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PERTURBATIVE FRAGMENTATION OF A SCALAR LEPTOQUARK INTO A HEAVY LEPTO-QUARKONIUM

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Abstract

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The fragmentation function of a scalar leptoquark into possible S-wave bound states with a heavy antiquark is calculated in the leading order of the perturbative QCD for the high energy processes at large transverse momenta. The one-loop equations for the q^2-evolution of moments of the fragmentation function due to the hard gluon emission by the leptoquark are derived. Integral probabilities of fragmentation are evaluated. The distribution of lepto-quarkonium over the transverse momentum with respect to the fragmentation axis is calculated in the scaling limit.

Аннотация

Киселев В.В. Пертурбативная фрагментация скалярного лептокварка в тяжелый лепто-кварконий: Препринт ИФВЭ 97-78. – Протвино, 1997. – 10 с., 5 рис., 1 табл., библиогр.: 16.

Функция фрагментации скалярного лептокварка в возможные связанные S-волновые состояния с тяжелым анти-кварком вычислена в ведущем порядке теории возмущений КХД для процессов высоких энергий при больших поперечных импульсах. Выведены однопетлевые уравнения для эволюции моментов функции фрагментации по q^2 из-за излучения лептокварком жестких глюонов. Сделаны оценки интегральных вероятностей фрагментации. Вычислено распределение лепто-кваркония по поперечному импульсу относительно оси фрагментации в скейлинговом пределе.

Introduction

An abundant discussion about a possible appearance of new physics at HERA [1,2,3] has provided an important and useful experience for a handling of almost-real properties attributed to interactions beyond the Standard Model. So, if a leptoquark [4] is the correct interpretation for the excess in the HERA data at high Q^2, x, then the former has the total width Γ, which is much less than the QCD-confinement scale, $\Gamma_{LQ} \ll \Lambda_{QCD}$ [5]. The latter fact means that before a decay, the color-triplet leptoquark LQ can be bound with quarks in the lepto-hadrons: $(\bar{q}LQ)$-baryons or $(q_1 q_2 LQ)$-mesons, which are quite exotic states as well as the double LQ-onia: $(\overline{LQ}_1 LQ_2)$.

It is attractive and interesting to study the spectroscopy, production and decays of such hadrons as a possible window of new physics independently of a success or failure of the treatment of the HERA events.

The description of leptoquarks bound with light quarks is a subject of Leptoquark Effective Theory, that can be developed as a straightforward continuation of analogous Heavy Quark Effective Theory [6] with taking care on the spin structure of leptoquark. The heavy lepto-quarkonia: $(\bar{b}LQ)$ and $(\bar{c}LQ)$, can be considered in the framework of Non-Relativistic QCD [7] keeping in mind again the spin features.

In this work we discuss the high energy production of heavy lepto-quarkonium containing a scalar leptoquark.

The model-independent pair production of free leptoquarks in hadronic collisions was considered in [8] with account for the next-to-leading order QCD corrections. Attaching the result to the Tevatron search for the scalar leptoquarks [9], the authors have found the constraint $m_{LQ} > 190$ GeV.

At high transverse momenta, the dominant production mechanism for the heavy lepto-quarkonium bound states is the leptoquark fragmentation, which can be calculated in perturbative QCD [10] after the isolation of soft-binding factor extracted from the non-relativistic potential models [11,12]. The corresponding fragmentation function is universal for any high energy process for the direct production of lepto-quarkonia.

In the leading α_s-order, the fragmentation function has a scaling form, which is the initial one for the perturbative QCD evolution caused by the emission of hard gluons by the leptoquark before the hadronization. The corresponding splitting function differs from that for the heavy quark because of the spin structure of gluon coupling to the leptoquark.

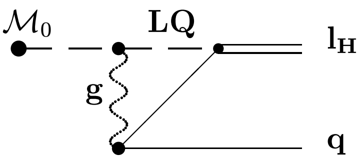

Fig. 1. The diagram of leptoquark fragmentation into the heavy lepto-quarkonium.

In this work, the LO-scaling fragmentation function is calculated in Section 1. The limit of infinitely heavy leptoquark, $m_{LQ} \to \infty$, is obtained from the full QCD consideration for the fragmentation. The distribution of the heavy lepto-quarkonium over the transverse momentum with respect to the fragmentation axis is analytically calculated to the leading order of perturbative QCD in Section 2. The splitting kernel of the DGLAP-evolution is derived in Section 3, where the one-loop equations of renormalization group for the moments of fragmentation function are obtained and solved. The mentioned equations are universal, since they do not depend on whether the leptoquark will be bound or free at low virtualities, where the perturbative evolution stops. The integrated probabilities of leptoquark fragmentation into the heavy lepto-quarkonia are evaluated in Section 4 with making the use of non-relativistic wave-functions for the bound states. The results are summarized in Conclusion.

1. Fragmentation function in leading order

The contribution of fragmentation into the direct production of heavy lepto-quarkonium has the form

$$d\sigma[l_H(p)] = \int_0^1 dz\; d\hat{\sigma}[LQ(p/z), \mu]\; D_{LQ\to l_H}(z, \mu),$$

where $d\sigma$ is the differential cross-section of lepto-quarkonium with the 4-momentum p, $d\hat{\sigma}$ is that of the hard production of leptoquark with the scaled momentum p/z, and D is interpreted as the fragmentation function depending on the fraction of momentum carried out by the bound state. The value of μ determines the factorization scale. In accordance with the general DGLAP-evolution, the μ-dependent fragmentation function satisfies the equation

$$\frac{\partial D_{LQ\to l_H}(z,\mu)}{\partial \ln \mu} = \int_z^1 \frac{dy}{y}\; P_{LQ\to LQ}(z/y, \mu)\; D_{LQ\to l_H}(y, \mu), \tag{1}$$

where P is the kernel caused by the emission of hard gluons off the leptoquark leg before the production of heavy quark pair. Therefore, the initial form of fragmentation function is determined by the diagram shown in Fig. 1, and, hence, the corresponding initial factorization scale is equal to $\mu = 2m_Q$. Furthermore, this function can be calculated as an expansion in $\alpha_s(2m_Q)$. The leading order contribution is evaluated in this Section.

Consider the fragmentation diagram in the system, where the momentum of initial leptoquark has the form $q = (q_0, 0, 0, q_3)$ and the lepto-quarkonium one is p, so that

$$q^2 = s, \quad p^2 = M^2.$$

In the static approximation for the bound state of leptoquark and heavy quark, the quark mass is expressed as $m_Q = rM$, and the leptoquark mass equals $m = (1-r)M$.

The matrix element has the form

$$\mathcal{M} = -\frac{2\sqrt{2\pi}\alpha_s}{3\sqrt{3M^3}} \frac{R(0)}{r(1-r)(s-m^2)^2} (q^\mu + (1-r)p^\mu)\rho_{\mu\nu}\, \bar{q}\gamma^\nu(\hat{p}-M)l_H\, \mathcal{M}_0, \tag{2}$$

where the sum over the gluon polarizations is written down in the axial gauge with $n = (1, 0, 0, -1)$

$$\rho_{\mu\nu}(k) = -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{k \cdot n},$$

with $k = q - (1-r)p$. The spinors of l_H and $\bar{q}$ correspond to the lepto-quarkonium and heavy quark associated to the fragmentation. $\mathcal{M}_0$ denotes the matrix element for the hard production of leptoquark at high energy, $R(0)$ is the radial wave-function at the origin.

Define

$$z = \frac{p \cdot n}{q \cdot n}.$$

The fragmentation function is determined by expression [13]

$$D(z) = \frac{1}{16\pi^2} \int ds \theta\Big(s - \frac{M^2}{z} - \frac{m_Q^2}{1-z}\Big) \frac{|\mathcal{M}|^2}{|\mathcal{M}_0|^2},$$

in the limit of high energies $q \cdot n \to \infty$. Then one can straightforwardly find

$$D(z) = \frac{8\alpha_s^2}{27\pi} \frac{|R(0)|^2}{M^3 r^2 (1-r)^2} \frac{z^2(1-z)^2}{(1-(1-r)z)^6} [(1+r^2)(1+(1-r)^2 z^2) - 2(1-r)^2(1+r)z], \tag{3}$$

which tends to

$$\tilde{D}(y) = \frac{8\alpha_s^2}{27\pi} \frac{|R(0)|^2}{m_Q^3} \frac{(y-1)^2}{r} \Big(\frac{4}{y^6} + \frac{1}{y^4}\Big), \tag{4}$$

at $r \to 0$ and $y = (1-(1-r)z)/(rz)$. The coefficient at the $(y-1)^2$ term is the same as in the fragmentation of heavy quark with the mass m into the S-wave states of the heavy quarkonium at $y \to 1$, if one excepts the factor related with the wave-function of final state. The limit of $\tilde{D}(y)$ is in agreement with the general consideration of $1/m$-expansion for the fragmentation function [14], where

$$\tilde{D}(y) = \frac{1}{r} a(y) + b(y).$$

Eq.(4) determines the $a(y)$-function explicitly.

2. Transversal momentum of lepto-quarkonium

In the system with the infinitely large initial momentum of the leptoquark, its invariant mass is expressed through both the fraction of longitudinal momentum carried out by the lepto-quarkonium, z, and the transverse momentum with respect to the axis of fragmentation, p_T, in the following form (see Fig. 1)

$$s = m^2 + \frac{M^2}{z(1-z)}[(1-(1-r)z)^2 + t^2],$$

where $t = p_T/M$. The calculation of diagram in Fig. 1 gives the double distribution for the fragmentation probability

$$\frac{d^2P}{ds\, dz} = \mathcal{D}(z,s),$$

where the function $\mathcal{D}$ has the following form

$$\mathcal{D}(z,s) = \frac{16\alpha_s^2}{27\pi}\,\frac{|R(0)|^2}{r^2(1-r)^2}\,\frac{M^3}{(1-(1-r)z)^2(s-m^2)^4}\Big\{(1-r)[1+r-2(1+r^2)z + (1+r)(1-r)^2z^2]\,\frac{s-m^2}{M^2} - z(1-z)\Big(\frac{s-m^2}{M^2}\Big)^2\Big\}. \qquad (5)$$

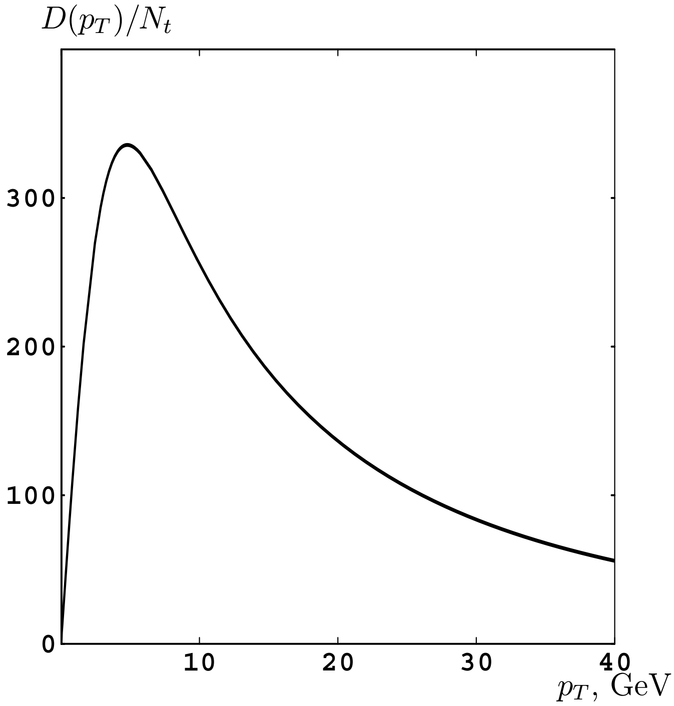

Fig. 2. The distribution over the transverse momentum with respect to the axis of leptoquark fragmentation into the heavy lepto-quarkonium, the factor N_t is defined by the expression: $N_t = \frac{4\alpha_s^2}{27\pi}\,\frac{|R(0)|^2}{M^4 r^2(1-r)^7}$, for the $(\bar{b}LQ)$-state with $r = 0.02$.

Then one can easily see, that the distribution over the transverse momentum can be obtained by the integration over z

$$D(t) = \int_0^1 dz\, \mathcal{D}(z,s)\,\frac{2M^2 t}{z(1-z)}.$$

Thus, one explicitely has

$$D(t) = \frac{4\alpha_s^2}{27\pi}\,\frac{|R(0)|^2}{r^2(1-r)^7M^3}\,\frac{1}{t^6}\Big\{(1-r)t[60r^3 + r(15-12r-11r^2)t^2 - (1+3r^2)t^4] +$$

$$[-60r^4 - 3r^2(1-24r-13r^2)t^2 + (3+12r+16r^2+12r^3+r^4)t^4 +$$
$$(1+3r^2)t^6] \operatorname{arctg}\Big(\frac{(1-r)t}{r+t^2}\Big) +$$
$$8rt[2r^2(2+3r)+(1+r)t^2] \ \ln\Big(\frac{r^2(1+t^2)}{r^2+t^2}\Big)\Big\}. \tag{6}$$

At large transverse momenta of the lepto-quarkonia, $t \to \infty$, the distribution $D(t)$ decreases as $1/t^3$

$$D(t) = \frac{32\alpha_s^2}{81\pi} \frac{|R(0)|^2}{r^2(1-r)^7 M^3} \frac{1}{t^3} (1+9r-9r^2-r^3+6r(1+r)\ln r). \tag{7}$$

At low transverse momenta, $t \to 0$, one has

$$D(t) = \frac{32\alpha_s^2}{27\pi} \frac{|R(0)|^2}{210 r^5(1-r)^2 M^3}\, t\,(8-2r+r^2). \tag{8}$$

An ordinary form of the distribution over the transverse momentum of lepto-quarkonium with respect to the axis of leptoquark fragmentation is shown in Fig. 2.

3. Hard gluon emission

The one-loop contribution of hard gluon emission can be calculated in the way described in the previous sections. Then the splitting kernel of the leptoquark is equal to

$$P_{LQ\to LQ}(x,\mu) = \frac{4\alpha_s(\mu)}{3\pi}\left[\frac{2x}{1-x}\right]_+, \tag{9}$$

where the "plus" denotes the ordinary action: $\int_0^1 dx f_+(x)\cdot g(x) = \int_0^1 dx f(x)\cdot[g(x)-g(1)]$. The scalar leptoquark splitting function can be compared with that of the heavy quark

$$P_{Q\to Q}(x,\mu) = \frac{4\alpha_s(\mu)}{3\pi}\left[\frac{1+x^2}{1-x}\right]_+,$$

which has the same normalization factor at $x \to 1$.

Further, multiplying the evolution equation by z^n and integrating over z, one can get from eq.(1) the μ-dependence of moments $a_{(n)}$ for the fragmentation function to the one-loop accuracy of renormalization group,

$$\frac{\partial a_{(n)}}{\partial \ln\mu} = -\frac{8\alpha_s(\mu)}{3\pi}\left[\frac{1}{2}+\ldots+\frac{1}{n+1}\right] a_{(n)}, \quad n \geq 1. \tag{10}$$

At $n=0$ the right hand side of (10) equals zero, which means that the integral probability of leptoquark fragmentation into the heavy lepto-quarkonium does not change during the evolution, and it is determined by the initial fragmentation function calculated perturbatively in the previous section.

The solution of eq.(10) has the form

$$\frac{a_{(n)}(\mu)}{a_{(n)}(\mu_0)} = \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\gamma_n}, \tag{11}$$
$$\gamma_n = \frac{16}{3\beta_0}\left[\frac{1}{2} + \ldots + \frac{1}{n+1}\right],$$

where one has used the one-loop expression for the QCD coupling constant

$$\alpha_s(\mu) = \frac{2\pi}{\beta_0 \ln(\mu/\Lambda_{QCD})},$$

where $\beta_0 = 11 - 2n_f/3$ with n_f being the number of quark flavors with $m_q < \mu < m_{LQ}$.

Relation (11) is the universal one, since it is independent whether the leptoquark is free or bound at the virtualities less than μ_0. In this work we include the evolution for the fragmentation into the heavy lepto-quarkonium.

As one can see in Fig. 3, the leptoquark can lose about 20% of its momentum before the hadronization.

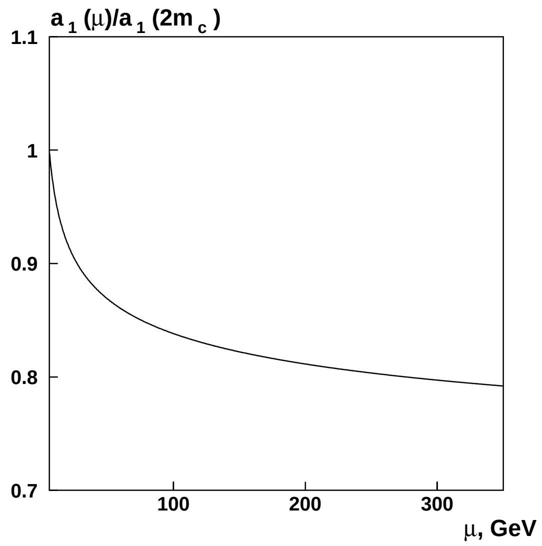

Fig. 3. The QCD-evolution for the averaged fraction of scalar leptoquark momentum, as it is developed in the fragmentation with account for the gluon emission to the scale μ, characterizing the hard production of leptoquark, from the initial value chosen $\mu_0 = 2m_c$.

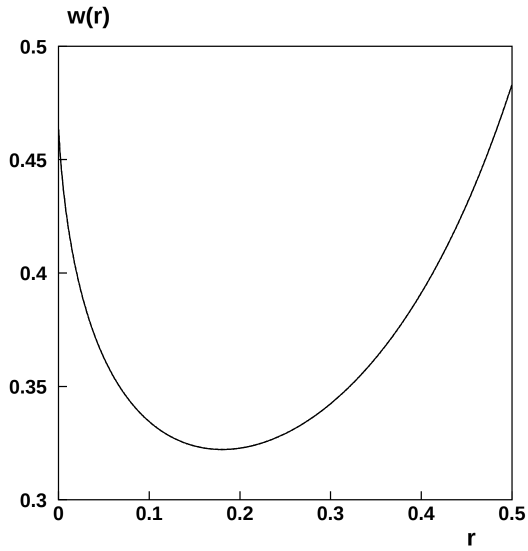

Fig. 4. The w-function for the leptoquark fragmentation into the heavy lepto-quarkonium versus the fraction of leptoquark mass, $r = m_Q/m$.

4. Integral probabilities of fragmentation

As has been mentioned above, the evolution conserves the integral probability of fragmentation, which can be calculated explicitly from eq.(3)

$$\int dz\ D(z) = \frac{8\alpha_s^2}{27\pi}\frac{|R(0)|^2}{m_Q^3}\,\mathrm{w}(r), \tag{12}$$

$$\mathrm{w}(r) = \frac{[7+30r+20r^2+20r^3-75r^4-2r^5+30r(1+r+3r^2+r^3)\ln r]}{15(1-r)^7}. \tag{13}$$

The function of $\mathrm{w}(r)$ is shown in Fig. 4 at low r.

To estimate numerically the yield of $(\bar{b}LQ)$ and $(\bar{c}LQ)$, one has to evaluate the radial wave function at the origin from the non-relativistic potential models.

Table 1. The radial wave-functions at the origin, level energies and average sizes of heavy lepto-quarkonia, as evaluated in the Martin potential.

Level	$R(0)$, GeV$^{3/2}$	E, GeV	$\langle r\rangle$, fm
1S($\bar{c}LQ$)	8 1.61	1.023	0.27
2S($\bar{c}LQ$)	1.20	1.606	0.58
1S($\bar{b}LQ$)	3.43	4.039	0.16
2S($\bar{b}LQ$)	2.56	4.594	0.35

As was found in [15,16], the Martin and Buchmüller–Tye potentials give close results for the heavy quarkonia with the reduced mass about $m_c \sim 1.5$ GeV. The characteristics of charmed and beauty lepto-quarkonia are presented in Tab.1, where we have used the Martin potential in the limit of infinitely heavy leptoquark, $m \gg m_Q$, so that the reduced mass equals the heavy quark mass, and the level energy is given by the sum of quark mass and the binding energy evaluated numerically from the Schrödinger equation.

Then one finds for the probabilities of leptoquark fragmentation into the 1S-states with b and c-quarks: $P_w(b) = 1.22\cdot 10^{-4}$ and $P_w(c) = 2.16\cdot 10^{-3}$, respectively, at $m_b = 4.9$ GeV, $m_c = 1.5$ GeV, $\alpha_s(2m_b) = 0.18$, $\alpha_s(2m_c) = 0.26$ and $m = 245$ GeV.

The perturbative fragmentation function in the leading α_s-order is shown in Fig. 5 at $r = 0.02$. It is quite a hard distribution, which becomes softer with the evolution (see Fig. 5).

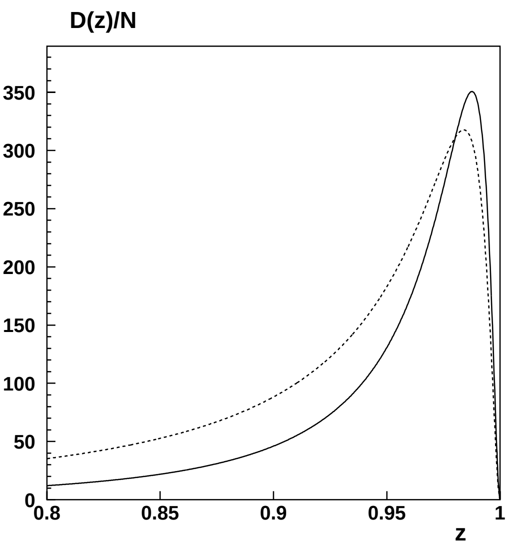

Fig. 5. The fragmentation function of leptoquark into the heavy lepto-quarkonium, the N-factor is determined by $N = \frac{8\alpha_s^2}{27\pi}\frac{|R(0)|^2}{M^3r^2(1-r)^2}$, the initial function – solid line, for the $(\bar{b}LQ)$-state with $r = 0.02$, the fragmentation function including the evolution – dashed line, at the scale μ: $\frac{8\alpha_s}{3\pi}\ln\frac{\mu}{\mu_0} = 0.25$.

Conclusion

In this paper the dominant mechanism for the production of possible bound states of a scalar leptoquark with a heavy anti-quark is considered for high energy processes at large transverse momenta, where the fragmentation contributes as the leading term.

The corresponding fragmentation function of scalar leptoquark into the heavy lepto-quarkonium can be calculated in the perturbative QCD, so that for the S-wave states one finds

$$D(z) = \frac{8\alpha_s^2}{27\pi}\,\frac{|R(0)|^2}{M^3 r^2(1-r)^2}\,\frac{z^2(1-z)^2}{(1-(1-r)z)^6}\,[(1+r^2)(1+(1-r)^2z^2) - 2(1-r)^2(1+r)z],$$

where r is the ratio of heavy quark mass to the mass of the bound state. In the infinitely heavy leptoquark limit, $D(z)$ has the form, which agrees with what is expected from the general consideration of $1/m$-expansion for the fragmentation functions.

The distribution over the transverse momentum of lepto-quarkonium with respect to the axis of leptoquark fragmentation can be calculated in the analytic form to the leading oreder of perturbative QCD, so that

$$\begin{aligned} D(t) &= \frac{4\alpha_s^2}{27\pi}\,\frac{|R(0)|^2}{r^2(1-r)^7M^3}\,\frac{1}{t^6} \\ &\Big\{(1-r)t[60r^3 + r(15-12r-11r^2)t^2 - (1+3r^2)t^4] + \\ &[-60r^4 - 3r^2(1-24r-13r^2)t^2 + (3+12r+16r^2+12r^3+r^4)t^4 + \\ &(1+3r^2)t^6]\,\mathrm{arctg}\Big(\frac{(1-r)t}{r+t^2}\Big) + \\ &8rt[2r^2(2+3r)+(1+r)t^2]\,\ln\Big(\frac{r^2(1+t^2)}{r^2+t^2}\Big)\Big\}, \end{aligned}$$

where $t = p_T/M$.

The hard gluon corrections caused by the splitting of scalar leptoquark are taken into account so that the evolution kernel has the form

$$P_{LQ\to LQ}(x,\mu) = \frac{4\alpha_s(\mu)}{3\pi}\left[\frac{2x}{1-x}\right]_+,$$

which results in the corresponding one-loop equations for the moments of fragmentation function (see Eqs.(10), (11)).

The integral probabilities of scalar leptoquark fragmentation into the charmed and beauty lepto-quarkonia are of the order of 10^{-3} and 10^{-4}, correspondingly.

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