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γ-BEAM PROPAGATION IN THE FIELD OF MONOCHROMATIC LASER WAVE

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Abstract

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Propagation of γ-beam in the field of monochromatic laser wave is considered. The optical properties of the laser wave are described with the use of the permittivity tensor. The refractive indices and polarization characteristics of normal electromagnetic waves propagating in a laser wave are found. The relations, describing variations of γ -beam intensity and Stokes parameters as functions of the propagation length, are obtained. The influence of laser wave intensity on the considered process is calculated.

Аннотация

Маишеев В.А. Распространение пучка γ-квантов в поле монохроматической лазерной волны: Препринт ИФВЭ 97-25. – Протвино, 1997. – 14 с., 4 рис., библиогр.: 16.

Рассмотрено встречное прохождение пучка γ-квантов через поле лазерной волны. Оптические свойства лазерной волны описаны тензором диэлектрической проницаемости. Найдены показатели преломления и исследованы поляризационные характеристики нормальных электромагнитных волн, которые могут распространяться в такой среде. Получены соотношения, описывающие изменение начальной поляризации и интенсивности γ-пучка при его распространении в поле лазерной волны. Обсуждается влияние интенсивности лазерной волны на исследуемый процесс.

Introduction

Polarization effects [1,2] arising from the visible light propagation in the anisotropic or gyrotropic medium are well-known. The theory [3] makes prediction about the analogous effects for γ-quanta with energy > 1 GeV propagating in single crystals, which present the anisotropic medium. The main absorption process of γ-quanta in the single crystals is the electron-positron pair production. The cross section of this process depends on the linear polarization of γ-beam with respect to crystallographic planes. As a result, the monochromatic linear polarized γ-beam may be presented as a superposition of two electromagnetic waves with different refractive indices due to which the transformation of linear (circular) polarization to circular (linear) one takes place.

On the other hand, the process of e^+e^--pair production in single crystals is similar to the same process in a linear polarized laser wave [4]. A possibility to use a bunch of linearly polarized laser photons as a "single crystal" is pointed in [5], but the concrete calculations of this possibility are not given.

In the recent paper [6] it has been shown that the polarization effects such as birefringence and rotation of polarization plane for γ- beams with energies of tens GeV or more take place for short (about some picosecunds) laser bunches and parameters of lasers, which may be provided by real techniques. In the cited paper the equations which determine the variation of Stokes parameters and intensity of γ-quanta traversing a bunch of arbitrary polarized laser photons are obtained. For these calculations the well-known scattering amplitudes for the process of elastic scattering light by light [7,8] were used.

In the present paper we study the propagation of high energy γ-quanta through the laser wave on the basis of traditional crystal optics of the anisotropic and gyrotropic medium [1,2]. In this case the interaction of γ-beam with the field of laser photons can be described by introducing a permittivity tensor. This tensor makes it possible to describe, from a unified standpoint, a number of processes occurring in laser waves, such as dichroism accompanying the passage of γ-quanta, the Cherenkov radiation from charged particles and others.

1. Permittivity tensor in laser wave

We write the equations of the electromagnetic field in a medium (γ-beam propagating in a laser wave) in the following form [1,2]

$$rot\vec{B} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t}\,, \quad div\vec{D} = 0\,,$$
$$rot\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\,, \quad div\vec{B} = 0\,, \tag{1}$$

where $\vec{E}$ is the intensity of electric field and $\vec{D}$ and $\vec{B}$ are the electric and magnetic induction vectors, t is the time , c is the speed of light. All the properties of the medium (the electromagnetic field of the laser wave is the medium) are reflected in the relation between $\vec{B}, \vec{E}$ and $\vec{D}$. Eqs.(1) would suffice to describe the γ-beam propagation in a medium and such a property as the intensity of magnetic field is not needed [1,2]. We represent the relation between $\vec{D}$ and $\vec{E}$ in the form

$$D_i(\omega) = \varepsilon_{ij} E_j(\omega)\,, \quad (i, j = 1, 2, 3)\,, \tag{2}$$

where $\varepsilon_{ij} = \varepsilon'_{ij} + i\varepsilon''_{ij}$ is the complex permittivity tensor and ω is the frequency of γ-quanta.

In order to determine the permittivity tensor in the case of a monochromatic field (high energy γ-beam) $\vec{E}_0 e^{i(\vec{k}\vec{r}-\omega t)}$, where $\vec{k}$ is the wave vector of the γ-quanta, we find the average energy lost by the electromagnetic wave per unit volume V and per unit time [1,2]

$$\tilde{q} = \frac{1}{4\pi V}\int_V \vec{E}\frac{\partial \vec{D}}{\partial t} dV = \frac{i\omega}{16\pi}(\varepsilon^*_{ij} - \varepsilon_{ji})E^*_j E_i\,. \tag{3}$$

The mechanism by which the wave loses energy is e^+e^-- pair production in the field of the laser wave [9]. The process is determined primarily by the transverse part of the permittivity tensor, while the longitudinal components of the tensor are higher-order infinitesimals in the interaction constant α [4,10]. Taking this into account, and in the coordinate system one axis of which is oriented parallel to the wave vector of γ-quanta, we have from (3)

$$\tilde{q} = \frac{i\omega J}{4}\{(\varepsilon_{11}{}^* - \varepsilon_{11})(1+\xi_3) + (\varepsilon_{12}{}^* - \varepsilon_{21})(\xi_1 - i\xi_2) + (\varepsilon_{21}{}^* - \varepsilon_{12})(\xi_1 + i\xi_2) + (\varepsilon_{22}{}^* - \varepsilon_{22})(1-\xi_3)\}\,, \tag{4}$$

where $J = (E_1 E_1{}^* + E_2 E_2{}^*)/8\pi, \xi_i$ are Stokes parameters of γ-beam. On the other hand, knowing the cross section $\sigma_{\gamma\gamma}$ of the pair production process, we can write

$$\tilde{q} = 2n_l\sigma_{\gamma\gamma}(cn_\gamma E_\gamma) = 2n_l\sigma_{\gamma\gamma}cJ\,, \tag{5}$$

where cn_γ is the flux density of γ-quanta with energy equal to E_γ, n_l is the number of photons per volume unit of the laser wave. Factor 2 in this formula is due to the counter-motion of the γ-beam and laser wave. Let us select two coordinate axes parallel and perpendicular with respect to the direction of linear polarization of laser wave (the third axis is directed parallel to the wave vector). Then in this system we can write the cross section of e^+e^- -pair production in the following form [4,6,7,9]:

$$\sigma_{\gamma\gamma}(z) = \sigma_0(z) + \sigma_c(z)\xi_2 P_c + \sigma_l(z)\xi_3 P_l\,,\ \ 0 < z \le 1\,, \tag{6}$$

where $z = \frac{m^2c^4}{E_\gamma E_l}$ is the invariant variable, m is the electron mass, E_l and P_c, P_l are the energy, circular and linear polarizations of the laser photon. It is well known that the pair production is a threshold process and, because of this, the laser wave is a transparent medium for γ-beam , when $E_\gamma E_l < m^2c^4$ or $z > 1$. The functions σ_0, σ_c , σ_l are defined in Refs.[4,6,7,9] and they are contained in Eqs.(17)-(19), (23)-(25) according to relations (9)-(11) (see below). Comparing Eqs.(4) and (5) we can find the components of permittivity tensor caused by γ-quanta absorption.

Then we can determine the other components of the tensor with the help of the following dispersion relations [2]:

$$\varepsilon'_{ij} - \delta_{ij} = \frac{2}{\pi}\mathcal{P}\int_0^\infty \frac{x\varepsilon''_{ij}(x)\,dx}{x^2-\omega^2}\,, \tag{7}$$

$$\varepsilon''_{ij} = -\frac{2\omega}{\pi}\mathcal{P}\int_0^\infty \frac{(\varepsilon'_{ij}-\delta_{ij})\,dx}{x^2-\omega^2}\,, \tag{8}$$

where δ_{ij} is the Kronecker δ-function. Comparing (4) and (5), we get

$$\varepsilon''_{11} + \varepsilon''_{22} = 4n_l c\sigma_0/\omega\,, \tag{9}$$

$$\varepsilon''_{11} - \varepsilon''_{22} = 4n_l c\sigma_l P_l/\omega\,, \tag{10}$$

$$\varepsilon'_{12} - \varepsilon'_{21} = 4n_l c\sigma_c P_c/\omega\,. \tag{11}$$

It easy to verify that $\varepsilon_{12}+\varepsilon_{21} = 0$. This conclusion follows immediately from the invariance of the components of the permittivity tensor relative to coordinate rotation around the wave vector for the circular polarization of laser wave ($P_c \ne 0, P_l = 0$). The same result is evident from the theory of generalized receptivity [11] for values, which change their own sign at the time inversion. In our case, the photon angular momentum is such a value.

Thus the components of permittivity tensor caused by γ-quanta absorption are determined from relations

$$\varepsilon''_{11} = 2n_l c(\sigma_0 + P_l\sigma_l)/\omega\,, \tag{12}$$

$$\varepsilon''_{22} = 2n_l c(\sigma_0 - P_l\sigma_l)/\omega\,, \tag{13}$$

$$\varepsilon'_{12} = -\varepsilon'_{21} = 2n_l c P_c\sigma_c/\omega\,. \tag{14}$$

The other components of the permittivity tensor can be calculated with the help of relations (7)-(8).

The results of calculations of the components ε_{ij} are presented below for the coordinate system with one axis oriented parallel to the wave vector of γ-beam and the other two axes oriented in parallel and perpendicular with respect to the linear polarization of the laser wave.

$$\varepsilon'_{11} - \varepsilon'_{22} = \frac{\alpha}{2\pi}\frac{< E^2 >}{E_o^2} P_l z^2 {F_1}'(z) \tag{15}$$

$$\varepsilon'_{11} + \varepsilon'_{22} = 2 + \frac{2\alpha}{\pi}\frac{< E^2 >}{E_o^2} z^2 {F_2}'(z,1)\,, \tag{16}$$

$$\varepsilon'_{12} = -\varepsilon'_{21} = \frac{\alpha < E^2 >}{2E_o^2} P_c z^2 {F_c}'(z)\,, \tag{17}$$

$$\varepsilon''_{11} - \varepsilon''_{22} = -\frac{\alpha}{4}\frac{< E^2 >}{E_o^2} P_l {F_1}''(z)\,, \tag{18}$$

$$\varepsilon''_{11} + \varepsilon''_{22} = \frac{\alpha < E^2 >}{E_o^2} {F_2}''(z,1)\,, \tag{19}$$

$$\varepsilon''_{12} = -\varepsilon''_{21} = \frac{\alpha < E^2 >}{\pi E_o^2} P_c {F_c}''(z)\,, \tag{20}$$

where $< E^2 >= 4\pi n_l E_l$ is the mean square of intensity electric laser field and the functions $F'_1, F'_2, F'_c, F''_1, F''_2, F''_c$ are equal to:

$$ {F_1}'(z) = \begin{cases} [\sqrt{1-z} + \frac{z}{2}L_-]^2 + [\sqrt{1+z} - \frac{z}{2}L_+]^2 - \frac{\pi^2 z^2}{4},\ 0 < z \le 1, \\ -[\sqrt{z-1} - z\,\mathrm{arcctg}\,\sqrt{z-1}]^2 + [\sqrt{1+z} - \frac{z}{2}L_+]^2,\ z > 1. \end{cases} \tag{21}$$

$$ {F_2}'(z,\mu) = \begin{cases} -2 - \mu - (1+\mu(z-\frac{z^2}{2}))\frac{1}{4}L_-^2 - (1-\mu(z+\frac{z^2}{2}))\frac{1}{4}L_+^2 + \\ +\frac{(1+\mu z)\sqrt{1-z}}{2}L_- - \frac{(\mu z-1)\sqrt{z+1}}{2}L_+ + \frac{\pi^2}{4}(1+\mu(z-\frac{z^2}{2})),\ 0 < z \le 1, \\ -2 - \mu + (1+\mu(z-\frac{z^2}{2}))\,\mathrm{arcctg}^2(\sqrt{z-1}) - (1-\mu(z+\frac{z^2}{2}))\frac{1}{4}L_+^2 + \\ +(1+\mu z)\sqrt{z-1}\,\mathrm{arcctg}\,\sqrt{z-1} - \frac{(\mu z-1)\sqrt{1+z}}{2}L_+,\ z > 1. \end{cases} \tag{22}$$

$$ {F_c}'(z) = \begin{cases} 3\sqrt{1-z} - L_-,\ 0 < z \le 1, \\ 0,\ z > 1. \end{cases} \tag{23}$$

$$ {F_1}''(z) = \begin{cases} z^4(L_- + \frac{2\sqrt{1-z}}{z}),\ 0 < z \le 1, \\ 0,\ z > 1. \end{cases} \tag{24}$$

$$ {F_2}''(z,\mu) = \begin{cases} z^2((1+\mu(z-\frac{z^2}{2}))L_- - \sqrt{1-z}(1+\mu z)),\ 0 < z \le 1, \\ 0,\ z > 1 \end{cases} \tag{25}$$

$$ {F_c}''(z) = \begin{cases} z^2(\frac{3}{2}\sqrt{1-z}L_- - \frac{3}{2}\sqrt{1+z}L_+ - \frac{1}{4}L_-^2 + \frac{1}{4}L_+^2 + \frac{\pi^2}{4}),\ 0 < z \le 1, \\ z^2(3\sqrt{z-1}\,\mathrm{arcctg}\,\sqrt{z-1} - \frac{3}{2}\sqrt{1+z}L_+ + \mathrm{arcctg}^2(\sqrt{z-1}) + \frac{1}{4}L_+^2),\ z > 1. \end{cases} \tag{26}$$

The functions L_-, L_+ are equal to:

$$L_- = ln\frac{1+\sqrt{1-z}}{1-\sqrt{1-z}}\,,$$

$$L_+ = ln\frac{\sqrt{1+z}+1}{\sqrt{1+z}-1}\,.$$

The constant $E_o = \frac{m^2c^3}{e\hbar}$ is the critical field of quantum electrodynamics. The presented here data completely determine the permittivity tensor for high energy γ-quanta traversing a bunch of laser photons. In a number of problems in crystal optics it is more convenient to employ the tensor η_{ij} , which is inverse of the tensor ε_{ij}. When $|\varepsilon_{ij} - \delta_{ij}| \ll 1$, these tensors are related to

$$\eta_{ij} + \varepsilon_{ij} = 2\delta_{ij}. \quad (27)$$

The behavior of permittivity tensor components as functions of the invariant variable z are illustrated in Fig. 1.

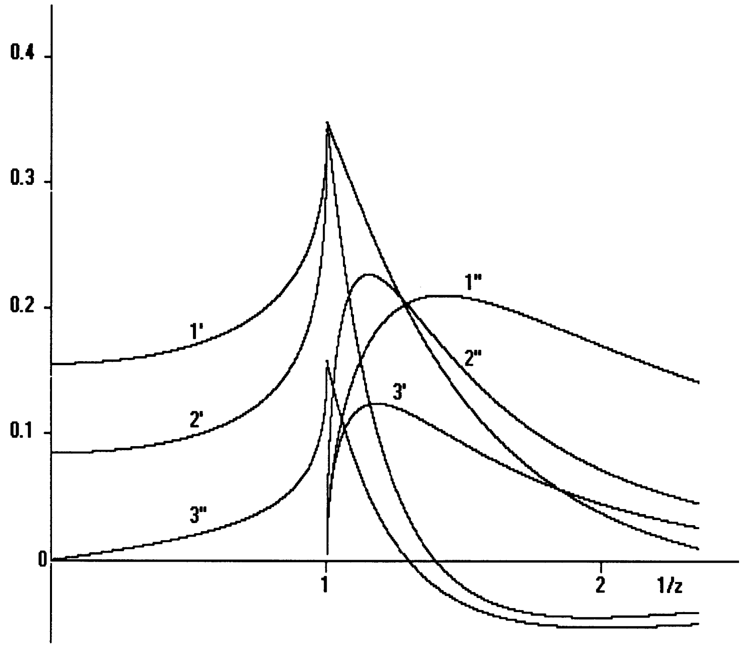

Fig. 1. Components of the permittivity tensor as functions of the invariant variable z; the curves $1'$, $1''$ are $k(\varepsilon'_{11}+\varepsilon'_{22}-2)/2$, $k(\varepsilon''_{11}+\tilde{\varepsilon}''_{22})/2$; 2',2" are $k(\varepsilon'_{22}-\varepsilon'_{11})/P_l$, $k(\varepsilon''_{22}-\varepsilon''_{11})/P_l$; 3',3" are $k\varepsilon'_{12}/P_c$, $k\varepsilon''_{21}/P_c$. Components $\varepsilon'_{21} = -\varepsilon'_{12}$, $\varepsilon''_{12} = -\varepsilon''_{21}$. Multiply $k^{-1} = \frac{\alpha<E^2>}{E_o^2}$.

2. Refractive indices of γ-quanta

The main problem of optic of anisotropic (gyrotropic) medium is to investigate the propagation of monochromatic plane waves, characterized by definite values of the frequency ω and wave vector $\vec{k}$. Such waves, satisfying a homogeneous wave equation, are called normal electromagnetic waves [2], and they have the form

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r}-\omega t)}, \vec{k} = \omega\tilde{n}\vec{s}/c\,,$$

where $\vec{E}_0$ is the complex vector, independent of coordinates $\vec{r}$ and time, $\tilde{n}$ is the complex index of refraction and $\vec{s} = \vec{k}/|k|$ is a real unit vector. The vectors $\vec{D}$ and $\vec{B}$ have the same form.

From Maxwell's equations (1) we obtain the wave equation

$$rotrot\vec{E} + \frac{1}{c}\frac{\partial^2\vec{D}}{\partial t^2} = 0\,.$$

Taking into account the relation between $\vec{D}$ and $\vec{E}$ in a system of coordinates in which the axis x is oriented parallel to the wave vector, we obtain

$$\begin{aligned} \eta_{11}\frac{\partial^2 D_1}{\partial x^2} + \eta_{12}\frac{\partial^2 D_2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 D_1}{\partial t^2} = 0, \\ \eta_{21}\frac{\partial^2 D_1}{\partial x^2} + \eta_{22}\frac{\partial^2 D_2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 D_2}{\partial t^2} = 0. \end{aligned} \tag{28}$$

For a monochromatic plane wave it follows from this equation that

$$(\tilde{n}^{-2}\delta_{ij} - \eta_{ij})D_j = 0,\ i,j = 1,2. \tag{29}$$

From the condition that the two homogeneous equations are compatible, we find the index of refraction of the γ-quanta

$$\tilde{n}^{-2} = \frac{S}{2} \pm \sqrt{\frac{S^2}{4} - D_\eta} = (\eta_{11} + \eta_{22})/2 \pm \sqrt{(\eta_{11} - \eta_{22})^2/4 + \eta_{12}\eta_{21}}\,, \tag{30}$$

where S and D_η are, respectively, the trace and determinant of the matrix η_{ij}. Thus, in the general case, the γ-beam propagates through the laser wave as the superposition of two electromagnetic waves with different refractive indices. Note, that the two roots of (30), which have form $-1 +$ *small quantity*, are superfluous. They correspond to the γ-quanta motion in the reverse direction.

In the general case the refractive indices are complex values. However, they are real values, when $z \geq 1$. It is easy to see from (30). Thus in this case the laser wave is a transparent medium.

The refractive indices for linear ($P_c = 0$) and circular ($P_l = 0$) polarizations of the laser wave are correspondingly equal to $n_1^2 = \varepsilon_{11}, n_2^2 = \varepsilon_{22}$ and $n_{1,2}^2 = (\varepsilon_{11} + \varepsilon_{22})/2 \pm i\varepsilon_{12}$, where the components of tensor are written in the selected previously coordinate system and the relation $\varepsilon_{12} = -\varepsilon_{21}$ is taken into account. The asymptotic behavior of these refractive indices at $z \to \infty$ correspondingly describe the following expressions

$$n_\parallel, n_\perp = 1 + \frac{\alpha}{\pi}\frac{< E^2 >}{E_o^2}(\frac{11 \pm 3P_l}{45})\,, \tag{31}$$

$$n_\Rightarrow, n_\Leftrightarrow = 1 + \frac{\alpha}{\pi}\frac{< E^2 >}{E_o^2}(\frac{11}{45} \pm \frac{16P_c}{315z})\,. \tag{32}$$

Figs. 2 and 3 illustrate the refractive indices as functions of the invariant variable z.

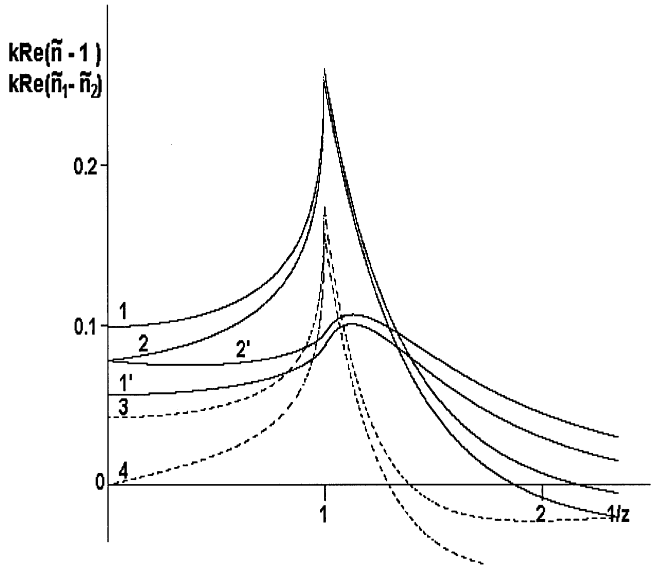

Fig. 2. The real parts of refractive indices $(1, 1')$ for linearly $(P_l = 1, P_c = 0)$ and $(2, 2')$ for circularly $(P_l = 0, P_c = 1)$ polarized laser waves, and the corresponding differences $(3, 4)$ of these values as functions of the invariant variable z. Multiply k^{-1} is equal to $\frac{\alpha<E^2>}{E_o^2}$.

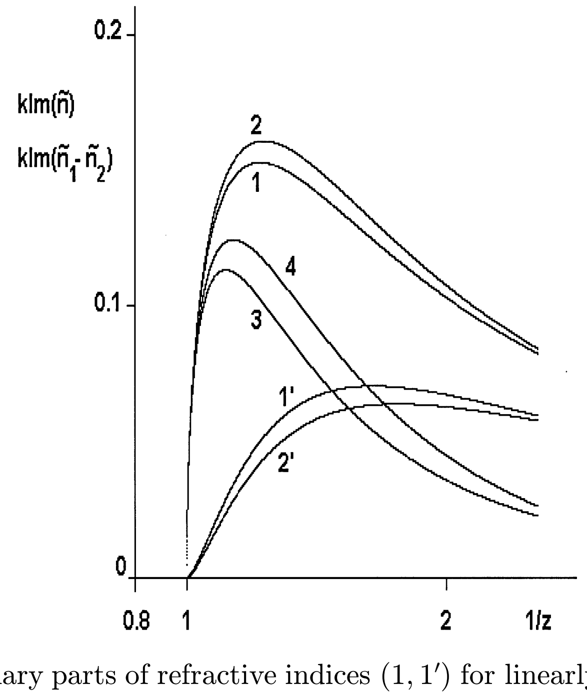

Fig. 3. The imaginary parts of refractive indices $(1, 1')$ for linearly $(P_l = 1, P_c = 0)$ and $(2, 2')$ circularly $(P_l = 0, P_c = 1)$ polarized laser waves, and the corresponding differences $(3, 4)$ of these values as functions of the invariant variable z. Multiply k^{-1} is equal to $\frac{\alpha<E^2>}{E_o^2}$.

3. Polarization properties of γ-beam propagation in laser wave

Here we consider the polarization properties one of two normal electromagnetic waves. From dispersion equations (29) we find the ratios of the components of the vector $\vec{D}$

$$\frac{D_1}{D_2} = \kappa = \frac{\tilde{n}^{-2} - \eta_{22}}{\eta_{21}} = \frac{|D_1|}{|D_2|} e^{i\delta}, \tag{33}$$

where δ is the phase shift between D_1 and D_2. This ratio κ can be reduced to zero or to the form $\kappa = i\rho$ (since $|D_1||D_2|\sin\delta = b_1 b_2$, where b_1 and b_2 are the semiaxes of the ellipse and $|\rho| = b_1/b_2$ [12]) by the rotation of the coordinate system around the wave vector of γ-quanta (the x-axis is constantly aligned with the wave vector). The first case corresponds to the propagation of a linearly polarized wave and the second case corresponds to an elliptically polarized wave; in addition, $\rho > 0 (\rho < 0)$ corresponds to left (right) - hand polarization of γ-quanta.

First we consider the case when $z > 1$ and laser wave is a transparent medium for γ-quanta. In the case when $\eta''_{12} \neq 0$ we can write

$$\kappa = i\rho = i \frac{(\eta'_{11} - \eta'_{22})/2 \pm \sqrt{(\eta'_{11} - \eta'_{22})^2/4 + {\eta''_{12}}^2}}{\eta''_{12}} \,. \tag{34}$$

We see that in general case the normal waves are elliptically polarized and the principal axes of ellipse are parallel to the axes of the selected coordinate system. The circular P_{circ} and linearP_{line} polarizations of this waves is calculated from the following well known relations

$$P_{circ} = 2\rho/(1+\rho^2)\,, \tag{35}$$

$$P_{line} = (1-\rho^2)/(1+\rho^2)\,. \tag{36}$$

It is easily seen that $P_{circ} \neq P_c$, $P_{line} \neq P_l$ in general. The normal waves are the linearly polarized along coordinate axes, when $\eta''_{12} = 0$.

Now we consider the case, when $z < 1$ and γ-quanta are absorbed in the laser wave. In this case the normal electromagnetic waves propagating in a linearly polarized laser wave ($P_c = 0$, $P_l \neq 0$) are linearly polarized ($P_{line} = \pm 1$) along the coordinate axes. In the general case the normal waves propagating in a laser wave of the arbitrary polarization ($P_c \neq 0$, $P_l \neq 0$) are elliptically polarized and the principal axes of these ellipses are turned relative to the axes of coordinate system. Let us denote the angle of this turn by φ. In the case under consideration the refractive indices are the complex values. Because of this, the value κ is also complex and we get the following relation between two normal waves

$$\kappa^{(1)}\kappa^{(2)} = 1\,, \tag{37}$$

where the indices in parentheses refer to the waves with refractive indices $\tilde{n}_1$ and $\tilde{n}_2$. In what follows we will use only one of two values, namely, the $\kappa = \kappa^{(1)}$ (without pointing any indices). In our case one can obtain

$$D_1^{(1)} D_1^{(2)} + D_2^{(1)} D_2^{(2)} = 2 D_2^{(1)} D_2^{(2)}\,, \tag{38}$$

$$D_1^{(1)} D_1^{*(2)} + D_2^{(1)} D_2^{*(2)} = D_2^{(1)} D_2^{(2)} (\kappa + \kappa^*)/\kappa^*\,, \tag{39}$$

where the indices in parentheses refer to waves with refractive indices $\tilde{n}_1$ and $\tilde{n}_2$. From here we can see that $\vec{D}^{(1)}$ and $\vec{D}^{(2)}$ vectors are not orthogonal. The same result is and for $\vec{D}^{(1)}$ and $\vec{D}^{*(2)}$ vectors[2].

Let us name the Stokes parameters of the normal wave with the refractive indices $\tilde{n}_1$ and $\tilde{n}_2$ respectively as X_1, X_2, X_3 and Y_1, Y_2, Y_3. Then we get

$$X_1 = \frac{\kappa + \kappa^*}{1 + \kappa\kappa^*}, \tag{40}$$

$$X_2 = -\frac{i(\kappa^* - \kappa)}{1 + \kappa\kappa^*}, \tag{41}$$

$$X_3 = \frac{\kappa\kappa^* - 1}{1 + \kappa\kappa^*}. \tag{42}$$

We have also the following relations $Y_1 = X_1, Y_2 = -X_2, Y_3 = -X_3$. The angle φ is found from relation $tg2\varphi = X_1/X_3$, and it is equal to $-\varphi$ for the second wave.

Fig. 4 illustrates the results of calculations of P_{circ}, P_{line} φ as functions of the invariant variable z for the laser wave provided $P_c = P_l = 1/\sqrt{2}$.

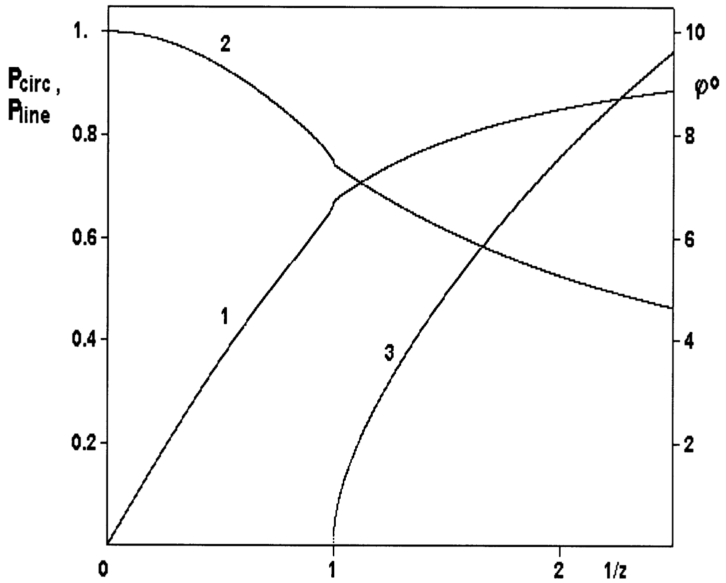

Fig. 4. The variations of absolute values P_{circ} and P_{line}, and the angle φ (in degree) as functions of the invariant variable z. The linear and circular polarization of laser wave are selected $P_l = P_c = 1/\sqrt{2}$. When $z > 1(z^{-1} < 1)$ the angle $\varphi = 0$.

4. γ-quanta propagation in the laser wave

Now we can find the relations describing the variations of intensity and Stokes parameters of γ-quanta propagating in the uniform ($n_l = const$) laser wave. Then representing the γ-beam as the superposition of two normal waves corresponding to the polarization state of a laser wave we get the following relations:

$$J_\gamma(x) = J_1(x) + J_2(x) + 2J_3(x)\,, \tag{43}$$
$$\xi_1(x) = (X_1 J_1(x) + Y_1 J_2(x) + p_1 J_3(x))/J_\gamma(x)\,, \tag{44}$$
$$\xi_2(x) = (X_2 J_1(x) + Y_2 J_2(x) + p_2 J_4(x))/J_\gamma(x)\,, \tag{45}$$
$$\xi_3(x) = (X_3 J_1(x) + Y_3 J_2(x) + p_3 J_4(x))/J_\gamma(x)\,, \tag{46}$$

where $J_\gamma, \xi_1, \xi_2, \xi_3$ are the intensity and Stokes parameters of γ-quanta on the laser bunch thickness equal to x. The partial intensities $J_i(x), (i = 1-4)$ have the following form (the physical sense of these values is easy to understand, if the relation $(\vec{D}^{(1)} + \vec{D}^{(2)})(\vec{D}^{*(1)} + \vec{D}^{*(2)})$ is written in the component-wise form)

$$J_1(x) = J_1(0)\exp(-2\,\mathrm{Im}(\tilde{n}_1)\omega x/c)\,, \tag{47}$$
$$J_2(x) = J_2(0)\exp(-2\,\mathrm{Im}(\tilde{n}_2)\omega x/c)\,, \tag{48}$$
$$J_3(x) = \exp(-\,\mathrm{Im}(\tilde{n}_1 + \tilde{n}_2)\omega x/c)\{J_3(0)\cos(\mathrm{Re}(\tilde{n}_1 - \tilde{n}_2)\omega x/c) - J_4(0)\sin(\mathrm{Re}(\tilde{n}_1 - \tilde{n}_2)\omega x/c)\}\,, \tag{49}$$
$$J_4(x) = \exp(-\,\mathrm{Im}(\tilde{n}_1 + \tilde{n}_2)\omega x/c)\{J_3(0)\sin(\mathrm{Re}(\tilde{n}_1 - \tilde{n}_2)\omega x/c) + J_4(0)\cos(\mathrm{Re}(\tilde{n}_1 - \tilde{n}_2)\omega x/c)\}\,. \tag{50}$$

The initial partial intensities are defined from the following relations

$$J_1(0) = \frac{\xi_2(0) - f\xi_3(0)}{2(X_2 - fX_3)} + \frac{\xi_1(0) - q}{2(X_1 - q)}\,, \tag{51}$$
$$J_2(0) = -\frac{\xi_2(0) - f\xi_3(0)}{2(X_2 - fX_3)} + \frac{\xi_1(0) - q}{2(X_1 - q)}\,, \tag{52}$$
$$J_3(0) = \frac{X_1 - \xi_1(0)}{2(X_1 - q)}\,, \tag{53}$$
$$J_4(0) = \frac{\xi_3(0)X_2 - \xi_2(0)X_3}{p_3(X_2 - fX_3)}\,. \tag{54}$$

The relations between X_i and Y_i values were used, because of this the Y_i-values are absent in Eqs. (51)-(54). Besides, we assume that $J_\gamma(0) = 1$. The parameters f, q, p_1, p_2, p_3 have the following form:

$$f = \frac{i(\kappa\kappa^* - 1)}{\kappa - \kappa^*}\,, \quad q = \frac{1 + \kappa\kappa^*}{\kappa + \kappa^*}\,,$$
$$p_1 = \frac{2(1 + \kappa\kappa^*)}{\kappa + \kappa^*}\,, \quad p_2 = \frac{2(1 - \kappa\kappa^*)}{\kappa + \kappa^*}\,, \quad p_3 = \frac{2i(\kappa - \kappa^*)}{\kappa + \kappa^*}\,.$$

Eqs. (43)-(46) describe the general case of γ-beam propagation, when the variations intensity and Stokes parameters are determined by the imaginary values of refractive indices and the difference of their real quantities. However, these equations do not describe such cases, when the relation $\kappa + \kappa^* = 0$ takes place.This condition applies to completely linearly or circularly polarized γ-beams and for transparent medium (for arbitrary polarization). In the case of $\kappa + \kappa^* = 0$, one can use the known relations from papers [6,13] or one can find limits of the obtained here Eqs.(43)-(46). For example, one can offer $\kappa = \delta + i\rho$ and δ direct to zero. The case $\kappa + \kappa^* = 0$ is described in Appendix.

5. Influence of the laser wave intensity on γ-quanta propagation

The influence of the laser wave intensity on the e^+e^--pair production was studied in a number of papers (see Ref.[4] and literature therein). The degree of this intensity can be characterized by the dimensionless parameter [4] $\xi^2 = \frac{<E^2>}{{E_o}^2}\frac{m^2c^4}{{E_l}^2}$. Here we have considered the case of the relatively not high intensity of a laser wave, when $\xi^2 \ll 1$. The results of papers [4,9] allowe one to write down the components of permittivity tensor taking into account the first terms of the expansion of the intensity in Taylor series. Thus, we have found that the already obtained components of the tensor should be transformed with the help of the following simple rules. Firstly, the variable z is substituted by the variable $\tilde{z} = z(1+\xi^2)$. Secondly, the value E_o (the critical field) in Eqs.(15)-(20) is substituted by the value $\tilde{E}_o = \frac{m^2c^3(1+\xi^2)}{e\hbar}$. Thirdly, the functions $F_2'(z,1), F_2''(z,1), F_1'(z), F_(''z), F_c'(z), F_c''(z)$ are substituted by the functions $F_2'(\tilde{z},\mu), F_2''(\tilde{z},\mu), F_1'(\tilde{z}), F_1''(\tilde{z})F_c'(\tilde{z}), F_c''(\tilde{z})$, where $\mu = 1/(1+\xi^2)$. The new condition for the pair production threshold follows from these rules. It is $\tilde{z} < 1$. It means that the threshold energy of γ-beam enhances (at the fixed frequency of laser photons). In the strict sense the field of application of these more refined relations satisfies the condition $\xi^2 \ll 1$. Nevertheless, we can receive the important information in this case. For example, in Ref.[6] some estimates were carried out for photon energy equal to 1.18 eV and $\xi^2 \approx 0.1$. Then, taking into account the intensity of the laser wave we can see that the threshold of pair production enhances from 221 GeV to 247 GeV. Considering the behavior of refractive indices (see Figs. 2,3) near the threshold, the more precise calculations are desirable to use.

In the case, when the parameter $\xi^2 \gg 1$, the pair production process is similar to an analogous process in the constant electromagnetic field. The permittivity tensor for these fields was found in Ref.[14] and some particular calculations of γ-quanta propagation are in [15].

Discussion

In recent paper [6] it has been shown that a strong variation of Stokes parameters of tens GeV or more γ-quanta traversing a laser wave take place for the short (about some picosecounds) laser bunches and parameters of lasers, which may be provided by real techniques. In the cited paper the elastic process of the light by light scattering was considered and it was shown that this interaction caused the variations of Stokes parameters. These variations were described by the system of differential equations. The solutions of this system were found for the arbitrary polarized laser wave represented a transparent medium and for the linearly ($P_c = 0$) and circularly $P_l = 0$ polarized laser wave, when γ-quanta are absorbed. In principle, this consideration is large enough for the description of the investigated process. However, some of useful facts are not reflected in Ref.[6]. So, there is not a description of the laser wave as a specific optic medium, due to, for example, the difference in refractive indices determined by the effect of polarization transform, whereas refractive indices are the property of the medium. A full description of the laser wave as a specific optic medium provides the subject matter for our paper.

Starting from the cross sections of e^+e^--pair production we have obtained the permittivity tensor of a laser wave. As is well known, optical theorem [8] relates the amplitude of elastic light by light scattering at zero angle with the total cross section for e^+e^- pair production. Because of this, it is believed that the relations for variations of the Stokes parameters under our consideration and the consideration of Ref.[6] should be numerically equal. Besides, we have considered the problem with the help of the methods of macroscopic electrodynamics and hence we should not investigate a particular mechanism of the phenomenon. With the point of view of these methods the polarization of a medium takes place, when a γ-beam propagates in the laser wave. In our case the polarization of medium is due to the presence of virtual e^+e^- pairs. The polarization of these pairs alters the electromagnetic field of laser wave and this process can be described with the use of permittivity tensor.

Using the dispersion equations we have found the refractive indices of the normal electromagnetic waves and the polarization characteristics of these waves. The normal electromagnetic waves represent the eigenfunctions of the problem about the propagation of γ-beam in a laser wave. Using this eigenfunctions we obtained the simple relations, described the variations of intensity and Stokes parameters of γ-beam moving in the laser wave.

We compared the results obtained with the use of Eqs.(43)-(46) with the analogous results of [6]. The results of numerical integration of the differential equations in [6] are in good agreement with computations using Eqs.(43)-(46). The difference between both results is approximatly equal to the accuracy of integration.

A knowledge of the quantities of refractive indices makes it possible to study some other processes in a laser wave. For example, we estimate that charged particles with the Lorentz factor about $\sim 10^6 - 10^7$ can emit Cherenkov radiation [5] in a laser wave under conditions, which may be provided by real techniques. However, the mass of particles should be such that their bremsstrahlung energy losses are negligible compared with the losses at Cherenkov radiation. The typical energy of γ-quanta is about some hundreds GeV.

The propagation of γ-quanta through a laser wave (when $\xi^2 \ll 1$ is similar to the same process in single crystals for the region of coherent pair production. For example, the permittivity tensor in single crystals [5] is determined by the functions F_1', F_2', F_1'', F_2'' as in a laser wave. However, the existence of some frequencies of pseudophotons and incoherent pair production in single crystals is the main difference between these two cases.

Note that there are no experiments yet in support of the transformation of γ-beam polarization in single crystals. Nevertheless, a number of proposals on the investigation and utilization of this phenomenon [10,16] is available.

The new possibility [6] of the observation of polarization effects in a laser beams allows one to test the main principles of the theory for a laser wave as well as single crystals.

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Relations for intensity and Stokes parameters of γ-beam in case $\kappa + \kappa^* = 0$

$$J = J_0 e^{-(n_1+n_2)}(\mathrm{ch}(n_2 - n_1) - a\,\mathrm{sh}(n_2 - n_1))$$

$$\xi_1 = \frac{b\sin(\Delta) + \xi_1^\circ \cos(\Delta)}{\mathrm{ch}(n_2 - n_1) - a\,\mathrm{sh}(n_2 - n_1)}$$

$$\xi_2 = \frac{P_{circ}\,\mathrm{sh}(n_2 - n_1) + aP_{circ}\,\mathrm{ch}(n_2 - n_1) + P_{line}[-b\cos(\Delta) + \xi_1^\circ \sin(\Delta)]}{\mathrm{ch}(n_2 - n_1) - a\,\mathrm{sh}(n_2 - n_1)}$$

$$\xi_3 = \frac{P_{line}\,\mathrm{sh}(n_2 - n_1) + aP_{line}\,\mathrm{ch}(n_2 - n_1) + P_{circ}[b\cos(\Delta) - \xi_1^\circ \sin(\Delta)]}{\mathrm{ch}(n_2 - n_1) - a\,\mathrm{sh}(n_2 - n_1)},$$

where $a = \xi_3^\circ P_{line} + \xi_2^\circ P_{circ}$, $b = \xi_3^\circ P_{circ} - \xi_2^\circ P_{line}$, $n_1 = \mathrm{Im}(\tilde{n}_1)\omega x/c$, $n_2 = \mathrm{Im}(\tilde{n}_2)\omega x/c$, $\Delta = \mathrm{Re}(\tilde{n}_1 - \tilde{n}_2)\omega x/c$, the values P_{line} and P_{circ} correspond to the polarization of a normal wave with the refractive indice $\tilde{n}_1$, ξ_i° are initial values of Stokes parameters and other notations are the same as in the text. It is obvious that $n_1 = n_2 = 0$ in a transparent medium.

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