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BOX ANOMALY AND $\eta' \to \pi^+\pi^-\gamma$ DECAY

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Abstract

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In the framework of the "current scheme" of $\eta - \eta'$ mixing we calculate the amplitude of the decay $\eta' \to \pi^+\pi^-\gamma$ in the soft limit and compare our results with recently reported values of the phenomenological additive terms to the ρ–meson background.

Аннотация

Киселев А.В., Петров В.А. "Квадратная" аномалия и распад $\eta' \to \pi^+\pi^-\gamma$: Препринт ИФВЭ 98-16. – Протвино, 1998. – 5 с., 2 табл., библиогр.: 15.

В рамках "токовой схемы" $\eta - \eta'$–смешивания мы вычисляем амплитуду распада $\eta' \to \pi^+\pi^-\gamma$ в мягком пределе и сравниваем наши результаты с недавно представленными значениями феноменологического вклада, аддитивного ко вкладу ρ–мезона.

1. Introduction

In the experiment on the study of $\eta' \to \pi^+\pi^-\gamma$ decay completed with a high accuracy by the LEPTON F Collaboration [1] it was found that certain discrepancy between the data and fits of the dipion spectrum based solely on the ρ–meson decay could be avoided if the ρ–meson contribution was coherently supplemented by a constant additive term. A subseqent thorough analysis of the world data at CERN [2] confirmed such a solution.

Recently the new measurements of the dipion mass distribution in $\eta' \to \pi^+\pi^-\gamma$ by the Crystal Barrel Collaboration [3] have also shown the existence of a non–resonant contribution to this decay. The analysis of the decay $\eta' \to \pi^+\pi^-\gamma$ from Ref. [4] has confirmed this conclusion.

It has been known for a long time (see e.g. [5]) that the amplitude of the decay $\eta' \to \pi^+\pi^-\gamma$ is related to the chiral anomalies (AVV, AAAV). In Ref. [6] (and references therein) this decay was also considered as an additional way of checking the quark charge assignment (the standard fractional charges [7] versus integral ones [8]).

One should note that in comparison with, say, $\gamma\gamma$ decays, the use of the soft limit of the anomalous term in the lowest order is complicated by the ρ–meson dominance (responsible for the bulk of the observed dipion mass spectrum). In Ref. [6] the account of the ρ–meson was made by multiplying the low–energy amplitude by the Breit–Wigner factor. In Refs. [2] it has been assumed that this is the term, additional to the ρ–meson term, which should be identified with anomalous contribution in the soft limit.

The detailed analysis of the significance of the ρ–resonance in η' decay can be found in Refs. [2].

In Ref. [9] we evaluated the anomalous terms related to $\eta/\eta' \to \pi^+\pi^-\gamma$ decays in the soft limit for two $\eta - \eta'$ mixing schemes [10].

In the present paper we calculate the anomalous term in $\eta' \to \pi^+\pi^-\gamma$ in the "current mixing" scheme and compare our theoretical results with the experimental values reported in Refs. [2,3].

2. $\eta - \eta'$ mixing

The amplitude we study contains anomalies that come through the relationship of the Heisenberg field operators of η, η' to the divergences of the $SU(3)$ octet and singlet axial currents. In Ref. [10] the following expressions for the interpolating fields of η and η' were obtained

$$
\begin{aligned}
\eta &= \frac{1}{m_\eta^2}\left(\frac{1}{F_8}\cos\theta D^8 - \frac{1}{F_0}\varepsilon\sin\theta D^0\right), \\
\eta' &= \frac{1}{m_{\eta'}^2}\left(\frac{1}{\varepsilon F_8}\sin\theta D^8 + \frac{1}{F_0}\cos\theta D^0\right),
\end{aligned} \tag{1}
$$

where $D_{8(0)} = \partial_\mu A_{8(0)}^{\mu 5}$ and $\langle\Omega|A_{8(0)}^{\mu 5}|\eta(\eta')\rangle = ip_\mu f_{8(0)}$ $(F_{8(0)} = f_{8(0)}/\cos\theta)$, $A_{8(0)}^{\mu 5}$ being octet (singlet) axial currents, and θ stands for the mixing angle. Equation (1) contains the factor ε which is equal to $m_\eta/m_{\eta'}$ in the so–called "current mixing scheme" (see details in Ref. [10]).

As usual, when applying formulae like Eq. (1) we mean the substraction of anomalous terms, so that in the soft limit Lorentz invariant formfactors of divergences disappear leaving us with the net anomalous contribution (up to the sign).

3. Basic formulae

The amplitude of the decay $\eta' \to \pi^+\pi^-\gamma$ depends on the "decay constants" $f_{8(0)}$ and the mixing angle which can be expressed in terms of the widths of $\gamma\gamma$ decays. Equation (1) allows us to obtain the following relations

$$
\begin{aligned}
R_\eta &\equiv \left[\frac{3\Gamma(\eta \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)}\right]^{1/2}\left(\frac{m_\pi}{m_\eta}\right)^{3/2} = \frac{f_\pi}{F_8}\cos\theta - \sqrt{8}\frac{f_\pi}{F_0}\varepsilon\sin\theta, \\
R_{\eta'} &\equiv \left[\frac{3\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)}\right]^{1/2}\left(\frac{m_\pi}{m_{\eta'}}\right)^{3/2} = \frac{f_\pi}{F_8}\frac{1}{\varepsilon}\sin\theta + \sqrt{8}\frac{f_\pi}{F_0}\cos\theta.
\end{aligned} \tag{2}
$$

One can also relate the parameters f_8, f_0 and θ to the decays $J/\psi \to \eta\gamma$ and $J/\psi \to \eta'\gamma$. Following [11], we suppose that the J/ψ radiative decays are dominated by $c\bar{c}$ annihilation into $gg\gamma$. The gluons g are in the pseudoscalar state, and the ratio R of the widths of the decays $J/\psi \to \eta\gamma$ and $J/\psi \to \eta'\gamma$ is defined by the ratio of the corresponding matrix elements of the gluon anomaly, $\langle 0|G\tilde{G}|\eta'\rangle$, $\langle 0|G\tilde{G}|\eta\rangle$ [11]. The ratio R was estimated in Ref. [10]:

$$
\begin{aligned}
R &\equiv \left[\frac{\Gamma(J/\psi \to \eta'\gamma)}{\Gamma(J/\psi \to \eta\gamma)}\right]^{1/2}\left(\frac{p_{\eta'}}{p_\eta}\right)^{3/2}\left(\frac{m_\eta}{m_{\eta'}}\right)^2 = \left|\frac{\langle 0|G\tilde{G}|\eta'\rangle}{\langle 0|G\tilde{G}|\eta\rangle}\right|\left(\frac{m_\eta}{m_{\eta'}}\right)^2 \\
&= \varepsilon\frac{\sqrt{2}f_0\cos\theta + \varepsilon f_8\sin\theta}{-\sqrt{2}f_0\sin\theta + \varepsilon f_8\cos\theta},
\end{aligned} \tag{3}
$$

where $p_{\eta'}/p_\eta = (1 - m_{\eta'}^2/m_{J/\psi}^2)/(1 - m_\eta^2/m_{J/\psi}^2)$.

One can thus express the mixing angle and the "decay constants" in terms of R_η, $R_{\eta'}$ and R. This was done and discussed in Ref. [10]. Experimental data on the decays $\eta/\eta' \to \gamma\gamma$ and $J/\psi \to \eta/\eta'\gamma$ [12] allow for the production of the numerical results summarized in Table 1.

The value $\theta = (-19.7 \pm 2.2)^\circ$ is in good agreement with the estimate $\theta \approx -(19 \div 20)^\circ$ found in Refs. [13].

We also get

Table 1. Mixing angle and "decay constants" extracted from $\eta/\eta' \to \gamma\gamma$ and $J/\psi \to \eta/\eta'\gamma$ decay.

θ°	-19.62 ± 2.33
f_8/f_π	0.84 ± 0.05
f_0/f_π	0.88 ± 0.07

$$
\begin{aligned}
\tan\theta &= [2\varepsilon(R_\eta - 4R_{\eta'}R)]^{-1}\{5(\varepsilon^2 R_{\eta'} + R_\eta R) - \\
&- [9(R_\eta R - \varepsilon^2 R_{\eta'})^2 + 16(R_\eta^2 + \varepsilon^2 R_{\eta'}^2)(\varepsilon^2 + R^2)]^{1/2}\},
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\frac{f_\pi}{f_8} &= R_\eta + \varepsilon R_{\eta'}\tan\theta, \\
\frac{f_\pi}{f_0} &= \frac{-R_\eta \tan\theta + \varepsilon R_{\eta'}}{\sqrt{8}\,\varepsilon}.
\end{aligned} \tag{5}
$$

Note that θ depends only on R and the ratio $R_\eta/R_{\eta'}$.

One should note that our formulas for the decay width ratios, which underlie the estimate of η–η' mixing angle and the decay constants, differ from conventionally used formulas (see Refs. [2] and references therein). The latters can be reproduced in our approach by means of the formal substitution $\varepsilon = 1$ in Eqs. (1)-(5).

4. Renormalization scale dependence

We now make some digression to discuss the question of the RG properties of the "decay constants" f_0 and f_8. Since we consider QED in the lowest orders, no question arises on the renormalization scale dependence of both anomalous divergences, D_8 and D_0. We, however, cannot neglect the QCD renormalization. "Hard" nonconservation (anomaly) of the singlet axial current leads to its RG scale dependence (the corresponding anomalous dimension was calculated in Ref. [14]) and, as a consequence, to the scale dependence of D_0 and f_0.

Due to multiplicative renormalization the matrix element of the "decay" of D_0 in $\pi^+\pi^-\gamma$ can be cast into the following form

$$
\begin{aligned}
\langle\pi(p_+)\pi(p_-)\gamma(k)|D_0(\mu^2)|\Omega\rangle &= \\
= Z(\mu^2/q^2)\langle\pi(p_+)\pi(p_-)\gamma(k)|D_0(q^2)|\Omega\rangle,
\end{aligned} \tag{6}
$$

where $q = p_+ + p_- + k$. The same factor $Z(\mu^2/q^2)$ renormalizes $f_0(q^2)$ to $f_0(m_{\eta'}^2)$, making the amplitude of the decay $\eta' \to \pi^+\pi^-\gamma$ RG invariant (because η' depends on the ratio D_0/f_0 as one can see from Eq. (1)).

3

The matrix element $\langle\pi\pi|D_0(q^2)|\Omega\rangle$ (with the pole at $m^2_{\eta'}$ subtracted) is calculated in the soft limit $q^2 \to 0$, while $f_0(q^2)$ is taken at $q^2 = m^2_{\eta'}$. It is usually assumed that one can put $\langle\pi\pi|D_0(0)|\Omega\rangle \simeq \langle\pi\pi|D_0(q^2_0)|\Omega\rangle$, where $q_0 \simeq 1$ GeV. Note that due to the slow evolution of $Z(\mu^2/q^2)$ (the anomalous dimension starts from the second order in α_s [14]) the difference between $D_0(q^2_0)$ and $D_0(m^2_{\eta'})$ is negligible. So, one get $\langle\pi\pi|D_0(0)|\Omega\rangle/f_0(m^2_{\eta'}) \simeq \langle\pi\pi|D_0(m^2_{\eta'})|\Omega\rangle/f_0(m^2_{\eta'})$.

The RG properties of other matrix elements of η' (the amplitude of $\eta' \to \gamma\gamma$ decay, in particular) was analysed in Ref. [15].

5. $\eta' \to \pi^+\pi^-\gamma$ decay in the soft limit

The amplitude of this decay has the following general Lorentz structure

$$M(\eta' \to \pi(p_+)\pi(p_-)\gamma(k)) = E_P(p_+k, p_-k)\varepsilon_{\mu\nu\rho\sigma}\varepsilon^\mu k^\nu p^\rho_+ p^\sigma_-, \qquad (7)$$

where ε^μ is the photon polarization vector.

Using Eq. (1), and with all reservations made, we obtain the expression in the soft limit (both "box" and "triangle" anomalies contribute [5], [6]):

$$E_{\eta'}(0) = -\frac{e}{4\pi^2\sqrt{3}f^2_\pi}\left(\frac{\sin\theta}{\varepsilon F_8} + \sqrt{2}\frac{\cos\theta}{F_0}\right), \qquad (8)$$

where $e^2 = 4\pi\alpha_{em}$.

With the parameters from Table 1 we find

$$E_{\eta'}(0) = -4.17 \pm 0.57. \qquad (9)$$

In obtaining this estimate, we accounted for all the data on $\eta \to \gamma\gamma$ [12] (with Primakoff–production measurements included). If we use only two–photon data, we get somewhat lower value of $E_{\eta'}$:

$$E_{\eta'}(0) = -4.01 \pm 0.38. \qquad (10)$$

Equations (9), (10) are the main result of our paper.

Table 2. The experimental values of the additive extra terms to the resonance background for two variants of the latter.

	Model M1	Model M2	Ref.
$E_{\eta'}$	$-5.06^{+0.53}_{-0.54}$	$-2.17^{+0.49}_{-0.46}$	[2]
$E_{\eta'}$	-4.46 ± 0.51	-1.78 ± 0.53	[3]

In Refs. [2,3] the phenomenological value $E_{\eta'}$ for the additive contribution (which according to these authors should be identified with $E_{\eta'}(0)$) to the ρ background depends on the choice of the latter, for which the freedom was reduced to two variants: model 1 (M1) and model 2 (M2). More details can be found in Refs. [2]. The corresponding results for $E_{\eta'}$ are reproduced in Table 2.

Thus, we have calculated the non–resonant contribution in $\eta' \to \pi^+\pi^-\gamma$ decay, $E_{\eta'}$. The values obtained say definitely in favour of model M1. It is somewhat surprising that our theoretical predictions obtained in the soft limit, which is rather far from being a physical one, appear to be very close to the experimental values of $E_{\eta'}$.

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