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QUARK MASSES
IN A RELATIVISTIC CONFINEMENT MODEL

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Abstract

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We consider a relativistic quantum model of confined massive quark and antiquark which describes leading Regge trajectories of mesons. A comparison with experimental meson masses makes it possible to determine the quark masses (in MeV) $m_s = 228 \pm 5$, $m_c = 1340 \pm 50$, $m_b = 4550 \pm 100$. We have used these numbers to calculate other meson masses in agreement with experiment.

Аннотация

Соловьев Л.Д. Релятивистская модель конфайнмента и массы кварков: Препринт ИФВЭ 97-7. – Протвино, 1997. – 9 с., библиогр.: 10.

Рассмотрена релятивистская квантовая модель конфайнмента массивного кварка и антикварка, описывающая главные реджевские траектории мезонов. Сравнение модели с экспериментом позволило определить массы кварков (в Мэв) $m_s = 228 \pm 5$, $m_c = 1340 \pm 50$, $m_b = 4550 \pm 100$. Использование этих масс для вычисления масс мезонов приводит к хорошему согласию с экспериментом.

It has been believed for a long time that properties of quarks confined in a meson are closely related to those of the relativistic string with Nambu-Goto self-interaction [1]. However, the anomaly in the quantized string theory in 4-dimensional space-time turned the main development of the string theory from hadron models to other directions [1]. Nevertheless, the hadron theory can try to use some simple particular configurations of the string for an approximate description of the hadrons if these configurations admit relativistic quantization. If the approximate hadron model obtained in this way appears to be acceptable for experiment, one can try the next, more complicated string configuration, having in mind that at some step the whole notion of string may fail, especially when more experimental information about hadron daughter trajectories are available.

The simplest string configuration, a straight-line string, was quantized in [2,3] in accord with the Poincaré invariance and in good agreement with the spectrum of the light-quark mesons lying on the leading Regge trajectory. The next approximation was to take into account the masses of the quarks attached to the ends of the string. This has been done in [4-7] with different assumptions, the common assumption being vanishing of the quark velocity along the string.

In this paper we get rid of this assumption [10]. We consider the Nambu-Goto straight-line string with point-like massive quarks attached to its ends. This is an extended relativistic object called a rotator for which the explicitly relativistic description introduces auxiliary variables resulting in a symmetry of the rotator Lagrangian. The rotator Hamiltonian is given by an implicit function which can be calculated numerically. For important particular cases (light or heavy quarks) series expansions for the Hamiltonian are obtained. Quantization of this system preserving the Poincaré invariance gives meson states with different spins lying on a Regge trajectory which depends on the quark masses. A comparison with experiment allows one to estimate the s-, c- and b-quark masses while the u- and d-quark masses are zero within error bars. As a check we have used the obtained quark masses to calculate the $s\bar{s}$-, $c\bar{c}$- and $b\bar{b}$-meson masses (not used in the input) in agreement with experiment.

We consider mesons with non-zero spins. Spin-zero-states demand special consideration because more complicated mechanisms may be involved in their formation.

So, let us consider a simplest extended relativistic object – a straight-line

$$x(\tau,\sigma) = r(\tau) + f(\tau,\sigma)q(\tau), \tag{1}$$

where r is a 4-vector corresponding to a point on the straight-line, q is an affine 4-vector of its direction, f is a scalar monotonic function of σ labelling points on the line and τ is a scalar evolution parameter. We shall not fix the coordinates $f_i(\tau) = f(\tau, \sigma_i(\tau))$ of the end points of the string considering them as dynamical variables to be determined from extremum of an action. Then the explicit Poincaré covariance of (1) introduces superfluous variables not necessary for the description of the straight-line as a physical object, so that theory in terms of (1) must be invariant under a group of three sets of τ-dependent transformations (gauge transformations)

1) shift of r along q:

$$r \to r + f(\tau)q, \tag{2}$$

2) multiplication of q by an arbitrary scalar function:

$$q \to g(\tau)q, \quad \text{and} \tag{3}$$

3) reparametrization of τ, which means that the Lagrangian must satisfy the condition

$$\mathcal{L}(h(\tau)\dot{z}, h(\tau)(h(\tau)\dot{z})^{\bullet}) = h(\tau)\mathcal{L}(\dot{z}, \ddot{z}), \tag{4}$$

where $\dot{z}$ and $\ddot{z}$ mean every τ-derivative in the Lagrangian.

This symmetry implies that the phase-space variables of our system obey three constraints which are in involution with respect to their Poisson brackets, the canonical Hamiltonian is zero and the total Hamiltonian is a linear combination of the constraint functions.

An important consequence of this symmetry comes from the observation that a shift of the end-point coordinates $f_i \to f_i + a$ or their velocities is equivalent to a transformation (2). The solutions of our problem do not depend on a and $\dot{a}$ and without loss of generality we can use Lagrangians satisfying these conditions explicitly:

$$\sum \mathcal{L}_{f_i} = 0, \quad \sum \mathcal{L}_{\dot{f}_i} = 0. \tag{5}$$

Here and below $\sum$ corresponds to summation over $i = 1, 2$ and a variable as an index denotes the partial derivative with respect to this variable.

Invariants of a symmetry play an important role in the description of a symmetric system. In our case they are orthonormal vectors along the line direction, velocity of the line rotation and velocity of its movement as a whole

$$n = cq, \quad v^1 = b^{-1}\dot{n}, \quad v^0 = (\dot{r}_\perp^2)^{-1/2}\dot{r}_\perp, \tag{6}$$

where

$$c = (-q^2)^{-1/2}, \quad b = (-\dot{n}^2)^{1/2} \tag{7}$$

and

$$\dot{r}_{\perp}^{k}=(g^{kl}+n^{k}n^{l}+v^{1k}v^{1l})\dot{r}_{l}. \tag{8}$$

The angular velocity b is invariant under (2) and (3) and transforms as the Lagrangian under (4). The scalar invariant of the symmetry is

$$l=b^{-1}(\dot{r}_{\perp}^{2})^{1/2}. \tag{9}$$

We shall label points on the string with respect to the instant center of its rotation z

$$f=z+y, \tag{10}$$

$$z=b^{-1}\dot{r}v^{1} \tag{11}$$

(velocity of the point $r+zn$, orthogonal to q, is orthogonal to v^{1}). The length of the rotator at fixed τ is $|y_2-y_1|$. From $\dot{x}_i^2\geq 0$ it follows that $|y_i|\leq l$.

The Lagrangian of our model is a sum of the Nambu-Goto Lagrangian for an open string and two Lagrangians for free point-like particles with masses m_1 and m_2 and velocities of the ends of the string

$$\mathcal{L}=-a\int_{\sigma_1}^{\sigma_2}g^{1/2}d\sigma-\frac{1}{2}\sum(\frac{1}{be_i}\dot{x}_i^2+be_im_i^2), \tag{12}$$

where $g=(\dot{x}x')^2-\dot{x}^2x'^2$ is minus determinant of the induced metric of the string world-sheet, $\dot{x}_i=dx(\tau,\sigma_i(\tau))/d\tau, i=1,2$ are velocities of the string ends and e_i are Lagrange multipliers determined from the condition $\mathcal{L}_{e_i}=0$. Using the notations introduced above we can rewright (12) for the straight-line string (1,10) in the form

$$\mathcal{L}=-bF, \tag{13}$$

where F is a gauge and Poincaré invariant function

$$F=a\int_{y_1}^{y_2}(l^2-x^2)^{1/2}+\frac{1}{2}\sum(e_i^{-1}(l^2-y_i^2-w_i^2)+e_im_i^2), \tag{14}$$

$$w_i=b^{-1}(\dot{y}_i+\dot{z}-\dot{r}n). \tag{15}$$

Let us denote

$$y_1=-y/2+d,\quad y_2=y/2+d, \tag{16}$$

where $y=y_2-y_1$ and $d=(y_1+y_2)/2$. Then the first condition (5)

$$\sum F_{y_i}=0 \tag{17}$$

makes it possible to express d through y and l (and e_i). The second condition (5) determines $\dot{d}$. Putting $\dot{d}$ into (14) we get

$$F=a\int_{y_1}^{y_2}(l^2-x^2)dx+\frac{1}{2}[\sum(e_i^{-1}(l^2-y_i^2)+e_im_i^2)-\frac{b^{-2}}{e_1+e_2}\dot{y}^2]. \tag{18}$$

If y depends on $\tau, \dot{y} \neq 0$, then there exists a gauge (a parametrization) in which

$$y = k_1 \tau + k_2, \tag{19}$$

where $k_{1,2}$ do not depend on τ. They must ensure the extremum of F

$$\partial F/\partial k_i = 0. \tag{20}$$

Since

$$\partial F/\partial k_i = \sum_j F_{y_j} y_{jk_i} + F_{k_i} \tag{21}$$

and

$$y_{jk_1} = (-1)^j \frac{\tau}{2} + d_{k_1}, \quad y_{jk_2} = (-1)^j \frac{1}{2} + d_{k_2}, \tag{22}$$

then because of (17)

$$\partial F/\partial k_1 = (F_{y_2} - F_{y_1})\frac{\tau}{2} - \frac{b^{-2}}{e_1 + e_2} k_1, \tag{23}$$

$$\partial F/\partial k_2 = (F_{y_2} - F_{y_1})\frac{1}{2}. \tag{24}$$

This means that

$$k_1 = 0, \tag{25}$$

or y does not depend on τ,

$$F = a \int_{y_1}^{y_2} (l^2 - x^2)^{1/2} dx + \frac{1}{2} \sum (e_i^{-1}(l^2 - y_i^2) + e_i m_i^2) \tag{26}$$

and from (17) and (24)

$$F_{y_i} = 0, \quad i = 1, 2. \tag{27}$$

Putting e_i satisfying $F_{e_i} = 0$ into (26) we get

$$F = a \int_{y_1}^{y_2} (l^2 - x^2)^{1/2} dx + \sum m_i (l^2 - y_i^2)^{1/2} \tag{28}$$

with y_i satisfying (27), so that

$$(-1)^i y_i = (l^2 + (m_i/2a)^2)^{1/2} - (m_i/2a). \tag{29}$$

Calculating the momenta p and π canonically conjugate to r and q

$$p = -\partial \mathcal{L}/\partial \dot{r}, \quad \pi = -\partial \mathcal{L}/\partial \dot{q} \tag{30}$$

we get three constraints $\phi_i = 0, \quad i = 1, 2, 3$, where the constraint functions are

$$\phi_1 = pq, \qquad \phi_2 = \pi q, \tag{31}$$

$$\phi_3 = L - K. \tag{32}$$

Here

$$L = ((q^2 - (qp)^2/p^2)\pi^2)^{1/2} \tag{33}$$

is the magnitude of the conserved orbital spin

$$L_\mu = \epsilon_{\mu\nu\rho\sigma} p^\nu M^{\rho\sigma}/2m, \tag{34}$$

where

$$M^{\mu\nu} = r^{[\mu} p^{\nu]} + q^{[\mu} \pi^{\nu]} \tag{35}$$

is the angular momentum tensor. K is a function of $m = (p^2)^{1/2}$, implicitly given by the equations

$$K = lF_l - F, \tag{36}$$

$$F_l = m. \tag{37}$$

The rotator Hamiltonian is a linear combination of the constraint functions

$$H = \sum_{i=1,2,3} c_i \phi_i. \tag{38}$$

It determines the dynamical equations for any variable X

$$\dot{X} = \{X, H\}, \tag{39}$$

$\phi_i = 0$ after calculating the brackets and the non-zero Poisson brackets are

$$\{p^\mu, r^\nu\} = \{q^\mu, \pi^\nu\} = g^{\mu\nu}. \tag{40}$$

We can choose gauge conditions to fix c_i in (38), or we can describe our symmetrical system by invariants of the symmetry

$$p, \quad r_0 = r + (p\pi)q/p^2, \quad v = (-q_p^2)^{-1/2} q_p, \quad L \tag{41}$$

($q_p^\mu = (g^{\mu\nu} - p^\mu p^\nu/p^2) q_\nu$) which have zero Poisson brackets with $\phi_{1,2}$. To have zero brackets of the external coordinate of the rotation center r_0 with the internal coordinates v and L we introduce four orthonormal vectors $e_\alpha, \alpha = 0, 1, 2, 3$

$$e_0 = p/m, \quad e_\alpha e_\beta = g_{\alpha\beta} \tag{42}$$

and define new variables

$$n^a = -e_a v, \quad L^a = -e_a L, \quad a = 1, 2, 3, \tag{43}$$

$$z = r_0 + \frac{1}{2} \epsilon_{abc} e_{a\nu} \frac{\partial e_b^\nu}{\partial p} L^c. \tag{44}$$

The non-zero Poisson brackets of the new variables are

$$\{p^k, z^l\} = g^{kl}, \quad \{L^a, L^b\} = \epsilon_{abc} L^c, \quad \{L^a, n^b\} = \epsilon_{abc} n^c. \tag{45}$$

The constraint function ϕ_3 in the new variables is

$$\phi_3 = ((L^a)^2)^{1/2} - K(m) \tag{46}$$

and the solution of the dynamical equations (38) has the form

$$z = z_0 + lVp/m, \tag{47}$$

$$n = n_0 \cos V - n_1 \sin V, \tag{48}$$

$$V = \int c_3 d\tau. \tag{49}$$

»From (47) the laboratory time of the rotation center

$$t = z^0 - z^0_0 = lVp^0/m \tag{50}$$

and the space coordinates of this point

$$z^a = z^a_0 + p^a t/p^0 \tag{51}$$

correspond to its movement in the laboratory with constant velocity p^a/p^0. The direction of the rotator rotates with constant angular velocity

$$\omega = \frac{m}{p^0 l}, \tag{52}$$

where $l = l(m)$ from (37).

The canonical quantization can now be performed quite easily. We replace our variables by operators and their Poisson brackets (45) by commutators. The constraint equation now holds for the wave function

$$[((L^a)^2)^{1/2} - K(m) - a_0]\psi = 0, \tag{53}$$

where in the operator form of (46) we have added a constant a_0 of order $\hbar$, which can always be done and in our case helps make the model better at small L's. So our model has two free constants, a and a_0, the latter being of phenomenological nature.

Our quantum system is relativistic because the quantization procedure transforms the classical Poisson brackets of p^μ and $M^{\nu\sigma}$ into commutators without any change in their form, so that the Pioncaré algebra is fully preserved.

Quark spins are important especially for small L. They were taken into account in [6,7] where the spinless-particle Lagrangians in eq.(12) were replaced by those of Berezin and Marinov [8], with the result that for the leading Regge trajectories one can simply replace the orbital spin L in (53) by the total meson spin J. For the physical eigenstates this gives

$$(J(J+1))^{1/2} = K(m) + a_0, \tag{54}$$

where we consider $J = 1, 2, ...$, while $J = 0$ demands special consideration.

The function $K(m)$ is given by eqs.(28,29,36,37). We must solve eq.(37) to find l as a function of m and put this function into (36). This can be done numerically for any quark masses. For important particular cases K can be expanded into series. For light quarks

$$y_i = \pi m_i/m \ll 1 \tag{55}$$

$$K(m) = \frac{m^2}{2\pi a}[1 - \frac{4}{3\pi}\sum y_i^{3/2}(1 - \frac{3}{20}y_i) + \frac{1}{(3\pi)^2}(\sum y_i^{3/2})^2 + O(y_i^{7/2})]. \tag{56}$$

For heavy quarks

$$D = m - m_1 - m_2 \ll m_i \tag{57}$$

$$K(m) = \frac{1}{a}(\frac{2}{3}D)^{3/2}\nu_1^{-1/2}(1 + \frac{7}{36}\frac{\nu_3}{\nu_1^2}D + O((\frac{D}{m_i})^2)), \tag{58}$$

$$\nu_n = \sum m_i^{-n}. \tag{59}$$

For light and heavy quarks

$$d = m - m_2, \quad y_1 = \frac{\pi m_1}{2d} \ll 1, \quad x_2 = \frac{2d}{\pi m_2} \ll 1, \tag{60}$$

$$\begin{aligned} K(m) &= \frac{d^2}{\pi a}\Big[1 - \frac{8}{3\pi}y_1^{3/2} - \frac{2}{\pi}x_2 + \frac{9}{\pi^2}x_2^2 - (\frac{54}{\pi^3} - \frac{7}{6\pi})x_2^3 + (\frac{270}{\pi^4} - \frac{35}{2\pi^2})x_2^4 + \\ &+ O(y_1^{5/2}) + O(y_1^{3/2}x_2) + O(x_2^5)\Big]. \end{aligned} \tag{61}$$

We see that the slope of the trajectory for mesons formed by a heavy and light quark (antiquark) is twice as large as for light-quark mesons.

Applying eq.(54) to the leading ρ - and $K^\star$-trajectories we have

$$K(m_{\rho J}) = K(m_{K^\star J}), \tag{62}$$

or, neglecting the u- and d-quark masses

$$\frac{m_s}{m_{K^\star J}} = \frac{1}{\pi}z_J^{2/3}(1 + \frac{1}{10}z_J^{2/3} + \frac{1}{18\pi}z_J + O(z_J^{4/3})), \tag{63}$$

$$z_J = \frac{3\pi}{4}(1 - \frac{m_{\rho J}^2}{m_{K^\star J}^2}). \tag{64}$$

Using the experimental data from [9] and putting into (63) for $J = 1$ $m_\rho = 768.5$ (all masses are in MeV) and $m_{K^\star} = 891.6$ we get the strange-quark mass $m_s = 220$. In the same way, for $J = 2$ ($m_{a_2} = 1318.1$, $m_{K_2^\star} = 1425.4$) we get $m_s = 234$. For $J = 3$ ($m_{\rho_3} = 1691$, $m_{K_3^\star} = 1770 \pm 10$) $m_s = 204 \pm 18$. The error in the last number comes mainly from that of the $K_3^\star$-mass. We conclude that

$$m_s = 228 \pm 5 \quad MeV \tag{65}$$

is a reasonable estimate for the strange quark mass. The error of 2% corresponds to the accuracy of calculations and, partly, to the accuracy of the model.

To check this result we can use it to calculate masses m of mesons consisting of $s\bar{s}$. For $J=1$ we get $m=1030$ ($m_\phi = 1019$), for $J=2$ $m=1480$ ($m_{f_2'} = 1525 \pm 5$) and for $J=3$ $m=1850$ ($m_{\phi_3} = 1854 \pm 7$), which can be compared with experimental masses in brackets, the biggest deviation of 3% being for $J=2$.

In eq.(62) we assumed that the constant a_0 is the same for the u-, d- and s-quarks. We shall see that this is correct up to 2%. We get the following values for the model parameters

$$a = .176, \quad 2\pi a \equiv \alpha'^{-1} = 1.11\ GeV^2, \tag{66}$$

$$a_0(q) = .88, \tag{67}$$

where q stands for light quarks. The parameter (67) corresponds to the intercept parameter (of J with the $K=0$ axis) $J_0(q) = .51$.

For the c-quark from (54)

$$K(m_{J/\psi}) = K(m_{D^\star}) = \sqrt{2} - a_0(c), \tag{68}$$

which allows one to calculate the c-quark mass through those of J/ψ and $m_{D^\star}$ and to estimate $a_0(c)$:

$$m_c = 1340, \tag{69}$$

$$a_0(c) = .72. \tag{70}$$

The corresponding value of intercept is $J_0(c) = .38$.

As a check we have calculated the $c\bar{c}$-meson mass for $J=2$ to be 3430, which is 4% smaller than the experimental mass 3556 of $\chi_{c2}(1P)$.

Quite similar, for the b-quark

$$K(m_\Upsilon) = K(m_{B^\star}) = \sqrt{2} - a_0(b) \tag{71}$$

and

$$m_b = 4550, \tag{72}$$

$$a_0(b) = .40, \tag{73}$$

the intercept being $J_0(b) = .14$.

The $b\bar{b}$-meson with $J=2$ has mass 9670, which is 2% smaller than the experimental mass 9913 of $\chi_{b2}(1P)$.

From eqs.(67,70,73) we see that a_0 decreases with quark masses. Approximating this dependence by a linear one

$$a_0(q) = .88 - .38 m_q/m_b \tag{74}$$

we see that for the u-, d- and s-quarks it remains the same within 2%.

In conclusion let us discuss, which quark masses correspond to our model. It is a quantum mechanical model of free quarks bound in mesons. If it describes an experiment and if we believe that QCD with usual perturbative renormalization procedure summed to all orders also describes experimental meson spectra, then our quark masses are the current masses entering as parameters the QCD Lagrangian.

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[10] Note. When this paper had been completed, the author became aware of the paper by V.V.Nesterenko. Z. Phys. C – Particles and Fields, 47, 111-114 (1990), where the conclusion about the absence of the radial quark motion on a straight-line string in the 3-dimensional Minkovsky space had been also made.

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