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THE UPPER LIMIT ON GRAVITON MASS

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Abstract

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Basing on the field theory of gravity and observable parameters of the expanding Universe, the upper limit of $m_g \leq 4.5 \cdot 10^{-66}$ g on the value of possible graviton mass has been derived.

Аннотация

Герштейн С.С., Логунов А.А., Мествиришвили М.А. О верхнем пределе на массу гравитона: Препринт ИФВЭ 97-57. – Протвино, 1997. – 5 с., библиогр.: 7.

Исходя из полевой теории гравитации и наблюдаемых параметров расширяющейся Вселенной, установлен верхний предел на возможную массу гравитона $m_g \leq 4.5 \cdot 10^{-66}$ г.

The problem of existence of nonzero invariant mass of the graviton can be of the fundamental significance. The estimate for the upper bound on the graviton mass ($m_g < 2 \cdot 10^{-62}$g) has been derived in Ref. [1], where the authors used the data on the existence of the gravitational coupling between the galaxy clusters, which is not cut off by the Yukawa potential, at least, up to distances ∼500 Kpc. In this paper we will present the estimates for the upper limit on the graviton mass basing on the observable parameters of the Universe expansion. The fact that in this case typical distances are 3-4 orders of magnitude larger than those between gravitationally bound galaxy clusters allows one to strengthen the estimates on the upper limit on the graviton mass by few orders of magnitude, respectively.

It should be noted that introduction of the nonzero invariant mass of the graviton requires going beyond the General Theory of Relativity (GTR). This can be done naturally by using the notions of the gravitational field in the Minkowsky space [2,3]. In Ref. [3] the complete energy-momentum tensor $t^{\mu\nu}$ (including the gravitational field), which is conserved in the Minkowsky space, is considered as a source of the gravitational field described by symmetric tensor $\Phi^{\mu\nu}$. In an arbitrary fixed (not necessarily inertial) frame of the Minkowsky space with metric tensor $\gamma_{\mu\nu}$, the equations for the density of the gravitational field $\tilde{\Phi}^{\mu\nu}$ can be written analogously to the Maxwell equations and Lorentz condition for the electromagnetic field as follows:

$$(\gamma^{\alpha\beta} D_\alpha D_\beta + m_g^2)\tilde{\Phi}^{\mu\nu} = 16\pi \tilde{t}^{\mu\nu}, \quad (1)$$

$$D_\nu \tilde{\Phi}^{\mu\nu} = 0 \ , \quad (2)$$

where D_α is the covariant derivative in the Minkowsky space, m_g is the graviton mass ($\hbar = c = G = 1$), and $\tilde{\Phi}^{\mu\nu}, \tilde{t}^{\mu\nu}$ are the densities of tensors

$$\tilde{\Phi}^{\mu\nu} = \sqrt{-\gamma}\Phi^{\mu\nu}, \quad \tilde{t}^{\mu\nu} = \sqrt{-\gamma} t^{\mu\nu}, \quad \gamma = \det(\gamma_{\mu\nu}) = \det(\tilde{\gamma}^{\mu\nu}) \ . \quad (3)$$

Condition (2) singles out polarization states with spin values of 2 and 0 and provides the conservation of the density of the energy-momentum tensor $D_\mu \tilde{t}^{\mu\nu} = 0$ in Eq. (1). The density of the energy-momentum tensor is defined, following Hilbert, by Euler's variation with $\gamma_{\mu\nu}$ metric of the Lagrangian density of the system

$$\tilde{L} = \tilde{L}_g(\gamma_{\mu\nu}, \Phi_{\mu\nu}) + \tilde{L}_M(\gamma_{\mu\nu}, \Phi_{\mu\nu}, \Phi_A) \ , \quad (4)$$

where $\tilde{L}_g$ is the density of the gravitational field Lagrangian, and $\tilde{L}_M$ corresponds to the density of Lagrangian of the matter described by the Φ_A fields

$$\tilde{t}^{\mu\nu} = -2\frac{\delta \tilde{L}}{\delta \gamma_{\mu\nu}} \ , \tag{5}$$

where Euler's variation is

$$\frac{\delta \tilde{L}}{\delta \gamma_{\mu\nu}} = \frac{\partial \tilde{L}}{\partial \gamma_{\mu\nu}} - \partial_\sigma \left(\frac{\partial \tilde{L}}{\partial \gamma_{\mu\nu,\sigma}} \right) \ ; \gamma_{\mu\nu,\sigma} = \frac{\partial \gamma_{\mu\nu}}{\partial x^\sigma} \ . \tag{6}$$

One can derive equations for the gravitational field and matter fields from the least action principle

$$\frac{\delta \tilde{L}}{\delta \tilde{\Phi}^{\mu\nu}} = 0, \quad \frac{\delta \tilde{L}}{\delta \Phi_A} = 0. \tag{7}$$

To get the form of (1) and (2) for these equations it is necessary to have the density of the gravitational field $\tilde{\Phi}^{\mu\nu}$ coming into the density of the matter Lagrangian $\tilde{L}_M$ in combination with the density of the metric tensor $\tilde{\gamma}^{\mu\nu}$

$$\tilde{g}^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \tilde{\Phi}^{\mu\nu}, \ \ \tilde{g}^{\mu\nu} = \sqrt{-g} g^{\mu\nu}, \ \ g = \det(g_{\mu\nu}) = \det(\tilde{g}^{\mu\nu}) \ , \tag{8}$$

i.e. $\tilde{L}_M(\tilde{g}^{\mu\nu}, \Phi_A)$. It means that the motion of the matter subjected to the gravitational field looks like as if this could take place in the Riemann space with the metric $g_{\mu\nu}$. The Lagrangian density resulting in Eqs. (1) and (2) has the form

$$\tilde{L} = \tilde{L}_g + \tilde{L}_M(\tilde{g}^{\mu\nu}, \Phi_A), \tag{9}$$

$$\tilde{L}_g = \frac{1}{16\pi} \tilde{g}^{\mu\nu} (G^\lambda_{\mu\nu} G^\sigma_{\lambda\sigma} - G^\lambda_{\mu\sigma} G^\sigma_{\nu\lambda}) - \frac{m^2}{16\pi} \left(\frac{1}{2} \gamma_{\mu\nu} \tilde{g}^{\mu\nu} - \sqrt{-g} - \sqrt{-\gamma} \right) \ , \tag{10}$$

where the $G^\lambda_{\mu\nu}$ values are the components of the <u>tensor</u>

$$G^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma} (D_\mu g_{\nu\sigma} + D_\nu g_{\mu\sigma} - D_\sigma g_{\mu\nu}) \ , \tag{11}$$

and due to this fact the $\tilde{L}_g$ value behaves as the density of the scalar under <u>any</u> coordinate transformations. Using (9) and (10) and taking into account (7), one can write the equations for the gravitational field in the form of [3]

$$\left(R^\mu_\nu - \frac{1}{2} \delta^\mu_\nu R \right) + \frac{m_g^2}{2} (\delta^\mu_\nu + g^{\mu\alpha} \gamma_{\alpha\nu} - \frac{1}{2} \delta^\mu_\nu g^{\alpha\beta} \gamma_{\alpha\beta}) = 8\pi T^\mu_\nu, \tag{12}$$

$$D_\nu \tilde{g}^{\mu\nu} = 0 \ , \tag{13}$$

where T^μ_ν is the matter energy-momentum tensor in the Riemann space.

From these equations one gets the equations for the matter

$$\nabla_\nu \tilde{T}^{\mu\nu} = 0, \quad \tilde{T}^{\mu\nu} = -2\frac{\delta \tilde{L}_M}{\delta g_{\mu\nu}} \ , \tag{14}$$

where ∇_ν is the covariant derivative in the effective Riemann space. Eqs. (12) and (13) are covariant with respect to any coordinate transformations and form-invariant under Lorentz's transformations.

Writing down the interval of the effective Riemann space for the homogeneous and isotropic Universe in the form of

$$ds^2 = U(t)dt^2 - V(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\Theta^2 + \sin^2\Theta d\Phi^2)\right] , \tag{15}$$

(where $k = 1, \ -1, \ 0$ for the closed, hyperbolic and "flat" Universe), one gets from Eqs. (13)

$$\frac{\partial}{\partial t}\sqrt{\frac{V^3}{U}} = 0, \quad \text{i.e. } V = aU^{1/3}, a = \text{const.} \tag{16}$$

$$\frac{\partial}{\partial r}[r^2(1-kr^2)^{1/2}] - 2r(1-kr^2)^{-1/2} = 0 \, . \tag{17}$$

Eq. (17) is valid only for $k = 0$. Thus, the Universe can be only "flat" (i.e. its space geometry is Euclidian). Using the proper time $d\tau = U^{1/2}dt$ and denoting $R^2 = U^{1/3}$, one can write down interval (15) in the form

$$ds^2 = d\tau^2 - aR^2(\tau)(dx^2 + dy^2 + dz^2) \, . \tag{18}$$

In this case Eqs. (12) in the inertial frame take the form[1]

$$\left(\frac{1}{R}\frac{dR}{d\tau}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\omega}{R^6}\left(1 - \frac{3R^4}{a} + 2R^6\right), \tag{19}$$

$$\frac{1}{R}\cdot\frac{d^2R}{d\tau^2} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) - 2\omega\left(1 - \frac{1}{R^6}\right) , \tag{20}$$

where

$$\omega = \frac{1}{12}\left(\frac{m_g c^2}{\hbar}\right)^2 . \tag{21}$$

It follows from Eq. (19) in the region $R >> 1$ that the density of the matter in the Universe is equal to

$$\rho(\tau) = \rho_c(\tau) + \frac{1}{16\pi G}\left(\frac{m_g c^2}{\hbar}\right)^2 , \tag{22}$$

[1]Note, that the metric of the Minkowsky space $\gamma_{\mu\nu}$ comes into Eq.(12). Due to this fact the Minkowsky space becomes observable, and the casualty principle for the gravitational field in the effective Riemann space should be fulfilled: the motion of the matter subjected to the gravitational field should not leave the light-cone limits in the Minkowsky space. This condition can be formulated in the form of $g_{\mu\nu}V^\mu V^\nu \leq 0$ for any isotropic vector V^μ on the light cone $\gamma_{\mu\nu}V^\mu V^\nu = 0$. This condition being applied to interval (15) with account for (16) and (17) leads to $R^2(R^4 - a) \leq 0$. Thus, the constant "a" has the notion of the fourth power of maximal value of the scale factor: $a = R^4_{\text{max}}$, and to describe the existing Universe, one should have $a >> 1$.

where $\rho_c(\tau)$ is the critical density determined by the Hubble "constant"

$$\rho_c = \frac{3H^2(\tau)}{8\pi G}, \quad H(\tau) = \frac{1}{R} \cdot \frac{dR}{d\tau} \ . \tag{23}$$

This conclusion inevitably requires the existence of the "dark" matter that agrees with current observations.

From (19) and (20) one can get the expression for the Universe deceleration parameter $q(\tau)$. At the present stage of the nonrelativistic matter dominance ($p = 0$)

$$q = -\frac{\ddot{R}}{R} \cdot \frac{1}{H^2} = \frac{1}{2} + \frac{1}{4H^2} \left(\frac{m_g c^2}{\hbar} \right)^2 . \tag{24}$$

Relation (24) gives the principal possibility to determine the graviton mass from two other observables, H and q. The sensitivity of q to the graviton mass is due to the fact that a small value $\frac{1}{\lambda_g} = \frac{m_g c}{\hbar}$ comes into (24) multiplied by a large value $\left(\frac{c}{H}\right) = 9.25 \cdot 10^{27} \cdot h^{-1}$ cm, which is the Hubble radius of the Universe. Though the q value has not been measured with high accuracy, its possible values do not exceed few units ($q \leq 5$, see [4]). This allows one to estimate from (24) the graviton mass

$$m_g \leq 1.7 \cdot 10^{-65} \cdot h \text{ (g)}, \quad \text{where} \ \ 0.4 \leq h \leq 1, \tag{25}$$

$$\frac{\hbar}{m_g c} > 0.2 \cdot \frac{c}{H} = 2 \cdot 10^{27} \cdot h^{-1} \text{ (cm)}. \tag{26}$$

Despite the smallness of the upper limit in (25), nonzero graviton mass can have principal influence on the character of the Universe evolution. One can see from Eq. (19) that for $R \to 0$ the negative term $\frac{\omega}{R^6}$ in the right-hand side of the equation grows in absolute value faster than the matter density ($\rho \sim \frac{1}{R^4}$ for radiatively dominant stage). Therefore, from the condition of the nonnegative left-hand side of (19), it follows that the expansion should begin from some minimal value R_{min}, which corresponds to $\frac{dR}{d\tau} = 0$. On the other side, the expansion should stop at $R >> 1$, when density (22) reaches its minimal value $\rho_{\text{min}} = \frac{1}{16\pi G} \left(\frac{m_g c^2}{\hbar} \right)^2$, and after that the expansion is replaced by the compression process up to R_{min}. So, nonzero graviton mass eliminates the cosmological singularity and leads to a cyclic character of the Universe evolution. Such a character of the Universe evolution seemed to be promising for a number of authors (see, for example, [5]). The time of the Universe expansion from the maximal density to the minimal one is determined mostly by the stage of the nonrelativistic matter dominance, and it is equal to [6]

$$\tau_{\text{max}} \simeq \sqrt{\frac{2}{3}} \cdot \frac{\pi \hbar}{m_g c^2} \ . \tag{27}$$

Accepting the value of $(10 - 15) \cdot 10^9$ years for the Universe age and using $\tau_{\text{max}} \geq 20 \cdot 10^9$ years, one gets more strict limit on the graviton mass

$$m_g \leq 4.5 \cdot 10^{-66} \text{ (g)} \ . \tag{28}$$

Equations (12) and (13) explain all known gravitational effects in the Solar system, which are attributed to the noninertial frame. It is well known that the introduction of the graviton mass in the linear tensor theory is accompanied by the difficulty: the presence of "ghosts". However, as it has been shown in Ref. [7], this difficulty is eliminated in the framework of the nonlinear tensor theory described by Eqs. (12) and (13) under condition that gravitons spread in the effective Riemann space, rather than Minkowsky's one (as it takes place in the linear theory). If this circumstance is sequently taken into consideration, one gets a positively determined flux of the gravitational energy, when calculating the intensity.

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