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**ESTIMATE OF $\alpha_s(m_Z)$
IN SUM RULES FOR BOTTOMONIUM**

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Abstract

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The value of $\alpha_s(m_Z) = 0.106 \pm 0.005$ is obtained in a scheme of sum rules for the leptonic constants of the vector nS -levels in the heavy $(\bar{b}b)$ -quarkonium.

Аннотация

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В рамках схемы правил сумм для лептонных констант векторных nS -уровней в тяжелом кварконии $(\bar{b}b)$ получена величина $\alpha_s(m_Z) = 0.106 \pm 0.005$.

Introduction

The coupling constant of strong interaction measured in the Z -boson pole at LEP in the framework of Standard Model, $\alpha_s(m_Z) \approx 0.125$ is significantly greater than the value of $\alpha_s(m_Z) \approx 0.11$, which is extracted from the low-energy data [1], such as the deep inelastic lepton-hadron scattering (the Gross–Llewellyn Smith sum rule and the Q^2 evolution of structure functions [2]), the Υ decays, in the QCD sum rules for the $(\bar{b}b)$ system, and in the lattice computations of mass spectrum for the heavy quarkonium. The mentioned deviation between the LEP measurements at $Q^2 \sim 10^4 \text{ GeV}^2$ and the value, expected from QCD at lower energies ($Q^2 \sim 10^2 \text{ GeV}^2$), can point out virtual effects of a "new" large-energy scale physics, which is beyond the scheme of Standard Model, at the energies of Z -boson [3].

The QCD sum rules for the leptonic constants of vector states in the heavy $(\bar{b}b)$ -quarkonium have been recently considered for the precision extraction of the strong-interaction coupling constant and the b -quark mass in [4]. M.Voloshin has used the scheme of moments for the spectral density of the transversal correlator of vector currents in the nonrelativistic approximation, so that he has found $\alpha_s(m_Z) = 0.109 \pm 0.001$, where only the statistical error is presented. To study a methodic stability of the result on the $\alpha_s(m_Z)$ measurement in the sum rules for $(\bar{b}b)$, a consideration of the problem in other schemes is of a great interest.

In this paper we find $\alpha_s(m_Z) = 0.106 \pm 0.005$ from the data on the constants of vector nS -levels of $(\bar{b}b)$ in the sum rule scheme offered in [5].

1. Evaluation of α_s

In [5] one has considered the scheme of sum rules [6] for the two-point transversal correlator of vector currents of heavy quarks. It allows one to use the nonrelativistic movement of heavy quarks inside the quarkonium, the suppression of nonperturbative corrections from the quark-gluon condensates in the expansion over the inverse heavy-quark mass as well as the regularity of the heavy quarkonium mass spectra and explicit

expressions for the energetic density of nS -levels. One has found the broad region of numbers n_{mom} for the moments of the spectral density, where the following conditions are valid: the contribution of gluon condensate is low and the terms from the excited resonances in the heavy quarkonium are still essential (for $(\bar{b}b)$ one has $2 < n_{mom} < 20$). In this region the moments from the contribution of the sum over resonances can be quite accurately represented in the integral form with the energetic density of levels dn/dM_n , where n is the number of nS -level in the quarkonium. So, the following relation for the leptonic constants of vector states takes place

$$\frac{f_n^2}{M_n} = \frac{\alpha_s}{\pi} \frac{dM_n}{dn} \left(\frac{4m_{red}}{M_n} \right)^2 H_V Z_{sys} , \quad (1)$$

where $m_{red} = m_1 m_2 / (m_1 + m_2)$ is the reduced mass in the system of two quarks with the masses $m_{1,2}$. In the $\overline{\text{MS}}$ renormalization scheme the α_s value is determined by the expressions

$$\alpha_s = \alpha_s^{\overline{\text{MS}}} (e^{-5/3} \bar{p}_Q^2) , \quad (2)$$

where \bar{p}_Q^2 is the average square of the momentum transfer between the quarks, so

$$\bar{p}_Q^2 = \langle (\mathbf{p}_1 - \mathbf{p}_2)^2 \rangle = 2 \langle \mathbf{p}_{1,2}^2 \rangle = 4 \langle T \rangle m_{red} . \quad (3)$$

The H_V factor is the result of the hard gluon correction to the correlator of vector currents. It takes the form

$$H_V = 1 + \frac{2\alpha_s^H}{\pi} \left(\frac{m_2 - m_1}{m_2 + m_1} \ln \frac{m_2}{m_1} - \frac{8}{3} \right) , \quad (4)$$

where for $m_1 = m_2 = m_Q$ the QCD coupling constant according to the BLM procedure [7] is determined by the expression obtained in [4]

$$\alpha_s^H = \alpha_s^{\overline{\text{MS}}} (e^{-11/12} m_Q^2) . \quad (5)$$

Further, the factor $Z_{sys} = Z_{nr}/Z_{int}$ is close to unit and it determines the systematic correction due to the nonrelativistic approximation for the contribution of quark loop with the account for the α_s/v -terms of the coulomb type (Z_{nr}) and due to the integral representation of the resonance contributions into the hadronic part of sum rules (Z_{int}). The Z_{sys} value weakly depends on the number of nS -resonance, so that for the basic state of $(\bar{b}b)$ one has

$$Z_{sys} = 0.90 \pm 0.03 . \quad (6)$$

The state density dn/dM_n for the heavy quarkonium possesses the empirical regularity, since with a good accuracy the differences between the state masses in the $(\bar{c}c)$ and $(\bar{b}b)$ systems do not depend on the flavors of heavy quarks. This regularity has found the most evident expression in the framework of potential models as the statement on the independence of the average kinetic energy of heavy quarks on the flavors [8], so that the corresponding equation for the density of energy levels takes place (see [5])

$$\frac{dM_n}{dn} = \frac{2T}{n} . \quad (7)$$

From (7) one has

$$T = \frac{M_2 - M_1}{\ln 4}, \quad (8)$$

where M_n is the mass of nS -level, $M_n = (3M_{Vn} + M_{Pn})/4$, $M_{V,P}$ are the masses of vector and pseudoscalar states, correspondingly. From the data on the masses of charmonium and bottomonium it follows that

$$T = 415 \pm 20 \text{ MeV.}$$

Taking into account (7), one has considered the spectroscopy leading to the estimates of quark masses [9]

$$m_b = 4.63 \pm 0.03 \text{ GeV}, \quad m_c = 1.18 \pm 0.07 \text{ GeV.} \quad (9)$$

The m_b value is close to that given by M.Voloshin [4], $m_b^* = m_b(\mu) - 0.56\alpha_s(\mu)\mu \approx 4.64 \text{ GeV}$. As for the c -quark mass, m_c obtained in the $1/m_c^2$ -order, its value has only a weak influence on the accuracy of the $\alpha_s(m_Z)$ determination from the data on $(\bar{b}b)$.

Note, that with a high accuracy the value

$$a_Q = \alpha_s H_V \left(\frac{2m_Q}{M_1} \right)^2 Z_{\text{sys}}$$

does not depend on the heavy quark flavor, which leads to the scaling relations for the leptonic constants [5]. In accordance with (1), the empirical value

$$a_Q = \pi \frac{f_1^2}{M_1} \frac{\ln 2}{M_2 - M_1}$$

gives a possibility to determine α_s . In this way, one has to take into account that the leptonic width of $\Upsilon(1S) \rightarrow l^+l^-$ is determined by the effective fine structure constant $\bar{\alpha}_{em}$ at the M_Υ scale [4], so that we suppose $f_1 = 700 \pm 15 \text{ MeV}$.

In the two-loop accuracy for the "running" α_s constant¹ (see prescriptions for the Λ_{QCD} dependence on the number of active flavors in [1]), one finds

$$\alpha_s^{\overline{\text{MS}}}(m_Z) = 0.106 \pm 0.005, \quad (10)$$

so that $\Lambda^{(5)} = 108 \pm 35 \text{ MeV}$. The error in (10) is basically systematic. The increase of the b -quark mass leads to the decrease of the mean value in (10). The uncertainty is determined by the variation of m_b in the broad region: $4.50 < m_b < 4.85 \text{ GeV}$. The restriction of the b -quark mass, as it stands in (9), results in the decrease of the systematic error from 0.005 to 0.002 in (10).

The result of the $\alpha_s(m_Z)$ extraction in the framework of the nonrelativistic sum rules for $(\bar{b}b)$ in the scheme of the spectral density moments [4] supposes to have a systematic

¹The analogous consideration at the one-loop accuracy for α_s is given in [10].

uncertainty close to 0.004. So, combining (10) with the result of [4], one gets the average value of coupling constant determined in the sum rules for ($\bar{b}b$)

$$\alpha_s^{\overline{\text{MS}}}(m_Z) = 0.108 \pm 0.003 . \quad (11)$$

Note, that the recent application of the BLM scale choice to the hadronic event shape method of the α_s measurement resulted in $\alpha_s^{\overline{\text{MS}}}(m_Z) = 0.109 \pm 0.008$ [11].

The high accuracy of the α_s value extraction in the sum rules for ($\bar{b}b$) is caused by the large role of coulomb α_s/v -corrections, which result in the linear dependence of the leptonic constant squared on the value in (2) at the scale of average momentum transfer between the quarks inside the heavy quarkonium. In the way under consideration, this scale is equal to

$$\mu_b = e^{-5/6} \sqrt{2Tm_b} = 0.85 \pm 0.03 \text{ GeV}. \quad (12)$$

In Voloshin's paper [4] $\mu_b = 1 \text{ GeV}$. From our point of view, the difference between the μ_b values does mainly lead to the difference between the α_s estimates given in these sum rules ([4] and the present work). The value inverse to (12) determines the average size of the ($\bar{b}b$) system. In the potential models it is close to 0.25 fm.

Finally, the analogous procedure for the ($\bar{c}c$) system gives the α_s value being in a good agreement with (10), but the corresponding uncertainty is much greater, because of the errors in the evaluation of m_c and Z_{sys} . On the other hand, the strong dependence of the a_c value on the charm quark mass and the α_s extraction from the ($\bar{b}b$) data can be used to evaluate m_c . One finds

$$m_c = 1.20 \pm 0.07 \text{ GeV}.$$

The uncertainty decreases to 0.02 GeV if one uses the b -quark mass in the strict region of (9).

Conclusion

Thus, we have shown that the use of different schemes of sum rules for the leptonic constants of vector states in the heavy ($\bar{b}b$)-quarkonium leads to the systematically stable result $\alpha_s^{\overline{\text{MS}}}(m_Z) = 0.108 \pm 0.003$, which is significantly lower than the $\alpha_s(m_Z)$ value measured in the Z -boson pole at LEP.

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