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ANALYTICITY AND MINIMALITY OF NONPERTURBATIVE CONTRIBUTIONS IN PERTURBATIVE REGION FOR $\bar{\alpha}_s$

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Abstract

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It is shown, that a possibility of freezing a QCD running coupling constant at zero in the approach with "forced analyticity" can not be in accord with the Schwinger-Dyson equation for the gluon propagator. We propose to add to the analytic expression the well-known infrared singular term $1/q^2$ as well as a pole term corresponding to "excited gluon". By this example we formulate the principle of minimality of nonperturbative contributions in the perturbative (ultraviolet) region, which allows us to fix ambiguities when introducing nonperturbative terms and maintain the finiteness of the gluon condensate. As a result we obtain estimates of the gluon condensate, which quite agree with the existing data. The nonzero effective mass of the "excited gluon" leads also to some interesting qualitative consequences.

Аннотация

Алексеев А.И., Арбузов Б.А. Аналитичность и принцип минимальности непертурбативных вкладов в пертурбативной области для $\bar{\alpha}_s$: Препринт ИФВЭ 97-26. – Протвино, 1997. – 8 с., 1 табл., библиогр.: 23.

Показано, что "замораживание" бегущей константы связи КХД вблизи нуля, получаемое методом "аналитизации", не согласуется с уравнением Швингера-Дайсона для глюонного пропагатора. Предложено добавить к аналитическому выражению известный сингулярный в инфракрасной области член вида $1/q^2$, а также полюсной член, соответствующий "возбужденному глюону". Сформулирован принцип минимальности непертурбативных вкладов в пертурбативной (ультрафиолетовой) области, который позволяет зафиксировать произвол введения непертурбативных членов и обеспечить конечность глюонного конденсата. Полученная оценка величины глюонного конденсата находится в согласии с экспериментальными данными. Ненулевое значение эффективной массы "возбужденного глюона" приводит также к представляющим интерес качественным следствиям.

The discovery of the asymptotic freedom property [1] in non-Abelian gauge theories turned out to be a decisive factor in the formation of QCD as a strong interaction theory. The negative sign of QCD β-function $\beta(g^2) = \beta_0 g^4 + ..., \beta_0 = -b_0/(16\pi^2), b_0 = 11C_2/3 - 2N_f/3$ in the vicinity of zero, provided a number of active quarks being not too large (for $SU_c(3)$ $N_f \leq 16$), gives a coupling constant, which describes quarks and gluons interaction at large Euclidean q^2, i.e. at small distances,

$$\bar{g}^2(q^2/\mu^2, g) = \frac{g^2}{1 - \beta_0 g^2 \ln(q^2/\mu^2)}, \tag{1}$$

tending towards zero. Therefore, in the deep Euclidean region we are allowed to use the perturbation theory. In expression (1), which takes into account the main logarithms, μ is a normalization point. An account of the next g^2 corrections does not change an asymptotic behaviour (1) for $q^2 \to \infty$. By introducing a dimensional constant $\Lambda^2 = \mu^2 \exp(-4\pi/(b_0\alpha_s)), \alpha_s = g^2/4\pi$, we turn from explicitly renormalization invariant expression (1) to the following formula

$$\bar{\alpha}_s(q^2) = \frac{4\pi}{b_0 \ln(q^2/\Lambda^2)}. \tag{2}$$

It is reasonable to estimate parameter Λ in approximate expression (2) to be around of few hundreds MeV. With decreasing q^2, effective constant (2) increases, which may indicate a tendency of unlimited growth of the interaction at large distances, leading to a confinement of coloured objects. However, at $q^2 = \Lambda^2$ in expression (2) the pole is present, which is nonphysical at least due to the failing of the perturbation theory, starting from which formula (2) has been obtained.

In recent work [2] a solution of the problem of a ghost pole was proposed with imposing a condition of analyticity in q^2. The idea of "forced analyticity" goes back to works [3,4] of the late fifties, which were dedicated to the problem of Landau-Pomeranchuk pole [5] in QED. Using for $\bar{\alpha}_s(q^2)$ a spectral representation without subtractions, the following expression for the running coupling constant was obtained in paper [2]

$$\bar{\alpha}_s^{(1)}(q^2) = \frac{4\pi}{b_0}\left[\frac{1}{\ln(q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - q^2}\right]. \tag{3}$$

This expression has the asymptotic freedom property and its analyticity in the infrared region is due to nonperturbative contributions. It does not contain any additional parameter and has a finite limit at zero, $\bar{\alpha}_s^{(1)}(0) = 4\pi/b_0 \simeq 1.40$ (freezing of the coupling constant), which depends only on symmetry factors. This limit turns out to be stable with respect to higher orders corrections.

As it is noted in work [4], a procedure of summation of leading logarithmic terms is not defined uniquely. A partial fixation of this ambiguity in QED is realized by using a method of summation of the perturbation theory series under the sign of the spectral integral of the Källen-Lehmann representation. Nevertheless after such summation a functional ambiguity remains, which on the one side, does not violate correct analytic properties of Green functions in a complex plane of a corresponding invariant variable and on the other side, contains nonanalytic dependence on constant g^2. In work [6] while investigating the photon propagator in QED it was shown, that ambiguities in summation procedure of the diagram series could be removed provided one demands not only the validity of spectral representation, but also the fulfillment of equations of motion.

In the present paper we consider a problem of consistency of the constant behaviour of the effective charge in the infrared region with Schwinger-Dyson (SD) equation for a gluon propagator. Further we include into consideration nonperturbative terms, the singular in the infrared region term $\sim 1/q^2$ in particular, the necessity of the renormalization invariance being taken into account. Then we discuss possibilities of an adjustment of demands of confinement, asymptotic freedom, analyticity, accordance with the perturbation theory and correspondence with estimates of the gluon condensate value.

To study the problem of a possibility of a constant behaviour of the running constant in the infrared region let us consider the integral SD equation for the gluon propagator in ghost-free axial gauge [7] $A^a_\mu \eta_\mu = 0, \quad \eta_\mu$ — gauge vector, $\eta^2 \neq 0$. In this gauge the effective charge is directly connected with the gluon propagator and Slavnov-Taylor identities [8] have the simplest form. The important preference of the axial gauge consists in a possibility to exclude the term from the SD equation, which contains the full four-gluon vertex by means of contraction of the equation with tensor $\eta_\mu \eta_\nu / \eta^2$.

In what follows we shall work in the Euclidean momentum space, where smallness of the momentum squared is immediately connected with smallness of its components. The equation to be considered has the form:

$$[D^{-1}_{\mu\nu}(p) - D^{-1}_{(0)\mu\nu}(p)]\frac{\eta_\mu \eta_\nu}{\eta^2} = \Pi_{\mu\nu}(p)\frac{\eta_\mu \eta_\nu}{\eta^2},$$

$$\Pi_{\mu\nu}(p) = -\frac{C_2 g^2 \mu^{4-n}}{2(2\pi)^n} \int d^n k \Gamma^{(0)}_{3\mu\lambda\rho}(p, -k, k-p) D_{\lambda\sigma}(k) D_{\rho\delta}(p-k) \times$$
$$\times \Gamma_{3\sigma\delta\nu}(k, p-k, -p), \tag{4}$$

where $\Pi_{\mu\nu}(p)$ is the one-loop part of the polarization operator, $D_{\mu\nu}(p)$ is the propagator, $\Gamma_{\sigma\delta\nu}(k, p-k, -p)$ is the one-particle irreducible three-gluon vertex function, $\Gamma^{(0)}_{\mu\lambda\rho}(p, -k, k-p)$ is the free three-gluon vertex.

We suppose the approximation $D_{\mu\nu}(p) = Z(p^2) D_{(0)\mu\nu}(p)$ to be appropriate to study the infrared region. Let us divide the momentum integration domain in expression (4) in two parts: $k^2 < \lambda^2$ and $k^2 > \lambda^2$, where λ is sufficiently small, but finite. Then domain $k^2 > \lambda^2$ in the case of absence of kinematic singularities in three-gluon vertex gives a contribution, which is regular in p^2 for $p^2 \to 0$, and in domain $k^2 < \lambda^2$ full Green functions can be approximated by free ones up to constant factors according to an assumption of a running constant to be frozen at zero. Then one can write

$$\Pi_{\mu\nu}(p)\frac{\eta_\mu\eta_\nu}{\eta^2} = -\frac{C_2 g^2 \mu^{4-n} Z(0)}{2(2\pi)^n}\int\limits_0^\lambda d^n k \Gamma^{(0)}_{3\mu\lambda\rho}(p,-k,k-p)\times$$

$$\times D^{(0)}_{\lambda\sigma}(k)D^{(0)}_{\rho\delta}(p-k)\Gamma^{(0)}_{3\sigma\delta\nu}(k,p-k,-p)\eta_\mu\eta_\nu/\eta^2 + Q(p^2;y,\lambda,n). \tag{5}$$

Here $y = (p\eta)^2/p^2\eta^2$ is the gauge parameter. The integration in formula (5) can be extended up to the entire domain of momentum, which results in a change of the regular in p^2 contribution Q. Thus one has

$$\Pi_{\mu\nu}(p)\frac{\eta_\mu\eta_\nu}{\eta^2} = Z(0)\Pi^{(1)}_{\mu\nu}(p)\frac{\eta_\mu\eta_\nu}{\eta^2} + Q(p^2;y,n), \tag{6}$$

where $\Pi^{(1)}_{\mu\nu}(p)$ is the one-loop perturbation theory contribution to the polarization operator. This contribution has been calculated in [9] and has rather a complicated structure. Let us present the expression for the leading terms of convolution (6) at $y \to 0$. We have

$$\Pi^{(1)}_{\mu\nu}(p)\frac{\eta_\mu\eta_\nu}{\eta^2} = Cp^2\left[-\frac{22}{3\epsilon} - \frac{22}{3}\left(\gamma - 2 + \ln\frac{p^2}{4\pi\mu^2}\right) - \frac{70}{9} + \right.$$

$$\left. + \frac{40}{3}y\ln y + O(y, y^2\ln y)\right]. \tag{7}$$

Here $C = g^2 C_2/32\pi^2$, γ is the Euler constant. From expression (7) we see, that singularity at $y = 0$ is smooth and the limit at $y = 0$ does exist. Term $\sim 1/\epsilon$ $(n = 4 + 2\epsilon)$ as well as constant ones could be absorbed into function Q, while the logarithm of the momentum squared necessarily persists. The equation for function $Z(p^2)$ takes the form

$$Z^{-1}(p^2) = 1 + Z(0)\frac{g^2 C_2}{16\pi^2}\frac{11}{3}\ln p^2 + Q(p^2;n). \tag{8}$$

We see, that behaviour $Z(p^2) \simeq Z(0) \neq 0$ for $p^2 \to 0$ does not agree with the SD equation.

This conclusion stimulate us to look for the possibilities different from the assumption on the finiteness of the coupling constant at zero. Recently a possibility of the soft singular power infrared behaviour of the gluon propagator has been discussed [10], $D(q) \sim (q^2)^{-\beta}$, $q^2 \to 0$, where β is a small positive non-integer number. In Ref. [11] the consistency of such behaviour with Eq. (4) was studied. A characteristic equation for the exponent β was obtained and this equation was shown not to have solutions in the region $0 < \beta < 1$. The authors of Ref. [12] also came to the conclusion on the inconsistency of the soft singular infrared behaviour of the gluon propagator. The case of possible interference of power terms was studied in Ref. [13] and it was shown that in a rather wide interval

$-1 < \beta < 3$ of the non-integer values of the exponent the characteristic equation had no solutions. At present a more singular, in comparison with free case, infrared behaviour of the form $D(q) \simeq M^2/(q^2)^2$, $q^2 \to 0$ seems to be most justified [14,15,16]. The physical consequences of such enhancement of zero modes are discussed in reviews [17,18]. Bearing in mind the remarks stated above let us consider the following expression for the running coupling:

$$\bar{\alpha}_s(q^2) = \frac{4\pi}{b_0}\left[\frac{1}{\ln q^2/\Lambda^2} + \frac{\Lambda^2}{\Lambda^2 - q^2} + c\frac{\Lambda^2}{q^2}\right]. \tag{9}$$

Let us represent this expression in explicitly renormalization invariant form. It can be done without solving the differential renormalization group equations. In this order we write $\bar{\alpha}_s(q^2) = \bar{g}^2(q^2/\mu^2,\ g^2)/4\pi$ and use the normalization condition $\bar{g}^2(1, g^2) = g^2$. Then we obtain the equation for wanted dependence of the parameter Λ^2 on g^2 and μ^2:

$$g^2/4\pi = \frac{4\pi}{b_0}\left[\frac{1}{\ln \mu^2/\Lambda^2} + \frac{\Lambda^2}{\Lambda^2 - \mu^2} + c\frac{\Lambda^2}{\mu^2}\right].$$

From dimensional reasons $\Lambda^2 = \mu^2 exp\{-\varphi(x)\}$, where $x = b_0 g^2/16\pi^2 = b_0\alpha_s/4\pi$, and for function $\varphi(x)$ we obtain the equation:

$$x = \frac{1}{\varphi(x)} + \frac{1}{1 - e^{\varphi(x)}} + ce^{-\varphi(x)}.$$

The solution of this equation at $c > 0$ is monotonously decreasing function $\varphi(x)$, which has the behaviour $\varphi(x) \simeq 1/x$ at $x \to 0$ and $\varphi(x) \simeq -\ln(x/c)$ at $x \to +\infty$. The relation obtained ensures the renormalization invariance of $\bar{\alpha}_s(q^2)$. At low g^2, we obtain $\Lambda^2 = \mu^2 \exp\{-4\pi/(b_0\alpha_s)\}$, which indicates the essentially nonperturbative character of both last terms of Eq. (9) and these terms are absent in the perturbation theory. With the given value of the QCD scale parameter Λ, the parameter c can be fixed by the string tension κ or the Regge slope $\alpha' = 1/(2\pi\kappa)$ assuming the linear confinement $V(r) \simeq \kappa r = a^2 r$ at $r \to \infty$. We define the potential $V(r)$ of static $q\bar{q}$ interaction [19,20] by means of three-dimensional Fourier transform of $\bar{\alpha}_s(\vec{q}^2)/\vec{q}^2$ with the contributions of only one dressed gluon exchange taken into account. This gives the following relation

$$c\Lambda^2 = (3b_0/8\pi)a^2 = (b_0/16\pi^2)g^2M^2. \tag{10}$$

At large q^2 from Eq. (9) one obtains

$$\bar{\alpha}_s(q^2) = \frac{4\pi}{b_0}\left[\frac{1}{\ln q^2/\Lambda^2} + (c-1)\frac{\Lambda^2}{q^2} - \frac{\Lambda^4}{(q^2)^2} + O((q^2)^{-3})\right]. \tag{11}$$

From Eq. (11) it is seen that in the ultraviolet region the nonperturbative contributions decrease more rapidly than all renormalization group improved perturbation theory corrections. The value $c = 1$ corresponds to the maximal suppression of nonperturbative contributions in the ultraviolet region. Accepting this condition, one obtains the connection of the QCD scale parameter Λ and string tension $\kappa = a^2$ of the form $\Lambda^2 = 3b_0\kappa/8\pi$.

Taking $a \simeq 0.42\,GeV$, one obtains for Λ a reasonable estimation, $\Lambda \simeq 0.434\,GeV$ ($b_0 = 9$ in the case of 3 light flavours).

Considering the nonperturbative contributions, the following arguments can be expressed. One knows QCD to be renormalizable in the perturbation theory and, as usual, the renormalization procedure can be developed to remove the divergences in all the orders. However, what about the nonperturbative contributions? If they bring in the additional divergences, then the problem of renormalization turns out to be unsolved. The situation, when nonperturbative contributions do not violate the perturbative renormalization properties seems to be more attractive. It takes place if the nonperturbative contributions decrease at momentum infinity sufficiently fast and do not introduce the divergences in observables. So, it is natural to demand their fastest of possible decrease at large momenta. An application of the principle of minimality of nonperturbative contributions in the ultraviolet region will be shown further by taking as an example the important physical quantity, namely, the gluon condensate, $K = <\alpha_s/\pi : G^a_{\mu\nu}\, G^a_{\mu\nu} :>$. According to the definition (see e.g., [17]) up to the quadratic approximation in the gluon fields, one has after the Wick rotation

$$K = \frac{48}{\pi} \int \frac{d^4k}{(2\pi)^4} \left(\bar{\alpha}_s(k^2) - \bar{\alpha}_s^{pert}(k^2)\right) = \frac{3}{\pi^3} \int_0^\infty \bar{\alpha}_s^{np}(y)\, y dy\,, \tag{12}$$

where $\bar{\alpha}_s^{np}$ is nonperturbative part of the running coupling constant. In our case the two last terms of Eq. (9) should be taken. By substituting these terms in Eq. (12), one can see the logarithmic divergences of the integral at infinity and at finite point $k^2 = \Lambda^2$.

The acceptance of the cancellation mechanism for the nonphysical perturbation theory singularities (2) by the nonperturbative contributions leads to the necessity of supplementary definition of the integral (12) near point $k^2 = \Lambda^2$. This problem can be reformulated as a problem of dividing perturbative and nonperturbative contributions in $\bar{\alpha}_s$ resulting in the introduction of some parameter $k_0 = 1 \div 2$ GeV. This provides the absence of the pole at $k^2 = \Lambda^2$ in both perturbative and nonperturbative parts. The divergence of the integral (12) at infinity stimulates a further modification of the running coupling constant. Going over from Eq. (3) to Eq. (9), the isolated singularity has been introduced. In this case the singularity corresponding to the unitary cut was not changed and in accordance with the approach of Refs. [3,4,2] is determined by the perturbation theory. Following to this logic, let us consider the expression for $\bar{\alpha}_s$ with one more isolated singularity in the time-like region. The tachion singularity in the space-like region is certainly prohibited.

The principle of minimality of nonperturbative contributions in ultraviolet region then leads to the following unique expression for the running coupling constant

$$\bar{\alpha}_s(q^2) = \frac{4\pi}{b_0}\left(\frac{1}{ln(q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - q^2} + \frac{c\Lambda^2}{q^2} + \frac{(1-c)\Lambda^2}{q^2 + m_g^2}\right), \tag{13}$$

with fixed residue and mass parameter m_g,

$$m_g^2 = \Lambda^2/(c-1), \tag{14}$$

for the newly introduced term. Expression (13) can be represented in explicitly renormalization invariant form similar to expression (9). Nonperturbative contributions in Eq. (13) decrease at infinity as $1/q^6$, the integral in Eq. (12) converges and we can obtain

$$K = \frac{4}{3\pi^2}\Lambda^4 \left\{ln(c-1) + k_0^2/\Lambda^2 + ln(k_0^2/\Lambda^2 - 1)\right\}. \tag{15}$$

Phenomenology gives the positive value of the gluon condensate K in the interval $(0.32\,GeV)^4$ – $(0.38\,GeV)^4$ [21,22]. As an example, we take values $k_0 = 1.2 \div 1.3$ GeV. If one regards the string tension parameter to be given, then from Eqs. (14), (15) and (10) one has the dependencies of all the values under consideration on the parameter c, which are presented in Table I.

Note that values $c = 1.063$, $\Lambda = 422\,MeV$, $m_g = 1.682\,GeV$, $k_0 = 1.265$ GeV correspond to the conventional value of the gluon condensate [21] $K = (0.33\,GeV)^4$. Certainly, these results should be considered as tentative, but nevertheless, they seem encouraging.

Table 1. Parameters of the running coupling constant (13) and gluon condensate as functions of parameter c.

c	Λ, GeV	m_g, GeV	$K^{1/4}$, GeV $k_0 = 1.2$ GeV	$K^{1/4}$, GeV $k_0 = 1.25$ GeV	$K^{1/4}$, GeV $k_0 = 1.3$ GeV
1.01	0.433	4.332	0.298	0.309	0.318
1.02	0.431	3.048	0.307	0.317	0.326
1.03	0.429	2.476	0.312	0.321	0.330
1.04	0.427	2.134	0.315	0.324	0.332
1.05	0.425	1.900	0.317	0.326	0.334
1.06	0.423	1.726	0.319	0.327	0.335
1.07	0.421	1.591	0.320	0.328	0.336
1.08	0.419	1.481	0.321	0.329	0.337
1.10	0.415	1.313	0.322	0.330	0.337
1.12	0.411	1.187	0.323	0.330	0.337
1.16	0.404	1.010	0.323	0.330	0.337
1.20	0.397	0.889	0.322	0.329	0.336
1.24	0.391	0.798	0.321	0.328	0.335
1.30	0.382	0.697	0.319	0.326	0.332

It is seen from Eq. (13) that the pole singularities are situated at two points $q^2 = 0$ and $q^2 = -m_g^2$. It corresponds to the two effective gluon masses, 0 and m_g. Therefore, the physical meaning of the parameter m_g is not the constituent gluon mass, but rather the mass of the exited state of the gluon. It is essential that the residue at m_g^2 is very small, so the states with the exited gluons should be quite narrow in contrast to the spectrum of the coupled massless gluons.

The qualitative picture of the glueball states corresponding to the running coupling constant (13) with $m_g \simeq 1.7$ GeV could be the following:

1) The states $g\,g$ — continuous spectrum and very wide resonances are probable;

2) The states $g\,g'$ — resonances with probable mass interval 1500 – 1800 MeV and with width suppression factor $(1-c)$;

3) The narrow states $g'\,g'$ — resonances with possible masses 3000 – 3600 MeV and with width suppression factor $(1-c)^2$.

Note that in region 2) there are the glueball candidates. Region 3) is insufficiently investigated, some indications in favour of the narrow states arise (see e.g., [23]).

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