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INSTITUTE FOR HIGH ENERGY PHYSICS

IHEP 96-56

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On partial wave analysis in the mass region of $a_2(1320)$

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Protvino 1996

UDK 539.12.142 M–24

Abstract

Arestov Yu.I. On Partial Wave Analysis in the Mass Region of $a_2(1320)$: IHEP Preprint 96-56. – Protvino, 1996. – p. 5, refs.: 3.

Some notes have been made to refine the partial wave analysis (PWA) in the $a_2(1320)$ region. It is argued that the helicity amplitudes T^{0m}_{bd} with $m = 2$ which are typically missed in PWA may play an important role for discovering waves with the small intensities. The analysis interplay in the Gottfried-Jackson and s-channel helicity rest frames has been discussed.

Аннотация

Арестов Ю.И. О парциально-волновом анализе в области масс $a_2(1320)$: Препринт ИФВЭ 96-56. – Протвино, 1996. – 5 с., библиогр.: 3.

Сделаны некоторые замечания с целью уточнить парциально-волновой анализ (ПВА) в области масс резонанса $\alpha_1(1320)$. Показано, что спиральные амплитуды T^{0m}_{bd} с $m = 2$, которые обычно опускаются в ПВА, могут играть важную роль при обнаружении волн с малой интенсивностью. Обсуждается взаимная связь результатов анализа в системах покоя Готфрида-Джексона и s-канальной.

Some experimental groups performed partial wave analysis (PWA) of $\eta\pi(\eta'\pi)$ system in the mass region 1-2 GeV and candidates to meson states with exotic quantum numbers were observed. Below I shall refer mainly to the results of [1] and [2] at p_{lab}=100 and 37 GeV/c, respectively. The reaction under study was

$$\pi^- + N \to (j,m) + N' \quad (1)$$

in the region of $t' \leq 1$ GeV^2, $t' = t_{max} - t$. Here (j,m) denotes $\eta\pi$ or $\eta'\pi$ system with spin j and helicity m, N and N' are nucleons. In ref. [1] $\eta\pi^o$ system was analysed and a new exotic state $J^{PC} = 1^{-+}$ with mass 1406 MeV was reported. In ref. [2] the broad bump with exotic quantum numbers $J^{PC} = 1^{-+}$ was discovered in $\eta\pi^-$ and $\eta'\pi^-$ states.

These interesting results were obtained through the conventional PWA, however under the condition of *absense* of the wave component (jm)=(22). At the same time no restrictions on the waves with m=1 are imposed. This way of handling amplitudes seems, generally speaking, to be inconsistent.

The matter is that in the binary process (1) some of the scattering helicity amplitudes $T^{om}_{bd}(t,s)$ with m=2 (and b,d standing for the initial and final nucleon helicities) have the same order of suppression at small t' as the amplitudes with m=1. For example, the amplitudes T^{02}_{++}, T^{02}_{--}, T^{01}_{+-} and $T^{0,-1}_{+-}$ have the same rate of vanishing (in terms of $\sqrt{t'}$) in the Gottfried-Jackson rest frame. Alternatively in the s-shannel frame the amplitudes T^{02}_{-+} and T^{01}_{+-} include the factors $\sqrt{t'}$ and t', respectively. So, *a priori* in a certain t' interval the first amplitude with m=2 is expected to play more essential role than the second amplitude with m=1.

It means that simplifications in PWA should be made by removing *all* amplitudes of the same order. Violation of this rule is the violation of the fit precision, and it may cause ghost signals.

In PWA two j rest frames are commonly used: s-channel frame in which the spin quantization axis of j is opposite to N' momentum and t-channel (Gottfried-Jackson) frame in which the quantization axis of j is along the direction of initial π^- as seen from the rest frame of j. The helicity amplitudes in these channels are related through the

Wigner rotation angles ω:

$$\langle jm, d|T|b\rangle^t = \sum_{m'd'b'} d^j_{m'm}(\omega_j) d^{1/2}_{d'd}(\omega_{N'}) d^{1/2}_{b'b}(\omega_b) \langle jm, d|T|b\rangle^s. \quad (2)$$

This expression includes the rotation d-functions which appear in the Jacob-Wick expansion of the helicity amplitudes. The rotation angles are defined as

$$cos\ \omega_j \;=\; (x(s + m_j^2 - m_{N'}^2)(t + m_j^2 - m_\pi) - 2m_j^2\Delta)/(S_{jN'}Q_{\pi j}) \;\equiv\; f(x, m_j, m_{N'}, m_\pi)$$

with x=+1 and $\Delta = m_N^2 - m_\pi^2 - m_{N'}^2 + m_j^2$, $S_{jN'}^2 = (s - (m_j + m_{N'})^2)(s - (m_j - m_{N'})^2)$, $Q_{\pi j}^2 = (t - (m_j + m_\pi)^2)(t - (m_j - m_\pi)^2)$.

Three other rotation angles are obtained by cyclic permutations, so that $cos\ \omega_{N'} = f(-1, m_{N'}, m_j, m_N)$, $cos\ \omega_N = f(+1, m_N, m_\pi, m_{N'})$, and finally for initial pion $cos\ \omega_\pi = f(-1, m_\pi, m_N, m_j)$ (not used in (2)).

Two values can be attributed to each s-channel helicity amplitude: the net helicity flip n that is defined as $n = |(0-b)-(m-d)|$, and the total number of flips N=$|(0+b)-(m+d)|$. In the above expressions the zero stands for the helicity of the beam particle to support the generality of consideration. As follows from the Jacob-Wick expansion the net helicity flip determines evidently the small-t' behaviour of the s-channel scattering amplitudes that vanish as

$$(\sqrt{t'})^{|(0-b)-(m-d)|} \;=\; (\sqrt{t'})^n.$$

Intuitively one may feel that N, the so called total number of flips, should play a dynamical role in the underlying production mechanism. However, a possible role of N is not discussed here because this quantity does not appear naturally in the Jacob-Wick expansion, and any usage of N would be model-dependent. A sample model for treating N can be found in ref. [3].

As is seen from (2) a t-channel scattering amplitude mixes the s-channel amplitudes, so its small-t' behaviour is not so transparent. Extra powers of $\sqrt{t'}$ appear, and t-channel amplitudes with definite parity exchange behave as $(\sqrt{t'})^{n+\delta}$, where $n+\delta \;=\; |j-0|+|d-b|$.

Factorizing the helicity amplitudes at small t' by extracting $(\sqrt{t'})^p$, one can plot the following table of p values:

helicity amplitudes	s-channel $p = n$	t-channel $p = n + \delta$
$T^{0m}_{\pm\pm}$	$\lvert m\rvert$	$\lvert m\rvert$
T^{0m}_{+-}	$\lvert m+1\rvert$	$\lvert m\rvert + 1$
T^{0m}_{-+}	$\lvert m-1\rvert$	$\lvert m\rvert + 1$

From this table one can make two observations. First, although the suppression at small t' grows, in general, with increasing m, some interplay between helicity indices may occur. So, some of the amplitudes with m=2 will be of the same order in terms of $\sqrt{t'}$ as some of the amplitudes with m=1. Second, the suppression of some amplitudes at small scattering angles is, in general, stronger in the t-channel frame. This may cause an additional insensitivity to the extra-suppressed waves in the real analysis within the limited statistics. To avoid this it is preferable to perform PWA in terms of the s-channel helicity amplitudes unless an advantage of the t-channel consideration is argued for the given process.

Assuming the limitation $j \leq 2$ and specifying exchanges conventionally as natural (N) and unnatural (U), one can get the following relations: $N_j^m = T_{bd}^{0m} - (-1)^m T_{bd}^{0,-m}$ and $U_j^m = T_{bd}^{0m} + (-1)^m T_{bd}^{0,-m}$. It follows from this that the amplitudes T_{bd}^{00} are purely unnatural and consist of U_S^0, U_P^0 and U_D^0 waves which were labeled as S, P_0 and D_0 in refs. [1],[2]. The helicity amplitudes which were neglected in those papers are N_D^2 and U_D^2. However, these amplitudes have $n+\delta$=2 at $b=d$ and they can, in general, compete with the amplitudes N_D^1, U_D^1, N_P^1 and U_P^1 at $b=+, d=-$. In refs. [1],[2] the latter four amplitudes were labeled as D_+, D_-, P_+ and P_-. Recall that ρ, the natural-parity exchange Regge trajectory, has the sizeable coupling with the flipping nucleon and it contributes to $a_2(1320)$ production in reaction (1). So, an original ratio between the wave intensities can be violated in PWA, if the m=2 waves are removed.

Let us discuss shortly how to access the $m=2$ terms.

Denoting the transition amplitude for reaction (1) as T_{bd}^{j0m} one can write the differential cross section in the form

$$\begin{aligned} d^4\sigma/dmdt'd\Omega^* &= \sum_{bd} \left| \sum_{j=0}^{2} \sum_{m=-j}^{j} T_{bd}^{j0m} \right|^2 \\ &= \sum_{bd} \Big| U_S + U_P^0 d_{00}^1(\theta) + (N_P^1 \sin\phi + U_P^1 \cos\phi) d_{10}^1(\theta) \\ &+ U_D^0 d_{00}^2(\theta) + (N_D^1 \sin\phi + U_D^1 \cos\phi) d_{10}^2(\theta) \\ &+ (N_D^2 \sin 2\phi + U_D^2 \cos 2\phi) d_{20}^2(\theta) \Big|^2, \end{aligned} \tag{3}$$

where $N_j^m = T^{j0m} - (-1)^m T^{j0,-m}$ and $U_j^m = T^{j0m} + (-1)^m T^{j0,-m}$ for m=0,1,2 represent contributions corresponding to the natural and unnatural exchanges. Here S, P and D subscripts stand for j=0,1 and 2, respectively.

To extract the partial waves, the experimental angular distributions in θ, ϕ are conventionally weighed only with the spherical harmonics $Re\ Y_{lm}$, if the target nucleon N is not polarized. The full and orthogonal system of the spherical harmonics is very useful in iterative procedures for rapid extractions of the large-intensity waves. However the simplified application of this method in refs. [1], [2] can be insensitive to the m=2 states.

Indeed, the m=2 wave intensities can be extracted with weighing by $Re\ Y_{44}$ from the moment $\langle Re\ Y_{44} \rangle = const \cdot (-|N_D^2|^2 + |U_D^2|^2)$. But these are the terms of the second order of smallness, and besides the difference in the last brackets can be insignificant if the intensities inside are comparable similarly to the N_D^1 and U_D^0 in ref. [1] (the two last waves are labeled as D_+ and D_0 in the quoted reference). Anyway, getting the difference of production probabilities is not the best way to extract them.

On the other hand, an attempt to access the m=2 terms seems more promising. The first order terms arise from the interference with large contributions. Certainly, this requires some *a prioi* knowledge about the large contributions. They can be found in the preliminary PWA fits with simplified assumptions, for example, $|m| \leq 1$ as was done in refs. [1],[2]. In the conventional method, the interference term $N_D^2 \cdot N_D^1$, which is proportional to $sin\ 2\phi \cdot sin\ \phi$ survives under weighing by $Re\ Y_{21}$. Unfortunately, a number of other contributions survive, particularly $N_P^1 \cdot N_D^1$ $(P_+ D_+)$. As the D_+ contribution appeared to be large in both references, the m=2 terms could not be seen against that background.

From the above consideration it follows that getting out the m=2 terms requires a special careful extraction. This is part of a general problem of extracting small terms. There can not be a unique recommendation because of different conditions in different experiments. Say, U_P^0 intensity is evaluated as a large one in ref. [1] and as a negligible one in ref. [2]. It is evident that testing various weighings, not only $Re\ Y_{lm}$, would be useful *before* the full PWA.

Within the problem considered simple convolutions of (3) with the following expressions can be used to test the m=2 terms: A – $sin\ 2\phi \cdot sin\ \phi \cdot d^2_{10}(\theta)$; B – $cos\ 3\phi \cdot d^1_{00}(\theta)$. Three interfering terms survive in (3) after convolution A: $N_D^2 N_D^1$, $U_S U_D^1$ and $U_P^1 U_P^0$, and two terms survive after convolution B: $N_D^2 N_D^1$ and $U_D^2 U_D^1$. In ref. [1] the wave intensities N_D^1, U_D^0 and U_P^0 are classified as large contributions, whereas in ref. [2] only N_D^1 is found to be large. The other terms can be regarded as small or negligible. Thus, the analysis of convolution B of the experimental angular distributions would be fruitful in both references. The same is true for convolution A, if it were explored in ref. [2]. The usage of convolution A in the data handling of ref. [1] would be also useful but not so instructive due the mixture of the large wave intensity U_P^0.

Below I summarize all the above considered.

• The reduced wave analysis with $|m| \leq 1$ of reaction (1) in the mass interval $1 \div 2$ Gev exhibits reliable results for the parameters connected with the large resonance signals of $a_2(1320)$ (D_0 and D_+ waves in ref. [1] and D_+ wave in ref. [2]). However, the small signals in the reduced analysis require more careful treating.

• Interesting indications for an exotic state with mass about 1400 GeV in the waves P_0 and P_+ have been obtained in the reduced analysis. As these signals are not intensive the confirmation is needed in the extended analysis up to $|m| = 2$.

• An advantage of the s-channel consideration has to be used because the helicity amplitudes with the flip along the nucleon line may have an additional suppression in the Gottfried- Jackson frame compared to that in s-channel. For example, the ratio N_D^2/N_D^1 at small t' is a constant in the s-channel helicity frame and is proportional to t' in the t-channel helicity frame. The parallel partial wave analysis in both frames would be the most instructive.

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Received July 11, 1996

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О парциально-волновом анализе в области масс $a_2(1320)$.

Оригинал-макет подготовлен с помощью системы LaTeX.
Редактор Е.Н.Горина.

Подписано к печати 11.07.1996. Формат $60 \times 84/8$.
Офсетная печать. Печ.л. 0.5. Уч.-изд.л. 0.4. Тираж 240. Заказ 724.
Индекс 3649. ЛР №020498 17.04.97.

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П Р Е П Р И Н Т 96-56, И Ф В Э, 1996