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TOWARDS A QUANTIZATION OF FOUR-FERMION INTERACTION

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Abstract

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Four-fermion interaction is considered in the representation of auxiliary field. It is shown that the conformal anomaly results in the emergence of kinetic term of the auxiliary field, the conventional compositeness conditions being broken. This kinetic term makes it possible to construct the renormalizable perturbation theory on the basis of the four-fermion interaction Lagrangian.

Аннотация

Куликов А.В. К квантованию четырехфермионного взаимодействия: Препринт ИФВЭ 98-12. – Протвино, 1998. – 4 с., библиогр.: 5.

Четырехфермионное взаимодействие рассматривается в представлении вспомогательного поля. Показано, что конформная аномалия приводит к появлению кинетического члена у вспомогательного поля. Этот член разрушает обычные условия композитности и делает возможным построение перенормированного ряда теории возмущений.

The concept of quantization in field theory implies the procedure that can be broken down into two subsequent stages. The first stage is called canonical quantization. At this stage, quantum canonical variables, space of states etc. are conventionally constructed. In most cases of practical significance, there are no difficulties at the first stage.

The second stage involves constructing the S-matrix (or the generating functional) and the like. The main problem is the infinities stemming from the definition of the S-matrix as the time-ordered exponential of the classical Lagrangian. The elimination of the infinities (in perturbation heory) consists in adding the counterterms to the classical Lagrangian with

$$L_{cl} \to L_{qu} = L_{cl} + \sum Z_i \Delta L_i, \tag{1}$$

where the counterterms are calculated by employing the R-operation.

The sum in formula (1) can be either finite or infinite. In the former case the theory is called renormalizable, theories of this type are widely used. In the latter case the theory is called nonrenormalizable. In this case, the calculation of counterterms cannot be completed and thus the quantization (in the sense as it was defined above) cannot be performed. Its limitation hinders a wide usage of such theories.

The question arises of whether the theory with four-fermion interaction is renormalizable. Let us consider the perturbation expansion in the theory with the simplest Lagrangian

$$L_\Psi = \bar{\Psi} i \partial \Psi + \frac{G}{2}(\bar{\Psi}\Psi)^2, \quad \partial = \gamma^\mu \partial_\mu. \tag{2}$$

Making use of counting rules for the diagrams in the perturbation theory one can show that the theory is not renormalizable. However, if we consider the representation that does not rely on the perturbation theory, the nonrenormalizability is not so obvious. In order to demonstrate this, it is sufficient to consider the vacuum average of the S-matrix

$$S = \int d\bar{\Psi} d\Psi exp(iL_\Psi) \tag{3}$$

(here and below, the integration over four-dimensional space is implied).

We can use change of variables [1] in order to eliminate integration with respect to fermion fields. First, we introduce the additional integration:

$$S = \int d\bar{\Psi} d\Psi d\sigma expi\{\bar{\Psi}(i\partial - \sigma)\Psi\} - \frac{1}{2G}\sigma^2. \tag{4}$$

The scalar variable σ introduced here represents an auxiliary field. The term "auxiliary" means that the transition from formula (3) to formula (4) does not endow the field σ with a definite physical sense.

Integration with respect to fermions in formula (4) yields the effective action for the auxiliary field

$$L_\sigma = -iTrln(i\partial - \sigma) - \frac{1}{2G}\sigma^2. \tag{5}$$

With the aid of this Lagrangian we can try to determine the renormalizable perturbation series [2]. In order to make certain of this, we compare representation (4) with the Yukawa model

$$L_Y = \bar{\Psi}(i\partial - g\varphi)\Psi - \frac{1}{2}\varphi(M^2 + \partial^2)\varphi. \tag{6}$$

Integration with respect to fermions in formula (6) yields the effective action:

$$L_\varphi = iTrln(i\partial - g\varphi) - \frac{1}{2}\varphi(M^2 + \partial^2)\varphi. \tag{7}$$

Comparing (7) with (5) and taking into account that $Trln(i\partial - \sigma)$ involves the kinetic term, we make sure that the Lagrangians L_σ and L_φ yield coincident perturbation expansions. By the analogy with the Yukawa model, renormalization of the Lagrangian L_σ consists in adding the counterterms

$$\Delta L = -\frac{Z}{2}\sigma\partial^2\sigma - \frac{Z_2}{2}\sigma^2 - \frac{Z_4}{4}\sigma^4. \tag{8}$$

As it has been mentioned, the counterterms may be added only in the case of quantum (that is, possessing asymptotic states and the respective Hilbert space) fields. The field σ has no of such attributes. For this reason it is highly questionable that the auxiliary field σ is equivalent to a field with the kinetic term. These speculations result in the conclusion that the field σ should not be renormalized:

$$Z = Z_4 = 0. \tag{9}$$

(The second term in formula and (8) is related to charge renormalization.)

Relations (9) are referred to as the compositeness conditions, which were proposed many years ago [1,3] in order to distinguish between elementary particles and bound states. Currently these conditions are widely used [4], mainly for the studies of four-fermion interactions.

In terms of the original interaction (2), conditions (9) look as follows:

$$L_{\Psi,cl} = L_{\Psi,qu}. \tag{10}$$

Strictly speaking, it is this requirement that has led to the compositeness conditions. The surprising thing is that the inconsistency of this requirement has not been noticed so far. Indeed, this requirement does not agree with the quantization procedure, as has been mentioned, consists in the calculations of counterterms. The procedure of calculation of counterterms to the Lagrangian L_Ψ is well defined in the perturbation theory, even though it is nonrenormalizable, whereas condition (10) (or, which is the same, conditions (9)) exclude any calculations beyond the tree approximation. To put it differently, conditions (9) and (10) rule out the possibility that the respective theory can be quantized.

The solution of this puzzle stems from noticing that one should renormalize the original action (2) instead of the auxiliary field σ.

Let us consider the renormalized version of the action in (2)

$$L_{\Psi,qu} = Z_\Psi \bar{\Psi} i\partial\Psi + Z_\Psi^2 \frac{G}{2}(\bar{\Psi}\Psi)^2. \tag{11}$$

By applying transformation (4) to the Lagrangian in (11), we obtain

$$L_{\sigma,qu} = -iTrln\{Z_\Psi(i\partial - \sigma)\} - \frac{1}{2G}\sigma^2. \tag{12}$$

In what follows, the crucial point is that the fermion determinant contains the conformal anomaly [5]. For this reason,

$$Trln\{Z_\Psi(i\partial - \sigma)\} = Trln(i\partial - \sigma) - lnZ_\Psi \cdot Tra_o + const, \tag{13}$$

where the Seeley–DeWitt coefficient for the operator $i\partial - \sigma$ is as follows:

$$a_o = -\frac{1}{(4\pi)^2}(\sigma\partial^2\sigma + \sigma^4). \tag{14}$$

In order to derive relation (14) one should change over to the Euclidean space, define the regularized determinant, and calculate the Seeley coefficients. In this case, we refer to the ξ-function as an example of regularization. A simpler way of deriving relation (14) is as follows: by making use of an arbitrary regularization ε, one should expand the trace $Trln(i\partial - \sigma)$ in powers of σ, and then use the obtained Lagrangian L^ε for the calculation of the Seeley coefficient

$$a_o = \lim_{\varepsilon\to 0} \varepsilon \cdot L^\varepsilon(\sigma).$$

The result is independent of regularization.

After substituting (13) in formula (12) we obtain the ultimate expression for the renormalized action:

$$L_{\sigma,qu} = -iTrln(i\partial - \sigma) - \frac{1}{2G}\sigma^2 - \frac{1}{2}Z\sigma^2\partial^2\sigma - \frac{1}{4}Z_4\sigma^4. \tag{15}$$

In this case, making use of the explicit form of the anomaly of (14) leads to the relation:

$$2Z = Z_4. \tag{16}$$

Formula (15) furnishes the main result of this study: the emergence of kinetic term of the auxiliary field is the consequence of the conformal anomaly, that arises from the renormalization of the fermion fields. Certainly, a similar result holds for the four-fermion interaction of the general form. Obviously, the greater the number of types of interaction (SS, VV, AA, etc.) is involved in the initial Lagrangian, the greater the number of coupling constants, which are connected with each other by the relations of form (16). The studies of these new compositeness conditions are very promising.

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