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S.M.Klishevich

**ELECTROMAGNETIC INTERACTION
OF MASSIVE SPIN-3 STATE
FROM STRING THEORY**

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Abstract

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In the given work we study an interaction of second massive state of an open boson string with the constant electromagnetic field. This state contains massive fields with spins 3 and 1. Using the method of an open string BRST quantization, we receive gauge-invariant lagrangian, describing the electromagnetic interaction of these fields. From the explicit form of transformations and lagrangian it follows that the presence of external constant e/m field leads to the mixing of the given level states. Most likely that the presence of the external field will lead to the mixing of the states at other mass string levels as well.

Аннотация

Клишевич С.М. Электромагнитное взаимодействие массивного спин-3 состояния из теории струн: Препринт ИФВЭ 98-20. – Протвино, 1998. – 16 с., библиогр.: 26.

В данной работе мы изучаем взаимодействие второго массивного состояния открытой возонной струны с постоянным электромагнитным полем. Это состояние содержит массивные поля со спинами 3 и 1. Используя метод BRST квантования открытой бозонной струны, мы получаем калибровочно-инвариантный лагранжиан, описывающий электромагнитное взаимодействие этих полей. Из явного вида преобразований и лагранжиана следует, что присутствие постоянного e/m поля приводит к перемешиванию состояний на данном уровне. Наиболее вероятно, это присутствие внешнего поля должно приводить к перемешиванию состояний и на других массовых уровнях струны.

1. Introduction

The problem of building the consistent interaction for high spin fields has already sufficiently great history [1,2,3]. However, and at present this problem is still far away from its completion.

The lagrangian description of free massive high spin fields was received by Singh and Hagen [4] in the 70's. Later it was realized [5,6] that for the covariant description of massless fields, they must necessarily be gauge fields. For such fields it was offered to enter an interaction as continuous deformation of the gauge algebra and lagrangian [7]. Such entering of the interaction is consistent since a continuous deformation of the gauge algebra preserves a type of the equations defining mass shell. At the same time it has been realized [8] that for massless fields of spins $s \geq 3/2$ in an asymptotically flat space-time it is impossible to build a consistent "minimal" interaction with an Abelian vector field. The same is valid and for the gravity interaction of fields with spins $s \geq 2$. It is possible to motivate the given statement as follows [8]: A free gauge-invariant lagrangian in the flat space has a structure $\mathcal{L}_0 = \partial\Phi\partial\Phi$ with transformations $\delta\Phi = \partial\xi$. The introduction of the interaction means a replacement of the usual derivative by a covariant one $\partial \rightarrow \mathcal{D}$. In this, the gauge invariance fails, and a residual of type $[\mathcal{D}, \mathcal{D}] \mathcal{D}\Phi\xi = \mathcal{R}\mathcal{D}\Phi\xi$ appears, where \mathcal{R} is a tensor of the tension of electromagnetic or gravity field. In the case of the electromagnetic interaction for the fields with spins $s \geq 3/2$, one cannot cancel the residual by any changes of the lagrangian and the transformations in the linear approximation. Therefore, in this case such approximation is absent, but since any linear approximation is independent of the presence in the system of any other fields, this means that a whole theory of interaction does not exist either. In the case of the gravity field a residual for the field spin $3/2$ is proportional to the gravity equations of motion: $\delta\mathcal{L}_0 \sim i(\bar{\psi}^\mu\gamma^\nu\eta)(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)$. One can eliminate such residual by modifying the lagrangian and the transformations. As a result, the theory of supergravity has appeared. For fields with spins $s > 2$ the residual contains terms proportional to Riemann tensor $R_{\mu\nu\alpha\beta}$. It is impossible to cancel such type terms in an asymptotically flat space. Hence, gravity interactions do not exist for any massless fields with spin $s > 3/2$.

It is possible to overcome similar difficulties in several ways. In the case of the gravity interaction, one can consider fields in a constant curvature space. Then the lagrangian of gravity would have an additional term $\Delta\mathcal{L} = \sqrt{-\det\|g_{\mu\nu}\|}\lambda$, where λ is a cosmological constant. A modification of the lagrangian and the transformations lead to a mixing of orders with the different derivative numbers. This allows one to eliminate a residual with terms proportional to $R_{\mu\nu\alpha\beta}$. A complete theory will be represented as a series with respect to inverse degrees of λ [9,10]. This means non-analyticity of the theory with respect to λ at zero, i.e. the impossibility of a smooth transition to the flat space. Such theory was considered in papers [9,10,11].

It is also possible to avoid these difficulties if one considers massive high spin fields [12]. But the situation with the consistent interaction of massive fields is significantly more complicated. The classical description [4] is not gauge invariant. The introduction of a "minimal" interactions lead to a change of the physical degree number of freedom. This problem can be solved by introducing non-minimal terms in the interactions but, in a general case, it results in the loss of causality. If we have not a gauge invariance, then the universal principle that allows one to get the consistent interaction of massive high spin fields is absent.

Early in the 80's the gauge invariant description of massive fields with arbitrary spins [13] was offered. In this case one can consider a consistent interaction as continuous deformation of the gauge algebra similar to a massless case.

In a natural way, a gauge description of massive high spin fields appears in the frame of free string BRST quantization¹ [16,17]. At present this is practically the unique consistent theory describing an interaction of high spin fields. However, in a general case, it is rather difficult to get any particular information on separate spins from the theory of interacting strings. But in some simple cases, such as a constant electromagnetic field, it is possible. In [18,19] the authors generalized [16,17] for the case of propagation of an open boson string in a homogeneous external e/m field and obtained lagrangians which describe the consistent interaction of massive fields of spins 2 and 1 with the constant electromagnetic fields.

Here we study an interaction of the second massive state of open boson string with the constant electromagnetic field. This state contains massive fields with spins 3 and 1. Using the BRST quantization method of the open string, we get a gauge invariant lagrangian describing the electromagnetic interaction of these fields. The action for gauge massive high spin fields available from BRST formulation of free string [16] is not canonical since a kinetic part contains cross terms. By means of redefinitions of the fields, one can diagonalize the action and consider each state independently. In the presence of the external field, it is already impossible to consider the states independently. From the explicit form of the transformations and the lagrangian for the second mass level, it follows that the presence of the constant e/m field leads to a mixing of the states at the given level. Most likely that the presence of the external field will lead to a mixing of the states at other mass levels of the string as well.

¹One can also receive a gauge description of free massive fields of arbitrary spins using dimensioned reductions [14,15].

2. Electromagnetic Interaction of High Spin Fields in Context Boson String

Let us consider the action describing a propagation of the open boson string with charges on the ends in a homogeneous electromagnetic field in Minkowski space \mathbf{M}_D in the conformal gauge [20,21,18,19]

$$S = \frac{1}{2\pi} \int d\tau \int_0^\pi d\sigma \left(\dot{X}_\mu \dot{X}^\mu - X'_\mu X'^\mu \right) - \frac{1}{2} \int d\tau d\sigma \mathcal{F}_{\mu\nu} \dot{X}^\mu X^\nu (q_0 \delta(\sigma) + q_\pi \delta(\sigma - \pi)), \quad (1)$$

where the point and the prime denote a derivative with respect to the world sheet coordinates of the string τ and σ , respectively. For the tension coefficient the agreement $\alpha' = \frac{1}{2}$ is accepted.

Equations of motion received from action (1) can be exactly solved [21,18], which allows one to canonically quantize the theory. Commutational relations for modes have the following form:

$$[a_m^\mu, a_n^\nu] = H_m^{\mu\nu} \delta_{m+n}, \quad (2)$$

with the usual Hermitian conjugation $a_n^{\mu\dagger} = a_{-n}^\mu$. Above we used the following notations

$$\begin{aligned} H_m^{\mu\nu} &= m\eta^{\mu\nu} + F^{\mu\nu}, \\ F^{\mu\nu} &= \frac{1}{i\pi} (\text{arctanh}(\pi q_0 \mathcal{F}) + \text{arctanh}(\pi q_\pi \mathcal{F}))^{\mu\nu}. \end{aligned} \quad (3)$$

Thus, the presence of the constant electromagnetic field leads to the deformation of the infinite Heisenberg algebra.

The Fok space, on which the representation of algebra (2) is realized, is built in the same manner as in the absence of the external field. The vacuum vector is defined by the relations

$$a_n^\mu |0\rangle = 0, \quad \forall n < 0,$$

and any vector of the Fok space has the form

$$|\Phi\rangle = \sum_k \sum_{\{n_i\}} \Phi_{\mu_1 \dots \mu_k}^{(n_1 \dots n_k)}(x) a_{-n_1}^{\mu_1} \dots a_{-n_k}^{\mu_k} |0\rangle, \quad \text{где } n_i > 0.$$

The coefficient functions $\Phi_{\mu_1 \dots \mu_k}^{(n_1 \dots n_k)}(x)$ are tensor fields on the space \mathbf{M}_D .

The theory with action (1), as in the case with the free string, contains constraints expressed in terms of Virasoro operators at the quantum level. In spite of the presence of the external field, the Virasoro operators corresponding to action (1) have the usual form

$$L_m = \frac{1}{4\pi} \int_{-\pi}^{\pi} e^{-im\sigma} (\dot{X} + X')^2 d\sigma = \frac{1}{2} \sum_p :a_{m-p} \cdot a_p: . \quad (4)$$

They form the same algebra but with other central charge [21]

$$[L_m, L_n] = (m - n)L_{m+n} + \left\{ \frac{D}{12}(m^3 - m) - \frac{1}{2}mF_{\mu\nu}^2 \right\} \delta_{m+n}.$$

These relations can be reduced to the same form as for the free string. For that it is necessary to redefine the operator L_0 including into its definition the term $-\frac{1}{4}F_{\mu\nu}^2$. This lead only to the redefining of a normal ordering constant².

A subspace of physical states is defined by means of the following conditions:

$$L_n|\Psi\rangle = 0, \quad \forall n \geq 0. \quad (5)$$

For the free string, a representation of the zero mode is, as usual, chosen in the form of the momentum operator $a_0^\mu = p^\mu = i\partial^\mu$ acting on the space of functions $C^\infty(\mathbf{M}_D)$. In such representation, one can get from (5) the differential equations for coefficient function of the physical states. As is well known, conditions (5) contain the equations for the massive states with arbitrary integer spins.

Let us consider the commutation relations for the zero modes in the presence of the external electromagnetic field

$$[a_0^\mu, a_0^\nu] = F^{\mu\nu}.$$

In the constant field the particular form of $F^{\mu\nu}$ (3) does not matter³, therefore, we "will forget" about the origin of algebra (2) and interpret the antisymmetric tensor $F^{\mu\nu}$ as a tension of the electromagnetic field. We consider that the coupling constant and the imaginary unity are included in the definition of the tension tensor. Then, a_0^μ can be interpreted as a covariant momentum operator

$$a_0^\mu = \mathcal{P}_\mu = i\mathcal{D}^\mu,$$

where \mathcal{D}^μ is $U(1)_{em}$ covariant derivative. Under such interpretation, conditions (5) define the equations of motion for the physical states with arbitrary spins in the constant electromagnetic field. But it is necessary to notice that the obtaining of a lagrangian formulation, which can give all these equations in such form, is sufficiently difficult.

In [16,17] gauge invariant equations for the free string states coming from the requirement of BRST invariance was proposed. In [18] the author showed that in the case of homogeneous electromagnetic field, the BRST charge is built in the same manner as in the case of the free string.

Let us increase the above considered Fok space by entering an infinite set of "ghost" and "antighost" operators c_n and b_n with anticommutation relations

$$\begin{aligned} [c_m, b_n]_+ &= \delta_{m+n}, \\ [c_m, c_n]_+ &= [b_m, b_n]_+ = 0 \end{aligned}$$

and with the usual Hermitian conjugation $c_n^\dagger = c_{-n}$, $b_n^\dagger = b_{-n}$. A "ghost" vacuum is defined as follows

$$\begin{aligned} c_n|0\rangle_{gh} &= 0, & \forall n > 1, \\ b_n|0\rangle_{gh} &= 0, & \forall n > -2. \end{aligned}$$

²Which we will include into the definition of L_0 as well.

³It is possible to take $F^{\mu\nu}$ as an arbitrary uneven function of a constant antisymmetric matrix.

A complete vacuum is defined as a tensor product of the two vacuum vectors

$$|0\rangle = |0\rangle_X \otimes |0\rangle_{gh},$$

where $|0\rangle_X$ is the boson vacuum vector defined by conditions (2). The complete vacuum is $sl(2, R)$ invariant.

In the theory of the free string BRST quantizing the states are classified by means of the "level" operator

$$\mathcal{N} = \sum_{p=1}^{\infty} (a_{-p} \cdot a_p + p c_{-p} b_p)$$

and the operator of a "ghost" number [22,23]

$$\mathcal{N}_{gh} = \frac{1}{2} [c_0, b_0] + \sum_{p=1}^{\infty} (c_{-p} b_p - b_{-p} c_p).$$

Physical states are eigenvectors of these operators. The vacuum is the physical state with "ghost" number $-3/2$. In the theory with an external field we shall use the same states as in the free theory, but the states will not be eigenvectors of the operators \mathcal{N} and \mathcal{N}_{gh} already because of the presence in the commutation relations (2) of the tension tensor of the electromagnetic field.

For the feature of the physical states in the presence of the external field we shall use the same terms as for the free string. We shall assume that its is defined in the limit $F_{\mu\nu} \rightarrow 0$.

Both for the free string and the one in a constant Abelian field, the BRST operator is defined as

$$Q_{BRST} = \sum_{-\infty}^{\infty} : \left(L_p^X + \frac{1}{2} L_p^{gh} \right) c_{-p} :, \quad (6)$$

where operators L_m^X are defined by expression (4), $L_m^{gh} = \sum_p (m-p) b_{p+m} c_{-p}$ and the normal ordering constant is included into the definition of L_0 . As for the free string, this Hermitian operator is nilpotent in the space with dimensionality 26. Equations (5) are equivalent to the requirement of the BRST invariance for physical states

$$Q_{BRST} |\Psi\rangle = 0. \quad (7)$$

The physical states $|\Psi\rangle$ and $|\Psi\rangle + Q_{BRST} |\Lambda\rangle$ are equivalent as a consequence of the nilpotency of the BRST operator. Here the "ghost" number of the vector $|\Lambda\rangle$ is by the unity less than the number of $|\Psi\rangle$. This means that equations (7) are gauge invariant.

Equations (7) can be derived from the lagrangian

$$\mathcal{L} = \langle \Psi | Q_{BRST} | \Psi \rangle, \quad (8)$$

where $\langle \Psi | = |\Psi\rangle^\dagger$, $\langle 0 | = |0\rangle^\dagger$ and an uncertainty in the determination of the vacuum average is fixed by taking the agreement

$$\langle 0 | c_{-1} c_0 c_1 | 0 \rangle = 1.$$

Obviously, this lagrangian is invariant under the gauge transformations

$$\delta|\Psi\rangle = Q_{BRST}|\Lambda\rangle. \quad (9)$$

Thus, the BRST formulation of the open boson string allows one to obtain the gauge invariant lagrangians describing the massive fields with arbitrary integer spins.

3. The Second Massive String State

In [18] the authors studied the first massive state of the open boson string and using the above method, they proposed the following lagrangian describing a propagation of the massive spin 2 field in a constant electromagnetic field

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}\bar{\phi}(\mathcal{P}H_1\mathcal{P}+2)\phi - \frac{1}{2}\bar{B}H_2 \cdot (\mathcal{P}H_1\mathcal{P}+2)B - \bar{B}H_2FB \\ & + \text{tr} [\bar{H}_1\bar{h}H_1 \cdot (\mathcal{P}H_1\mathcal{P}+2)h] + 4\text{tr} [\bar{H}_1\bar{h}H_1Fh] \\ & + \frac{1}{2} \left(2\mathcal{P}\bar{H}_1\bar{h} + \bar{H}_2\bar{B} - \mathcal{P}\bar{\phi} \right) \cdot H_1 \cdot (2\mathcal{P}H_1h + H_2B - \mathcal{P}\phi) \\ & - \frac{1}{4} \left(\text{tr} [\bar{H}_1\bar{h}H_1] + \mathcal{P}\bar{H}_2\bar{B} - 3\bar{\phi} \right) \left(\text{tr} [H_1h\bar{H}_1] + \mathcal{P}H_2B - 3\phi \right), \end{aligned} \quad (10)$$

which is invariant with respect to the gauge transformations

$$\begin{aligned} \delta h_{\mu\nu} &= \mathcal{P}_\mu \xi_\nu^1 + \mathcal{P}_\nu \xi_\mu^1 - \frac{1}{2}g_{\mu\nu}\xi^0, \\ \delta B_\mu &= -\mathcal{P}_\mu \xi^0 + 2 \left(H_1 \cdot \xi^1 \right)_\mu, \\ \delta \phi &= 2 \left(\mathcal{P} \cdot H_1 \cdot \xi^1 \right) - 3\xi^0 \end{aligned} \quad (11)$$

Here, the bar means the complex conjugation, $h_{\mu\nu}$ is a symmetrical tensor. We used the matrix notations $(\mathcal{P}H_1\mathcal{P}) = \mathcal{P}_\mu H_1^{\mu\nu} \mathcal{P}_\nu$, $(\mathcal{P}H_1h)_\mu = \mathcal{P}_\alpha H_1^{\alpha\beta} h_{\beta\mu}$ and etc. as well.

In [18] it has been also proved that the propagation of the massive spin 2 field in the constant field described by lagrangian (10) is causal. This allows us to hope that (8) gives consistent description of the interaction with the homogeneous electromagnetic field for the massive ones of arbitrary integer spins.

The second massive state of the open boson string contains the massive fields with spins 1 and 3. This means that the average (8) of the BRST operator with respect to the present state gives us a gauge invariant lagrangian describing the interaction of these fields with the constant electromagnetic field.

The second massive string level state is defined by the conditions

$$\mathcal{N}|\Psi, 3\rangle \xrightarrow{F \rightarrow 0} 2|\Psi, 3\rangle, \quad \mathcal{N}_{gh}|\Psi, 3\rangle = |\Psi, 3\rangle$$

and has the following form

$$\begin{aligned} |\Phi, 3\rangle = & \{ \Phi_{\mu\nu\alpha} a_{-1}^\mu a_{-1}^\nu a_{-1}^\alpha c_1 + (h_{\mu\nu} + A_{\mu\nu}) a_{-2}^\mu a_{-1}^\nu c_1 + B_\mu a_{-3}^\mu c_1 \\ & + A_\mu a_{-1}^\mu c_{-1} + \varphi c_{-2} + \sigma b_{-2} c_1 c_{-1} + \omega_{\mu\nu} a_{-1}^\mu a_{-1}^\nu c_0 + \theta_\mu a_{-2}^\mu c_0 \\ & + \xi_\mu a_{-1}^\mu b_{-2} c_0 c_1 + \gamma b_{-3} c_0 c_1 \} |0\rangle. \end{aligned} \quad (12)$$

Here $A_{\mu\nu}$ is an antisymmetric and $\Phi_{\mu\nu\alpha}, h_{\mu\nu}, \omega_{\mu\nu}$ are symmetrical tensor fields.

The gauge transformation for these states in terms of the BRST operator is

$$\delta|\Phi, 3\rangle = Q_{BRST}|\Lambda, 3\rangle,$$

where

$$|\Lambda, 3\rangle = \{\Lambda_{\mu\nu}a_{-1}^\mu a_{-1}^\nu + x_\mu a_{-2}^\mu + \lambda_\mu a_{-1}^\mu b_{-2}c_1 + \eta b_{-3}c_1 + v b_{-2}c_0\}|0\rangle.$$

Here $\Lambda_{\mu\nu}$ is an arbitrary symmetrical tensor. Hereafter we shall not distinguish the upper and lower indexes.

In terms of the coefficient functions, the gauge transformations take the following form

$$\begin{aligned}\delta\Phi_{\alpha\beta\gamma} &= i\mathcal{D}_{(\gamma}\Lambda_{\alpha\beta)} + \frac{1}{2}g_{(\alpha\beta}\lambda_{\gamma)}, \\ \delta h_{\alpha\beta} &= i\mathcal{D}_{(\alpha}x_{\beta)} + i\mathcal{D}_{(\alpha}\lambda_{\beta)} + 2H_{1(\alpha|\gamma}\Lambda_{\gamma|\beta)} + g_{\alpha\beta}\eta, \\ \delta A_{\alpha\beta} &= i\mathcal{D}_{[\alpha}\lambda_{\beta]} - i\mathcal{D}_{[\alpha}x_{\beta]} + 2F_{[\alpha|\gamma}\Lambda_{\gamma|\beta]}, \\ \delta b_\alpha &= i\mathcal{D}_\alpha\eta + H_{2\alpha\beta}x_\beta + H_{1\alpha\beta}\lambda_\beta, \\ \delta A_\alpha &= i2H_{1\beta\gamma}\mathcal{D}_\beta\Lambda_{\gamma\alpha} + H_{2\alpha\beta}x_\beta + 3\lambda_\alpha, \\ \delta\varphi &= iH_{2\alpha\beta}\mathcal{D}_\alpha x_\beta + \bar{H}_{1\alpha\beta}H_{1\beta\gamma}\Lambda_{\alpha\gamma} + 4v + 5\eta, \\ \delta\sigma &= -iH_{1\alpha\beta}\mathcal{D}_\alpha\lambda_\beta + 2v - 4\eta,\end{aligned}\tag{13}$$

$$\begin{aligned}\delta\omega_{\alpha\beta} &= -\frac{1}{2}H_{1\gamma\delta}\mathcal{D}_\gamma^2\Lambda_{\alpha\beta} + 2H_{1(\alpha|\gamma}\Lambda_{\gamma|\beta)} + \frac{1}{2}g_{\alpha\beta}v, \\ \delta\theta_\alpha &= -\frac{1}{2}H_{1\beta\gamma}\mathcal{D}_\beta^2x_\alpha + i\mathcal{D}_\alpha v + H_{2\alpha\beta}x_\beta, \\ \delta\chi_\alpha &= \frac{1}{2}H_{1\beta\gamma}\mathcal{D}_\beta^2\lambda_\alpha + i\mathcal{D}_\alpha v - H_{2\alpha\beta}\lambda_\beta, \\ \delta\gamma &= \frac{1}{2}H_{1\alpha\beta}\mathcal{D}_\alpha^2\eta - 2\eta + v.\end{aligned}\tag{14}$$

The fields $\omega_{\alpha\beta}, \theta_\alpha, \chi_\alpha, \gamma$ are auxiliary since they are coefficient functions at the combinations of operators containing c_0 but the BRST operator has the structure

$$Q_{BRST} = -\frac{1}{2}\mathcal{D}^2c_0 + \dots,$$

where the dots are marked terms which do not exceed the first order for the derivatives. Therefore, the auxiliary fields have an algebraic equations on mass surfaces. Having solved this equation, one can to exclude all the auxiliary fields from our consideration. However, it is the auxiliary fields that allow one to write down a lagrangian for the second massive state in the most compact form.

From the transformations (13) it is not difficult to see that the scalar parameter v appears only as a translation. This means that we can discard one scalar field, for instance σ , using this parameter. Herewith the gauge transformations are changed only for field φ

$$\delta_{new}\varphi = iH_{2\alpha\beta}\mathcal{D}_\alpha x_\beta + 2iH_{1\alpha\beta}\mathcal{D}_\alpha \lambda_\beta + \bar{H}_{1\alpha\beta}H_{1\beta\gamma}\Lambda_{\alpha\gamma} + 13\eta$$

and for the auxiliary fields.

After excluding the field σ , the gauge lagrangian of the second massive state will take the form

$$\begin{aligned} \mathcal{L} &= \langle \Phi, 3 | Q_{BRST} | \Phi, 3 \rangle \Big|_{\sigma \rightarrow 0} \\ &= 3\bar{H}_{1\alpha\mu}\bar{H}_{1\beta\nu}\bar{H}_{1\gamma\rho}\bar{H}_{1\delta\lambda}\mathcal{D}_\delta\Phi_{\alpha\beta\gamma}\mathcal{D}_\lambda\bar{\Phi}_{\mu\nu\rho} \\ &\quad + 18\bar{\Phi}_{\alpha\beta\gamma}H_{1\alpha\delta}H_{1\beta\rho}H_{1\rho\mu}H_{1\gamma\nu}\Phi_{\delta\mu\nu} - 6\bar{\Phi}_{\alpha\beta\gamma}H_{1\alpha\delta}H_{1\beta\mu}H_{1\gamma\nu}\Phi_{\delta\mu\nu} \\ &\quad - 6\bar{H}_{1\mu\beta}\bar{\Phi}_{\alpha\beta\gamma}H_{1\alpha\delta}H_{1\gamma\nu}\mathcal{D}_\nu\omega_{\delta\mu}i + 3\bar{\Phi}_{\alpha\beta\gamma}H_{1\alpha\delta}\xi_\delta H_{1\beta\mu}\bar{H}_{1\mu\gamma} \\ &\quad + \frac{1}{2}\bar{H}_{1\gamma\nu}\bar{H}_{1\beta\mu}\bar{H}_{2\alpha\delta}\mathcal{D}_\gamma h_{\alpha\beta}\mathcal{D}_\nu\bar{h}_{\delta\mu} + \bar{h}_{\alpha\beta}H_{2\beta\gamma}H_{2\gamma\delta}\bar{H}_{1\mu\alpha}h_{\delta\mu} \\ &\quad - \bar{h}_{\alpha\beta}H_{2\beta\gamma}h_{\gamma\delta}F_{\delta\mu}\bar{H}_{1\mu\alpha} + \frac{1}{2}\bar{H}_{2\alpha\delta}\bar{H}_{1\beta\mu}\bar{H}_{1\gamma\nu}\mathcal{D}_\gamma A_{\alpha\beta}\mathcal{D}_\nu\bar{h}_{\delta\mu} \\ &\quad + \bar{h}_{\alpha\beta}H_{2\beta\gamma}H_{2\gamma\delta}A_{\delta\mu}\bar{H}_{1\mu\alpha} - \bar{h}_{\alpha\beta}H_{2\beta\gamma}A_{\gamma\delta}F_{\delta\mu}\bar{H}_{1\mu\alpha} \\ &\quad - 2\bar{h}_{\alpha\beta}H_{2\beta\gamma}H_{1\gamma\delta}\omega_{\delta\mu}\bar{H}_{1\mu\alpha} - \bar{H}_{2\alpha\beta}\bar{h}_{\beta\gamma}H_{1\gamma\delta}\mathcal{D}_\delta\theta_\alpha i \\ &\quad + \bar{H}_{1\alpha\beta}\bar{h}_{\beta\gamma}H_{2\gamma\delta}\mathcal{D}_\delta\xi_\alpha i + \bar{h}_{\alpha\beta}H_{1\beta\gamma}\bar{H}_{2\gamma\alpha}\gamma \\ &\quad - \left\{ \frac{1}{2}H_{1\alpha\delta}H_{1\gamma\nu}H_{2\beta\mu}\mathcal{D}_\gamma\bar{A}_{\alpha\beta}\mathcal{D}_\nu h_{\delta\mu} + \bar{A}_{\alpha\beta}H_{2\beta\gamma}H_{2\gamma\delta}h_{\delta\mu}\bar{H}_{1\mu\alpha} \right. \\ &\quad - \bar{A}_{\alpha\beta}H_{2\beta\gamma}h_{\gamma\delta}F_{\delta\mu}\bar{H}_{1\mu\alpha} + \frac{1}{2}\bar{H}_{1\gamma\nu}\bar{H}_{2\alpha\mu}\bar{H}_{1\beta\delta}\mathcal{D}_\gamma A_{\alpha\beta}\mathcal{D}_\nu\bar{A}_{\delta\mu} \\ &\quad + \bar{A}_{\alpha\beta}H_{2\beta\gamma}H_{2\gamma\delta}A_{\delta\mu}\bar{H}_{1\mu\alpha} - \bar{A}_{\alpha\beta}H_{2\beta\gamma}A_{\gamma\delta}F_{\delta\mu}\bar{H}_{1\mu\alpha} \\ &\quad - 2\bar{A}_{\alpha\beta}H_{2\beta\gamma}H_{1\gamma\delta}\omega_{\delta\mu}\bar{H}_{1\mu\alpha} + \bar{H}_{2\alpha\beta}\bar{A}_{\beta\gamma}H_{1\gamma\delta}\mathcal{D}_\delta\theta_\alpha i \\ &\quad \left. + \bar{H}_{1\alpha\beta}\bar{A}_{\beta\gamma}H_{2\gamma\delta}\mathcal{D}_\delta\xi_\alpha i \right\} + \bar{A}_{\alpha\beta}H_{1\beta\gamma}\bar{H}_{2\gamma\alpha}\gamma \\ &\quad + \frac{1}{2}\bar{H}_{3\alpha\gamma}\bar{H}_{1\beta\delta}\mathcal{D}_\beta b_\alpha\mathcal{D}_\delta\bar{b}_\gamma + \bar{b}_\alpha H_{3\alpha\beta}H_{2\beta\gamma}b_\gamma \\ &\quad - \bar{b}_\alpha H_{3\alpha\beta}H_{2\beta\gamma}\theta_\gamma + \bar{b}_\alpha H_{3\alpha\beta}H_{1\beta\gamma}\xi_\gamma + \bar{b}_\alpha H_{3\alpha\beta}\mathcal{D}_\beta\gamma i \\ &\quad - \bar{\varphi}\{H_{1\alpha\beta}\mathcal{D}_\alpha\xi_\beta i + 4\gamma\} - \{6\bar{H}_{1\alpha\beta}\bar{\omega}_{\beta\gamma}H_{1\gamma\delta}H_{1\mu\nu}\mathcal{D}_\mu\Phi_{\alpha\delta\nu}i \\ &\quad + 2\bar{\omega}_{\alpha\beta}H_{1\beta\gamma}H_{2\gamma\delta}(h_{\delta\mu} + A_{\delta\mu})\bar{H}_{1\mu\alpha} - 2\bar{H}_{1\alpha\beta}\bar{\omega}_{\beta\gamma}H_{1\gamma\delta}\mathcal{D}_\delta A_{\alpha}i \end{aligned} \tag{15}$$

$$\begin{aligned}
& -4\bar{\omega}_{\alpha\beta}H_{1\beta\gamma}\omega_{\gamma\delta}\bar{H}_{1\delta\alpha}\} - H_{1\alpha\beta}(\mathcal{D}_\alpha A_{\gamma\beta} + \mathcal{D}_\alpha h_{\gamma\beta})\bar{H}_{2\gamma\delta}\bar{\theta}_\delta i \\
& + \bar{\theta}_\alpha H_{2\alpha\beta}H_{1\beta\gamma}A_\gamma - \bar{\theta}_\alpha H_{2\alpha\beta}H_{3\beta\gamma}b_\gamma + 2\bar{\theta}_\alpha H_{2\alpha\beta}\theta_\beta \\
& - \frac{1}{2}\bar{H}_{1\beta\delta}\bar{H}_{1\alpha\gamma}\mathcal{D}_\beta A_\alpha \mathcal{D}_\delta \bar{A}_\gamma - \bar{A}_\alpha H_{1\alpha\beta}H_{2\beta\gamma}A_\gamma \\
& + 2H_{1\alpha\beta}\mathcal{D}_\alpha \omega_{\beta\gamma}\bar{H}_{1\gamma\delta}\bar{A}_\delta i + \bar{A}_\alpha H_{1\alpha\beta}H_{2\beta\gamma}\theta_\gamma - 3\bar{A}_\alpha H_{1\alpha\beta}\xi_\beta \\
& + H_{2\alpha\beta}(\mathcal{D}_\alpha A_{\beta\gamma} + \mathcal{D}_\alpha h_{\beta\gamma})\bar{H}_{1\gamma\delta}\bar{\xi}_\delta i + 3\bar{\xi}_\alpha H_{1\alpha\beta}\Phi_{\beta\gamma\mu}H_{1\gamma\delta}\bar{H}_{1\delta\mu} \\
& + \bar{\xi}_\alpha H_{1\alpha\beta}H_{3\beta\gamma}b_\gamma - \bar{\xi}_\alpha H_{1\alpha\beta}\mathcal{D}_\beta \varphi i + 4\bar{\xi}_\alpha H_{1\alpha\beta}\xi_\beta - 3\bar{\xi}_\alpha H_{1\alpha\beta}A_\beta \\
& + \bar{\gamma}\{H_{3\alpha\beta}\mathcal{D}_\alpha b_\beta i + H_{2\alpha\beta}(h_{\beta\gamma} + A_{\beta\gamma})\bar{H}_{1\gamma\alpha} - 4\varphi + 6\gamma\}.
\end{aligned}$$

Varying this lagrangian with respect to auxiliary fields, it is not difficult to make sure that the equations of motion for them are really algebraic. Using this equations, one can exclude the auxiliary fields from the lagrangian (15). We have placed the obtained lagrangian into Appendix because it is rather long. It is necessary to note that the received result is complete with respect to the tension of the electromagnetic field.

Lagrangian in (Appendix) for the interacting fields of spins 3 and 1 is non-canonical, since the free part of the lagrangian contains the cross kinetic terms. To diagonalize them, it is necessary to do the following replacement of the variables

$$\begin{aligned}
\Phi_{\alpha\beta\gamma} & \rightarrow \Phi_{\alpha\beta\gamma} + \frac{1}{416}g_{(\alpha\beta}(13A_\gamma) + 3b_\gamma), \\
h_{\alpha\beta} & \rightarrow h_{\alpha\beta} - \frac{3}{4}g_{\alpha\beta}\varphi, \\
A_\alpha & \rightarrow \frac{1}{208}(13A_\alpha + 3b_\alpha) + 3\Phi_{\alpha\beta\beta}, \\
b_\alpha & \rightarrow \frac{1}{16}(b_\alpha - A_\alpha), \\
\varphi & \rightarrow 2h_{\alpha\alpha} - 3\varphi.
\end{aligned} \tag{16}$$

After such changing, the fields $\Phi_{\alpha\beta\gamma}, h_{\alpha\beta}, b_\alpha, \varphi$ will describe the pure massive spin 3 (see, for instance, [12,16]). The gauge transformations for these fields will acquire the form

$$\begin{aligned}
\delta\Phi_{\alpha\beta\gamma} & = i\mathcal{D}_{(\alpha}\tilde{\Lambda}_{\beta\gamma)} + \frac{i}{13}g_{(\alpha\beta}F_{\mu\nu}\mathcal{D}_\mu\tilde{\Lambda}_{\nu|\gamma)} - \frac{i}{5408}g_{(\alpha\beta}F_{\gamma)\mu}\mathcal{D}_\mu(3\tilde{u} + 13\tilde{\eta}) \\
& + \frac{1}{52}g_{(\alpha\beta}(H_{4\gamma)\mu}\tilde{x}_\mu - F_{\gamma)\mu}\tilde{\lambda}_\mu) \\
\delta h_{\alpha\beta} & = i\mathcal{D}_{(\alpha}\tilde{x}_{\beta)} + 2H_{1(\alpha|\mu}\tilde{\Lambda}_{\mu|\beta)} + \frac{43}{52}g_{\alpha\beta}\tilde{u} - \frac{1}{8}g_{\alpha\beta}(3iF_{\mu\nu}\mathcal{D}_\mu\tilde{x}_\nu \\
& + iF_{\mu\nu}\mathcal{D}_\mu\tilde{\lambda}_\nu - 2F_{\mu\nu}F_{\nu\gamma}\tilde{\Lambda}_{\mu\gamma} + \frac{1}{208}F_{\mu\nu}^2(3\tilde{u} + 13\tilde{\eta})) \\
\delta b_\alpha & = i\mathcal{D}_\alpha\tilde{u} + 24\tilde{x}_\alpha - 2iF_{\mu\nu}\mathcal{D}_\mu\tilde{\Lambda}_{\nu\alpha} + \frac{i}{208}F_{\alpha\mu}\mathcal{D}_\mu(3\tilde{u} + 13\tilde{\eta})
\end{aligned} \tag{17}$$

$$\begin{aligned}
& + \frac{1}{2} F_{\alpha\mu} (25\tilde{x}_\mu + \tilde{\lambda}_\mu) \\
\delta\varphi = & \tilde{u} - \frac{i}{6} F_{\alpha\beta} (3\mathcal{D}_\alpha \tilde{x}_\beta + \mathcal{D}_\alpha \tilde{\lambda}_\beta) + \frac{1}{3} F_{\alpha\beta} F_{\beta\gamma} \tilde{\Lambda}_{\alpha\gamma} \\
& - \frac{1}{1248} F_{\alpha\beta}^2 (3\tilde{u} + 13\tilde{\eta}).
\end{aligned}$$

The gauge parameters are redefined as follows:

$$\begin{aligned}
\tilde{\Lambda}_{\alpha\beta} &= \Lambda_{\alpha\beta} - \frac{1}{26} g_{\alpha\beta} \Lambda_{\gamma\gamma}, \\
\tilde{x}_\alpha &= x_\alpha + \lambda_\alpha, \\
\tilde{u} &= \Lambda_{\alpha\alpha} + 13\eta, \\
\tilde{\lambda}_\alpha &= \lambda_\alpha - x_\alpha, \\
\tilde{\eta} &= \Lambda_{\alpha\alpha} - 3\eta.
\end{aligned}$$

In turn, the fields $A_{\alpha\beta}$, A_α will relate to the massive spin 1 with the gauge transformations

$$\begin{aligned}
\delta A_{\alpha\beta} &= i\mathcal{D}_{[\alpha} \tilde{\lambda}_{\beta]} + 2F_{[\alpha|\gamma} \tilde{\Lambda}_{\gamma|\beta]}, \\
\delta A_\alpha &= i\mathcal{D}_\alpha \tilde{\eta} + 8\tilde{\lambda}_\alpha - 2iF_{\beta\gamma} \mathcal{D}_\beta \tilde{\Lambda}_{\gamma\alpha} + \frac{i}{208} F_{\alpha\beta} \mathcal{D}_\beta (3\tilde{x} + 13\tilde{\eta}) \\
&\quad - \frac{1}{2} F_{\alpha\beta} (7\tilde{x}_\beta - \tilde{\lambda}_\beta).
\end{aligned} \tag{18}$$

Under such substitution lagrangian (23) increases approximately three times. For this reason we do not display it here.

From transformations (17) and (18) it is clear that in the presence of the constant electromagnetic field, a mixing of the states with spins 3 and 1 occurs, i.e. in the presence of the interaction we cannot any longer consider these states independently from each other. It would appear reasonable that the presence of the constant Abelian field leads to the mixing of the states for any other higher mass level of the open boson string as well.

At the end of this section we notice that the massive spin 1 state appears at the second massive string level in the non-standard form (see also [16]). The free lagrangian corresponding to a noncharged massive spin 1 can be reduced to the form

$$\mathcal{L}_{S=1} = \frac{1}{12} \mathcal{H}_{\alpha\beta\gamma} \mathcal{H}^{\alpha\beta\gamma} - \frac{1}{4} \mathcal{H}_{\alpha\beta} \mathcal{H}^{\alpha\beta} + \frac{m}{2} \mathcal{H}_{\alpha\beta} A^{\alpha\beta} - \frac{m^2}{4} A_{\alpha\beta} A^{\alpha\beta} \tag{19}$$

by a change of field normalizations. Here $\mathcal{H}_{\alpha\beta\gamma} = \partial_\alpha A_{\beta\gamma} + \partial_\beta A_{\gamma\alpha} + \partial_\gamma A_{\alpha\beta}$, $\mathcal{H}_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ and the dependency on the dimensional parameter was restored as well. This lagrangian is invariant under the gauge transformations

$$\begin{aligned}
\delta A_{\alpha\beta} &= \partial_{[\alpha} \lambda_{\beta]m} \\
\delta A_\alpha &= \partial_\alpha \eta + \lambda_\alpha.
\end{aligned}$$

The field A_α is the Goldstone one for the antisymmetric tensor. Herewith the gauge algebra is reducible as in the case with the vector-tensor models with the topological coupling [24,25]. However, unlike the last mentioned, it is obvious that the form of the lagrangian (19) does not depend on the dimensionality of the space-time.

4. About High Spin Interactions in Non-critical Dimensionality

Lagrangians (10) and (23) obtained in the context of the boson string are gauge invariant in the space-time of critical dimensionality 26 only. However, this does not mean that high spin interaction is consistent for the critical dimensionality only. So, for instance, in [12] using the Noether procedure a complete gauge invariant lagrangian describing interaction of massive spin 2 field with the constant electromagnetic field in the space-time of any dimensionality was derived. Similarly, one can consider the massive spin 3 field as well. Besides, it is possible to get an interaction of the high spin fields with the constant field in a non-critical dimensionality from the BRST quantization of "massive" strings [26].

The transition from the usual boson string to the "massive" one is realized by the following modification of action (1) (refer to [26])

$$\tilde{S} = -\frac{\alpha'}{2\pi} \int \sqrt{-g} d^2 \xi g^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu} - \frac{\beta}{2\pi} \int \sqrt{-g} d^2 \xi \left(g^{ab} \partial_a \varphi \partial_b \varphi + 2R_g \varphi \right).$$

Here β is an arbitrary non-negative parameter.

Under quantization, such modification leads to the expansion of the Fock space of the usual string by adding a infinite set of the scalar operators b_m with commutation relations

$$[b_m, b_n] = m \delta_{m+n} \quad (20)$$

and to the change of the definition for the Virasoro operators

$$L_m^X = \frac{1}{2} \sum :a_{m+p} \cdot a_{-p}: + \frac{1}{2} \sum :b_{m+p} b_{-p}: + 2\sqrt{\beta} i m b_m + 2\beta \delta_m. \quad (21)$$

We are always able to include the last term into the definition of L_0 . Besides, a new constraint on the physical states appears

$$(b_0 - q) |\Psi\rangle = 0,$$

where q is an arbitrary real parameter. This requirement leads to a shift of the theory spectrum. To see it we write down the condition $L_0 |\Psi\rangle$ in the absence of the external field

$$\left(-\frac{1}{2} M^2 + \frac{1}{2} \sum_{p=1}^{\infty} (a_{-p} \cdot a_p + b_{-p} b_p) + \frac{1}{2} b_0^2 - 1 \right) |\Psi\rangle = 0,$$

i.e. the mass of any string level gets difference q^2 . Hence it is clear that if $q^2 \geq 2$ the tachyons will be absent in the spectrum.

Operators (21) satisfy the Virasoro algebra with the other central charge

$$[L_m, L_n] = (m - n) L_{m+n} + \frac{D + 1 + 48\beta}{12} (m^3 - m) \delta_{m+n}. \quad (22)$$

From these relations and the requirement of the nilpotency of the BRST operator (6) we have

$$\beta = \frac{25 - D}{48}.$$

This means that we can construct the gauge invariant lagrangian (8) in the space-time with dimensionality $D \leq 25$ for any level. This statement is valid both for the free string and for the string in the constant electromagnetic field. Thereby, we can describe the interaction of high spin fields with the constant electromagnetic field in the space-time with the non-critical dimensionality. A "charge" for such possibility will be an increase of a state number at each string level⁴

Conclusion

In the given work we have considered the electromagnetic interaction of the second massive state of the open boson string. According to the general theory this state contains the massive fields with the spins 3 и 1. Using the BRST quantization method for the open string, we get the gauge invariant lagrangian describing the interaction of these fields with the constant electromagnetic field. From the explicit form of transformations and lagrangian it follows that the presence of the external constant e/m field lead to the state mixing at the given level. Most likely, the presence of the external field will lead to a mixing of states at other mass string levels as well. However, it is not clear if the states of each particular level will mixed all together or some clusterization of states will occur. This question requires a separate study.

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⁴The state number will be the same as in the case of the dimensional reduction $D \rightarrow D - 1$.

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Appendix 1

Here we display the complete non-diagonal lagrangian describing a propagation of the fields with spins 3 and 1 in the homogeneous electromagnetic field without the auxiliary fields

$$\begin{aligned}
L = & 3\bar{H}_{1\alpha\mu}\bar{H}_{1\beta\nu}\bar{H}_{1\gamma\rho}\bar{H}_{1\delta\lambda}\mathcal{D}_\delta\Phi_{\alpha\beta\gamma}\mathcal{D}_\lambda\bar{\Phi}_{\mu\nu\rho} \\
& - 9\bar{H}_{1\alpha\mu}\bar{H}_{1\beta\nu}\bar{H}_{1\gamma\lambda}\bar{H}_{1\delta\rho}\mathcal{D}_\delta\Phi_{\alpha\beta\gamma}\mathcal{D}_\lambda\bar{\Phi}_{\mu\nu\rho} - 3H_{1\beta\mu}H_{1\alpha\delta}H_{1\gamma\nu}\mathcal{D}_{\beta\gamma}^2\bar{A}_\alpha\Phi_{\delta\mu\nu} \\
& - 3\bar{H}_{1\gamma\nu}\bar{H}_{1\alpha\delta}\bar{H}_{1\beta\mu}\mathcal{D}_{\gamma\beta}^2A_\alpha\bar{\Phi}_{\delta\mu\nu} - \frac{1}{4}\bar{H}_{1\beta\mu}\bar{H}_{2\gamma\delta}\bar{H}_{2\alpha\nu}\mathcal{D}_\gamma h_{\alpha\beta}\mathcal{D}_\nu\bar{h}_{\delta\mu} \\
& - \frac{1}{2}\bar{H}_{1\beta\nu}\bar{H}_{1\gamma\mu}\bar{H}_{2\alpha\delta}\mathcal{D}_\gamma h_{\alpha\beta}\mathcal{D}_\nu\bar{h}_{\delta\mu} + \frac{1}{2}\bar{H}_{1\gamma\nu}\bar{H}_{1\beta\mu}\bar{H}_{2\alpha\delta}\mathcal{D}_\gamma h_{\alpha\beta}\mathcal{D}_\nu\bar{h}_{\delta\mu} \\
& + \frac{1}{4}H_{1\beta\nu}H_{2\delta\mu}H_{2\alpha\gamma}\mathcal{D}_{\delta\gamma}^2\bar{h}_{\alpha\beta}A_{\mu\nu} + \frac{1}{4}\bar{H}_{1\beta\nu}\bar{H}_{2\delta\mu}\bar{H}_{2\alpha\gamma}\mathcal{D}_{\delta\gamma}^2h_{\alpha\beta}\bar{A}_{\mu\nu} \\
& + \frac{1}{2}H_{1\beta\gamma}H_{2\alpha\mu}H_{1\delta\nu}\mathcal{D}_{\delta\gamma}^2\bar{h}_{\alpha\beta}A_{\mu\nu} - \frac{1}{2}H_{2\alpha\mu}H_{1\beta\nu}H_{1\gamma\delta}\mathcal{D}_{\delta\gamma}^2\bar{h}_{\alpha\beta}A_{\mu\nu} \\
& - \frac{1}{2}\bar{H}_{1\beta\nu}\bar{H}_{2\alpha\mu}\bar{H}_{1\gamma\delta}\mathcal{D}_{\delta\gamma}^2h_{\alpha\beta}\bar{A}_{\mu\nu} + \frac{1}{2}\bar{H}_{2\alpha\mu}\bar{H}_{1\beta\gamma}\bar{H}_{1\delta\nu}\mathcal{D}_{\delta\gamma}^2h_{\alpha\beta}\bar{A}_{\mu\nu} \\
& - \frac{1}{4}H_{2\alpha\gamma}H_{1\beta\delta}\mathcal{D}_{\beta\alpha}^2\bar{\varphi}h_{\gamma\delta} - \frac{1}{4}\bar{H}_{2\alpha\gamma}\bar{H}_{1\beta\delta}\mathcal{D}_{\beta\alpha}^2\varphi\bar{h}_{\gamma\delta} \\
& + \frac{1}{4}\bar{H}_{1\beta\nu}\bar{H}_{2\delta\mu}\bar{H}_{2\alpha\gamma}\mathcal{D}_{\delta\gamma}^2A_{\alpha\beta}\bar{A}_{\mu\nu} - \frac{1}{2}\bar{H}_{1\beta\nu}\bar{H}_{2\alpha\mu}\bar{H}_{1\gamma\delta}\mathcal{D}_{\delta\gamma}^2A_{\alpha\beta}\bar{A}_{\mu\nu} \\
& + \frac{1}{2}\bar{H}_{2\alpha\mu}\bar{H}_{1\beta\gamma}\bar{H}_{1\delta\nu}\mathcal{D}_{\delta\gamma}^2A_{\alpha\beta}\bar{A}_{\mu\nu} - \frac{1}{4}H_{2\alpha\gamma}H_{1\beta\delta}\mathcal{D}_{\beta\alpha}^2\bar{\varphi}A_{\gamma\delta} \\
& - \frac{1}{4}\bar{H}_{2\alpha\gamma}\bar{H}_{1\beta\delta}\mathcal{D}_{\beta\alpha}^2\varphi\bar{A}_{\gamma\delta} - \frac{1}{2}H_{1\gamma\delta}\bar{H}_{1\alpha\beta}\mathcal{D}_\beta A_\alpha\mathcal{D}_\delta\bar{A}_\gamma \\
& + \bar{H}_{1\alpha\gamma}\mathcal{D}^2A_\alpha\bar{A}_\gamma + \frac{1}{2}\bar{H}_{1\beta\delta}\bar{H}_{3\alpha\gamma}\mathcal{D}_\beta b_\alpha\mathcal{D}_\delta\bar{b}_\gamma - \frac{1}{6}\bar{H}_{3\alpha\delta}\bar{H}_{3\beta\gamma}\mathcal{D}_\beta b_\alpha\mathcal{D}_\delta\bar{b}_\gamma \\
& + \frac{1}{4}H_{1\alpha\beta}\mathcal{D}_{\beta\alpha}^2\varphi\bar{\varphi} - \frac{3}{4}\bar{H}_{2\alpha\gamma}\bar{H}_{1\beta\delta}\bar{H}_{1\mu\nu}\bar{H}_{1\mu\rho}\mathcal{D}_\gamma h_{\alpha\beta}\bar{\Phi}_{\delta\nu\rho}i \\
& + \frac{3}{4}H_{2\alpha\gamma}H_{1\beta\delta}\bar{H}_{1\mu\nu}\bar{H}_{1\mu\rho}\mathcal{D}_\gamma\bar{h}_{\alpha\beta}\Phi_{\delta\nu\rho}i + 3H_{1\beta\mu}H_{1\gamma\nu}H_{2\alpha\delta}H_{1\delta\rho}\mathcal{D}_\gamma\bar{h}_{\alpha\beta}\Phi_{\mu\nu\rho}i \\
& - 3\bar{H}_{1\delta\rho}\bar{H}_{1\gamma\nu}\bar{H}_{1\beta\mu}\bar{H}_{2\alpha\delta}\mathcal{D}_\gamma h_{\alpha\beta}\bar{\Phi}_{\mu\nu\rho}i + \frac{3}{4}H_{2\alpha\gamma}H_{1\beta\delta}\bar{H}_{1\mu\nu}\bar{H}_{1\mu\rho}\mathcal{D}_\gamma\bar{A}_{\alpha\beta}\Phi_{\delta\nu\rho}i \\
& - \frac{3}{4}\bar{H}_{2\alpha\gamma}\bar{H}_{1\beta\delta}\bar{H}_{1\mu\nu}\bar{H}_{1\mu\rho}\mathcal{D}_\gamma A_{\alpha\beta}\bar{\Phi}_{\delta\nu\rho}i + 3H_{1\beta\mu}H_{1\gamma\nu}H_{2\alpha\delta}H_{1\delta\rho}\mathcal{D}_\gamma\bar{A}_{\alpha\beta}\Phi_{\mu\nu\rho}i \\
& - 3\bar{H}_{1\delta\rho}\bar{H}_{1\gamma\nu}\bar{H}_{1\beta\mu}\bar{H}_{2\alpha\delta}\mathcal{D}_\gamma A_{\alpha\beta}\bar{\Phi}_{\mu\nu\rho}i - \frac{3}{4}H_{1\alpha\beta}\bar{H}_{1\gamma\mu}\bar{H}_{1\gamma\delta}\mathcal{D}_\alpha\bar{\varphi}\Phi_{\beta\delta\mu}i \\
& + \frac{3}{4}\bar{H}_{1\gamma\mu}\bar{H}_{1\gamma\delta}\bar{H}_{1\alpha\beta}\mathcal{D}_\alpha\varphi\bar{\Phi}_{\beta\delta\mu}i + \frac{1}{4}H_{1\gamma\mu}H_{2\beta\delta}H_{3\alpha\gamma}\mathcal{D}_\beta\bar{b}_\alpha h_{\delta\mu}i \\
& - \frac{1}{4}\bar{H}_{1\gamma\mu}\bar{H}_{3\alpha\gamma}\bar{H}_{2\beta\delta}\mathcal{D}_\beta b_\alpha\bar{h}_{\delta\mu}i + \frac{1}{2}H_{1\beta\mu}H_{2\gamma\delta}H_{3\alpha\gamma}\mathcal{D}_\beta\bar{b}_\alpha h_{\delta\mu}i \\
& - \frac{1}{2}\bar{H}_{1\beta\mu}\bar{H}_{2\gamma\delta}\bar{H}_{3\alpha\gamma}\mathcal{D}_\beta b_\alpha\bar{h}_{\delta\mu}i + \frac{1}{6}H_{1\gamma\mu}H_{2\gamma\delta}H_{3\alpha\beta}\mathcal{D}_\beta\bar{b}_\alpha h_{\delta\mu}i
\end{aligned} \tag{23}$$

$$\begin{aligned}
& -\frac{1}{6}\bar{H}_{1\gamma\mu}\bar{H}_{2\gamma\delta}\bar{H}_{3\alpha\beta}\mathcal{D}_\beta b_\alpha\bar{h}_{\delta\mu}i - \frac{3}{4}H_{1\alpha\delta}H_{2\beta\gamma}\mathcal{D}_\beta\bar{A}_\alpha h_{\gamma\delta}i \\
& + \frac{3}{4}\bar{H}_{1\alpha\delta}\bar{H}_{2\beta\gamma}\mathcal{D}_\beta A_\alpha\bar{h}_{\gamma\delta}i - \frac{1}{2}H_{1\beta\gamma}H_{1\alpha\mu}H_{2\gamma\delta}\mathcal{D}_\beta\bar{A}_\alpha h_{\delta\mu}i \\
& + \frac{1}{2}\bar{H}_{1\alpha\mu}\bar{H}_{2\gamma\delta}\bar{H}_{1\beta\gamma}\mathcal{D}_\beta A_\alpha\bar{h}_{\delta\mu}i - H_{1\beta\mu}H_{2\gamma\delta}H_{1\alpha\gamma}\mathcal{D}_\beta\bar{A}_\alpha h_{\delta\mu}i \\
& + \bar{H}_{1\beta\mu}\bar{H}_{2\gamma\delta}\bar{H}_{1\alpha\gamma}\mathcal{D}_\beta A_\alpha\bar{h}_{\delta\mu}i + \frac{1}{4}H_{1\gamma\mu}H_{2\beta\delta}H_{3\alpha\gamma}\mathcal{D}_\beta\bar{b}_\alpha A_{\delta\mu}i \\
& - \frac{1}{4}\bar{H}_{1\gamma\mu}\bar{H}_{3\alpha\gamma}\bar{H}_{2\beta\delta}\mathcal{D}_\beta b_\alpha\bar{A}_{\delta\mu}i + \frac{1}{2}H_{1\beta\mu}H_{2\gamma\delta}H_{3\alpha\gamma}\mathcal{D}_\beta\bar{b}_\alpha A_{\delta\mu}i \\
& - \frac{1}{2}\bar{H}_{1\beta\mu}\bar{H}_{2\gamma\delta}\bar{H}_{3\alpha\gamma}\mathcal{D}_\beta b_\alpha\bar{A}_{\delta\mu}i + \frac{1}{6}H_{1\gamma\mu}H_{2\gamma\delta}H_{3\alpha\beta}\mathcal{D}_\beta\bar{b}_\alpha A_{\delta\mu}i \\
& - \frac{1}{6}\bar{H}_{1\gamma\mu}\bar{H}_{2\gamma\delta}\bar{H}_{3\alpha\beta}\mathcal{D}_\beta b_\alpha\bar{A}_{\delta\mu}i - \frac{3}{4}H_{1\alpha\delta}H_{2\beta\gamma}\mathcal{D}_\beta\bar{A}_\alpha A_{\gamma\delta}i \\
& + \frac{3}{4}\bar{H}_{1\alpha\delta}\bar{H}_{2\beta\gamma}\mathcal{D}_\beta A_\alpha\bar{A}_{\gamma\delta}i - \frac{1}{2}H_{1\beta\gamma}H_{1\alpha\mu}H_{2\gamma\delta}\mathcal{D}_\beta\bar{A}_\alpha A_{\delta\mu}i \\
& + \frac{1}{2}\bar{H}_{1\alpha\mu}\bar{H}_{2\gamma\delta}\bar{H}_{1\beta\gamma}\mathcal{D}_\beta A_\alpha\bar{A}_{\delta\mu}i - H_{1\beta\mu}H_{2\gamma\delta}H_{1\alpha\gamma}\mathcal{D}_\beta\bar{A}_\alpha A_{\delta\mu}i \\
& + \bar{H}_{1\beta\mu}\bar{H}_{2\gamma\delta}\bar{H}_{1\alpha\gamma}\mathcal{D}_\beta A_\alpha\bar{A}_{\delta\mu}i - \frac{2}{3}H_{3\alpha\beta}\mathcal{D}_\alpha\bar{\varphi}b_{\beta\gamma}i + \frac{2}{3}\bar{H}_{3\alpha\beta}\mathcal{D}_\alpha\varphi\bar{b}_{\beta\gamma}i \\
& - \frac{1}{4}H_{1\alpha\beta}H_{3\beta\gamma}\mathcal{D}_\alpha\bar{\varphi}b_{\gamma\delta}i + \frac{1}{4}\bar{H}_{3\beta\gamma}\bar{H}_{1\alpha\beta}\mathcal{D}_\alpha\varphi\bar{b}_{\gamma\delta}i + \frac{3}{4}H_{1\alpha\beta}\mathcal{D}_\alpha\bar{\varphi}A_{\beta\gamma}i \\
& - \frac{3}{4}\bar{H}_{1\alpha\beta}\mathcal{D}_\alpha\varphi\bar{A}_{\beta\gamma}i - 6\bar{H}_{1\gamma\delta}\bar{H}_{1\alpha\beta}\bar{H}_{1\mu\nu}\Phi_{\alpha\gamma\mu}\bar{\Phi}_{\beta\delta\nu} \\
& + 18H_{1\alpha\beta}\bar{H}_{1\alpha\gamma}\bar{H}_{1\delta\mu}\bar{H}_{1\nu\rho}\Phi_{\beta\delta\nu}\bar{\Phi}_{\gamma\mu\rho} \\
& - \frac{9}{4}\bar{H}_{1\delta\nu}\bar{H}_{1\alpha\gamma}\bar{H}_{1\alpha\beta}\bar{H}_{1\delta\mu}\bar{H}_{1\rho\lambda}\Phi_{\beta\gamma\rho}\bar{\Phi}_{\mu\nu\lambda} \\
& - 9F_{\beta\lambda}H_{1\alpha\beta}\bar{H}_{1\gamma\delta}\bar{H}_{1\mu\nu}\bar{H}_{1\rho\lambda}\Phi_{\gamma\mu\rho}\bar{\Phi}_{\alpha\delta\nu} \\
& - \frac{3}{4}H_{1\alpha\beta}\bar{H}_{1\delta\nu}\bar{H}_{3\alpha\gamma}\bar{H}_{1\delta\mu}\Phi_{\beta\mu\nu}\bar{b}_\gamma - \frac{3}{4}H_{3\alpha\beta}\bar{H}_{1\delta\nu}\bar{H}_{1\alpha\gamma}\bar{H}_{1\delta\mu}\bar{\Phi}_{\gamma\mu\nu}b_\beta \\
& + \frac{9}{4}H_{1\alpha\beta}\bar{H}_{1\gamma\mu}\bar{H}_{1\gamma\delta}\bar{\Phi}_{\alpha\delta\mu}A_\beta + \frac{9}{4}\bar{H}_{1\alpha\gamma}\bar{H}_{1\alpha\beta}\bar{H}_{1\delta\mu}\Phi_{\beta\gamma\delta}\bar{A}_\mu \\
& + H_{2\alpha\beta}\bar{H}_{2\alpha\gamma}\bar{H}_{1\delta\mu}h_{\beta\delta}\bar{h}_{\gamma\mu} + F_{\gamma\mu}\bar{H}_{1\gamma\delta}\bar{H}_{2\alpha\beta}h_{\alpha\mu}\bar{h}_{\beta\delta} \\
& - \frac{1}{2}H_{1\gamma\delta}H_{2\alpha\beta}\bar{H}_{1\alpha\nu}\bar{H}_{2\gamma\mu}h_{\beta\delta}\bar{h}_{\mu\nu} - \frac{1}{2}H_{2\alpha\beta}\bar{H}_{2\gamma\delta}\bar{H}_{1\alpha\gamma}\bar{H}_{1\mu\nu}h_{\beta\mu}\bar{h}_{\delta\nu} \\
& - \frac{1}{6}H_{2\alpha\beta}H_{1\alpha\gamma}\bar{H}_{2\delta\mu}\bar{H}_{1\delta\nu}h_{\beta\gamma}\bar{h}_{\mu\nu} + \frac{1}{4}F_{\alpha\gamma}H_{2\alpha\beta}\bar{H}_{2\gamma\delta}\bar{H}_{1\mu\nu}h_{\beta\mu}\bar{h}_{\delta\nu} \\
& + \frac{1}{2}F_{\alpha\mu}H_{1\alpha\beta}\bar{H}_{2\gamma\delta}\bar{H}_{1\mu\nu}h_{\beta\gamma}\bar{h}_{\delta\nu} + F_{\gamma\mu}\bar{H}_{1\gamma\delta}\bar{H}_{2\alpha\beta}\bar{h}_{\beta\delta}A_{\alpha\mu} \\
& + F_{\delta\mu}H_{1\gamma\delta}H_{2\alpha\beta}h_{\beta\mu}\bar{A}_{\alpha\gamma} - H_{1\gamma\delta}H_{2\alpha\beta}\bar{H}_{2\alpha\mu}h_{\beta\delta}\bar{A}_{\gamma\mu} \\
& + H_{2\alpha\beta}\bar{H}_{2\alpha\gamma}\bar{H}_{1\delta\mu}\bar{h}_{\gamma\mu}A_{\beta\delta} - \frac{1}{2}H_{1\gamma\delta}H_{2\alpha\beta}\bar{H}_{1\alpha\nu}\bar{H}_{2\gamma\mu}h_{\beta\delta}\bar{A}_{\mu\nu} \\
& - \frac{1}{2}H_{1\gamma\delta}H_{2\alpha\beta}\bar{H}_{1\alpha\nu}\bar{H}_{2\gamma\mu}\bar{h}_{\mu\nu}A_{\beta\delta} + \frac{1}{2}H_{2\alpha\beta}H_{1\delta\mu}\bar{H}_{1\alpha\gamma}\bar{H}_{2\gamma\nu}h_{\beta\mu}\bar{A}_{\delta\nu}
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2}H_{2\alpha\beta}\bar{H}_{2\gamma\delta}\bar{H}_{1\alpha\gamma}\bar{H}_{1\mu\nu}\bar{h}_{\delta\nu}A_{\beta\mu}-\frac{1}{6}H_{2\alpha\beta}H_{1\alpha\gamma}\bar{H}_{2\delta\mu}\bar{H}_{1\delta\nu}h_{\beta\gamma}\bar{A}_{\mu\nu} \\
& -\frac{1}{6}H_{2\alpha\beta}H_{1\alpha\gamma}\bar{H}_{2\delta\mu}\bar{H}_{1\delta\nu}\bar{h}_{\mu\nu}A_{\beta\gamma}+\frac{2}{3}H_{2\alpha\beta}H_{1\alpha\gamma}h_{\beta\gamma}\bar{\varphi} \\
& +\frac{2}{3}\bar{H}_{1\alpha\gamma}\bar{H}_{2\alpha\beta}\bar{h}_{\beta\gamma}\varphi-\frac{1}{4}F_{\alpha\gamma}H_{1\gamma\delta}H_{2\alpha\beta}h_{\beta\delta}\bar{\varphi}+\frac{1}{4}F_{\alpha\gamma}\bar{H}_{1\gamma\delta}\bar{H}_{2\alpha\beta}\bar{h}_{\beta\delta}\varphi \\
& +H_{2\alpha\beta}\bar{H}_{2\alpha\gamma}\bar{H}_{1\delta\mu}A_{\beta\delta}\bar{A}_{\gamma\mu}-F_{\beta\mu}H_{1\alpha\beta}\bar{H}_{2\gamma\delta}A_{\gamma\mu}\bar{A}_{\alpha\delta} \\
& -\frac{1}{2}H_{1\gamma\delta}H_{2\alpha\beta}\bar{H}_{1\alpha\nu}\bar{H}_{2\gamma\mu}A_{\beta\delta}\bar{A}_{\mu\nu}-\frac{1}{2}H_{2\alpha\beta}\bar{H}_{2\gamma\delta}\bar{H}_{1\alpha\gamma}\bar{H}_{1\mu\nu}A_{\beta\mu}\bar{A}_{\delta\nu} \\
& -\frac{1}{6}H_{2\alpha\beta}H_{1\alpha\gamma}\bar{H}_{2\delta\mu}\bar{H}_{1\delta\nu}A_{\beta\gamma}\bar{A}_{\mu\nu}+\frac{2}{3}H_{2\alpha\beta}H_{1\alpha\gamma}A_{\beta\gamma}\bar{\varphi} \\
& +\frac{2}{3}\bar{H}_{1\alpha\gamma}\bar{H}_{2\alpha\beta}\bar{A}_{\beta\gamma}\varphi+\frac{1}{4}F_{\alpha\gamma}\bar{H}_{1\gamma\delta}\bar{H}_{2\alpha\beta}\bar{A}_{\beta\delta}\varphi \\
& -\frac{1}{4}F_{\beta\delta}\bar{H}_{1\gamma\delta}\bar{H}_{2\alpha\beta}A_{\alpha\gamma}\bar{\varphi}+H_{2\alpha\beta}\bar{H}_{3\alpha\gamma}b_{\beta}\bar{b}_{\gamma}-\frac{1}{4}H_{3\alpha\beta}\bar{H}_{1\alpha\gamma}\bar{H}_{3\gamma\delta}b_{\beta}\bar{b}_{\delta} \\
& -\frac{1}{2}H_{3\alpha\beta}\bar{H}_{2\alpha\gamma}\bar{H}_{3\gamma\delta}b_{\beta}\bar{b}_{\delta}+\frac{1}{6}F_{\alpha\gamma}H_{3\alpha\beta}\bar{H}_{3\gamma\delta}b_{\beta}\bar{b}_{\delta} \\
& +\frac{3}{4}H_{1\alpha\beta}\bar{H}_{3\alpha\gamma}\bar{b}_{\gamma}A_{\beta}+\frac{3}{4}H_{3\alpha\beta}\bar{H}_{1\alpha\gamma}b_{\beta}\bar{A}_{\gamma}+\frac{1}{2}H_{1\beta\gamma}H_{2\alpha\beta}\bar{H}_{3\alpha\delta}\bar{b}_{\delta}A_{\gamma} \\
& +\frac{1}{2}H_{3\alpha\beta}\bar{H}_{2\alpha\gamma}\bar{H}_{1\gamma\delta}b_{\beta}\bar{A}_{\delta}-\frac{9}{4}\bar{H}_{1\alpha\beta}A_{\alpha}\bar{A}_{\beta}-H_{2\alpha\beta}\bar{H}_{1\alpha\gamma}A_{\beta}\bar{A}_{\gamma} \\
& -\frac{1}{2}F_{\alpha\beta}^2\bar{H}_{1\gamma\delta}A_{\gamma}\bar{A}_{\delta}-\frac{1}{2}F_{\alpha\gamma}H_{1\alpha\beta}\bar{H}_{1\gamma\delta}A_{\beta}\bar{A}_{\delta} \\
& -\frac{1}{2}H_{1\beta\gamma}H_{2\alpha\beta}\bar{H}_{1\alpha\delta}A_{\gamma}\bar{A}_{\delta}-\frac{8}{3}\varphi\bar{\varphi}-\frac{1}{4}F_{\alpha\beta}^2\varphi\bar{\varphi}
\end{aligned}$$

С.М.Клишевич

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