^2) &= \omega(s, b, Q^2)U(s, b). \end{aligned} \quad (10)$$

Such factorization is valid, e.g. in the Regge model with factorizable residues and the Q^2 -independent trajectory.

Eq. (9) is also valid in VDM if the fluctuation length d_{fluct} is significantly longer than the characteristic size of strong interactions, i.e.

$$d_{fluct} \simeq \frac{2\nu}{Q^2 + m_V^2} \gg 1 fm. \quad (11)$$

The fluctuation length is proportional to the inverse of minimum energy needed to put the vector meson on its mass shell. If Eq. (11) is valid we can treat an interacting vector meson as an on-shell hadron. Then Eq. (10) is also valid and the function ω is proportional to the vector meson propagator, i.e.

$$\omega(Q^2) = D_V(Q^2) = \frac{m_V^2}{Q^2 + m_V^2}. \quad (12)$$

Thus, we will have for F^* and F^{**} the following representations

$$F^*(s, b, Q^2) = \frac{U^*(s, b, Q^2)}{1 - iU(s, b)} = D_V(Q^2) \frac{U(s, b)}{1 - iU(s, b)}, \quad (13)$$

$$F^{**}(s, b, Q^2) = \frac{U^{**}(s, b, Q^2)}{1 - iU(s, b)} = D_V^2(Q^2) \frac{U(s, b)}{1 - iU(s, b)} \quad (14)$$

and the unitarity will provide

$$\begin{aligned} |F^*(s, b, Q^2)| &\leq D_V(Q^2), \\ |F^{**}(s, b, Q^2)| &\leq D_V^2(Q^2). \end{aligned} \quad (15)$$

Thus, the unitarity does restrict the amplitudes F^{**} and F^* in the kinematical region where Eq. (11) is valid, i.e. at not too high Q^2 -values. Therefore in the case of γ^*p -interactions there is a range of Q^2 where the general solution of unitarity Eqs. (7) and

(8) are reduced to a simpler form. It seems relevant and important for the behavior of the structure function F_2 at small values of x .

To discuss this issue we consider a specific model for hadron scattering based on the ideas of chiral quark models [8]. In the model valence quarks located in the central part of hadron are supposed to scatter in a quasi-independent way by the mean field generated by the virtual massive quarks and by the selfconsistent field of valence quarks themselves. In accordance with the quasi-independence of valence quarks the U -matrix is represented as the product:

$$U(s, b) = \prod_{q=1}^N \langle f_q(s, b) \rangle \quad (16)$$

in the impact parameter representation. Factors $\langle f_q(s, b) \rangle$ correspond to the averaged individual quark scattering amplitude in the mean field. Eq. (16) implies that all valence quarks are scattered in the mean field simultaneously. Such factorization could be considered as an effective implementation of constituent quarks' confinement. This mechanism resembles the Landshoff mechanism of quark-quark independent scattering [14]. However, in our case we refer not to the pair interaction of valence quarks from the two colliding hadrons, but rather to the Hartree-Fock approximation for the scattering of valence quark in the mean field.

The b -dependence of the function $\langle f_q \rangle$ related to the constituent quark formfactor has a simple form $\langle f_q \rangle \propto \exp(-m_q b/\xi)$. Following such considerations, the explicit form for the generalized reaction matrix (U -matrix) can be constructed and it allows one to obtain the scattering amplitude reproducing the main regularities observed in elastic scattering at small and large angles [8].

The total cross-section has the following energy and quark mass dependencies

$$\sigma_{tot}(s) = \frac{\pi \xi^2}{\langle m_q \rangle^2} \Phi(s, N), \quad (17)$$

where $\langle m_q \rangle = \frac{1}{N} \sum_{q=1}^N m_q$ is the mean value of the constituent quark masses in the colliding hadrons. The function Φ has the following energy dependence:

$$\Phi(s, N) = \begin{cases} (8\tilde{g}/N^2) [1 + N\alpha\sqrt{s}/\langle m_q \rangle], & s \ll s_0, \\ \ln^2 s, & s \gg s_0. \end{cases} \quad (18)$$

Thus, the s -dependence of total cross-section at $s \ll s_0$ is described by a simple linear function of \sqrt{s} . It has been shown that such dependence is consistent with the experimental data for the hadron total cross-sections up to $\sqrt{s} \sim 0.5$ TeV [6]. This is a preasymptotic dependence and it has nothing to do with the true asymptotics of the total cross-sections. In the considered model such behavior of the hadronic cross-sections reflects the energy dependence of the number of virtual quarks generated at the intermediate transient stage of hadronic interaction.

Eqs. (2), (14) and (18) provide the following dependence of $\sigma_{tot}^{\gamma^*p}(W)$:

$$\sigma_{tot}^{\gamma^*p}(W) = \begin{cases} a(Q^2) + b(Q^2)W & W \ll W_0, \\ c(Q^2) \ln^2 W, & W \gg W_0, \end{cases} \quad (19)$$

where the functions $a(Q^2)$, $b(Q^2)$ and $c(Q^2)$ depend on Q^2 and the parameters related to the quark scattering in the mean field. W_0 separates preasymptotic and asymptotic region. It was estimated [6] at the value $W_0 \simeq 2$ TeV. Using the relations between $\sigma_{tot}^{\gamma^*p}(W)$ and $F_2(x, Q^2)$:

$$\sigma_{tot}^{\gamma^*p}(W) = \frac{4\pi\alpha_{em}}{Q^4} \frac{4m_p^2 x^2 + Q^2}{1-x} F_2(x, Q^2) \quad (20)$$

and

$$W^2 = Q^2 \left(\frac{1}{x} - 1 \right) + m_p^2$$

it is easily to get the explicit form for $F_2(x, Q^2)$, which is similar to Eq. (19). Thus, we can conclude that the s -channel unitarity does provide the constraint for $F_2(x, Q^2)$. In the above unitary approach the preasymptotic behavior

$$F_2(x, Q^2) \propto 1/\sqrt{x} \quad (21)$$

at small x is to be converted into

$$F_2(x, Q^2) \propto \ln^2(1/x) \quad (22)$$

at $x \rightarrow 0$. The line $Q^2 = xW_0^2$ separates the asymptotic and preasymptotic regions.

Conclusion

We have demonstrated that under reasonable assumptions the unitarity restricts the behavior of the structure function $F_2(x, Q^2)$ at $x \rightarrow 0$. This result is relevant for the region of not too high Q^2 when the fluctuation length is long enough to form the bound state of the two quarks produced by a fluctuating virtual photon. Usually, the direct extension of the on-shell unitarity for the virtual photon-hadron scattering by analogy with hadron-hadron scattering is tacitly implied. It could not be valid in the whole kinematical region due to off-shell effects. We have argued that it can be justified for the limited region of Q^2 -values.

The treatment of photon-induced reactions similar to the hadron-induced ones is consistent with the fact that the hadronic as well as (real and virtual) photon-hadron total cross-sections can be fitted by the same functional energy dependence in the preasymptotic energy range [6]. The original dynamics of hadronic interactions could be manifested directly in the preasymptotic energy region while in the region of very high energies the unitarity plays the most important role. It screens the dynamical mechanism and provides a similar behavior for the cross-sections in the different dynamical approaches.

At high Q^2 the photon also can split into $\bar{q}q$ pair, but this pair will not have enough time to form the bound state before interacting with a proton. In the latter case quarks cannot still be considered as a coherent pair [15] and due to that the function $U(s, b)$ is not to be represented as the product of the smeared quark scattering amplitudes.

At such high Q^2 values the interaction can be treated by the perturbative QCD (cf. e.g. [16]). Here the off-shell effects could play a significant role and the asymptotic behavior of the structure functions at $x \rightarrow 0$ will not be affected by the screening corrections. Those corrections will be suppressed according to Eq. (8).

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