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HADRONIC PRODUCTION OF HEAVY MESONS
IN PERTURBATIVE QCD

Abstract

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In the framework of perturbative QCD in the fourth order over the α_s coupling constant for the production of two pairs of quarks and in the model of weak binding of the quarks into meson, the analysis of the hadronic production of mesons, containing b-quark, is performed, and a minimal transverse momentum is determined, so that at the momenta greater than the found one, the meson differential spectrum can be reliably described in the model of factorization of the hard b-quark production and the subsequent fragmentation into the mesons. At low transverse momenta of the meson, nonfragmentational contributions are essential. They are determined by the complete set of QCD diagrams in the given order over α_s, so that the latters result in the effect of destructive interference with the diagrams of fragmentation at $z = 2E/\sqrt{s}$ close to 1.

Аннотация

Бережной А.В., Киселев В.В., Лиходед А.К. Адронное образование тяжелых мезонов в пертурбативной КХД: Препринт ИФВЭ 96-31. – Протвино, 1996. – 9 с., 6 рис., библиогр.: 13.

В рамках пертурбативной КХД в четвертом порядке по константе связи α_s для образования двух пар кварков и в модели слабой связи кварков в мезоне проведен анализ адронного рождения мезонов, содержащих b-кварк, и определен минимальный поперечный импульс, больше которого спектр мезонов может надежно описываться в модели факторизации жесткого образования b-кварков и последующей фрагментации в мезоны. При малых поперечных импульсах мезона существенны нефрагментационные вклады, определяемые полным набором диаграмм КХД в заданном порядке по α_s, которые приводят к эффекту деструктивной интерференции с диаграммами фрагментации при $z = 2E/\sqrt{s}$, близких к 1.

Introduction

In the study of the heavy quark production mechanisms, the consideration within the perturbative theory is basic. This is related with the fact that the heavy quark mass m_Q determines the scale being much greater than the confinement scale in QCD, $m_Q \gg \Lambda_{QCD}$. Then the cross-section of the heavy quark production is factorized as the partonic cross-section of the hard production of $Q\bar{Q}$ quarks and the distribution functions of quarks and gluons, in the interaction of which the $Q\bar{Q}$ pair is produced.

Next step is connected with the heavy quark transform into hadrons containing these quarks. Ordinary in this consideration, one uses the model of the heavy quark fragmentation into the quarkonium $Q \to (Q\bar{q}) + X$ with the fragmentation functions measured in the $(Q\bar{q})$-hadron production in e^+e^--annihilation, for instance. Such approach is the generalization of the factorization theorem on the case of hadronic production [1]. Apparently, the scheme mentioned above does not satisfactory work as one can see, say, in the B-meson production at the FNAL collider energy [2]. Indeed, the result of theoretical predictions for the b-quark production in the $O(\alpha_s^3)$-calculations is in the systematic disagreement with the experiment. On the other hand, the production of mixed flavor heavy quarkonium, B_c, considered in refs. [3,4,5], where the hadronization is already included as the composite element of the model, points out the deviation from the naive intuitive expectation of the fragmentation regime dominance.

The detailed consideration of the B_c-meson production processes shows, for instance, that in a broad kinematical region these processes can not be reduced to the simple consequence: 1) the b-quark production, 2) the b-quark fragmentation into the B_c-mesons with the fragmentation function $D_{b \to B_c}(z)$ known from the calculations in e^+e^--annihilation. So, for the B_c production in $\gamma\gamma$-collisions considered in [6], the c-quark fragmentation into B_c is greater than the $b \to B_c$ fragmentation. Remember, that in e^+e^--annihilation the $c \to B_c$ fragmentation is suppressed by two orders of magnitude with respect to the $b \to B_c$ fragmentation [7].

In the hadronic production, where there is no such enforcement of the c-quark contribution due to the $(Q_c/Q_b)^4 = 16$ factor, the b-quark fragmentation into B_c is still not

dominant. The naive expectations of the fragmentation dominance are just realized at $p_T \gg M_{B_c}$ ($p_T > 30$ GeV for B_c and $p_T > 40$ GeV for B_c^*). By the way, the contribution of diagrams different from those of the fragmentation type on the topology (the recombination contribution in the rest of the paper) is basic.

In this paper we investigate in what measure these additional contributions can be essential in the B-meson production. We evaluate the transverse momentum, so that at momenta greater than the determined one, the heavy meson production can be reliably described by the fragmentation model. We study these problems in the framework of model for the production of weakly bound quarkonium with the following standard assumptions. 1) The calculation of the production cross-section for two pairs of quarks is performed in the leading Born approximation (diagrams of the fourth order over α_s). 2) We neglect the binding energy and relative quark motion inside the meson. The latter means that the quarks inside the $(Q\bar{q})$-meson move with the parallel momenta $p_{\bar{q}} = p_Q \frac{m_q}{m_Q}$. Since the derivation of analytic expressions for the considered processes looks to be problematic, we use a numerical way for the evaluation of amplitudes corresponding to 36 Feynman diagrams in the hadronic production and to 20 diagrams in the photonic production of quarkonium.

1. Factorization of inclusive spectrum of heavy meson

The basic subject of our discussion below will be the factorization of the inclusive cross-section for the hadronic production of heavy meson in the region of high transverse momenta. In present Section we consider the subprocess of the gluon-gluon production of the meson. The physical basis for the construction of the factorization model is the appearance of two energetic scales determining the hard production of quarks at high transverse momenta and a soft nonperturbative forming of the bound state. The case, when the conditions $\sqrt{s} \gg m_Q \gg m_q \gg \Lambda_{\text{QCD}}$ are valid, is of a special interest. At the given ordering of scales, the quark production allows the description in the framework of perturbative QCD, and their hadronization into the meson is described by the nonrelativistic model of weakly bound state: the energy of the quark binding is low, $\epsilon_{Q\bar{q}} \ll m_q, \ m_Q$, and the perturbative production of quarks takes place in the region, where the quarks composing the meson move with the same velocity v_μ equal to the meson velocity. Then the quark momenta are equal to $p_Q^\mu = m_Q \cdot v^\mu, \ \ p_q^\mu = m_q \cdot v^\mu$. The $m_Q \gg m_q$ condition provides the appearance of two scales in the production process of quarks with different flavors, so that we can talk on the Q-quark fragmentation into the $(Q\bar{q})$-meson.

The technique of such calculations was given in refs. [3,6,7], in details.

As in the model of fragmentation, in the $O(\alpha_s^4)$-order over the QCD coupling constant the cross-section of the subprocess $gg \to (Q\bar{q}) + \bar{Q} + q$ is proportional to the $\alpha_s^4 |\Psi(0)|^2$ value, which determines an overall normalization of the cross-section and it includes all numeric effects of large distances (Λ_{QCD} and the meson size $< r_{Q\bar{q}} >$). In concrete calculations we have supposed $m_Q/m_q \simeq 17$, $\sqrt{s}/m_Q \simeq 20$, $m_q/\Lambda_{\text{QCD}} \simeq 10$.

By the general theorem on factorization [1] it is clear that at high transverse momenta the fragmentation of the heavier quark $Q \to (Q\bar{q}) + q$ must dominate. It is described by the factorized formula

$$\frac{d\sigma}{dp_T} = \int \frac{d\hat{\sigma}(\mu; gg \to Q\bar{Q})}{dk_T}|_{k_T = p_T/x} \cdot D^{Q \to (Q\bar{q})}(x; \mu) \frac{dx}{x} , \qquad (1)$$

where μ is the factorization scale, $d\hat{\sigma}/dk_T$ is the cross-section for the gluon-gluon production of quarks $Q+\bar{Q}$, D is the fragmentation function. In what follows, we consider the production of the S-wave states in the $(Q\bar{q})$ system: the vector 1^- state and pseudoscalar 0^- one. For these states the scaling analytic expressions of the fragmentation functions were derived at $M^2/s \ll 1$ in the leading order of perturbative QCD [7,8,9].

The distributions over the meson transverse momentum in the gluon-gluon production are shown in Fig. 1, in which one can see that at $p_T > p_T^{\min} \gg M_{(Q\bar{q})}$ the exact perturbative calculations of the $O(\alpha_s^4)$-order prove the validity of the factorization theorem, and they fix the scale, from which the production mechanism reaches the regime of fragmentation. Note, that numerically at the chosen ratios of the quark masses and the total energy of the partonic subprocess we have $p_T^{\min} \sim (5 \div 6) \cdot m_Q$, and the latter value is greater for the vector state. The analysis shows also that the low boundary for the applicability of the factorization approach shifts to the region of lower momenta at the decrease of the m_q/m_Q ratio.
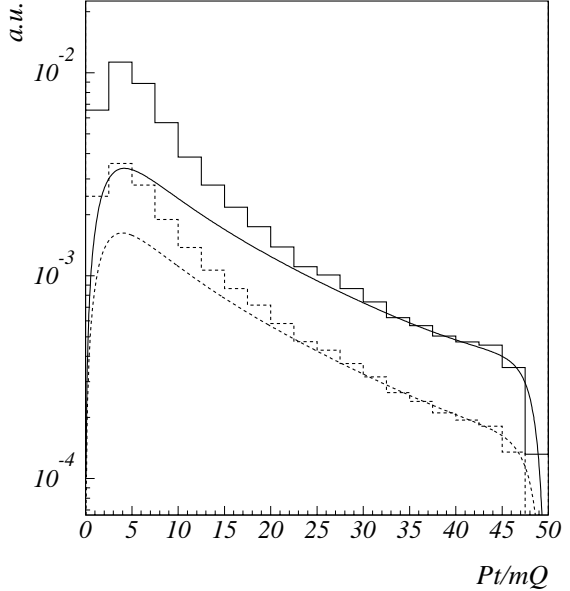

Fig. 1. The distributions over the transverse momenta of pseudoscalar and vector $(Q\bar{q})$-states, described in the $O(\alpha_s^4)$-order of QCD (the dashed and solid line histograms, correspondingly) and in the fragmentation model (dashed and solid curves) in gluon-gluon collisions. The cross-section is given in arbitrary units, the transverse momentum is in masses of $m_Q/5$.

Compare now the result of the exact perturbative $O(\alpha_s^4)$-calculation for the differential cross-section $d\sigma/dz$, $z = 2E/\sqrt{\hat{s}}$, with the result of the factorization model analogous to (1). Remember, that in e^+e^--annihilation the distribution under consideration coincides with the fragmentation function. One can see in Fig. 2, that, in contrast to the fragmentation model, there is a sizable additional contribution of the recombination type in the region of low z (i.e. at low energies and, hence, low p_T). We have found that the recombination contribution is generally given by the diagrams of the evolution type for the gluon splitting into the $q\bar{q}$ or gg pairs. However, the advantage of the given perturbative approach at $m_q \gg \Lambda_{\rm QCD}$ is the exact, process dependent account for the finite values

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of masses, virtualities and transverse momenta, in contrast to the universal approximate method of Altarelli–Parisi.
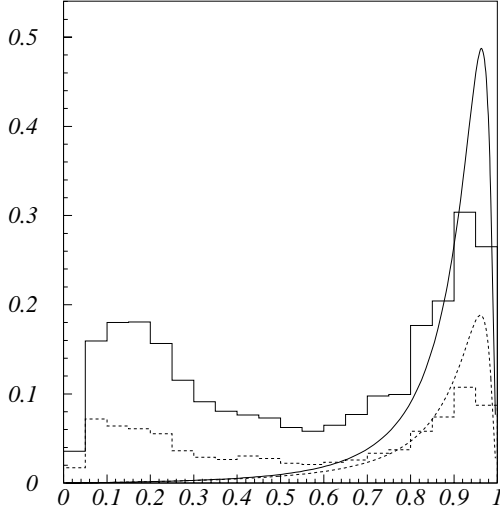

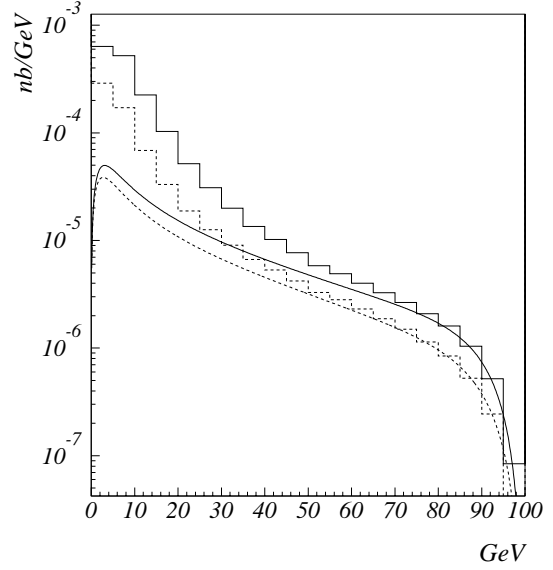Fig. 2. $d\hat{\sigma}/dz$ for the $(Q\bar{q})$-system (notations as in Fig. 1).

Fig. 3. $d\hat{\sigma}/dp_T$ for the $B_c^{(*)}$-mesons at the gluon interaction energy of 200 GeV (notations as in Fig. 1).

We have in details considered the contributions of each diagram in the region of $z \to 1$. In the covariant Feynman gauge the diagrams of the gluon-gluon production of $Q + \bar{Q}$ with the subsequent $Q \to (Q\bar{q})$ fragmentation dominate as well as the diagrams, when the $q\bar{q}$ pair is produced in the region of the initial gluon splitting, are large. However, as one can see in Fig. 2, the contribution of the latters diagrams leads to the destructive interference with the fragmentation amplitude, and this results in the "reduction" of the production cross-section in the region of z close to 1. So, $\Delta\sigma/\sigma$ is about 50%. In the axial gauge with the vector $n^\mu = p_{\bar{Q}}^\mu$ this effect of the interference has still a more bright appearance, since the diagrams like the splitting of gluons dominate by several orders of magnitude over the fragmentation, but the destructive interference results in the cancellation of such extremely large contributions. By the way, we have to note that discussions and speculations given in ref. [4] are certainly misleading, since the authors draw conclusions by the consideration of the singularities in the virtual particle propagators, which could determine a dominating contribution. As we have just seen, some separately large contributions can destructively interfere to cancel each to other. This interference is caused by the nonabelian nature of QCD, i.e. by the presence of the gluon selfaction vertices. The comparison with the abelian case will be in details given in next Section.

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2. Gluon production of B_c

The model for the production of the mixed flavor heavy quarkonium, as it stands in previous Section, is the most realistic in the description of the differential spectra for the B_c-mesons [10], for which, however, the m_c/m_b ratio can not be yet considered as the small parameter.

As it was shown in [3,4,5] and one can see in Fig. 3, the fragmentation regime for the B_c-mesons is strongly delayed into the region of high transverse momenta, $p_T^{\min} \sim 35 \div 40$ GeV. The last fact is related with the comparatively large value of the c-quark mass. Thus, one can again talk on the confirmation of the factorization theorem for the hard gluonic production of b-quarks at high transverse momenta $p_T \gg M_{B_c}$ and the less hard fragmentation of b-quark into $B_c^{(*)}$-meson ($m_c \ll p_T$). However, the large numeric value of $p_T^{\min}$ points out the fact that the basic amount of events of the hadronic $B_c^{(*)}$-production do not certainly allows the description in the framework of the fragmentation model. This conclusion looks more evident, if one considers the B_c-meson spectrum over the energy (see Fig. 4).
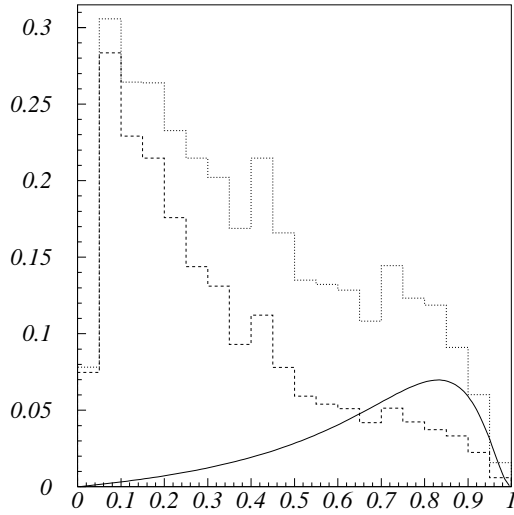

Fig. 4. $d\hat{\sigma}/dz$ for the B_c-mesons at the gluon interaction energy of 100 GeV. The dashed histogram presents the $gg \to B_c + \bar{b} + c$ process, the dotted one is the abelian case. The curve shows the result of the fragmentation model. The cross-sections are normalized over the cross-section of the $b\bar{b}$ pair production.
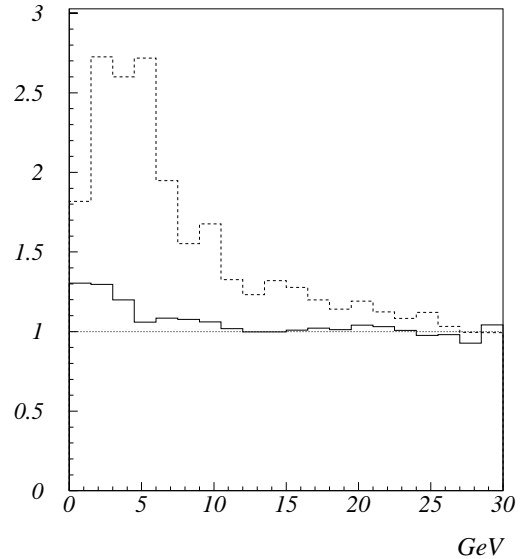
Fig. 5. The ratio of spectra for the summed contributions of B^* and B-mesons in $p\bar{p}$-collisions at the hadron interaction energy of 1.8 TeV, as they are calculated in the $O(\alpha_s^4)$-model of the weakly bound quarkonium and in the fragmentation model. $\sigma_{B_c^{(*)}}/\sigma_{B_c^{(*)}}^{\mathrm{frag}}(p_T)$ is given as the solid histogram, the dashed histogram is the same but with the cut off the gluon interaction energy > 60 GeV.

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The basic part of events for the gluon-gluon production of B_c is accumulated in the region of low z close to 0, where the recombination being essentially greater than the fragmentation, dominates. This is the significant difference between the B_c production and the $(Q\bar{q})$-meson one, considered in previous Section, where both the fragmentation and recombination have given comparable contributions. As it stands for $m_Q \gg m_q$, one can draw the conclusion on the essential destructive interference in the region of z close to 1, for the pseudoscalar state.

To stress the role of the interference diagrams related to the nonabelian selfaction of gluons, we have considered the process with abelian currents. To avoid effects of an enforcement for the fragmentation contributions by different quarks due to the difference between their charges [6], we have supposed the abelian charges of produced quarks to be equal each to other. As one can see in Fig. 4, where the spectrum over the energy of the pseudoscalar state is shown, in the abelian case the effect of the destructive interference due to the additional contribution by the selfaction of gauge quanta, is absent. So, the agreement between the factorized model of fragmentation and the exact perturbative calculation is quite good at z close to 1.

The direct verification of the given mechanism for the B_c-meson production could be the comparison of the B_c-meson spectra in two semispheres in the region of the gluon fragmentation and in the photon one, in the photonic production of B_c at nucleons.

3. Hadronic production of B-mesons

The model of the gluon-gluon production of quarkonium, as it is described in Section 2, can be, after some notes, expanded to the hadronic production of mesons with a heavy quark Q ($m_Q \gg \Lambda_{\text{QCD}}$) and a light antiquark $\bar{q}$ ($m_q \sim \Lambda_{\text{QCD}}$). Then one can talk on the model of meson composed of the constituent quarks, whose motions inside the meson are neglected, and, hence, one can apply the nonrelativistic model of quarkonium with weakly bound constituents. As for the nonperturbative parameters of model, m_q and $|\Psi(0)|$, they can be phenomenologically fixed over the spectrum form and total cross-section of the B-meson production in e^+e^--annihilation at the Z-boson peak, where the fragmentation regime dominates certainly. Considering the $\sigma^{-1} d\sigma/dz$ spectrum over $z = 2E/\sqrt{s}$, one has to take into account the perturbative evolution breaking the scaling for the function of the b-quark fragmentation into the B-meson, $D^{b \to B}(\mu = \sqrt{s}; z)$. After that, the $m_q = 0.3$ GeV mass value agrees with the observed form of the fragmentation function, still the accuracy of such estimate is low because of large experimental errors in parameters. So, in the one-loop approximation for the perturbative evolution the average fraction of energy taken by the B-meson, equals

$$\langle z(\sqrt{s}) \rangle = \left(\frac{\ln(m_b/\Lambda_{\text{QCD}})}{\ln(\sqrt{s}/\Lambda_{\text{QCD}})} \right)^n \langle z(m_b) \rangle \,, \tag{2}$$

where $n = 32/(9b)$, $b = 11 - 2n_f/3$, $n_f = 5$, and $\langle z(m_b) \rangle$ is the scaling value of the momentum fraction determined over the $b \to B^{(*)}$ fragmentation functions derived in [7,8,9]. From (2), the experimental value [11]

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$$\langle z_B(m_Z) \rangle = 0.708 \pm 0.003 \pm 0.015 \ ,$$

and $\Lambda_{\mathrm{QCD}} = 85 \pm 20$ MeV corresponding to $\alpha_s(m_Z) = 0.117 \pm 0.005$ [11] in the one-loop approximation, one finds $m_q = 0.2 \div 0.3$ GeV. In the following calculations we suppose $m_q = 0.3$ GeV. As for the normalization of the total cross-section of the B-meson production at the Z-boson peak, it is determined by the $\alpha_s^2 |\Psi(0)|^2$ value, where $\Psi(0)$ is expressed through the leptonic constant of the B-meson in the limit of static b-quark, $\tilde{f}_B^{\mathrm{stat}}$,

$$|\Psi(0)| = \tilde{f}_B^{\mathrm{stat}} \sqrt{\frac{M_B}{12}} \ .$$

In various papers [12] one gives estimates corresponding to $\tilde{f}_B^{\mathrm{stat}} = 200 \div 320$ MeV. Supposing the relative yields of B^0, B^+ and B_s^0-mesons to be equal to 0.4, 0.4 and 0.2, we get $\alpha_s = 0.8 \pm 0.2$. Of course, this phenomenological value of QCD coupling constant is large for one can talk on the reliability of perturbative calculations of the B-meson spectrum in the model of Section 2. Nevertheless, one can believe that, being phenomenological, such model is able to describe the basic differential characteristics in the hadronic production of B-mesons, as it is in e^+e^--annihilation.

In Section 2 we have chosen the quark mass ratios such that they correspond to $m_Q = m_b = 5.0$ GeV, $m_q = 0.3$ GeV and $\sqrt{s} = 100$ GeV. The only difference between the real case of the B-meson production and the consideration given in Section 2, is the change of the overall normalization of the total cross-section. The ratios of spectra over the B^* and B-meson transverse momenta are presented in Fig. 5, where they are calculated in the model of fragmentation and in the $O(\alpha_s^4)$-order of QCD in $p\bar{p}$-collisions at $\sqrt{s} = 1.8$ TeV with the use of gluon distributions given in [13]. We see that the cross-section has the significant additional contribution of recombination in the region of $p_T < 15$ GeV, that qualitatively agrees with the experimental data from CDF [2] (see Fig. 6).
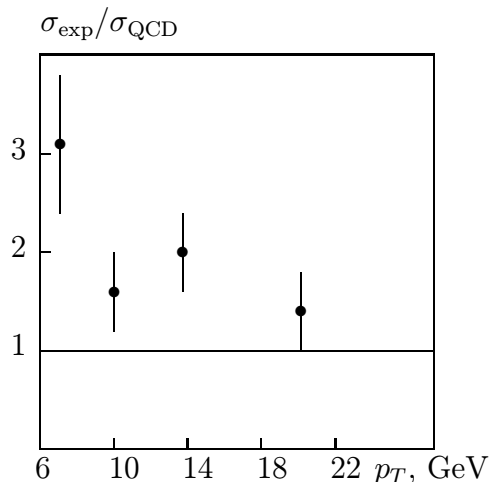

Fig. 6. The CDF data on the differential cross-section of the B-meson production at the FNAL Tevatron, in comparison with the $O(\alpha_s^3)$-prediction of perturbative QCD for the b-quark production, $\sigma_{\mathrm{exp}}/\sigma_{\mathrm{QCD}}$.

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One has to note that the gluonic luminosity of the $gg \to B + \bar{b} + q$ process enforces the contribution of the region near the threshold, $\sqrt{s_{th}} = 2(m_Q + m_q)$, where, strictly speaking, the fragmentation approach for the b-quarks at low momenta has uncertainties of the principal character, since the physical phase space of the subprocess is suppressed in comparison with the two-particle phase space of free b-quark pair. The enforcement of the near-threshold region due to the luminosities and phase space leads to the fact that the fragmentation model evidently gives overestimating values for the differential cross-sections at low p_T.

The uncertainty with the phase space decreases rapidly, if one limits the total energy of the gluon subprocess from the down. So, at $\sqrt{\hat{s}} > 60$ GeV one can clear see in Fig. 5, that the fragmentation picture of the B-meson production does not correspond to the exact perturbative $O(\alpha_s^4)$-calculations of the meson spectrum in the model of hadronization for the weakly bound constituent quarks, at $p_T < 25$ GeV. It is clear that at high energies, the multiple production of light quark pairs can take place in addition to the single one. The large value of the phenomenological constant $\alpha_s \approx 0.8$ points out the latter possibility. In the fourth order over α_s this means that the invariant mass of the $(b\bar{q})$ pair can be large in accordance with the $(q\bar{q})$ pair mass can be close to the threshold. Such contribution could make softer the b-quark spectrum, i.e. it could result in the additional contribution into the region of low $p_T < 15$ GeV, where the fragmentation regime does not still work.

Thus, we have shown that the perturbative $O(\alpha_s^4)$-calculations of QCD, according to the reasonable model of the b-quark hadronization, confirm the applicability of the factorization theorem at $p_T^B > 15$ GeV. They point out the appearance of additional, nonfragmentational contributions at low transverse momenta of the meson, $p_T^B < 15$ GeV, so that these corrections appropriately reflect the observed experimental disagreement with the fragmentation model.

4. Conclusion

In the framework of perturbative QCD up to $O(\alpha_s^4)$-terms and in the simplest model of quark hadronization, we have analyzed the differential cross-sections of the heavy mesons over the transverse momenta and the energies. This analysis shows the following.

1) As one could expect by the theorem of factorization for hard processes, in the region of $p_T > p_T^{\min} \gg M_{(Q\bar{q})}$ the fragmentation contribution dominates. At $p_T < p_T^{\min}$ the quark recombination into the heavy meson is basic, so that its relative value depends on the m_Q/m_q ratio. One has $p_T^{\min} \approx 40$ GeV for the B_c-mesons, and the recombination contribution in the total cross-section is much greater than the fragmentation. One has $p_T^{\min} \approx 15$ GeV for the B-meson production in hadron collisions, and the relative contributions by both the recombination and fragmentation are comparable with each other.

2) The exact perturbative $O(\alpha_s^4)$-calculations point out the nonfragmentational contributions, which were not yet taken into account elsewhere previously, and they dominate in the region of low $z = 2E/\sqrt{s}$. These contributions increase by their relative magnitude in the total cross-section versus the decrease of the m_Q/m_q ratio.

3) In the region of low transverse momenta and at z close to 1, there is the destructive interference of the fragmentation type diagrams with the diagrams of the initial gluon splitting into the $q\bar{q}$ pair, so that the presence of the interference effect is determined by the nonabelian selfaction of gluons in QCD, and the effect is absent for the abelian currents.

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