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S. M. Klishevich, Yu. M. Zinoviev <sup>1</sup>

ON ELECTROMAGNETIC INTERACTION  
OF MASSIVE SPIN-2 PARTICLE

Submitted to *Yad. Fiz*

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<sup>1</sup> E-mail address: ZINOVIEV@TH.IHEP.SU

**Abstract**

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In this work we construct a gauge invariant description of free massive particle with an arbitrary integer spin. Such description allows one to investigate the problem of consistent interactions for massive high spin particles using the requirement that the whole interaction Lagrangian must be gauge invariant. As a first example of such approach, we consider the case of electromagnetic interaction of massive spin-2 particle: a linear approximation in a case of the arbitrary field and a full theory for the homogeneous electromagnetic field in the space-time of any dimensionality.

**Аннотация**

Зиновьев Ю.М., Клишевич С.М. Об электромагнитном взаимодействии массивной частицы со спином 2: Препринт ИФВЭ 97-41. – Протвино, 1997. – 18 с., библиогр.: 15.

В данной работе построено калибровочно-инвариантное описание свободных массивных частиц произвольного целого спина. Такое описание позволяет исследовать проблему непротиворечивых взаимодействий для массивных частиц с высокими спинами, используя требование калибровочной инвариантности полного лагранжиана взаимодействия. В качестве первого примера такого подхода мы рассмотрели электромагнитное взаимодействие массивной частицы со спином 2: линейное приближение для произвольного поля и полную теорию для однородного электромагнитного поля в пространстве-времени произвольной размерности.

## Introduction

The interactions of the fields with spins  $s \leq 2$  are quite well studied, while about those of higher spins we know much less. In all investigations of high spin fields the Poincare group plays a special role. It not only determines the most important kinematical properties of the particles, but also fixes, to a great extent, their interactions. Namely, the covariant description of particles with spins  $s \geq 1$  necessarily requires the presence of the invariance under the gauge transformations and one has to keep this invariance at the switching on the interaction. Using such a requirement, in the investigations of gauge invariant interactions of spin-1 particles one unambiguously comes to the Yang-Mills theory, while starting from free spin-2 field and switching on the interaction, one can reproduce the usual gravity theory, see, for example, [1,2]. In the same way, the requirement of gauge invariance for the spin-2 and spin-3/2 particles interactions leads to supergravity [3].

It is known, that the minimal interactions of any massless spin  $s > 2$  particles with vector fields (abelian or non-abelian) turns out to be inconsistent<sup>1</sup>. Analogously, all the attempts to switch on the gravitational interaction for the massless spin  $s > 2$  particles lead to the inconsistencies. It seems, that there are no consistent theories for the interactions of such particles (in flat space). Till now, the only consistent theory with the high spin particles interactions is the superstring theory. The massless sector of this theory is restricted with the states of spins  $s \leq 2$ , while the massive sectors contain the particles of arbitrary spins. All these particles with all spins enter the string interaction, but it would be interesting to know the peculiarity of the interaction for concrete particle with definite spin. Unfortunately, it is hard to extract the information on the gauge symmetries or interactions for particular state in the string spectrum and no essential results in this direction were achieved.

Note, that there exist consistent theories of the interaction of high spin massless particles in the space of constant nonzero curvature, which were studied in [5]. In principle, starting from such theories one could obtain more physically interesting case of massive

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<sup>1</sup>This result has been formulated as a theorem in [4].

particles in the flat space. For that, one has to solve the problem of spontaneous gauge symmetry breaking in these theories, but it is a very complicated task and it has not been solved up to now.

The description for the free massive particles with arbitrary spin was given in a well known paper [6]. But the Lagrangian for the massive particle in this approach, unlike the one for the massless particles, does not possess gauge invariance. So, for the construction of the consistent interaction theory one has to introduce, besides the usual requirement of the Lorentz invariance, some additional considerations. For example, in paper [7], where the electromagnetic interactions of high spin particles were investigated, the requirement that the tree level amplitudes must have smooth massless limit with fixed charge was used. In turn, in papers [8,9] it was proposed to construct gravitational interaction for high spin particles using a tree level unitarity as such additional requirement.

For the spin-1 and spin-3/2 particles there is a well known mechanism — mechanism of spontaneous symmetry breaking. In this, one starts with the consistent massless theory and then introduces masses. But as we have already mentioned, it seems that there are no consistent theories for massless spin  $s > 2$  particles (in flat space). The main property of spontaneous symmetry breaking in both cases is the possibility to have gauge invariant description of massive spin-1 or spin-3/2 particles due to the introduction of Goldstone fields with inhomogeneous transformation laws. In paper [10] the method for the construction of massive high spin gauge fields interactions based on the gauge invariant description of such particles was proposed. It allows one to keep the principle of gauge invariance as the fundamental one for the construction of the interaction of massive particles in the same way as it was used for the massless ones. In paper [10] the gauge invariant description for massive particles with spins up to  $s = 3$  was given and as a demonstration the simplest possible case of spontaneous symmetry breaking in the Yang-Mills theory with  $SO(3)$  group was considered. It was shown that the approach proposed allows one to reproduce two well known possibilities: non-linear  $\sigma$ -model, see e.g. [11], and, with the help of the introduction of additional scalar field, the usual model of spontaneous symmetry breaking with the doublet of Higgs fields.

In this paper, in the next Section, using the method proposed in [10] we construct a gauge invariant description of free massive particles with arbitrary integer spin.

Among all the interactions, the electromagnetic interaction is one of the most well studied and for a long time it serves as a polygon for the investigation of different models. Thus, it seems natural to start with the investigations of electromagnetic interactions for the massive high spin particles. In Section 2 we begin with the linear approximation for the e/m interaction of massive spin-2 particle. The construction of the linear approximation is a very important step in studying any theory of high spin interactions, because this approximation does not depend on the presence of any other fields in the theory, while all higher approximations are heavily model dependent. Thus, the linear approximation turns out to be universal for any theory (with the given number of derivatives, of course).

The next to linear approximation requires a lot of calculations even in the minimal model without the introduction of any additional fields. The calculations become much easier, then one considers the case of homogeneous electromagnetic field. Recently, in [12]

the model describing massive spin-2 particle moving in homogeneous e/m field has been constructed. But the authors of [12] started from the bosonic string, so, their results hold for the  $d = 26$  dimension only. In Section 3, using constructive approach, we obtain an analogous result, but for the space of arbitrary dimension.

## 1. Gauge Invariant Description of Free Massive Particle with Arbitrary Integer Spin

### 1.1. Massless Particle Lagrangian

For the description of massless spin-S particle we will use the formalism proposed in [13]. Let us consider a symmetric tensor field of rank  $s$  —  $\Phi^s = \Phi^{\mu_1 \dots \mu_s}$ , where Greek indices take the following values:  $\mu, \nu, \dots = 0, \dots, 3$  and require this field be double traceless

$$\tilde{\Phi} = Sp(Sp \Phi^s) = 0,$$

here  $\tilde{\Phi} \stackrel{def}{=} Sp \Phi^s$ ,  $\tilde{\tilde{\Phi}} = Sp Sp \Phi^s$  and so on,  $Sp$  is a contraction of two indices by metric tensor.

The gauge transformation for  $\Phi^s$  is taken as

$$\delta_o \Phi^s = \frac{1}{s} \cdot \left\{ \partial \Lambda^{s-1} \right\}_s \quad (1)$$

where  $\{ \dots \}_s$  means symmetrization over all the indices (without normalization) and  $\Lambda^{s-1}$  is symmetric traceless tensor field of rank  $s - 1$ .

Let us write the most general quadratic Lagrangian with two derivatives for field  $\Phi^s$ . Using the freedom in the choice of field normalization, we can fix the coefficient at  $(\partial_\mu \Phi^s)^2$  so that this term has the standard form

$$\begin{aligned} \mathcal{L}_o = & \frac{(-1)^s}{2} (\partial_\mu \Phi^s) \cdot (\partial_\mu \Phi^s) + a_1 (\partial \cdot \Phi^s) \cdot (\partial \cdot \Phi^s) + a_2 (\partial_\mu \tilde{\Phi}^s) \cdot (\partial_\mu \tilde{\Phi}^s) \\ & + a_3 (\partial \cdot \Phi^s) \cdot (\overleftarrow{\partial} \tilde{\Phi}^s) + a_4 (\partial \cdot \tilde{\Phi}^s) \cdot (\partial \cdot \tilde{\Phi}^s), \end{aligned} \quad (2)$$

where  $(\partial \cdot \Phi^s) \stackrel{def}{=} \partial_\mu \Phi^{\mu \mu_2 \dots \mu_s}$ ,  $(\partial \cdot \partial \cdot \Phi^s) \stackrel{def}{=} \partial_\mu \partial_\nu \Phi^{\mu \nu \mu_3 \dots \mu_s}$ , and the point denotes contraction of all the indices between tensor objects, for example,  $\Phi^s \cdot \Phi^s \stackrel{def}{=} \Phi_{\mu_1 \dots \mu_s} \Phi^{\mu_1 \dots \mu_s}$ .

Calculating a variation<sup>2</sup> of the Lagrangian under transformation (1) one obtains

$$\begin{aligned} \delta_o \mathcal{L}_o = & \left( (-1)^s + \frac{2a_1}{s} \right) \partial^2 (\partial \cdot \Phi^s) \Lambda^{s-1} + \frac{2}{s} ((s-1) a_1 + a_3) \partial (\partial \cdot \partial \cdot \Phi^s) \Lambda^{s-1} \\ & + \frac{2}{s} (a_3 + 2a_2) \partial^2 (\partial \tilde{\Phi}^s) \Lambda^{s-1} + \frac{1}{s} ((s-2) a_3 + 4a_4) \partial \partial (\partial \cdot \tilde{\Phi}^s) \Lambda^{s-1}. \end{aligned}$$

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<sup>2</sup>We neglect the terms proportional to a total derivative.

From the condition of the invariance of Lagrangian (2) under the gauge transformation (1), one get simple equations on arbitrary coefficients in the Lagrangian. Solving these equations, we obtain a final form for the Lagrangian of a free massless spin- $s$  particle

$$\begin{aligned} \mathcal{L}_o = & (-1)^s \left\{ \frac{1}{2} (\partial_\mu \Phi^s) (\partial_\mu \Phi^s) - \frac{s}{2} (\partial \cdot \Phi^s) (\partial \cdot \Phi^s) - \frac{s(s-1)}{4} (\partial_\mu \tilde{\Phi}^s) (\partial_\mu \tilde{\Phi}^s) \right. \\ & \left. - \frac{s(s-1)}{2} (\partial \cdot \partial \cdot \Phi^s) \tilde{\Phi}^s - \frac{1}{8} s(s-1)(s-2) (\partial \cdot \tilde{\Phi}^s) (\partial \cdot \tilde{\Phi}^s) \right\} \end{aligned}$$

One can get the same result from the Lagrangian for massive particle proposed in [6], if one let  $m \rightarrow 0$  and make some field redefinition.

## 1.2. Lagrangian for Massive Particle

As is well known, it is not enough to have only one field  $\Phi^s$  for the correct description of massive spin- $s$  particle. One has to introduce some lower spin auxiliary fields. For instance, in [6] the authors considered a set of a symmetric traceless fields of ranks  $0, 1, \dots, s-2, s$  and demanded that the auxiliary fields with spins  $0, 1, \dots, s-2$  would vanish on the equations of motion, while the equations for the field  $\Phi^s$  had the usual form

$$(\partial^2 + m^2) \Phi^{\mu_1 \dots \mu_s} = 0, \quad \partial_\mu \Phi^{\mu \mu_2 \dots \mu_s} = 0.$$

But the resulting Lagrangian of massive particle was not gauge invariant, moreover, in the massless limit  $m \rightarrow 0$  the number of physical degrees of freedom changes from  $2s+1$  to 2.

Below we construct an alternative gauge invariant formalism for the description of massive spin- $s$  particles, whose massless limit gives a sum of the massless particles of spins  $s, s-1, \dots, 0$ . This approach, which was offered in [10], is based on the possibility to have gauge invariance for the massive free particles due to the introduction of additional fields, corresponding to all lower spins. As it has already been mentioned in the Introduction, such approach allows one to investigate the interactions of massive particles with arbitrary spins<sup>3</sup> in the same way as the ones of the massless particles.

We will start with the Lagrangian describing the sum of free massless particles with spins  $0, 1, \dots, s$  and we will add quadratic terms proportional to  $m$  (with one derivative) and to  $m^2$  (without derivatives) keeping all the gauge invariances of initial massless fields. As a result, one will get the description of free massive spin- $s$  particle, where, by construction, in the massless limit  $m \rightarrow 0$  the Lagrangian will break into the sum of Lagrangians corresponding to massless particles with spins  $0, 1, \dots, s$ , so that the number of physical degrees of freedom remains to be the same and the gauge invariance is present both on the massive as well as on the massless level.

Following the program described above, we introduce a set of symmetric double traceless fields  $\{\Phi^0, \Phi^1, \dots, \Phi^{s-1}, \Phi^s\}$  and, correspondingly, a set of traceless gauge parameters

<sup>3</sup>In this paper we will deal with the integer spin particles only.

$\{\Lambda^0, \Lambda^1, \dots, \Lambda^{s-2}, \Lambda^{s-1}\}$ . Let us write the most general form of gauge transformations for fields  $\{\Phi^0, \Phi^1, \dots, \Phi^{s-1}\}$  in the presence of a dimensional parameter  $m$ :

$$\delta_o \Phi^k = \frac{1}{k} \left\{ \partial \Lambda^{k-1} \right\}_s + m \left[ c_1(k) \Lambda^k + c_2(k) \left\{ g^2 \Lambda^{k-2} \right\}_s \right], \quad (3)$$

where the first term is absent at  $k = 0$  and the third one at  $k < 2$ , while  $g^2$  is a metric tensor.

Now we write the most general quadratic Lagrangian with, at most, two derivatives:

$$\begin{aligned} \mathcal{L}_o = & \sum_{k=0}^s \left\{ \mathcal{L}_o(k) + m [a_1(k) \Phi^{k-1} (\partial \cdot \Phi^k) + a_2(k) \tilde{\Phi}^k (\partial \cdot \Phi^{k-1}) \right. \\ & + a_3(k) (\partial \cdot \tilde{\Phi}^k) \Phi^{k-3} + a_4(k) (\partial \cdot \tilde{\Phi}^k) \tilde{\Phi}^{k-1}] \\ & \left. + m^2 [b_1(k) (\Phi^k)^2 + b_2(k) (\tilde{\Phi}^k)^2 + b_3(k) \tilde{\Phi}^k \Phi^{k-2}] \right\}, \end{aligned} \quad (4)$$

where  $\mathcal{L}_o(k)$  is the Lagrangian for a massless spin- $k$  particle. Requiring that Lagrangian (4) be invariant under transformations (3), one obtains the algebraic system of homogeneous equations for the arbitrary coefficients

$$\begin{aligned} a_3(k) &= 0, \\ (-1)^k c_1(k) - \frac{a_1(k+1)}{k+1} &= 0, \\ (-1)^{k+1} k c(k) - \frac{1}{k+1} (k a_1(k+1) - 2 a_2(k+1)) &= 0, \\ \frac{(-1)^k}{2} (k-1) k c_1(k) - \frac{2}{k+1} a_4(k+1) &= 0, \\ (-1)^{k+1} (k-1)^2 k c_2(k) - \frac{a_2(k)}{k-1} &= 0, \\ (-1)^k k (k-1) k c_2(k) + a_1(k) &= 0, \\ \frac{(-1)^{k+1}}{2} (k-2) (k-1)^2 k c_2(k) - \frac{k-2}{k-1} a_2(k) + \frac{2}{k-1} a_4(k) &= 0, \\ a_1(k+1) c_1(k) - 2 b_1(k+1) &= 0, \\ a_2(k) c_1(k) + \frac{2}{k+1} b_3(k+1) &= 0, \\ a_2(k+1) c_1(k) + \frac{4}{k+1} b_2(k+1) &= 0, \\ c_2(k) \left[ \frac{(k-1)k}{2} a_1(k+1) - (k-1) a_2(k+1) + 2k a_4(k+1) \right] \\ - b_3(k+1) &= 0, \end{aligned}$$

which allows one to express all the coefficients through the parameters  $c_1(k)$ :

$$c_2(k) = \frac{c_1(k-1)}{(k-1)^2},$$

$$\begin{aligned}
a_1(k) &= (-1)^{k+1} k c_1(k-1), \\
a_2(k) &= (-1)^{k+1} (k-1) k c_1(k-1), \\
a_3(k) &= 0, \\
a_4(k) &= \frac{(-1)^{k+1}}{4} (k-2)(k-1) k c_1(k-1), \\
b_1(k) &= \frac{(-1)^{k+1}}{2} \left( k c_1^2(k-1) - \frac{(k+1)(2k+1)}{k} c_1^2(k) \right), \\
b_2(k) &= \frac{(-1)^{k+1}}{4} \left( \frac{(k^2-1)(k+2)}{2} c_1^2(k) - (k-1) k^2 c_1^2(k-1) \right), \\
b_3(k) &= \frac{(-1)^{k+1}}{2} (k-1) k c_1(k-2) c_1(k-1),
\end{aligned} \tag{5}$$

in this, we have the recurrent relation for the  $c_1(k)$ :

$$c_1^2(k-1) - \frac{(k+1)(2k+1)}{k^2} c_1^2(k) + \frac{(k+2)^2}{k(k+1)} c_1^2(k+1) = 0.$$

In order to solve this relation unambiguously, we must fix one of the parameters  $c_1(k)$ . Let us require that the dimensional parameter  $m$  be a mass of the particle, i.e. let us put  $b_1(s) = \frac{(-1)^{s+1}}{2}$ . Then solving the recurrent relation, we obtain

$$c_1(k) = \frac{1}{k+1} \sqrt{\frac{(s-k)(s+k+1)}{2}}. \tag{6}$$

Putting (6) in (5) yields the final result. It is not difficult to calculate all the coefficients in (4) for each concrete case, but the general formula for an arbitrary spin is rather cumbersome, therefore, we will not write it here.

Thus, we have constructed the Lagrangian for a free massive spin- $s$  particle which is invariant under the gauge transformations (3), in this, the Lagrangian has the correct massless limit. Let us note that one can, in principle, rewrite all the formulas in terms of the unconstrained tensors. Namely, one can join double traceless field  $\Phi^4$  with  $\Phi^0$  into one unconstrained rank-4 tensor, then  $\Phi^5$  with  $\Phi^1$  and so on. Analogously, combining traceless gauge parameter  $\Lambda^2$  with  $\Lambda^0$ , one gets unconstrained rank-2 tensor and so on. It is easy to see that one ends with just four fields  $\Phi^s$ ,  $\Phi^{s-1}$ ,  $\Phi^{s-2}$  and  $\Phi^{s-3}$  and two gauge parameters  $\Lambda^{s-1}$  and  $\Lambda^{s-2}$ , exactly as in [14].

From the description of massive spin- $s$  particle given above one can reproduce the well-known result obtained in [6]. In order to show this, we represent the symmetric double traceless tensor field  $\Phi^k$  as

$$\Phi^k = \Phi'^k + \frac{1}{2k} \{g^2 \tilde{\Phi}^k\}, \quad k = 0, 1, \dots, s-1,$$

where  $\Phi'^k$  is a symmetric traceless tensor field. Using the gauge transformations we can always exclude the fields  $\{\Phi'^0, \Phi'^1, \dots, \Phi'^{s-1}\}$  by some choice of parameters  $\Lambda^k$ . Redefining

$\tilde{\Phi}^k = \Phi^{k-2}$ , we obtain the set of the symmetric traceless tensor fields  $\{\Phi^s, \Phi^{s-2}, \dots, \Phi^0\}$ , which is equivalent to the one in [6]. In this, the Lagrangian has the following form:

$$\begin{aligned} \mathcal{L}_o = & (-1)^s \left\{ \frac{1}{2} (\partial_\mu \Phi^s)^2 - \frac{s}{2} (\partial \cdot \Phi^s)^2 - \frac{(s-1)^2}{2} (\partial \cdot \partial \cdot \Phi^s) \Phi^{s-2} \right. \\ & - \frac{(s-1)^2 (2s-1)}{8s} (\partial_\mu \Phi^{s-2})^2 - \frac{(s-1)^2 (s-2)^2}{8s} (\partial \cdot \Phi^{s-2})^2 \\ & - \left[ \frac{1}{2} (\Phi^s)^2 - \frac{(s-1)(2s-1)}{8} (\Phi^{s-2})^2 \right] + \sum_{q=3}^s \left( A_1^{s-q} (\partial_\mu \Phi^{s-q})^2 \right. \\ & \left. \left. + A_2^{s-q} (\partial \cdot \Phi^{s-q})^2 + m B^{s-q+1} \Phi^{s-q} (\partial \cdot \Phi^{s-q+1}) + m^2 C^{s-q} (\Phi^{s-q})^2 \right) \right\}, \end{aligned}$$

where

$$\begin{aligned} A_1^k &= \frac{(-1)^{k+1} (k+1)^2 (2k+3)}{8(k+2)}, \\ A_2^k &= \frac{(-1)^{k+1} (k+1)^2 k^2}{8(k+2)}, \\ B^k &= \frac{(-1)^{k+1}}{4} k^2 \sqrt{\frac{(s-k-1)(s+k+2)}{2}}, \\ C^k &= \frac{(-1)^k}{16} (k+1)(s-k)(s+k+1). \end{aligned}$$

This Lagrangian differs from the one in ref. [6] only by the choice of normalization of the fields  $\{\Phi^s, \Phi^{s-2}, \Phi^{s-3}, \dots, \Phi^0\}$ .

## 2. Electromagnetic Interaction of Spin 2 Particle

In this section we will investigate the electromagnetic interaction of massive spin-2 particle in a linear approximation using the constructive approach offered in ref. [2,10].

Let us introduce the following notations. The superscripts will denote a number of derivatives (both for the transformations and the Lagrangians) while the subscripts will denote a number of fields for the transformations and number of fields minus two for the Lagrangians, i.e.:  $\delta_n^k \sim m^{1-(n+k)} \partial^k \Phi^n \Lambda$ ,  $\mathcal{L}_n^k \sim m^{2-(n+k)} \partial^k \Phi^{2+n}$ . In this notations a Lagrangian and transformations of any theory have the general structure

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_0^0 + \mathcal{L}_0^1 + \mathcal{L}_0^2 + \mathcal{L}_1^0 + \mathcal{L}_1^1 + \dots \\ \delta &= \delta_0^0 + \delta_0^1 + \delta_1^0 + \delta_1^1 + \dots \end{aligned}$$

In this, a variation of the Lagrangian has the form

$$\begin{aligned} \delta \mathcal{L} &= \delta_0^0 \mathcal{L}_0^0 + (\delta_0^0 \mathcal{L}_0^1 + \delta_1^0 \mathcal{L}_0^0) + (\delta_0^0 \mathcal{L}_0^2 + \delta_0^1 \mathcal{L}_0^1) + \delta_0^1 \mathcal{L}_0^2 + (\delta_0^0 \mathcal{L}_1^0 + \delta_1^0 \mathcal{L}_0^0) \\ &+ (\delta_0^0 \mathcal{L}_1^1 + \delta_0^1 \mathcal{L}_1^0 + \delta_1^0 \mathcal{L}_1^0 + \delta_1^1 \mathcal{L}_0^0) + \dots \end{aligned}$$

It is easy to see that  $\delta\mathcal{L}$  breaks into the sum of the independent groups for which the sums of the superscripts and subscripts are the same for every term of the group (in the linear approximation we are going to consider the sum of subscripts which are less or equal to one). The different groups contain the terms with different numbers of fields and/or derivatives, so the condition  $\delta\mathcal{L} = 0$  means that each group should vanish independently and this allows one to build an interaction by iterations on the number of fields.

It is convenient to describe charged particles by complex fields. Therefore, to begin with let us write the Lagrangian for complex free spin-2 particle with mass  $m$

$$\begin{aligned}
\mathcal{L}_0 = & \frac{1}{2}\partial_\mu\bar{h}_{\alpha\beta}\partial^\mu h^{\alpha\beta} - (\partial\bar{h})_\mu(\partial h)^\mu + \frac{1}{2}\left((\partial\bar{h})_\mu\partial^\mu h + h.c.\right) - \frac{1}{2}\partial_\mu\bar{h}\partial^\mu h \\
& - \frac{1}{4}\bar{B}_{\mu\nu}B^{\mu\nu} + \frac{1}{2}\partial_\mu\bar{\varphi}\partial^\mu\varphi + \frac{m}{\sqrt{2}}\left(\bar{h}_{\mu\nu}\partial^\mu b^\nu - \bar{h}(\partial b) + h.c.\right) \\
& + \frac{m\sqrt{3}}{2}(\bar{b}_\mu\partial^\mu\varphi + h.c.) - \frac{m^2}{2}(\bar{h}_{\mu\nu}h^{\mu\nu} - \bar{h}h) \\
& + \frac{\sqrt{3}}{2\sqrt{2}}m^2\bar{h}\varphi + m^2\bar{\varphi}\varphi, \tag{7}
\end{aligned}$$

where  $h = g^{\mu\nu}h_{\mu\nu}$ ,  $B_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu$  and the bar denotes a complex conjugation. In this, the gauge transformations have the form:

$$\begin{aligned}
\delta_0 h_{\mu\nu} &= \partial_\mu\xi_\nu + \partial_\nu\xi_\mu + \frac{m}{\sqrt{2}}g_{\mu\nu}\eta, \tag{8} \\
\delta_0 b_\mu &= \partial_\mu\eta + m\sqrt{2}\xi_\mu, \\
\delta_0\varphi &= -m\sqrt{3}\eta.
\end{aligned}$$

In the notations given above the Lagrangian for free massive particle has the structure  $\mathcal{L}_0 = \mathcal{L}_0^0 + \mathcal{L}_0^1 + \mathcal{L}_0^2$ , while the transformations have the form  $\delta_0 = \delta_0^0 + \delta_0^1$ .

Let us switch on the electromagnetic interaction using the minimal coupling prescription, i.e. we make the substitution  $\partial_\mu \rightarrow \mathcal{D}_\mu$ , where  $\mathcal{D}_\mu = \partial_\mu - iqA_\mu$  is the covariant derivative. In this, we have to add  $-\frac{1}{4}(F_{\mu\nu})^2$  to (7), where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ . As usual, for the theory to be invariant under the  $U(1)_{em}$  transformations, the field  $A_\mu$  must enter through the covariant derivative or the  $F_{\mu\nu}$  tensor only. Such covariantization of the derivatives means the addition of the terms of the form  $\mathcal{L}_1^1, \mathcal{L}_1^0, \mathcal{L}_2^0$  to Lagrangian (7) and the ones of the form  $\delta_1^0$  to transformations (9). But as a result of such substitutions, due to the noncommutativity of the covariant derivatives, the Lagrangian lost its invariance under the gauge transformations:

$$\begin{aligned}
\delta_1^0\mathcal{L}_0^2 + (\delta_1^0\mathcal{L}_0^1 + \delta_0^0\mathcal{L}_1^1) = & -iq\left\{-\sqrt{2}mF^{\mu\nu}\bar{b}_\mu\xi_\nu + \frac{1}{2}\partial_\mu F^{\mu\nu}\bar{h}\xi_\nu\right. \\
& + \frac{3}{2}F^{\mu\nu}\mathcal{D}_\mu\bar{h}\xi_\nu - \partial_\mu F^{\mu\nu}\bar{h}_{\nu\alpha}\xi^\alpha - 2F^{\mu\nu}\mathcal{D}_\mu\bar{h}_{\nu\alpha}\xi^\alpha \\
& \left. - F^{\mu\nu}(\mathcal{D}\bar{h})_\mu\xi_\nu + \frac{1}{2}F^{\mu\nu}\mathcal{D}_\mu\bar{b}_\nu\eta\right\} + h.c.
\end{aligned}$$

In order to recover the invariance of the Lagrangian let us add to it and to the transformations new terms. We study the linear approximation and it means that we add the linear terms to the transformations and the cubic ones to the Lagrangian only. The number of derivatives in the additional terms to the Lagrangian and to the transformations must be consistent. For example, introducing the new transformations of the form  $\delta_1^n$ , one has to add to the Lagrangian the terms of the form  $L_1^{n+1}$ , because in calculation of the variation they give a contribution of the same order<sup>4</sup>. In this, the transformations for  $A_\mu$  is defined up to the terms of the kind  $\delta A_\mu \sim \partial_\mu(\dots)$  because the contribution of such terms to the variations vanishes due to the  $U(1)_{em}$  gauge invariance.

We will start with the minimal number of derivatives in the additional terms and will increase this number until the gauge invariance is recovered.

Let us add all possible linear terms without derivatives to the transformations:

$$\delta_1^0 A_\mu = iq \left\{ \alpha_1 \bar{h}_{\mu\nu} \xi^\nu + \alpha_2 \bar{h} \xi_\mu + \alpha_3 \bar{b}_\mu \eta + \alpha_4 \bar{\varphi} \xi_\mu \right\} + h.c.$$

For the fields  $h_{\mu\nu}, b_\mu$  the appropriate terms have already been added, because they are part of the covariant derivatives and for the field  $\varphi$  they are absent in this order. A requirement of the closure of the algebra on the field  $A_\mu$  imposes a nontrivial condition on the unknown coefficients:

$$\alpha_1 + 4\alpha_2 + 2\alpha_3 - \sqrt{6}\alpha_4 = 0$$

while on the fields  $h_{\mu\nu}, b_\mu, \varphi$  the closure of algebra is trivial.

Correspondingly, the only possible in this order additional terms to the Lagrangian have the form:

$$\mathcal{L}_1^1 = iq F_{\mu\nu} \left\{ a_1 \bar{h}_{\mu\alpha} h^\alpha{}_\nu + a_2 \bar{b}_\mu b_\nu \right\}.$$

The requirement of the gauge invariance:

$$\begin{aligned} \delta_1^0 \mathcal{L}_0^1 + \delta_0^0 \mathcal{L}_1^1 &= 0, \\ \delta_1^0 \mathcal{L}_0^2 + \delta_0^0 \mathcal{L}_1^1 &= 0 \end{aligned} \tag{9}$$

yields a non-homogeneous system of linear equations on the coefficients in  $\mathcal{L}_1^1$  and  $\delta_1^0 A_\mu$ . But the system has no solution, therefore, one needs add linear terms with one derivative to the transformations and cubic terms with two derivatives to the Lagrangian.

Let us write all such nontrivial additional terms with two derivatives:

$$\begin{aligned} \mathcal{L}_1^2 &= \frac{iq}{m} F^{\mu\nu} \left\{ b_1 (\partial \bar{h})_\mu b_\nu + b_2 \bar{h}_{\mu\alpha} \partial^\alpha b_\nu + b_3 \partial_\mu \bar{h} b_\nu + b_4 \bar{h} \partial_\mu b_\nu + b_5 \partial_\mu \bar{b}_\nu \varphi \right. \\ &\quad \left. + b_6 \bar{b}_\nu \partial_\mu \varphi + b_7 \partial_\mu \bar{h}_{\nu\alpha} b^\alpha + b_8 \bar{h}_{\mu\alpha} \partial_\nu b^\alpha \right\} + h.c. \end{aligned}$$

Correspondingly, all possible additional terms to the transformations of the  $A_\mu$  field with one derivative have the form:

$$\begin{aligned} \delta_1^1 A_\mu &= \frac{iq}{m} \left\{ s_{h1} (\partial \bar{h})_\mu \eta + s_{h2} \bar{h}_{\mu\nu} \partial_\nu \eta + s_{h3} \partial_\mu \bar{h} \eta + s_{b1} (\partial \bar{b}) \xi_\mu + s_{b2} \bar{b}_\nu \partial^\nu \xi_\mu \right. \\ &\quad \left. + s_{b3} \partial^\nu \bar{b}_\mu \xi_\nu + s_{b4} \partial_\mu \bar{b}_\nu \xi^\nu + s_\varphi \bar{\varphi} \partial_\mu \eta \right\} + h.c. \end{aligned}$$

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<sup>4</sup>The order is defined by the number of derivatives.

while among the fields  $h_{\mu\nu}, b_m u, \varphi$  only vector one has such a term:

$$\delta_1^1 b_\mu = \frac{iq}{m} dF_{\mu\nu} \xi^\nu + h.c.$$

Here in calculations we use the ordinary derivatives because in the linear approximation they give the same result as the covariant ones<sup>5</sup>.

The closure of the algebra on the electromagnetic field in this order gives the nontrivial conditions for the unknown coefficients in the transformations

$$\begin{aligned} s_{b1} &= s_{h1} = 0, \\ \alpha_1 + \sqrt{2}(s_{b2} + s_{b3}) &= 0, \\ \sqrt{2}\alpha_2 + s_{b5} &= 0, \\ s_{b5} - 2s_{h3} &= 0, \\ s_{h3} + s_{b2} &= 0, \\ s_{h2} - s_{b3} - s_{b4} &= 0. \end{aligned}$$

The gauge invariance requires that condition (9) be supplemented with:

$$\delta_0^1 \mathcal{L}_1^2 + \delta_1^1 \mathcal{L}_0^2 = 0.$$

Besides, in this order new additional terms of the kind  $\delta_1^1 \mathcal{L}_{-1}^2$  appear, therefore, one has to modify the second equation in (9):

$$\delta_0^1 \mathcal{L}_1^1 + \delta_1^0 \mathcal{L}_0^2 + \delta_1^1 \mathcal{L}_0^1 = 0.$$

One needs to take into account that not all the terms obtained from the calculation of variation are independent. For example, it's necessary to use the Bianchi identity for the terms, which have the derivative on the  $F_{\mu\nu}$  tensor

$$\partial_\alpha F_{\mu\nu} + \partial_\mu F_{\nu\alpha} + \partial_\nu F_{\alpha\mu} = 0.$$

All these conditions give a non-homogeneous system of linear equations on the arbitrary coefficients. The solution of the system of equations yields the result:

$$\mathcal{L}_{total} = \mathcal{L}_{free} + \mathcal{L}_{int},$$

where

$$\begin{aligned} \mathcal{L}_{int} &= iqF^{\mu\nu} \bar{h}_{\mu\alpha} h^\alpha{}_\nu + iqF^{\mu\nu} \bar{b}_\mu b_\nu \\ &\quad - \frac{iq}{m} F^{\mu\nu} \left( \frac{1}{\sqrt{2}} \bar{h}_{\mu\alpha} \partial^\alpha b_\nu - \frac{\alpha_2}{\sqrt{2}} \partial_\mu \bar{h} b_\nu - \frac{\alpha_2 + \frac{1}{2}}{\sqrt{2}} \bar{h} \partial_\mu b_\nu \right. \\ &\quad + \frac{\alpha_4}{\sqrt{2}} \partial_\mu \bar{b}_\nu \varphi + \frac{\sqrt{\frac{3}{2}} + \alpha_4}{\sqrt{2}} \bar{b}_\nu \partial_\mu \varphi - \frac{\alpha_1 + 1}{\sqrt{2}} \partial_\mu \bar{h}_{\nu\alpha} b^\alpha \\ &\quad \left. + \frac{\alpha_1}{\sqrt{2}} \bar{h}_{\mu\alpha} \partial_\nu b^\alpha - h.c. \right). \end{aligned}$$

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<sup>5</sup>The difference between the usual and the covariant derivatives will be essential only in the next approximations.

In this, we still have the relation for  $\alpha_i$

$$\alpha_1 + 4\alpha_2 + 2\alpha_3 - \sqrt{6}\alpha_4 = 0.$$

The transformations of the fields have the following form:

$$\begin{aligned} \delta_1^1 b_\mu &= -\frac{iq}{m}\sqrt{2}F_{\mu\nu}\xi^\nu + h.c. \\ \delta_1^1 A_\mu &= iq(\alpha_1 \bar{h}_{\mu\nu}\xi^\nu + \alpha_2 \bar{h}\xi_\mu + \alpha_3 \bar{b}_\mu\eta + \alpha_4 \bar{\varphi}\xi_\mu) \\ &\quad + \frac{iq}{m}\left(\frac{\alpha_1+1}{\sqrt{2}}\bar{h}_{\mu\nu}\partial^\nu\eta - \frac{\alpha_2}{\sqrt{2}}\partial_\mu\bar{h}\eta - \frac{1+\alpha_1}{\sqrt{2}}\bar{b}_\nu\partial^\nu\xi_\mu\right. \\ &\quad + \frac{1}{\sqrt{2}}\partial^\nu\bar{b}_\mu\xi_\nu + \frac{\alpha_1}{\sqrt{2}}\partial_\mu\bar{b}_\nu\xi^\nu - \sqrt{2}\alpha_2\bar{b}_\mu(\partial\eta) \\ &\quad \left. + \frac{\alpha_4 + \sqrt{\frac{3}{2}}}{\sqrt{2}}\bar{\varphi}\partial_\mu\eta\right) + h.c. \end{aligned}$$

The presence of an arbitrariness in the Lagrangian and in the transformations is related to a possibility to make a redefinition of the field  $A_\mu$ , so that the physical content of theory remains unchanged. This is the general situation for the theories which have in an interaction the number of derivatives, which is equal to or greater than the number of derivatives in a free Lagrangian. In this order one has the three-parametric freedom in the definition of the field  $A_\mu$ :

$$A_\mu \longrightarrow A_\mu + \frac{iq}{m}\left(c_1\bar{h}_{\mu\nu}b^\nu + c_2\bar{h}b_\mu + c_3\bar{b}_\mu\varphi\right) + h.c.$$

Using this freedom one can choose the most convenient form for the Lagrangian

$$\begin{aligned} \mathcal{L}_{int} &= iqF^{\mu\nu}\bar{h}_{\mu\alpha}h^\alpha{}_\nu + iqF^{\mu\nu}\bar{b}_\mu b_\nu \\ &\quad - \frac{iq}{m}F^{\mu\nu}\left(\frac{1}{\sqrt{2}}\bar{h}_\mu{}^\alpha B_{\alpha\nu} - \frac{1}{4\sqrt{2}}\bar{h}B_{\mu\nu} - \frac{\sqrt{3}}{4}B_{\mu\nu}\varphi - h.c.\right) \end{aligned} \quad (10)$$

and the transformations

$$\begin{aligned} \delta_1^1 b_\mu &= -\frac{iq}{m}\sqrt{2}F_{\mu\nu}\xi^\nu + h.c. \\ \delta_1^1 A_\mu &= -iq\left(\bar{h}_{\mu\nu}\xi^\nu + \bar{b}_\mu\eta + \sqrt{\frac{3}{2}}\bar{\varphi}\xi_\mu\right) - \frac{iq}{m\sqrt{2}}\bar{B}_{\mu\nu}\xi^\nu + h.c. \end{aligned}$$

The following choice of the parameters corresponds to such result:

$$\alpha_1 = -1, \quad \alpha_2 = 0, \quad \alpha_3 = -1, \quad \alpha_4 = -\sqrt{\frac{3}{2}}.$$

Further, one can proceed in one of the three ways. Firstly, one can evade the introduction of any additional fields, but the most probable situation in this case is that one

will have to deal with essentially non-linear theory. Secondly, one can introduce in the system a finite number of additional fields and try to stop iterations at some finite order. Thirdly, one can try to get a linear theory introducing an infinite number of additional fields and using the properties of infinite-dimensional algebras of the Kac-Moody type.

Up to now only the third scenario has been realized in the theories of Kaluza-Klein type. The simplest example is the reduction of fifth-dimensional theory of gravity (see Ref. [15]). A linear approximation of this theory coincides with the result obtained in this section. It is natural, because (10) contains a linear approximation of any theory with the number of derivatives less than or equal to two. It is this universality of the linear approximations that makes possible to consider this important step independently of the scenario one could choose to proceed further on.

The next quadratic approximation requires rather cumbersome calculations even without introducing any additional fields. Therefore in the next Section we will consider the simpler case of homogeneous electromagnetic field.

### 3. Massive Spin-2 Particle in the Homogeneous Electromagnetic Field

Let us consider a massive spin-2 particle moving in a constant homogeneous electromagnetic field.

Up to now we worked in the four-dimensional space. From this point on we will use the flat Minkowski space of arbitrary dimension  $n$  with the signature of a metric  $(+, - - \dots)$ . Let Latin indices take the values  $0, 1, \dots, n-1$ . For convenience we will not make difference between upper and lower indices, while the summation over repeated the indices will be understood, as usual, i.e.

$$A_{k\dots}B_{k\dots} \equiv g^{kl}A_{k\dots}B_{l\dots}.$$

This time we will choose the gauge invariant Lagrangian describing free complex field with mass  $m$  and spin 2 in the following form:

$$\begin{aligned} \mathcal{L}_0 = & \partial_m \bar{h}_{kl} \partial_m h_{kl} - 2 \partial_k h_{kl} \partial_m \bar{h}_{lm} + \left( \partial_k h_{kl} \partial_l \bar{h} + h.c. \right) - \partial_k \bar{h} \partial_k h \\ & + 2 \left( \partial_k \bar{h}_{kl} \partial_l \varphi - \partial_l \bar{h} \partial_l \varphi + h.c. \right) - 2 \left( \partial_l \bar{b}_k \partial_l b_k - \partial_l b_k \partial_k \bar{b}_l \right) \\ & + 2m \left( \partial_l \bar{b}_k h_{kl} - \partial_k \bar{b}_k h + h.c. \right) - m^2 \left( \bar{h}_{kl} h_{kl} - \bar{h} h \right), \end{aligned} \quad (11)$$

The gauge transformations for this Lagrangian look like:

$$\begin{aligned} \delta h_{kl} &= 2 \partial_{(k} \xi_{l)} \\ \delta b_k &= \partial_k \eta + m \xi_k \\ \delta \varphi &= m \eta. \end{aligned} \quad (12)$$

The Lagrangian has been chosen in a non canonical form (with the off-diagonal kinetic terms) in order that the Goldstone part (proportional to mass) for the field  $h_{kl}$  was absent

in transformations (12). Lagrangian (11) can be obtained from (7) by redefining the second rank field

$$h_{kl} \rightarrow h_{kl} + \frac{1}{\sqrt{6}}g_{kl}\varphi$$

and changing the normalizations of the fields and the gauge parameters. Besides, unlike canonical form (7), Lagrangian (11) does not depend on dimensionality of the space-time.

As before we will start with switching on the electromagnetic interaction in (11) in the "minimal" way, i.e., we will substitute the covariant derivatives instead of the ordinary ones. For convenience, we put  $m = 1$  and change the definition of electromagnetic field tensor by including the imaginary unit and the charge  $q$  in its definition, i.e.  $iqF_{kl} \rightarrow F_{kl}$ .

As usual, after switching on the minimal interactions we lose the gauge invariance of Lagrangian (11) under transformations (12). The residual appears

$$\begin{aligned} \delta_{\bar{0}}\mathcal{L}_{\bar{0}} = & 4F_{kl}\mathcal{D}_l\bar{h}_{km}\xi_m - 2F_{lm}\mathcal{D}_k\bar{h}_{kl}\xi_m + 3F_{kl}\mathcal{D}_k\bar{h}\xi_l + 6F_{km}\mathcal{D}_k\bar{\varphi}\xi_m \\ & - 4F_{km}\bar{b}_k\xi_m - 2F_{kl}\mathcal{D}_l\bar{b}_k\eta + h.c. , \end{aligned} \quad (13)$$

where the bar over an index denotes the replacement of partial derivatives by the covariant ones. To compensate for this residual one has to add new terms to the Lagrangian and the transformations.

Let us consider the linear approximation, i.e., we neglect the terms, which are quadratic or higher in  $F$ . We will add to transformations (12) all possible terms containing up to one derivative<sup>6</sup>

$$\begin{aligned} \delta_1 h & \sim F\partial\xi, \\ \delta_1 b & \sim F\partial\eta + F\xi, \\ \delta_1 \varphi & \sim F\partial\xi. \end{aligned} \quad (14)$$

For the case of homogeneous electromagnetic field the transformations must not include more than one derivative, because in such case all the derivatives act upon parameters of gauge transformations and if the number of derivatives is more than one, the number of physical degrees of freedom will change.

New transformations (14) give a contribution for the variation in the following form<sup>7</sup>

$$\delta_1\mathcal{L}_0 = F\partial^3\bar{h}\xi + F\partial^3\bar{b}\eta + F\partial^3\bar{\varphi}\xi + F\partial^2\bar{h}\eta + F\partial^2\bar{b}\xi + F\partial\bar{h}\xi + h.c. \quad (15)$$

The most general ansatz for additional terms to the Lagrangian, which give a contribution like (15) contains terms with, at most, two derivatives

$$\mathcal{L}_1 = F\partial\bar{h}\partial h + F\partial\bar{h}\partial\varphi + F\partial\bar{b}\partial b + F\bar{h}\partial b + F\bar{b}\partial\varphi + F\bar{h}h + F\bar{b}b + h.c. \quad (16)$$

We will not consider the terms containing more than two derivatives. To support this limitation, we could offer two arguments:

<sup>6</sup>We have explicitly checked that if we restrict ourselves by derivativeless transformations, we shall receive an inconsistent system of equations in the next approximation.

<sup>7</sup>The calculations are rather cumbersome and were made with the help of the "REDUCE" system. Therefore, we will, as a rule, omit intermediate results and give only their schematic representation.

- i. In the case of homogeneous electromagnetic field,  $F$  is just a constant matrix, that does not depend on the spatial coordinates. Therefore, in all orders of the iterations the Lagrangian will remain quadratic in the nontrivial fields  $h_{\mu\nu}, b_\mu, \varphi$  and the transformations will be always Abelian. Hence the Lagrangian will be quasi-free and two derivatives are natural for it.
- ii. As we have already explained, there are no terms in the transformations, which correspond to the terms in the Lagrangian with more than two derivatives.

From the requirement of the gauge invariance in the linear approximation

$$\delta_0 \mathcal{L}_0 + \delta_1 \mathcal{L}_0 + \delta_0 \mathcal{L}_1 = 0$$

we obtain a nonhomogeneous system of linear equations for the arbitrary coefficients in (14) and (16)<sup>8</sup>.

Solving the system of linear equations, we obtain the following result for the transformations in the linear approximation

$$\begin{aligned} \delta_1 h_{kl} &= \alpha_1 g_{kl} F_{mn} \partial_n \xi_m, \\ \delta_1 b_k &= \alpha_2 F_{kl} \xi_l, \\ \delta_1 \varphi &= \alpha_3 F_{kl} \partial_l \xi_k. \end{aligned} \tag{17}$$

while for the Lagrangian we get:

$$\begin{aligned} \mathcal{L}_1 &= (\alpha_1 (n-1) - \alpha_2 + 1) F_{kl} \bar{h}_{km} h_{lm} + 4 F_{kl} \bar{b}_k b_l \\ &+ \left\{ (\alpha_1 (n-1) - \alpha_2 - 3) F_{kl} \partial_l \bar{b}_m h_{km} \right. \\ &+ (\alpha_1 (n-1) + \alpha_2 + 3) F_{kl} \partial_m \bar{b}_l h_{km} \\ &+ (\alpha_1 (n-1) + 2\alpha_2 + 3) F_{kl} \partial_l \bar{b}_k h_{mm} - 6 F_{kl} \partial_l b_k \bar{\varphi} + h.c. \left. \right\} \\ &+ (\alpha_1 (n-2) + 2\alpha_3) \left( F_{ln} \partial_m h_{kl} \partial_m \bar{h}_{kn} - F_{ln} \partial_k h_{kl} \partial_m \bar{h}_{mn} \right) \\ &+ \left\{ (\alpha_1 (n-2) + 2\alpha_3) \left( F_{mn} \partial_k h_{kl} \partial_n \bar{h}_{lm} + F_{ln} \partial_k h_{kl} \partial_n \bar{h} \right) \right. \\ &- 2\alpha_1 (n-1) F_{km} \partial_k \bar{\varphi} \partial_l h_{lm} - (\alpha_1 (n-1) + 3\alpha_2 + 3) F_{kl} \partial_l \bar{b}_k \partial_m b_m \\ &\left. + h.c. \right\} - (\alpha_1 (n-1) + 3\alpha_2 + 3) F_{km} \partial_l b_k \partial_l \bar{b}_m. \end{aligned}$$

In this, we have used a two-parametric freedom related to a possibility of field redefinitions:

$$\begin{aligned} h_{kl} &\rightarrow h_{kl} + s_h F_{(k|m} h_{l)m}, \\ b_k &\rightarrow b_k + s_b F_{kl} b_l. \end{aligned}$$

Now, going to a quadratic approximation, we get the residual, containing the terms quadratic in  $F$ , which appears from the  $\delta_1 \mathcal{L}_0 + \delta_0 \mathcal{L}_1 + \delta_1 \mathcal{L}_1$ . To compensate the residual we proceed in the same manner as in the linear approximation i.e. we add the terms of the form

$$\delta_2 \Phi \sim FF\partial\Lambda + FF\Lambda \tag{18}$$

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<sup>8</sup>As for all orders in  $F$  the algebra is Abelian, its closure does not give any additional conditions.

to the transformations, where  $\Phi \sim \{h_{kl}, b_k, \varphi\}$ , and  $\Lambda \sim \{\xi_k, \eta\}$ . At the same time, we add all the possible terms of the form

$$\mathcal{L}_2 = FF\partial\bar{\Phi}\partial\Phi + FF\partial\bar{\Phi}\Phi + FF\bar{\Phi}\Phi \quad (19)$$

to the Lagrangian. Imposing the condition of the gauge invariance in the quadratic approximation, i.e.

$$\delta_{\bar{1}}\mathcal{L}_0 + \delta_{\bar{0}}\mathcal{L}_{\bar{1}} + \delta_1\mathcal{L}_1 + \delta_2\mathcal{L}_0 + \delta_0\mathcal{L}_2 = 0 ,$$

one obtains a system of quadratic equations for the coefficients in (18), (17) and (19). Solving the system we get an answer to the given order.

Note, that in this order<sup>9</sup> one can exclude all the terms in the transformations proportional to  $F^2$  using the freedom in the field redefinition  $\Phi \rightarrow \Phi + FF\Phi$ .

In some sense the quadratic approximation is crucial, because if the coefficients  $\alpha_i$  in (17) are not equal to zero, then one can exclude all the transformations in the higher orders using the freedom in the field redefinition for every order as well as an arbitrariness in the definition of the  $F$  tensor  $F \rightarrow F + F^3 + F^6 + \dots$

Let us try to stop the iterations. For this we will decrease the number of derivatives for each next order of the iteration. That is we are adding the terms of the kind

$$\begin{aligned} \mathcal{L}_3 &\sim FFF\bar{\Phi}\Phi + FFF\bar{\Phi}\Phi , \\ \mathcal{L}_4 &\sim FFFFF\bar{\Phi}\Phi . \end{aligned}$$

Requiring the gauge invariance for all orders, we obtain a system of non-homogeneous algebraic equations of the fourth degree. Solving this system, we get the final answer for the Lagrangian describing a charged massive spin-2 particle moving in the constant homogeneous electromagnetic field

$$\mathcal{L}_{full} = \mathcal{L}_{\bar{0}} + \mathcal{L}_{\bar{1}} + \mathcal{L}_{\bar{2}} + \mathcal{L}_{\bar{3}} + \mathcal{L}_{\bar{4}} , \quad (20)$$

where

$$\begin{aligned} \mathcal{L}_1 &= \frac{5}{4}F_{kl}\bar{h}_{km}h_{lm} + 2F_{kl}\bar{b}_kb_l - \frac{3}{2}F_{kl}\mathcal{D}_l\bar{b}_mh_{km} + \frac{3}{2}F_{kl}\mathcal{D}_m\bar{b}_lh_{km} \\ &\quad + 6F_{kl}\mathcal{D}_l\bar{b}_k\varphi + \frac{3}{4}F_{km}\mathcal{D}_lb_k\mathcal{D}_l\bar{b}_m + \frac{3}{2}F_{kl}\mathcal{D}_l\bar{b}_k\mathcal{D}_mb_m + h.c. \\ \mathcal{L}_2 &= \frac{9}{4}F_{kl}F_{mn}\bar{h}_{ln}h_{km} + \frac{3}{2}\left(\frac{3}{8}F_{kl}F_{km}\bar{h}_{lm}h - F_{kl}F_{km}\bar{h}_{lm}\varphi + h.c.\right) \\ &\quad - 3\left(F_{kl}F_{km}\bar{b}_mb_l + \frac{1}{4}F_{kl}^2\bar{b}_mb_m - F_{kl}^2\bar{\varphi}\varphi\right) - \frac{9}{4}\left(F_{km}F_{ln}\mathcal{D}_l\bar{b}_kh_{mn} \right. \\ &\quad \left. + F_{km}F_{mn}\mathcal{D}_l\bar{b}_kh_{ln} + \frac{1}{2}F_{lm}F_{ln}\mathcal{D}_k\bar{b}_kh_{mn} + \frac{1}{2}F_{km}F_{lm}\mathcal{D}_l\bar{b}_kh + h.c.\right) \\ &\quad - 6(F_{kl}F_{lm}\mathcal{D}_k\bar{\varphi}b_m + h.c.) + \frac{9}{40}\left(\frac{3}{2}F_{np}^2\mathcal{D}_k\bar{h}_{kl}\mathcal{D}_mh_{lm} - \frac{3}{4}F_{np}^2\mathcal{D}_m\bar{h}_{kl}\mathcal{D}_mh_{kl} \right) \end{aligned}$$

<sup>9</sup>This holds for any even order.

$$\begin{aligned}
& -\frac{3}{2}F_{np}^2\mathcal{D}_m h_{lm}\mathcal{D}_l\bar{h} + \frac{3}{4}F_{np}^2\mathcal{D}_l\bar{h}\mathcal{D}_l h + \frac{1}{4}F_{np}F_{mp}\mathcal{D}_m h_{kl}\mathcal{D}_n\bar{h}_{kl} \\
& -\frac{1}{2}F_{np}F_{mp}\mathcal{D}_l h\mathcal{D}_l\bar{h}_{mn} + \frac{1}{2}F_{np}F_{mp}\mathcal{D}_k\bar{h}_{mn}\mathcal{D}_l h_{kl} - F_{np}F_{mp}\mathcal{D}_l h_{kl}\mathcal{D}_n\bar{h}_{km} \\
& +\frac{1}{2}F_{np}F_{lp}\mathcal{D}_m\bar{h}_{kl}\mathcal{D}_m h_{kn} + F_{np}F_{lp}\mathcal{D}_k\bar{h}_{kl}\mathcal{D}_n h - \frac{1}{2}F_{np}F_{lp}\mathcal{D}_k\bar{h}_{kl}\mathcal{D}_m h_{mn} \\
& -\frac{1}{4}F_{np}F_{lp}\mathcal{D}_l\bar{h}\mathcal{D}_n h + \frac{9}{2}F_{np}F_{lm}\mathcal{D}_m\bar{h}_{kl}\mathcal{D}_p h_{kn} - 9F_{np}F_{lm}\mathcal{D}_k\bar{h}_{kl}\mathcal{D}_p h_{mn} \\
& +\frac{9}{2}F_{np}F_{lm}\mathcal{D}_l\bar{h}\mathcal{D}_p h_{mn} - \frac{9}{4}F_{lp}F_{kn}\mathcal{D}_m\bar{h}_{kl}\mathcal{D}_m h_{np} + h.c.) \\
& +\frac{9}{4}\left(F_{ln}F_{mn}\mathcal{D}_k h_{kl}\mathcal{D}_m\bar{\varphi} - F_{lm}F_{kn}\mathcal{D}_m h_{kl}\mathcal{D}_n\bar{\varphi} - \frac{1}{2}F_{kn}F_{ln}\mathcal{D}_m h_{kl}\mathcal{D}_m\bar{\varphi} \right. \\
& \left. -\frac{1}{2}F_{km}F_{lm}\mathcal{D}_l h\mathcal{D}_k\bar{\varphi} + h.c.\right) - \frac{9}{2}F_{kl}F_{mn}\mathcal{D}_l b_k\mathcal{D}_n\bar{b}_m - 6F_{km}F_{lm}\mathcal{D}_l\bar{\varphi}\mathcal{D}_k\varphi, \\
\mathcal{L}_3 = & \frac{27}{80}F_{kl}^2F_{mn}\bar{h}_{np}h_{mp} - \frac{153}{40}F_{kl}F_{km}F_{np}\bar{h}_{mp}h_{ln} + \frac{9}{10}F_{kl}F_{km}F_{ln}\bar{h}_{mp}h_{np} \\
& +\frac{9}{4}\left(\frac{3}{2}F_{kl}F_{mp}F_{mn}\mathcal{D}_l b_k\bar{h}_{np} + F_{lm}^2F_{kn}\mathcal{D}_k\varphi\bar{b}_n + h.c.\right), \\
\mathcal{L}_4 = & -\frac{27}{16}\left(F_{kl}^2F_{mq}F_{mn}h_{nq}\bar{\varphi} + h.c.\right) + \frac{9}{8}F_{kl}^2F_{mn}^2\bar{\varphi}\varphi \\
& +\frac{81}{32}F_{kl}F_{km}F_{np}F_{nq}\bar{h}_{lm}h_{pq}.
\end{aligned}$$

In this, only field  $b_\mu$  has the nontrivial transformation

$$\delta_1 b_k = -\frac{3}{2}F_{kl}\xi_l. \quad (21)$$

It is easy to see that the result obtained does not depend on the dimensionality of the space-time<sup>10</sup>. This is a consequence of our choice of noncanonical form for the free Lagrangian and the absence of the term proportional to the metric tensor in the transformations.

As it has already been mentioned in the Introduction a similar problem in a context of the bosonic string theory was discussed in paper [12]. Comparing the result obtained there with lagrangian (20) and transformations<sup>11</sup> (12) and (21), one can make the conclusion that both models have the same structure. That is, the transformations of fields are linear for electromagnetic field  $F$ , the number of the derivatives in the Lagrangians does not exceed two and a maximal order in  $F$  equals four. But since the authors of ref. [12] started from the bosonic string they obtained the gauge invariant description of massive spin-2 field moving in the homogeneous electromagnetic field in the form, which is valid only for the 26-dimensional space-time.

<sup>10</sup>We have to remark that one should consider the case of two-dimensional space-time separately.

<sup>11</sup>One must replace the ordinary derivatives by the covariant ones.

## Conclusion

Thus, in this work the gauge invariant description of free massive fields with arbitrary integer spins was constructed. Basing on such description, one can investigate consistent theories of interactions of massive particles with high spins. As an example of such constructive approach, we considered the electromagnetic interaction of massive spin-2 field and obtained: a) the linear approximation with minimal number of the derivatives in the case of the arbitrary electromagnetic field; b) the full answer for the homogeneous field in the space-time of any dimensionality.

Later, using the offered method we are planning to consider the electromagnetic interaction of a massive spin-3 field and also to investigate possibilities to go beyond the linear approximation for the spin-2 field in a case of the arbitrary electromagnetic field.

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Зиновьев Ю.М., Клишевич С.М.

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142284, Протвино Московской обл.

