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**ON THE RENORMALIZATION-SCHEME DEPENDENCE
IN QUANTUM FIELD THEORY**

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Abstract

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Using quantum electrodynamics as an example, a dependence of physical predictions of quantum field theory in a finite perturbation theory order on the choice of renormalization scheme is studied. It is shown that On-Mass-Shell renormalization scheme is distinguished in quantum electrodynamics not only due to the agreement of the theory predictions with experimental results but also due to a set of specific theoretical properties. Thus, performing renormalization procedure in other theories, it seems reasonable to use some of On-Mass-Shell renormalization scheme prescriptions.

Аннотация

Коньчев А.В. К вопросу о схемной зависимости в квантовой теории поля: Препринт ИФВЭ 97-72. – Протвино, 1997. – 10 с., библиогр.: 10.

На примере квантовой электродинамики изучается зависимость физических предсказаний теории в конечном порядке теории возмущений от выбора ренормировочной схемы. Показывается, что схема вычитаний на массовой поверхности выделена в квантовой электродинамике не только благодаря согласию предсказаний теории с экспериментом, но и благодаря ряду особых теоретических свойств. Таким образом, выполняя процедуру перенормировок в других теориях, представляется разумным использовать некоторые из предписаний схемы вычитаний на массовой поверхности.

Introduction

It is known that renormalized expressions for the divergent Feynman diagrams in quantum field theory are defined ambiguously. The ambiguities arise due to a freedom in the choice of renormalization scheme (RS), e.g. the choice of subtraction point in momentum space in *MOM*-scheme, the choice of particular value for scale parameter μ in \overline{MS} -scheme, etc. These, ultraviolet, ambiguities are fixed in quantum electrodynamics (QED) by the use of On-Mass-Shell renormalization scheme (see e.g. [1] §110, where the physical arguments in favor of this scheme are put forward). Such a distinguished scheme is seemingly absent in quantum chromodynamics (QCD) because of the absence of free quarks and gluons (see [2] §9, §16). Usually one fixes the ultraviolet ambiguities in QCD putting scale parameter μ in \overline{MS} -scheme to be equal to the typical energy value for the process in question (see e.g. [3]). It is not well-defined quantity and one can not expect the high accuracy predictions working in such a manner. Moreover, an estimation of the uncertainty in the theory predictions is, in fact, quite arbitrary.

The aim of this work is, using QED as an example, to explore the influence of ultraviolet ambiguities on the theory predictions and show that On-Mass-Shell renormalization scheme is distinguished in QED not only due to the phenomenological reasons but also due to a number of specific theoretical properties. It seems reasonable to use some of On-Mass-Shell renormalization scheme prescriptions in other theories, e.g. in QCD.

1. General statements and definitions

Let us start with some general statements and definitions. Coefficient functions appearing in the expansion of S-matrix in powers of interaction contain, in general, products of free propagators. Free propagators are the singular distributions and their products can appear to be ill-defined on the whole space of basic functions. The corresponding integrals in momentum space, the convolutions of Fourier transforms of free propagators, seem to be ultravioletly divergent in this case. These products, or corresponding convolutions,

need an additional definition. This procedure is non-unique, the choice of renormalization scheme is equivalent to the choice of particular definition for these products (or for corresponding convolutions in momentum space).

Exact, i.e. containing all radiative corrections, Green functions calculated in two different renormalization schemes are related to each other by the Dyson transformation:

$$\Gamma(p | e, m; C) = z(C, C')\Gamma(p | e', m'; C') \quad (1)$$

Here $p = (p_1, p_2, \dots)$ are momentum arguments of Green function. e, e' and m, m' are the coupling constants and masses in renormalization schemes C and C' . Renormalization scheme is determined by a set of real parameters: $C = (c_1, c_2, \dots)$. These parameters appear as uncertain coefficients in Taylor expansions of renormalized self-energies and vertex functions and reflect the ambiguity of renormalization procedure (see [4] §27.2, §27.4, §28.2, [5] §24.1; see also formulas (5) and (11) in the following section). Relation (1) implies that Green function calculated in scheme C with coupling constant e and mass m is equal as a function of momenta, up to the factor $z(C, C')$, to the one calculated in scheme C' with coupling constant e' and mass m' . In other words, a change of renormalization scheme (a change of $C = (c_1, c_2, \dots)$ in fact) can be compensated by the change of e and m . The deduction of relation (1) can be found in §34 [4]¹.

An important remark should be made. Relation (1) is valid for exact Green functions only. It is not valid in any finite order of perturbation theory. This leads to the dependence of physical predictions in a finite order of perturbation theory on the choice of renormalization scheme. One can often meet with a statement that calculation of higher-order encounter corrections reduces RS dependence. This is, in general, a wrong statement. Conversely, higher-order corrections introduce additional uncertainties into a final answer. Let one have calculated Green functions and S-matrix elements up to the n -th order of perturbation theory so that all singular expressions in the second, third, ... n -th orders are already defined. Obtaining renormalized expressions for the Feynman diagrams of the $n+1$ -th order, one has to use the prescriptions adopted in low orders to define the divergent subgraphs of these diagrams. This statement is equivalent to the following one in counterterm approach: subdivergencies of the $n + 1$ -th order diagrams are removed by the counterterms of low orders. However, the expressions for Feynman diagrams of the $n+1$ -th order contain their own divergencies. These divergencies arise when all integration momenta tend to infinity. To remove these divergencies one needs the $n + 1$ -th order counterterms. These counterterms are, of course, of the same operator type as the counterterms of low orders but have independent coefficients². Thus, calculation of each additional perturbation theory order increases the number of free renormalization parameters and, hence, the uncertainty of the total result.

An apparent contradiction with relation (1), i.e. with the actual absence of RS dependence for exact Green functions, is due to the fact that the passage to the limit of exact

¹The parameters m and m' are called masses arbitrarily. In general, they are not equal to the physical mass M , the pole position of renormalized propagator. Like any other observable quantity, M is a function of the theory parameters and renormalization scheme: $M = M(e, m; C)$.

²In coordinate representation one needs to define additionally the expression for $S_{n+1}(x_1, \dots, x_{n+1}) = i^{n+1} T(L(x_1) \dots L(x_{n+1}))$ when all the arguments are equal to each other (see [4] §29.2).

Green functions is performed *formally* when relation (1) is obtained. The higher-order corrections partially reduce the uncertainties induced by low orders but introduce new uncertainties which are partially compensated by the further higher-order corrections and so on. The validity of relation (1) for the formally obtained exact Green functions containing *all* radiative corrections does not mean that RS dependence is being reduced when increase but still finite number of higher-order corrections is being taken into account³.

Nevertheless, relation (1) is useful for theoretical investigations. Using it, one can obtain renormalization group (RG) equations and demand that Green functions of the theory satisfy these equations while the terms of low orders in expansions of these Green functions in powers of interaction coincide with the results of perturbation theory. One can expect that such a RG-invariant Green functions will exhibit improved approximating properties. However, this procedure does not remove RS dependence since using perturbative approximations calculated in different renormalization schemes to construct RG-invariant expressions one obtains, in general, different RG-invariant results (see [4] §48.5)⁴. Thus RG invariance is not equivalent to RS independence. The only way to avoid uncertainties in such a situation is to find convincing arguments in favor of the particular renormalization prescription.

S-matrix elements are obtained from Green functions by means of reduction formula:

$$S(p_1 \cdots p_n; p'_1 \cdots p'_m) = \prod_1^n \{z_k^{-1/2} f^+(p_k) \Delta^{-1}(p_k)\} \prod_1^m \{z_l^{-1/2} f^-(p'_l) \Delta^{-1}(p'_l)\} \times \\ \times G^{(m+n)}(p_1 \cdots p_n; p'_1 \cdots p'_m) \quad (2)$$

(see e.g. [6] ch.4 §9). Here f^- and f^+ are wave functions of initial and final particles, $\Delta^{-1}(p)$ is the inverse free propagator with physical mass, e.g. $\Delta^{-1}(p) = p^2 - M^2$ for the scalar particle. Factor $z^{-1/2}$ is the square root of the pole residue of renormalized propagator, $z = (p^2 - M^2)G^{(2)}(p) |_{p^2=M^2}$ for the scalar particle. The presence of $z_{k,l}^{-1/2}$ factors is an an important peculiarity of formula (2). Their origin is related to the fact that when $z \neq 1$, wave functions of external particles are effectively renormalized. A probability of transition from the initial state to the final one is related to the S-matrix element by the formula

$$W_{i \rightarrow f} = \frac{|\langle \Phi_f | S | \Phi_i \rangle|^2}{\langle \Phi_f | \Phi_f \rangle \langle \Phi_i | \Phi_i \rangle}. \quad (3)$$

It is convenient to remove factors $z_{k,l}^{1/2}$ from the denominator of formula (3) to reduction formula (2). For wave functions f^- and f^+ in formula (2) conventional normalization remains.

³To illustrate the situation let us consider function f determined by the formal series:

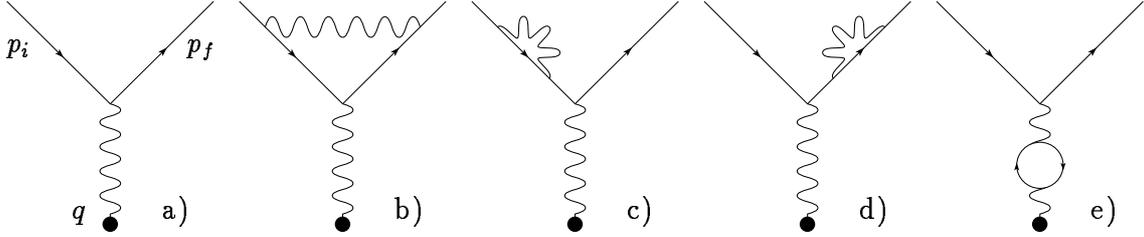
$$f(a_1, \cdots, a_n, \cdots) = a_1 + (a_2 - \frac{a_1}{2}) + (a_3 - \frac{a_2}{2} - \frac{a_1}{4}) + \cdots + (a_n - \frac{a_{n-1}}{2} - \cdots - \frac{a_{n-k}}{2^k} - \cdots - \frac{a_1}{2^{n-1}}) + \cdots$$

Truncation of this series at any finite order leads to the non-trivial dependence of function f on its arguments. However, being considered as the sum of actually infinite series, it does not depend on them.

⁴Recall that the coefficient of term proportional α_s^4 in the β -function expansion in powers of α_s in massless QCD depends on the choice of renormalization schemes. If quark masses are not equal to zero, even the coefficient of term proportional α_s^2 is RS-dependent. As a consequence, the expression for invariant coupling constant also appears to be RS-dependent.

2. RS dependence in perturbation theory

Let us turn now to a detailed consideration of the RS dependence in finite order of perturbation theory. As an example, the scattering of an electron from the external source in the third order of expansion in powers of interaction constant and in the first one of external field will be considered (see also [7] ch.15 §3). Being simple enough, this example makes it possible to observe in detail the influence of ultraviolet ambiguities on the physical predictions of the theory. Diagrams corresponding to the process are shown in the figure:



Renormalization of electron mass

Diagrams c) and d) make the contribution to the renormalization of the electron mass parameter. The following expression corresponds to diagram c)⁵:

$$M_c = \bar{u}(p_f) e \gamma^\mu A_\mu \frac{1}{\hat{p}_i - m} \Sigma(p_i) u(p_i), \quad (4)$$

where

$$\Sigma(p) = \frac{\alpha}{4\pi^3 i} \int \gamma^\nu \frac{\hat{p} - \hat{k} + m}{(p-k)^2 - m^2 + i\epsilon} \gamma^\nu \frac{dk}{k^2 + i\epsilon}.$$

Analogous expression corresponds to diagram d). $\Sigma(p)$, the electron self-energy, is determined up to two real coefficients (see [4] §27.2, §35.2):

$$\Sigma(p) = c_0 + c_1(\hat{p} - m) + \tilde{\Sigma}(p). \quad (5)$$

One can choose the following normalization for $\tilde{\Sigma}(p)$:

$$\tilde{\Sigma}(m) = 0, \quad \left. \frac{d\tilde{\Sigma}(p)}{d\hat{p}} \right|_{\hat{p}=m} = 0, \quad (6)$$

⁵Factor $2\pi i$ common for diagrams a) — e) is omitted.

where $\frac{d\tilde{\Sigma}(p)}{d\hat{p}} = \frac{\partial\tilde{\Sigma}(\hat{p}, p^2)}{\partial\hat{p}} + 2\hat{p}\frac{\partial\tilde{\Sigma}(\hat{p}, p^2)}{\partial p^2}$. Let us substitute (5) into expression (4):

$$M_c = \bar{u}(p_f)e\gamma^\mu A_\mu \frac{c_0}{\hat{p}_i - m} u(p_i) + \bar{u}(p_f)e\gamma^\mu A_\mu \frac{c_1}{\hat{p}_i - m} (\hat{p}_i - m)u(p_i) + \\ + \bar{u}(p_f)e\gamma^\mu A_\mu \frac{1}{\hat{p}_i - m} \tilde{\Sigma}(p_i)u(p_i). \quad (7)$$

If one supposes that $u(p_i)$ satisfies the equation $(\hat{p}_i - m)u(p_i) = 0$, the first term in the r.h.s. of equation (7) will contain a singularity when $c_0 \neq 0$. One can not calculate the total transition amplitude by the direct summation of diagrams a) — e) if $c_0 \neq 0$. However, the $c_0 \neq 0$ condition can be treated in the following manner. Let us consider electron propagator containing appropriate radiative corrections:

$$G^{(2)} = \frac{1}{\hat{p} - m} + \frac{1}{\hat{p} - m} \Sigma(p) \frac{1}{\hat{p} - m} = \frac{1}{\hat{p} - m} \left(1 + \frac{\Sigma(p)}{\hat{p} - m} \right) \approx \frac{1}{\hat{p} - m} \left(\frac{1}{1 - \frac{\Sigma(p)}{\hat{p} - m}} \right) = \\ = \frac{1}{\hat{p} - m - \Sigma(p)} = \frac{1}{\hat{p} - m - c_0 - c_1(\hat{p} - m) - \tilde{\Sigma}(p)}. \quad (8)$$

One sees from the r.h.s. of equation (8) that if $c_0 \neq 0$, the denominator is not equal to zero when $\hat{p} = m$. Thus, parameter m no longer coincides with physical mass M . The $u(p)$ should satisfy the equation $(\hat{p} - M)u(p) = 0$. To obtain the total transition amplitude when $c_0 \neq 0$, one should calculate vertex function $\Gamma^\mu = \gamma^\mu + \Lambda^\mu(p_i, p_f; m)$, propagators $G^{(2)}$ (see equation (8)) and $D_{\mu\nu}^{(2)}$ ⁷

$$D_{\mu\nu}^{(2)} = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \frac{1}{q^2 - q^2 \Pi(q^2; m)} + \frac{1}{q^2} \frac{q_\mu q_\nu}{q^2}. \quad (9)$$

Then, one should construct full (connected) Green function $G^{(3)} = G^{(2)} e \Gamma^\mu G^{(2)} D_{\mu\nu}^{(2)}$ and use reduction formula (2). Thus one obtains

$$M_{total} = \bar{u}(p_f) e \left(\gamma^\mu + \Lambda^\mu(p_i, p_f; m) \right) u(p_i) \frac{1}{1 - \Pi(q^2; m)} A_\mu. \quad (10)$$

Factors arising when the pole residues of propagators are different from unity are temporarily omitted in formula (10). It is important here that $\Lambda^\mu(p_i, p_f; m)$ and $\Pi(q^2; m)$ depend on parameter m rather than physical mass M . Thus, the total result contains the dependence on arbitrary parameter m . In contrast to the case of exact transition

⁶ $\Sigma(p)$ contains an infrared regulator. The general form for $\Sigma(p)$ is $\Sigma(\hat{p}, p^2) = F_0(p^2) + \hat{p}F_1(p^2)$, where F_0 and F_1 are the some functions. Expression (5) is equivalent to the fact that F_0 and F_1 are known up to the independent of p^2 additive values. Renormalization scheme is determined by the particular choice of c_0 and c_1 . It should be recognized that in perturbation theory order being considered, the expression for finite part of electron self-energy obtained in *any* renormalization scheme (\overline{MS} , MOM , etc.) can be reduced to the form (5), where $\tilde{\Sigma}(p)$ is normalized according to, e.g. (6).

⁷Function $\Pi(q^2; m)$ is related to the polarization operator $\Pi^{\mu\nu}(q^2; m)$ by the formula $\Pi^{\mu\nu}(q^2; m) = (g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) q^2 \Pi(q^2; m)$. The explicit expressions for $\Pi(q^2; m)$ and $\Lambda^\mu(p_i, p_f; m)$ see in §27, 28 [4].

amplitude, a change of parameter m in expression (10) can not be compensated by the change of renormalization scheme. A predictive power of the theory is lost.

If one puts $c_0 = 0$ (the requirement of On-Mass-Shell renormalization scheme), calculations can be performed both by the first method (the direct summation of diagrams a) — e)) and by the second one (the construction of full Green function followed by the application of reduction formula (2)). The results agree with each other up to the higher-order corrections.

It is to be noted that imaginary parts of $\Pi(q^2; m)$ and $\Sigma(p; m)$ are ultravioletly finite. Imaginary part of $\Pi(q^2; m)$ is different from zero when $q^2 > 4m^2$. Thus, the threshold for real electron-positron pair production is determined by the parameter m which, hence, should be equal to physical mass M . In this case the renormalized S-matrix will be unitary in each perturbation theory order. Otherwise, the exact S-matrix only will be unitary whereas in finite orders the unitarity will be broken.

Renormalization of electron propagator pole residue

Let us turn now to the consideration of the second term in the r.h.s. of equation (7). In what follows, to avoid difficulties arising when $m \neq M$, c_0 in expression (5) will be put to be equal to zero, i.e. $\Sigma(m) = 0$. The second term in r.h.s. of (7) contains an uncertainty: one can act by the operator $(\hat{p}_i - m)$ on the spinor $u(p_i)$ and obtain zero, on the other hand, one can "cancel" operators $(\hat{p}_i - m)$ in the numerator and the denominator and obtain the expression $c_1 \bar{u}(p_f) e \gamma^\mu A_\mu u(p_i)$. To resolve this uncertainty one can use the adiabatic hypothesis (for more details, see [7] ch.15 §3). As a result, one obtains for the second term the expression $\frac{1}{2} c_1 \bar{u}(p_f) e \gamma^\mu A_\mu u(p_i)$ ⁸. Factor $\frac{1}{2}$ is of great importance. Owing to gauge invariance of quantum electrodynamics, Λ^μ and Σ are related by the Ward identity:

$$\Lambda^\mu(p, p) = -\frac{\partial \Sigma(p)}{\partial p_\mu}.$$

An ambiguous renormalization coefficient c_2 in Λ^μ is therefore not independent of c_1 . If one writes Λ^μ as

$$\Lambda^\mu(p_i, p_f) = \tilde{\Lambda}^\mu(p_i, p_f) - c_2 \gamma^\mu, \quad (11)$$

where

$$\bar{u}(p) \tilde{\Lambda}^\mu(p, p) u(p) = 0 \quad (12)$$

then $\tilde{\Lambda}^\mu(p, p) = -\frac{\partial \tilde{\Sigma}(p)}{\partial p_\mu}$ where $\tilde{\Sigma}(p)$ is normalized according to (6) and, hence, $c_2 = c_1 = c$. Taking into account factor $\frac{1}{2}$ for diagrams c) and d) and calculating the sum of b)⁹, c) and d), one sees that terms containing c cancel from the total amplitude. Thus the total

⁸The question of the external lines renormalization of Feynman diagrams is not simple. See in this connection [8]. See also the end of §38 [9].

⁹Expression $\bar{u}(p_f) e \Lambda^\mu(p_i, p_f) u(p_i) A_\mu$ corresponds to diagram b).

result is free from the ambiguities connected with renormalization of the vertex function and the electron propagator pole residue¹⁰

If one uses the second method to obtain the total amplitude, the following expression is arrived at:

$$M_{total} = \bar{u}(p_f) \sqrt{\frac{1}{1-c}} e \left(\gamma^\mu + \tilde{\Lambda}^\mu(p_i, p_f) - c\gamma^\mu \right) \sqrt{\frac{1}{1-c}} u(p_i) \frac{1}{1-\Pi(q^2)} A_\mu. \quad (13)$$

Let us rewrite (13) as

$$M_{total} = \bar{u}(p_f) e \left(\gamma^\mu + \frac{1}{1-c} \tilde{\Lambda}^\mu(p_i, p_f) \right) u(p_i) \frac{1}{1-\Pi(q^2)} A_\mu. \quad (14)$$

Due to relation (12), M_{total} is independent of c when $q^2 = 0$ ($p_i = p_f = p$). However, when $q^2 \neq 0$, M_{total} depends on c . It is quite a disagreeable fact. Coefficient c can depend on an infrared regulator and gauge parameter, e.g. this is the case if one normalizes $\Sigma(p)$ as follows: $\Sigma(m) = 0$, $\frac{\partial \Sigma(p)}{\partial \hat{p}}|_{\hat{p}=m} = 0$ (see [4] §35.2; one should take into account that normalization for $\tilde{\Sigma}(p)$ and, hence, the definition of coefficient c_1 in this paper are different from those in the reference). In \overline{MS} -scheme c depends on an infrared regulator, gauge parameter and scale parameter μ . Thus, in general, expression (14) leads to an incorrect result. However, if one puts $c = 0$ in (14) (On-Mass-Shell renormalization scheme), the result will coincide, up to the higher-order corrections, with the one obtained in the first method where there is no ambiguity due to coefficient c ¹¹

Renormalization of photon propagator pole residue

Finally, let us turn to the consideration of the ambiguity connected with normalization of the photon propagator pole residue. One can write $\Pi(q^2)$ as $\Pi(q^2) = c_3 + \tilde{\Pi}(q^2)$ where $\tilde{\Pi}(0) = 0$ ¹². In exact Green functions and S-matrix elements a change of c_3 can be compensated by the change of e . This is not the case in the finite order of perturbation theory. Let us consider the expression for the total amplitude obtained in the second method. To take into account the charge renormalization correctly, one should write A_μ

¹⁰In QCD coefficient c_1 also cancels from the total result though a simple relation $c_2 = c_1$ is not valid there (see [2] §9.2). The choice of particular normalization for c_1 is immaterial when one works in the first method.

¹¹In the pure transversal gauge (Landau gauge) ultraviolet divergencies related to the coefficients c_1 and c_2 are absent. If one writes the expression similar to (5) for $\Sigma(p)$, the coefficient before $(\hat{p} - m)$ will not be uncertain. It will depend on an infrared regulator. When one uses the first method, this coefficient cancels from the sum of diagrams b), c) and d) and, hence, from the total amplitude. (Of course, vertex function contains some other terms depending on infrared regulator.) Working in the second method, one has to demand that this coefficient should be equal to zero to obtain a reasonable result despite the absence of a freedom in its normalization in the Landau gauge. A detailed consideration of infrared divergencies is beyond the scope of this paper. It should be still emphasized once again that if one works in the first method, one has no need to take care of the particular normalization for coefficient c_1 .

¹²As in the case of $\Sigma(p)$, this is general expression for $\Pi(q^2)$ explicitly distinguishing the ambiguity contained in it.

as $A_\mu = D_{\mu\nu} e J_\nu^{ext}$ where J_ν^{ext} is the external current responsible for field A_μ , $D_{\mu\nu}$ is free photon propagator. The expression for the total amplitude is

$$M_{total} = \bar{u}(p_f) e \left(\gamma^\mu + e^2 \bar{\Lambda}^\mu(p_i, p_f) \right) u(p_i) \frac{1}{1 - c_3 - e^2 \bar{\Pi}(q^2)} e J_\mu^{ext}. \quad (15)$$

Factors e^2 contained in $\bar{\Lambda}^\mu(p_i, p_f)$ and $\bar{\Pi}(q^2)$ are explicitly distinguished in formula (15). One can rewrite (15) as

$$M_{total} = \bar{u}(p_f) \left(\gamma^\mu + e^2 \bar{\Lambda}^\mu(p_i, p_f) \right) u(p_i) \frac{1}{\frac{1-c_3}{e^2} - \bar{\Pi}(q^2)} J_\mu^{ext}. \quad (16)$$

If one were allowed to neglect term $e^2 \bar{\Lambda}^\mu(p_i, p_f)$ in (16), the expression for the total amplitude would possess the desired property. A change of c_3 would be compensated by the change of e . However, it is this term that is the dominant radiative correction when $-q^2 \gg m^2$ (see e.g. [1] §122) so that it can not be ignored.

Working in the first method and choosing the appropriate value for coefficient c_3 , one can make a contribution to the total amplitude from diagram e) at some $q^2 \neq 0$ to be equal to zero. However, there is little sense in doing so because, as it has been just noted, the sum of diagrams b), c) and d) yields the major contribution to the total amplitude when $-q^2 \gg m^2$ rather than diagram e).

The value $c_3 = 0$ (i.e. $\Pi(0) = 0$, On-Mass-Shell renormalization scheme) is distinguished. In this case *all* the radiative corrections to scattering process in question tend to zero when $q^2 \rightarrow 0$ and the process is described with tree diagram a) only. Recall that the Compton scattering of the photon by the electron in the limit of low-energy photon is also described with the tree diagrams only in this scheme. Thus the numerical value of electric charge can be easily extracted from the experiment¹³

S-matrix calculated in On-Mass-Shell renormalization scheme is unitary in each perturbation theory order whereas in other schemes the exact S-matrix only is unitary. An important property of the scheme is the absence of radiative corrections to external lines of Feynman diagrams. Effects of self-interaction already taken into account by the initial approximation no longer occur in the theory in this scheme. The first and the second methods of obtaining the transition amplitude are equivalent in finite orders of perturbation theory in On-Mass-Shell renormalization scheme only.

Additional remarks

It has been demonstrated above that due to gauge structure of quantum electrodynamics an uncertain parameter, arising when renormalization of the vertex function is performed, is related to the appropriate one of electron self-energy. If one has a fixed

¹³It is to be noted that if the numerical value of a coupling constant is extracted from large- q^2 experimental data where higher-order corrections are significant, one needs to verify the stability of this value when the number of perturbation theory orders taken into account is changed. Such a situation is typical in QCD where low- q^2 domain is experimentally inaccessible. The last fact does not mean, of course, that On-Mass-Shell renormalization scheme can not be applied there.

renormalization prescription for the electron self-energy, the prescription for the vertex function has been also fixed. In a theory without gauge symmetry, e.g. $\lambda\phi^4$ -theory, uncertain renormalization parameter of 4-point vertex function is independent of renormalization parameters of self-energy. When one has defined the renormalized self-energy according to the requirements of On-Mass-Shell renormalization scheme, a freedom in normalization for the vertex function can be used to make radiative corrections at the point where the interaction constant is measured to be equal to zero. In fact, it is possible in $\lambda\phi^4$ -theory at the point $s = 4m^2$, $t = u = 0$ (threshold point) only. At other points in physical domain an imaginary part of the vertex function is different from zero (see [5] appendix 7, §24.1). Thus, there is a distinguished renormalization scheme in $\lambda\phi^4$ -theory too¹⁴.

Conclusion

Let us summarize the main statements of this paper.

Renormalized expressions for the divergent Feynman diagrams in quantum field theory are defined ambiguously. Physical predictions in a finite order of perturbation theory depend on the choice of renormalization scheme. Calculation of higher-order corrections does not lead, in general, to the diminishing of renormalization-scheme dependence.

On-Mass-Shell renormalization scheme is distinguished in quantum electrodynamics among other possible ones not only due to the phenomenological reasons but also due to a number of specific theoretical properties. Thus, it seems reasonable to use some of On-Mass-Shell renormalization scheme prescriptions performing renormalization procedure in other theories.

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¹⁴If one uses another renormalization prescription for 4-point vertex function in $\lambda\phi^4$ -theory, a numerical value of the interaction constant corresponding to this prescription can be related to the threshold value of the interaction constant (see [4] §36.2). However, the vertex functions obtained in two different renormalization schemes *functionally* differ from one another so that their numerical values at points distinct from the point $s = 4m^2$, $t = u = 0$ (and, hence, physical predictions of the theory) are different.

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