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V.V.Kiselev<br>DECAY OF $B_{c}^{*+}(3 S) \rightarrow B^{+} D^{0}$

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#### Abstract

Kiselev V.V. Decay of $\boldsymbol{B}_{c}^{*+}(\mathbf{3 S}) \rightarrow \boldsymbol{B}^{+} \boldsymbol{D}^{\mathbf{0}}$ : IHEP Preprint 96-63. - Protvino, 1996. - p. 6, tables 1, refs.: 11.

The decay constant for the vector state of $3 S$-level in the heavy ( $\bar{b} c)$-quarkonium is evaluated in the framework of sum rules for the mesonic currents. A scaling relation for the constants of vector quarkonia with different quark contents is derived. The numerical estime gives $\Gamma\left(B_{c}^{*+}(3 S) \rightarrow B^{+} D^{0}\right)=90 \pm 35 \mathrm{MeV}$.


## Аннотация

Киселев В.В. Распад $B_{c}^{*+}(3 S) \rightarrow B^{+} D^{0}$ : Препринт ИФВЭ 96-63. - Протвино, 1996. - 6 с., 1 табл., библиогр.: 11.

Константа распада векторного $3 S$-уровня тяжелого кваркония ( $\bar{b} c$ ) рассчитана в правилах сумм для мезонных токов. Получено масштабное соотношение для констант векторных кваркониев с различным кварковым составом. Численная оценка дает $\Gamma\left(B_{c}^{*+}(3 S) \rightarrow\right.$ $\left.B^{+} D^{0}\right)=90 \pm 35 \mathrm{M}$ В .

## Introduction

The experimental search for the $B_{c}^{+}$meson in the facilities with the vertex detectors (OPAL [1], ALEPH [2], DELPHI[3] and CDF [4]) stimulated the theoretical studies on the spectroscopy of the heavy ( $\bar{b} c)$-quarkonium [5], mechanisms of its production in different interactions [6] and on estimates of different decay widths for the both basic state [7] and excited levels $[5,8]$. The feature of the $(\bar{b} c)$-system is the absense of the annihilation decay modes caused by the strong or electromagnetic interactions. So, the basic pseudoscalar $B_{c}^{+}$state decays due to the weak interaction, and it is the long-lived particle, $\tau\left(B_{c}^{+}\right)=$ $0.55 \pm 0.15 \mathrm{ps}[7,9]$. The excited ( $\bar{b} c)$-quarkonium levels lying below the threshold of the decay to the heavy meson $B D$ pair, radiatively transform into the $(\bar{b} c)$-states with the smaller masses. The $B_{c}^{*+}(3 S)$ state is above the $B D$ thershold, so its decay is analogous to $\Upsilon(4 S) \rightarrow B^{+} B^{-}$. The constant of the latter decay was considered in ref.[10] in the framework of the sum rules for the mesonic currents.

In this work we consider the $g$ constant for the decay of the vector quarkonium, generally containing the quarks of different flavors, say, ( $\bar{b} c$ ) for the definite notations. This heavy quarkonium with the mass $M$, satisfying the condition $m_{B}+m_{D}<M<m_{B^{*}}+m_{D^{*}}$, decays to the heavy meson pair $B^{+} D^{0}$. We derive the scaling relation

$$
\frac{g^{2}}{M}\left(\frac{4 \mu_{B D}}{M}\right)=\text { const. }
$$

where $\mu_{B D}=m_{B} m_{D} /\left(m_{B}+m_{D}\right)$ is the reduced mass of the heavy meson pair. The constant value in the right hand side of the relation is the same for the decays of $\Upsilon(4 S) \rightarrow$ $B^{+} B^{-}, B_{c}^{*+}(3 S) \rightarrow B^{+} D^{0}$ and $\psi(3770) \rightarrow D^{+} D^{-}$, where $\mu_{B B}=M_{\Upsilon(4 S)} / 4, \mu_{D D}=$ $M_{\psi(3770)} / 4$.

In Section 1 we consider the sum rules for the mesonic currents. In Section 2 the scaling relation is derived and numerical estimates are performed. In the Conclusion the obtained results are summarized.

## 1. Sum rules

Let us consider the vector current of mesons

$$
J_{\mu}^{B D}(x)=\frac{i}{2}\left[B^{+}(x) \cdot \partial_{\mu} D^{0}(x)-\partial_{\mu} B^{+}(x) \cdot D^{0}(x)\right]
$$

and define the contribution of this current into the leptonic $f_{B D}$ constant of the vector $(\bar{b} c)$-quarkonium lying above the $B D$-threshold

$$
\begin{equation*}
i f_{B D} M \epsilon_{\mu}^{(\lambda)} e^{i p x}=\langle 0| J_{\mu}^{\dagger B D}(x)\left|V_{(\bar{b} c)}, \lambda\right\rangle \tag{1}
\end{equation*}
$$

where $\lambda$ is the polarization of the $V_{(\bar{b} c)}$ state, $\epsilon_{\mu}^{(\lambda)}$ is its vector of polarization, $p$ is the $V_{(\bar{b} c)}$ momentum, $p^{2}=M^{2}$.

Further, introduce the $\mathcal{F}$ form factor for the transversal interaction of the $B D$ pair with the vector $\mathcal{A}_{\mu}$ current due to the vertex

$$
\begin{equation*}
\mathcal{L}_{J \mathcal{A}}^{t r}=\mathcal{F}\left(q^{2}\right) \mathcal{A}_{\mu} \cdot k^{\mu} \tag{2}
\end{equation*}
$$

where $q=p_{B}+p_{D}, p_{B, D}$ are the momenta of the meson lines directed out the vertex, and $p_{B}=q_{B}+k, p_{D}=q_{D}-k, q_{B, D} \cdot k=0$. Thus, one has

$$
\left(g^{\mu \nu}-\frac{q^{\mu} q^{\nu}}{q^{2}}\right)\langle 0| J_{\nu}^{\dagger B D}(0)\left|B^{+}\left(p_{B}\right) D^{0}\left(p_{D}\right)\right\rangle=i \mathcal{F}\left(q^{2}\right) k^{\mu}
$$

Consider the transversal part of the current correlator,

$$
\Pi_{J J}^{t r}\left(q^{2}\right)=\frac{1}{3}\left(-g^{\mu \nu}+\frac{q^{\mu} q^{\nu}}{q^{2}}\right) \int d^{4} x e^{i q x}\langle 0| T J_{\mu}^{\dagger B D}(x) J_{\nu}^{B D}(0)|0\rangle
$$

One can isolate the contribution of the resonance lying above the kinematical threshold of the $B D$ pair, so that

$$
\Pi_{J J}^{t r}\left(q^{2}\right)=\frac{f_{B D}^{2} M^{2}}{M^{2}-q^{2}}+\int_{s_{t h}}^{\infty} \frac{d s}{s-q^{2}} \rho(s)
$$

where $\rho(s)$ is the density of the nonresonant contribution. On the other hand, the form factor in (2) determines the value

$$
\begin{equation*}
\Im m \Pi_{\mathcal{F} F}^{t r}\left(q^{2}\right)=\frac{1}{8 \pi} \frac{|\mathbf{k}|^{3}}{3 \sqrt{q^{2}}} \mathcal{F}^{2}\left(q^{2}\right) \tag{3}
\end{equation*}
$$

where $|\mathbf{k}|^{2}=-k^{2}=\left(q^{2}+m_{B}^{2}-m_{D}^{2}\right)^{2} /\left(4 q^{2}\right)-m_{B}^{2}$. Write down the sum rules for the mesonic currents

$$
\Pi_{J J}^{t r}\left(q^{2}\right)=\frac{1}{\pi} \int_{s_{i}}^{\infty} \frac{d s}{s-q^{2}} \Im m \Pi_{\mathcal{F F}}^{t r}(s)
$$

where $s_{i}=\left(m_{B}+m_{D}\right)^{2}$. One can consider the following model for the continuum density in the form

$$
\rho(s)=\frac{1}{\pi} \Im m \Pi_{\mathcal{F} \mathcal{F}}^{t r}(s) \theta\left(s-s_{t h}\right) .
$$

Then the sum rules are given by the following expression

$$
\begin{equation*}
\frac{f_{B D}^{2} M^{2}}{M^{2}-q^{2}}=\frac{1}{\pi} \int_{s_{i}}^{s_{t h}} \frac{d s}{s-q^{2}} \Im m \Pi_{\mathcal{F F}}^{t r}(s) . \tag{4}
\end{equation*}
$$

The value of the continuum threshold is determined by the energy of new channels in the particle production by the $J_{\mu}$ current. As was shown in [10] for the $\Upsilon(4 S) \rightarrow B^{+} B^{-}$and $\psi(3770) \rightarrow D^{+} D^{-}$decays, this value is given by the threshold of production of the vector $B^{*+} B^{*-}$ and $D^{*+} D^{*-}$ states, so that we suppose

$$
s_{t h}=\left(m_{B^{*}}+m_{D^{*}}\right)^{2} .
$$

Define

$$
v^{2}(s)=1-\frac{4 m_{B} m_{D}}{s-\left(m_{B}-m_{D}\right)^{2}} .
$$

Then one has $v_{t h}^{2} \ll 1$.
Further, the consideration of the $\mathcal{F}$ form factor in a model for the $B^{+} B^{-}$and $D^{+} D^{-}$ currents [10] resulted in the fact that relation (4) and its initial four derivatives over $q^{2}$ at $q^{2}=0$ give the stable value of $f$ with the accuiracy of $5 \%$ to $25 \%$, correspondingly. Allowing for the mentioned region of applicability (the number of the spectral density moment is less than 5), one can transform the integration in (4) to the variable of $v^{2}(s)$ and suppose $q^{2}=0$ and $\mathcal{F}(s) \approx \mathcal{F}\left(s_{i}\right)=F$. Then at $v_{\text {th }}^{2} \ll 1$ and $|\mathbf{k}| \approx 2 \mu_{B D} v$, one has

$$
f_{B D}^{2} \approx \frac{1}{\pi} \int_{0}^{v_{t h}} d v^{2} \cdot v^{3}\left(\frac{4 \mu_{B D}}{M}\right)^{4} \frac{F^{2}}{64 \pi} \frac{M^{2}}{3} .
$$

So

$$
\begin{equation*}
f_{B D}=\frac{F M}{4 \pi}\left(\frac{4 \mu_{B D}}{M}\right)^{2} \sqrt{\frac{v_{t h}^{5}}{30}} . \tag{5}
\end{equation*}
$$

Introduce the transversal vertex of the $V_{(\bar{b} c)}$ state decay to the $B^{+} D^{0}$ pair

$$
\begin{equation*}
\mathcal{L}_{g}=g \epsilon_{\mu}^{(\lambda)} \cdot k^{\mu} . \tag{6}
\end{equation*}
$$

Vertex (6) results in the imaginary part of the $f_{B D}$ constant, so that $\Im m f_{B D}\left(q^{2}\right) \rightarrow 0$ at $q^{2} \rightarrow s_{i}$, and, hence, $\Im m f_{B D} \ll \Re e f_{B D}$. Using the vector dominance, one can easily get the relation between $\Im m f_{B D}$ and the transversal correlator determined by the $\epsilon_{\mu}^{(\lambda)}$ current of decay and the mesonic current of $J_{\nu}$ [10]

$$
\Im m \Pi_{F g}^{t r}\left(q^{2}\right)=-\frac{M}{2} \Im m f_{B D}
$$

where $\Im m \Pi_{F g}^{t r}$ coincides the expression in (3) with the substitution $F^{2} \rightarrow F g$. Then the dispersion relation for the $f_{B D}$ function at $q^{2}=s_{i}=\left(m_{B}+m_{D}\right)^{2}$ gives

$$
\begin{equation*}
f_{B D}=\frac{1}{16 \pi^{2}} \frac{F g}{9}\left(\frac{4 \mu_{B D}}{M}\right)^{3} M v_{t h}^{3} \tag{7}
\end{equation*}
$$

Comparing (5) with (7), one finds

$$
\begin{equation*}
g=\left(\frac{M}{4 \mu_{B D}}\right) 12 \pi \sqrt{\frac{3}{10 v_{t h}}} . \tag{8}
\end{equation*}
$$

## 2. Scaling relation and numerical estimates

As has been mentioned, the $v_{t h}$ value is determined by the threshold of production of the vector excitations for the heavy mesons, $B^{*+}$ and $D^{* 0}$, so

$$
v_{t h}^{2} \approx \frac{1}{2 \mu_{B D}}\left(\Delta m_{B}+\Delta m_{D}\right),
$$

where $\Delta m_{B}=m_{B^{*}}-m_{B}, \Delta m_{D}=m_{D^{*}}-m_{D}$. In the Heavy Quark Effective Theory (see review in [11]), one has

$$
m_{B} \Delta m_{B}=m_{D} \Delta m_{D}=\text { const. }
$$

independently of the heavy quark flavor with the accuracy up to corrections over $\Lambda_{Q C D} / m_{B, D}$. Hence, one gets

$$
\begin{equation*}
v_{t h} \cdot \mu_{B D}=\text { const } . \tag{9}
\end{equation*}
$$

Using (9) and (8), one can easily obtain the scaling relation for the decay constant of the heavy vector quarkonium with the mass $m_{B}+m_{D}<M<m_{B^{*}}+m_{D^{*}}$

$$
\begin{equation*}
\frac{g^{2}}{M}\left(\frac{4 \mu_{B D}}{M}\right)=\text { const. } \tag{10}
\end{equation*}
$$

Relation (10) is in a good agreement with the experimenal data on the ratio of constants for the decays of $\Upsilon(4 S) \rightarrow B^{+} B^{-}$and $\psi(3770) \rightarrow D^{+} D^{-}$, where one has the accuracy of $\Delta g \simeq 3$ (see table I). Note, that the estimate due to (8) giving $g_{\Upsilon B \bar{B}}=57$ agrees the experimental value taken as the input parameter for the scaling relation. The latter fact points out the self-consistency of the method resulting in (10). As for the accuracy of the scaling relation, it is determined by the uncertainty in the sum rules, where eq.(8) has been derived. Remember, that the stability of the $f$ constant calculation over the initial 5 moments of the spectral density changes from $5 \%$ for $\Upsilon(4 S)$ to $25 \%$ for $\psi(3770)$ with the decrease of the vector state mass. This must be included in the systematic uncertainty of the method used. We evaluate $\Delta g / g \sim 15-20 \%$ for $B_{c}^{*+}(3 S)$, so that

$$
g_{B_{c} B D}=49 \pm 8 .
$$

The decay width is determined by the expression

$$
\begin{equation*}
\Gamma\left(B_{c}^{*+}(3 S) \rightarrow B^{+} D^{0}\right)=\frac{1}{24 \pi} g^{2} \frac{|\mathbf{k}|^{3}}{M^{2}} \approx 90 \pm 35 \mathrm{MeV} \tag{11}
\end{equation*}
$$

We assume that the channel of decay to $B^{*} D$ can be neglected, since it is suppressed by the third power of the momentum of the decay final states due to the greater mass of $B^{*}$ in comparison with the $B$ mass. Then taking into account the channel $B^{0} D^{+}$, the total width of $B_{c}^{*+}(3 S)$ is equal to $\Gamma_{\text {tot }}=180 \pm 70 \mathrm{MeV}$. We have supposed $M\left(B_{c}^{*+}(3 S)\right)=7.250$ GeV [5] in the numerical estimate of (11). Note, that the width strongly depends on the difference of masses, $\Delta M=M-\left(m_{B}+m_{D}\right)$ determining $|\mathbf{k}|$. At the used value of the quarkonium mass, one has $\Delta M \sim 110 \mathrm{MeV}$, which differs from $\Delta M \sim 30 \mathrm{MeV}$ for the decays of $\Upsilon(4 S) \rightarrow B^{+} B^{-}$and $\psi(3770) \rightarrow D^{+} D^{-}$. The larger phase space results in the fact that the total $B_{c}^{*+}(3 S)$ width is one order of magnitude greater than the total widths of $\Upsilon(4 S)$ and $\psi(3770)$ having $\Gamma_{\text {tot }} \simeq 24 \mathrm{MeV}$.

Table 1. The predictions of scaling relation in comparison with the current experimental data

| value | exp. | scaling rel. |
| :--- | :---: | :---: |
| $g_{\Upsilon(4 S) \rightarrow B^{+} B^{-}}$ | 52 | input |
| $g_{\psi(3770) \rightarrow D^{+} D^{-}}$ | 31 | 31 |
| $g_{B_{c}^{*+}(3 S) \rightarrow B^{+} D^{0}}$ | - | 49 |

## Conclusion

In this paper we have considered the sum rules for the mesonic currents. These sum rules allow one to determine the coupling constant of the heavy vector ( $\bar{b} c$ )-quarkonium decaying to the heavy meson pair,

$$
g=\left(\frac{M}{4 \mu_{B D}}\right) 12 \pi \sqrt{\frac{3}{10 v_{t h}}},
$$

where $m_{B}+m_{D}<M<m_{B^{*}}+m_{D^{*}}$. The value of $v_{t h}$ determining the threshold of the nonresonant contribution into the transversal correlator of currents, is given by the mass splitting between the vector and pseudoscalar states of heavy mesons, and it possesses the definite scaling property, so that one has derived the relation

$$
\frac{g^{2}}{M}\left(\frac{4 \mu_{B D}}{M}\right)=\text { const. }
$$

which is in a good agreement with the experimental data on the constants of decays of $\Upsilon(4 S) \rightarrow B^{+} B^{-}$and $\psi(3770) \rightarrow D^{+} D^{-}$. The numerical estimate of the $B_{c}^{*+}(3 S) \rightarrow$ $B^{+} D^{0}$ decay width strongly depends on the mass difference $\Delta M=M-\left(m_{B}+m_{D}\right)$ determining the phase space, so that at $M\left(B_{c}^{*+}(3 S)\right)=7.250 \mathrm{GeV}$ one has found $\Gamma=$ $90 \pm 35 \mathrm{MeV}$.

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