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MODEL FOR THREE GENERATIONS OF FERMIONS

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Abstract

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A model is constructed for a chiral abelian gauge-interaction of fermions and a potential of three higgses, so that the potential possesses a discrete symmetry of the vacuum state, which provides the introduction of three generations for the fermions. For the model including charged currents, a matrix of the flavor mixing is described, and a phase giving the violation of CP-invariance is determined. Some approximate relations connecting the values of mixing matrix to the mass ratios for the fermions in different generations are derived.

Аннотация

Киселев В.В. Модель трех поколений фермионов: Препринт ИФВЭ 98-21. – Протвино, 1998. – 20 с., 2 рис., 1 табл., библиогр.: 14.

Построена модель с киральным абелевым калибровочным взаимодействием фермионов и потенциалом трех хиггсовских полей, который обладает дискретной симметрией вакуумного состояния, что обеспечивает введение трех поколений фермионов. В случае включения в модель заряженных токов построена матрица смешивания ароматов для этих токов, определена комплексная фаза, задающая нарушение CP-инвариантности, и выведены приближенные соотношения, связывающие величины элементов матрицы с отношениями масс фермионов различных поколений.

Introduction

One problem among those standing beyond the carefully studied Standard Model of elementary particles [1] is the determination of an origin for the fermion generations, which possess identical quantum numbers with respect to gauge interactions and differ just in values of masses because of various couplings to the Higgs fields [2].

At present there are two approaches to this problem. According to the former, the fermionic flavors are postulated as charges of a new “horizontal” gauge interaction, an observation probability of which is suppressed at current energies due to both small values of these charges and large masses of corresponding intermediate bosons [3].

In the letter, the appearance of flavors is caused by some dynamical reasons. The following three ways are usually pointed out [4]:

1. Dynamical origin of flavors. In the framework of models along this way, fermions and Higgs fields are considered as composite states of fundamental “preons”, so that the generations differ because of an internal preonic structure [5].

2. Random lattice model of space and gauge interactions [6]. In such models the flavors appear as effective low-energy fields caused by some nontrivial random interactions on the lattice in a deep region of the Plank scale.

3. Geometric origin of generations: the superstring models with a compactification of an extended-dimensions space [7]. In this way the flavors naturally appear due to an appropriate choice of compactified manifold [8].

A common point of the mentioned ways is an essential extension for a set of fundamental fields with respect to the Standard Model. A replication of Higgs fields¹alike the number of generations seems to be a usual peculiarity of such extensions.

¹In supersymmetric theories an additional higgs appears even in the case of single generation because of the necessary introduction of both a scalar field determining the masses of “up” kind electroweak fermions at spontaneous breaking of the symmetry and a partner of the conjugated scalar field determining the masses of “down” kind electroweak fermions [9].

In the present paper we show how a model of potential for three Higgs fields can be constructed in a manner which results in an appearance of vacuum state possessing a discrete symmetry for higgses at spontaneous breaking of chiral gauge symmetry, so that the discrete symmetry provides the introduction of three generations of fermions. The advantage of such picture of generations is the absence of direct postulation of “preons”, random lattice dynamics or superstring compactifications². Therefore, the offered model can be considered as quite a self-consistent description for the introduction of fermion generations. On the other hand, in the framework of mentioned ways for the construction of fermion flavors, the model can be taken as a common low-energy limit for three effective higgses interacting with the fermion field.

The higgs potential under consideration possesses a discrete symmetry of three vacuum configurations which are determined by a minimum of energy for constant fields. In accordance with transformations of this symmetry, a state of the minimum can be obtained from another by a discrete change of complex phases for the vacuum expectation values (VEV) of higgses. These transformations are the representation of discrete group Z_3 , whereas the Higgs fields have the charges $\{-1, 0, 1\}$. The physical vacuum invariant by the action of discrete group can be constructed as a direct product of minimal energy states for the higgses. Then, the fermionic fields over each of the three minima have different mass values along with the identical properties with respect to the gauge interaction, which means the introduction of three generations of fermions. In a matrix representation of the direct product for the vacuum configurations of higgses, the mass matrix of fermions has a symmetric texture determined by the discrete group. For the presence of charged currents the representation of mass matrix can be fixed by the set of physical parameters in the model: The products of higgs VEV by three Yukawa coupling constants for fermions, whereas the only coupling constant is a complex one. The necessary condition for the fixing of the mass matrix is the introduction of complex phase causing the violation of CP-invariance. In real situation, when the fermion masses of major generation are much greater than the masses of the junior ones, the relation of elements for the matrix of charged current mixing [10] with the physical parameters of model is significantly simplified, so that up to small corrections over the mass ratios of generations one can determine the phase of CP-violation, and the values of mixing angles in the parameterization by Fritzsch-Xing [11] are expressed through the mass ratios.

To clarify both the presentation of method used for the construction of model and its physical meaning, we consider the potential for two higgses with the discrete Z_2 -symmetry of vacuum configurations in Section 1. So, the fermion field has two generations, and the mixing matrix of charged currents is determined by the only parameter, the Cabibbo angle, which is expressed in the known way of “see-saw” kind [12]. The potential

²Each of these ways has its own disadvantage. So, the “preonic” way looks like a straightforward continuation of the atomic idea for an embedding of more and more elementary particles inside bigger nonfundamental objects, so that it is not clear whether a last stage is reached or not, which is weakly attractive. In the random lattice dynamics, it is problematic to build realistic models giving the fermion masses close to the observed ones. In the superstring models a set of compactifications is very broad, so that one can describe practically an arbitrary situation in the low-energy region, but, unfortunately, one still has not found a principal physical basis determining the parameters of the compactification.

model with three higgses is constructed in Section 2. The vacuum is symmetric over the transformations of discrete cyclic group of the third order. This symmetry provides the introduction of three generations for the fermions. Relations for the elements of charged-current mixing matrix with the model parameters are derived. Numerical estimates for the former ones are performed with the use of current experimental data on the fermion masses and Cabibbo–Kobayashi–Maskawa matrix. The obtained results are summarized in the Conclusion.

1. Two generations

Consider a chiral interaction of dirac spinor with an abelian gauge field \mathcal{A}_μ and a higgs h

$$L = \bar{\psi}_R p_\mu \gamma^\mu \psi_R + \bar{\psi}_L (p_\mu - e\mathcal{A}_\mu) \gamma^\mu \psi_L + g\bar{\psi}_R \psi_L h + g\bar{\psi}_L \psi_R h^* + L_{GF} + L_0(h, \mathcal{A}), \quad (1)$$

where L_{GF} is the Lagrangian of free gauge field with a possible introduction of both a term fixing the gauge and a corresponding Lagrangian for Faddeev–Popov ghosts, L_0 is the higgs Lagrangian with a self-action and the gauge interaction, g is the Yukawa constant which can be taken real and positive because of an arbitrary definition of complex phases for the Higgs field and the product of spinors in the vertex of their interaction. At the spontaneous breaking of gauge symmetry the vacuum expectation value of higgs is not equal to zero

$$\langle 0|h|0\rangle = \eta e^{i\phi}. \quad (2)$$

In the unitary gauge $\phi = 0$, the fermion gets the mass³ $m = g\eta$

$$L_m = g\eta(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R). \quad (3)$$

In an arbitrary gauge, it is convenient to use the method of auxiliary fields, so that, for instance,

$$f_L = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_L \\ e^{i\phi} \psi_L \end{pmatrix}, \quad f_R = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\phi} \psi_R \\ \psi_R \end{pmatrix}, \quad (4)$$

and

$$L_m = \bar{f}_R M_{RL} f_L + \bar{f}_L M_{LR} f_R, \quad (5)$$

where

$$M_{RL} = M_{LR} = g\eta \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (6)$$

By the way, the kinetic term of Lagrangian and the gauge interaction of f with the field \mathcal{A} do not change. Thus, one can see from (6), that the field has the mass $m = g\eta$, as one must expect from the unitary gauge in (3).

³The appearance of mass for the gauge field is not a subject of consideration here.

1.1. Potential of two higgses and generations

Now consider the analogous interaction of fermions with two higgses h_1 and h_2 . For the latter ones at the spontaneous breaking of gauge symmetry, the vacuum expectation values are equal to

$$\langle 0|h_1|0\rangle = \eta_1 e^{i\alpha}, \quad \langle 0|h_2|0\rangle = \eta_2 e^{i\alpha+i\phi}, \quad (7)$$

where α is an arbitrary gauge parameter, ϕ is the difference between the phases of VEV, so that this difference can be observable. It is quite evident that the complex phase leads to the fermion mass depending on ϕ

$$\frac{1}{g^2} m^2 = (\eta_1 + \eta_2 \cos \phi)^2 + \eta_2^2 \sin^2 \phi = (\eta_1 + \eta_2)^2 - 4\eta_1 \eta_2 \sin^2 \frac{\phi}{2}. \quad (8)$$

The following situation is of special interest: The phase difference ϕ is constrained due to a form of higgs self-action, so that the vacuum minimum of potential takes place at some discrete values of ϕ .

In the model under consideration, the higgs potential has the form

$$V = \frac{\lambda_{11}}{4} (h_1 h_1^*)^2 - \frac{\mu^2}{2} h_1 h_1^* + \frac{\lambda_{22}}{4} (h_2 h_2^*)^2 - \frac{\lambda_{12}}{8} [h_1 h_2^* + h_2 h_1^*]^2, \quad (9)$$

where all of λ_{ij} and μ^2 are greater than zero, so that the constraint of the potential stability ($V > 0$ at infinity) gives $\lambda_{11}\lambda_{22} > \lambda_{12}^2$.

Then, the equations for the energy minimum

$$\frac{\partial V}{\partial |h_1|} = 0, \quad \frac{\partial V}{\partial |h_2|} = 0,$$

result in the VEV equal to

$$\eta_1^2 = \frac{\mu^2}{\lambda_{11} - \frac{\lambda_{12}^2}{\lambda_{22}} \cos^4 \phi}, \quad (10)$$

$$\eta_2^2 = \frac{\lambda_{12}}{\lambda_{22}} \eta_1^2 \cos^2 \phi. \quad (11)$$

Then, the minimal energy can be written down in the form

$$V_{min} = -\frac{1}{4} \frac{\mu^4}{\lambda_{11} - \frac{\lambda_{12}^2}{\lambda_{22}} \cos^4 \phi}. \quad (12)$$

Therefore, in the vacuum state one has $\cos^2 \phi = 1$ (see Figs.1 and 2). Thus, in the specified model of higgs self-action, the phase difference between the vacuum expectations runs over the discrete values

$$\phi = \pi n, \quad n \in \mathbb{Z}. \quad (13)$$

It is convenient to use the representation for the cyclic group of second order Z_2 , wherein the fields $\{h_1, h_2\}$ have the charges $q = \{0, 1\}$, correspondingly, and the group elements,

the relative phases $\exp\{iqn\pi\}$, are the integer powers of basis $a = \exp\{i\pi\}$. Thus, the configurations with the various relative phases of VEV can be obtained by the transformations of discrete group.

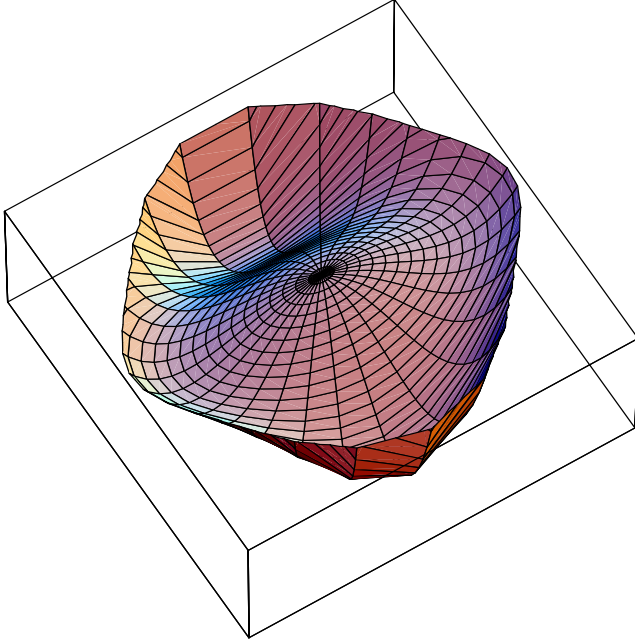


Fig. 1. The potential of two constant Higgs fields in the model of (9) at the fixed value of VEV for the h_2 field in accordance with (11) versus the relative phase ϕ and field $|h_1|$ in the polar coordinates. The measure units are arbitrary, $\lambda_{11} : \lambda_{12} : \lambda_{22} = 10 : 7 : 7$.

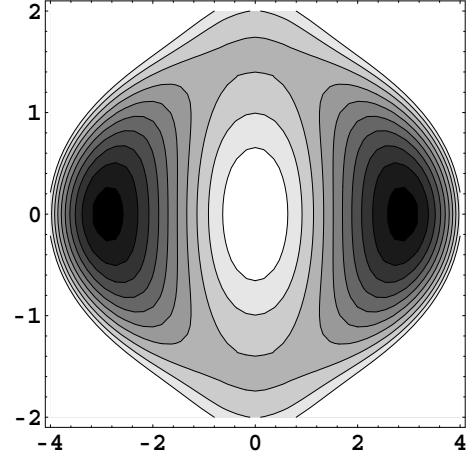


Fig. 2. The levels of potential for two constant Higgs fields in the model of (9) at the fixed value of VEV for the h_2 field in accordance with (11) versus the relative phase ϕ and field $|h_1|$ in the polar coordinates. The measure units are arbitrary, $\lambda_{11} : \lambda_{12} : \lambda_{22} = 10 : 7 : 7$.

Then, introducing fields (4), one gets

$$M_{RL} = g \begin{pmatrix} \eta_2 & \eta_1 e^{-2i\phi} \\ \eta_1 & \eta_2 \end{pmatrix}, \quad (14)$$

and at fixed ϕ (13) the real matrix $M_{RL} = M_{LR}$ has the eigenvalues corresponding to the fermion masses

$$m_{1,2} = g|\eta_1 \pm \eta_2|. \quad (15)$$

Further, introducing

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \quad (16)$$

one gets the diagonal form of the mass matrix

$$M_{LR}^U = M_{RL}^U = U \cdot M_{LR} \cdot U^\dagger$$

$$= g \begin{pmatrix} \eta_2 + \eta_1 & 0 \\ 0 & \eta_2 - \eta_1 \end{pmatrix}$$

with the eigen-vectors $f_{L,R}^U = U f_{L,R}$.

Let us emphasize that, first, the introduced auxiliary fields have the particular form

$$f_L^U = \begin{pmatrix} \psi_L^{(1)} \\ 0 \end{pmatrix}, \quad f_R^U = \begin{pmatrix} \psi_R^{(1)} \\ 0 \end{pmatrix},$$

at $\phi = \phi_{(1)} = 0$, so that acting on the minimal energy configuration $|0_{(1)}\rangle$, the field $\psi^{(1)}$ results in the state with the mass $m^{(1)} = g(\eta_1 + \eta_2)$. Second, one obtains

$$f_L^U = \begin{pmatrix} 0 \\ -\psi_L^{(2)} \end{pmatrix}, \quad f_R^U = \begin{pmatrix} 0 \\ \psi_R^{(2)} \end{pmatrix},$$

at $\phi = \phi_{(2)} = \pi$, so that the $\psi^{(2)}$ field has the mass $m^{(2)} = g|\eta_1 - \eta_2|$ over the configuration $|0_{(2)}\rangle$. Therefore, if one constructs the model vacuum as

$$|\text{vac}\rangle = |0_{(1)}\rangle \otimes |0_{(2)}\rangle = \begin{pmatrix} |0_{(1)}\rangle \\ |0_{(2)}\rangle \end{pmatrix}, \quad (17)$$

then one can use two kinds of auxiliary fields at $\phi = 0$ and $\phi = \pi$, f^U , having the mentioned particular form⁴ and the common mass-matrix independent of the chosen values of ϕ , to introduce the common physical field with the components corresponding to the kind of generation. So, the extended Lagrangian contains the physical field f over the vacuum $|\text{vac}\rangle$ with two generations. Note that the vacuum conserves the discrete symmetry of the second-order cyclic group, which does provide the introduction of fermionic generations in the given model⁵.

The structure of the Higgs fields vacuum cannot change the number of fermionic degrees of freedom, so, it cannot be the origin of generations, since it cannot produce these fermion fields. Two generations are introduced by construction. However, the considered model of Higgs vacuum certainly provides the introduction of two generations: the Higgs vacuum is arranged for the introduction of two generations of fermions. So, the number of fermionic degrees of freedom is not changed in an original Lagrangian, since it is introduced in the self-consistent way.

It is correct that in a gauge theory the only possible vacuum configuration can be chosen because the physical meaning of gauge invariance is the equivalence of those configurations: The physics in each configuration is the same, and the only configuration must be chosen, the others are produced by the gauge transformations. So, the physical

⁴One can easily find that the auxiliary fields have been chosen in the way, where for each of the given phase differences the eigen-vectors of mass matrix have been formed.

⁵We have defined the number of fermion fields to be equal to the number of configurations for the vacuum state, i.e. we have fixed the number of fermionic degrees of freedom in accordance with the vacuum structure.

principle for the restriction by the only configuration is the gauge invariance. However, that is not the case with the model under consideration. The constructed configurations of minimal energy are not gauge equivalent, since they distinguish by the phase difference which is the observable physical quantity. Therefore, the full vacuum of the theory must contain all of the configurations because there is no physical principle (“a rule of superselection”) to cancel some configuration and to restrict the state. So, the paper describes the theory, where all of the configurations are symmetrically presented in the vacuum.

The minimal energy of potential is symmetric, but the quantum numbers, which are the gauge independent differences between the VEV phases, are changed under the action of discrete group. Hence, this symmetry cannot be broken spontaneously, since the latter results in forbidding of some values for the quantum numbers, with no physical reasons.

The vacuum expectation values are equal to

$$\langle \text{vac} | h | \text{vac} \rangle = \langle 0_{(1)} | \otimes \langle 0_{(2)} | h | 0_{(1)} \rangle \otimes | 0_{(2)} \rangle = \begin{pmatrix} \langle 0_{(1)} | h | 0_{(1)} \rangle & 0 \\ 0 & \langle 0_{(2)} | h | 0_{(2)} \rangle \end{pmatrix},$$

which is the matrix due to the vacuum definition. So, the higgs VEV, which is a simple classic field, converts to the matrix with the classic elements. One can simply recognize that the same representation can be equivalently obtained by the following construction:

1). Introduce the replication of higgs sector for each generation, i.e. introduce the matrix for each higgs field

$$h_i \rightarrow \begin{pmatrix} h_i^{(1)} & 0 \\ 0 & h_i^{(2)} \end{pmatrix}.$$

2). Construct the new vacuum as

$$|0\rangle = |0_{(1)}\rangle |0_{(2)}\rangle,$$

where $|0_{(1)}\rangle$ is the single chosen vacuum configuration for the first generation with its own higgs sector, and $|0_{(2)}\rangle$ is the single vacuum configuration for the second generation with its higgs sector, respectively.

Then, the higgs VEV is the same matrix

$$\langle 0 | h | 0 \rangle = \begin{pmatrix} \langle 0_{(1)} | h^{(1)} | 0_{(1)} \rangle & 0 \\ 0 & \langle 0_{(2)} | h^{(2)} | 0_{(2)} \rangle \end{pmatrix}.$$

Thus, this construction clarifies the situation in the usual way.

The author prefers for the complex definition of the vacuum, when it carries an internal structure, than the replication of higgs sector and, hence, the undesirable breaking of discrete symmetry in each isolated sector, together with the additional observable scalar particles. The first economic way is more attractive to me, but the second one is more evident, simple, usual and clear.

In the matrix representation of vacuum state, there is the permutation symmetry for the vacuum configurations, which is related to the discrete group. These permutations contain two elements:

$$S_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

which correspond to the arbitrary arrangement of generations: Whether one puts the first generation to be light or heavy, so that in the initial representation these permutations result in the substitutions $\eta_1 \rightarrow \pm\eta_1$ in the mass matrix of (14).

Thus, two species of fermions possessing the identical properties with respect to the gauge interaction and different mass values, are introduced because of two kinds of vacuum configurations for the pair of higgses, so that in the model under consideration, the effect of introduction of two generations is provided by the vacuum structure of Higgs fields.

From the practical point of view, equation (15) shows that the large splitting of fermion masses for two generations (e, μ) can be caused by no difference between the values of Yukawa constants for the interaction between the fermions and higgses. The mass difference can be the result of small splitting between the vacuum expectation values⁶ (at $\eta = \eta_1 > \eta_2 > 0, \Delta\eta = \eta_1 - \eta_2 \ll \eta$)

$$\frac{m_1}{m_2} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} \approx \frac{\Delta\eta}{2\eta} \ll 1. \quad (18)$$

Thus, we have shown that at the spontaneous breaking of gauge symmetry the structure of vacuum for the pair of Higgs fields interacting with the fermions can provide the introduction of two generations of fermions as two kinds of the relation with the vacuum symmetric over the discrete group.

1.2. Mixing matrix. Cabibbo angle.

The introduction of auxiliary fields (4) with the symmetric mass-matrix at $\phi = 0, \pi$ contains the uncertainty related to an additional term

$$\Delta M = \begin{pmatrix} -a & 0 \\ 0 & a \end{pmatrix},$$

which does not contribute to the determination of the mass values, since it is canceled in the Lagrangian expressed through the initial single-generation fields. This uncertainty corresponds to the rotation of the introduced physical fields in the Lagrangian with two generations. So, considering the general form of extended Lagrangian with different Yukawa constants g_1 and g_2 for the Higgs fields h_1 and h_2 , one gets the following expression for the mass matrix⁷:

$$M = \begin{pmatrix} v_2 - v_1 \sin 2\theta & v_1 \cos 2\theta \\ v_1 \cos 2\theta & v_2 + v_1 \sin 2\theta \end{pmatrix}, \quad (19)$$

where $v_i = g_i \eta_i$, so that the eigenvalues of the matrix are the same $m_{1,2} = |v_1 \pm v_2|$, which are independent of the θ value. However, there is a particular state, when θ is determined

⁶Supposing $m_1/m_2 = m_e/m_\mu$, one finds that $\Delta\eta/\eta \approx 1/103 \sim \alpha_{em}$, so that, probably, the splitting is of the order of radiative corrections.

⁷The number of relative phases, which cannot be removed due to redefinitions in the set of couplings $\{g_1, g_2, \dots, g_n\}$ and fields $\{h_1, h_2, \dots, h_n\}$, is equal to $n_\delta = (n-1)(n-2)/2$, so that for $n=2$, we have the situation, when $n_\delta = 0$, and $\{g_1, g_2\}$ are real and positive.

by the physical quantities of model. So, we find⁸

$$M = \begin{pmatrix} v - a & v \\ v & v + a \end{pmatrix}, \quad (20)$$

where $v = v_2$, $a = v \tan 2\theta$,

$$\cos 2\theta = \frac{v_2}{v_1} = \frac{g_2}{g_1} \sqrt{\frac{\lambda_{12}}{\lambda_{22}}}.$$

The θ value is related to the masses of fermions

$$\tan 2\theta = 2 \frac{\sqrt{m_1 m_2}}{m_2 - m_1}. \quad (21)$$

The matrix gets the “see-saw” form in the “heavy” basis [13]

$$M^U = U \cdot M \cdot U^\dagger = \begin{pmatrix} 2v & a \\ a & 0 \end{pmatrix}, \quad (22)$$

where U is defined in eq.(16). At $v_1 = v_2$ we have $a = 0$, and the mass matrix of (20) is a “democratic” one [13].

Next, the model matrix of (22) takes the diagonal form after the action by the rotation to the angle θ .

Let us consider now the model with two kinds of the chiral fields $\psi^{(u)}$ and $\psi^{(d)}$ possessing different charges, so that the charged current interaction between the latter ones has the form

$$L_{cc} = e \bar{\psi}_L^{(u)} W_\mu \gamma^\mu \psi_L^{(d)} + \text{h.c.},$$

where the initial mass-matrices have the form of (20) with

$$\cos 2\theta^{(u)} = \frac{g_2^{(u)}}{g_1^{(u)}} \sqrt{\frac{\lambda_{12}}{\lambda_{22}}}, \quad \cos 2\theta^{(d)} = \frac{g_2^{(d)}}{g_1^{(d)}} \sqrt{\frac{\lambda_{12}}{\lambda_{22}}}.$$

Then, the Cabibbo mixing matrix has the form

$$V = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix},$$

where $\theta_c = \theta^{(u)} - \theta^{(d)}$. At $\frac{m_2^{(u)}}{m_1^{(u)}} \ll \frac{m_2^{(d)}}{m_1^{(d)}} \ll 1$, one gets the Cabibbo angle

$$\sin \theta_c \approx \sqrt{\frac{m_1^{(d)}}{m_2^{(d)}}}.$$

Thus, the offered model provides the introduction of two generations due to the structure of vacuum for two Higgs fields. It describes also the mixing matrix of charged currents through the physical quantities, which allows one to relate the mixing angle with the masses of fermions.

⁸We consider the case of $v_2/v_1 \leq 1$. A description of the inverse condition is quite evident after a suitable transformation of ψ_R .

2. Three generations

Constructing the model in this section, we will follow the scheme presented above in the case of two generations.

Consider the potential of three higgses

$$\begin{aligned}
 V = & \frac{\lambda_{11}}{4}(h_1^*h_1)^2 - \frac{\mu^2}{2}h_1^*h_1 + \frac{\lambda_{22}}{4}[(h_2^*h_2)^2 + (h_3^*h_3)^2] - \\
 & \frac{\lambda_{12}}{8}[h_1^*h_2 + h_2^*h_1 + h_1^*h_3 + h_3^*h_1 + h_2^*h_3 + h_3^*h_2 - \\
 & \frac{k}{2}(h_1^*h_1 + h_2^*h_2 + h_3^*h_3)]^2.
 \end{aligned} \tag{23}$$

As it has been mentioned in the previous section, the relative phases in the sum of pair products of three higgses in the expression for their potential can be removed by the redefinitions of fields so that there are no additional complex phases in the second string of (23). Moreover, in the contact terms of higgs interaction with the fermion field one can make two quantities of three Yukawa constants to be positive by the same procedure. So, there is the only complex parameter, a single Yukawa constant.

The vacuum configuration corresponds to constant fields determined by five parameters: three absolute values of VEV η_i and two phase differences between them, $\phi_j = \arg[h_j h_1^*]$. Putting the first derivatives of potential over these quantities to be equal to zero, we find

$$V_{min} = -\frac{\mu^2}{4}\eta_1^2$$

from the condition $\sum \eta_i \cdot \frac{\partial V}{\partial \eta_i} = 0$. Further, the derivatives over the phase differences give

$$\eta_1 \sin(\phi_1) + \eta_3 \sin(\phi_1 - \phi_2) = 0, \tag{24}$$

$$\eta_1 \sin(\phi_2) - \eta_2 \sin(\phi_1 - \phi_2) = 0, \tag{25}$$

for nonzero values of η_i . A consideration of $\phi_j = \{0, \pi\}$ can be easily performed, so that we study the configuration, wherein the latter ones are excluded. Then, we get

$$\phi_1 = -\phi_2, \tag{26}$$

$$\cos^2 \phi_1 = \frac{2\lambda_{22}}{\lambda_{12}(2+k)^2} - \frac{1}{2}, \tag{27}$$

$$\eta_1^2 = \frac{\mu^2}{\lambda_{11} - \frac{\lambda_{12}}{16 \cos^2 \phi_1} (4 + 4k + k^2)(2 \cos^2 \phi_1 + 1)}. \tag{28}$$

In the rest of paper we use the following set of potential parameters

$$k = 1, \tag{29}$$

$$\lambda_{11} = 20\lambda_{12}, \tag{30}$$

$$\lambda_{22} = \frac{27}{8}\lambda_{12}, \tag{31}$$

which leads to

$$\eta_1 = \eta_2 = \eta_3, \quad (32)$$

$$\phi_1 = \pm \frac{2\pi}{3}, \quad (33)$$

where we have taken into account the condition $\eta_i > 0$. For the chosen set of parameters the same absolute values of both VEV and minimal energy are reached at $\phi_j = 0$, too.

Thus, in the described model of potential for three Higgs fields the vacuum configuration corresponds to three possible sets of phase differences: $\phi_1 = \{0; \pm 2\pi/3\}$, $\phi_2 = -\phi_1$. The fact means that the model vacuum is symmetric over the action of third-order cyclic group Z_3 , whereas the fields $\{h_1, h_2, h_3\}$ possess the charges $\{0, 1, -1\}$, correspondingly. As it was in the case of two generations, it is convenient to use the matrix representation for the vacuum state defined as the direct product of three given configurations of minimal energy

$$|\text{vac}\rangle = |0_{(1)}\rangle \otimes |0_{(2)}\rangle \otimes |0_{(3)}\rangle = \begin{pmatrix} |0_{(1)}\rangle \\ |0_{(2)}\rangle \\ |0_{(3)}\rangle \end{pmatrix}. \quad (34)$$

Acting on each element of vacuum $|\text{vac}\rangle$, the fermionic operator produces the field with different masses: three configurations lead to three values of masses. Thus, the discrete symmetry of vacuum provides the introduction of three generations of fermionic field.

In the Lagrangian for the interaction of higgses with the fermion field

$$L_m = g_1 \bar{\psi}_R \psi_L h_1 + g_1 \bar{\psi}_L \psi_R h_1^* + g_2 \bar{\psi}_R \psi_L h_2 + g_2 \bar{\psi}_L \psi_R h_2^* + g_3 \bar{\psi}_R \psi_L h_3 + g_3 \bar{\psi}_L \psi_R h_3^*, \quad (35)$$

we will suppose that $g_{2,3} > 0$, and

$$g_1 = |g_1| e^{i\epsilon}.$$

Introduce the notations

$$g_1 \cdot \eta_1 = v + i \cdot \bar{v}, \quad (36)$$

$$g_2 \cdot \eta_2 = v_2, \quad (37)$$

$$g_3 \cdot \eta_3 = v_3, \quad (38)$$

and

$$v' = \frac{1}{2}(v_2 + v_3), \quad (39)$$

$$\bar{v}' = \frac{\sqrt{3}}{2}(v_2 - v_3). \quad (40)$$

Then the masses of generations are determined by the expressions

$$m_3^2 = (v + 2v')^2 + \bar{v}^2, \quad (41)$$

$$m_2^2 = (v - v')^2 + (\bar{v} + \bar{v}')^2, \quad (42)$$

$$m_1^2 = (v - v')^2 + (\bar{v} - \bar{v}')^2. \quad (43)$$

Construct the auxiliary fields

$$f_L = \frac{1}{\sqrt{3}} \begin{pmatrix} \psi_L \\ e^{-2i\phi_1}\psi_L \\ e^{-2i\phi_2}\psi_L \end{pmatrix}, \quad f_R = \frac{1}{\sqrt{3}} \begin{pmatrix} \psi_R \\ e^{-i\phi_1}\psi_R \\ e^{-i\phi_2}\psi_R \end{pmatrix}. \quad (44)$$

Then the Lagrangian of (35) can be written down in terms of fields (44) with the use of mass matrix

$$\tilde{M}_{RL} = \begin{pmatrix} v_1 & v_2 e^{3i\phi_1} & v_3 e^{3i\phi_2} \\ v_2 & v_3 e^{i(\phi_1+\phi_2)} & v_1 e^{i(2\phi_2-\phi_1)} \\ v_3 & v_1 e^{i(2\phi_1-\phi_2)} & v_2 e^{i(\phi_1+\phi_2)} \end{pmatrix}. \quad (45)$$

In the considered model of potential the admissible set of phase differences results in the mass matrix, which does not depend on the choice of vacuum configuration

$$M_{RL} = \begin{pmatrix} v_1 & v_2 & v_3 \\ v_2 & v_3 & v_1 \\ v_3 & v_1 & v_2 \end{pmatrix}. \quad (46)$$

In a general case, matrix (46) is not hermitian because of the complex value of v_1 . It possesses the permutation symmetry over the indices of higgses in the way that its eigenvalues do not change through these permutations. The elements of the permutation group are given by the matrices

$$S_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (47)$$

$$S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (48)$$

The mass matrix can be diagonalized by the action of unitary matrices

$$U_R = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad U_L^\dagger = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

where $\omega = \exp\{i2\pi/3\}$, so that

$$U_R M_{RL} U_L^\dagger = \begin{pmatrix} M_3 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_1 \end{pmatrix},$$

where $|M_i| = m_i$,

$$M_3 = v_1 + v_2 + v_3, \quad (49)$$

$$M_2 = v_1 + v_2\omega + v_3\omega^2, \quad (50)$$

$$M_1 = v_1\omega^2 + v_2\omega + v_3. \quad (51)$$

Further, note that the unitary matrices U^\dagger diagonalizing M_{RL} , are constructed from the corresponding vectors, which are given by the auxiliary fields of (44) at three fixed phase differences of higgs VEV. The fact means that, as it was in the model with two generations, each configuration of auxiliary field in the diagonal representation corresponds to the only generation, exactly, i.e. the only element of vector is not equal to zero. Thus, we can introduce the physical field f , whose components correspond to the generations of fermion field, with the common mass-matrix of (46).

2.1. Mixing matrix of charged currents. Phase of CP-violation

Having considered the example of two fermionic generations, we see that the initial mass-matrix for the auxiliary fields contains the uncertainty, which does not change the values of masses and can be related to the rotations of components for the extended physical field. For three generations, a displacement of relative phase between the generations is added to the rotations. The rotation angles and the complex phase cannot be any external additional parameters of the model, and they have to be related to the physical quantities: v, v', \bar{v}, \bar{v}' . So, the transformation parameters can be determined due to a fixing of a symmetric form of the mass matrix. For the system of two generations, the symmetry condition for the 2×2 matrix has looked like:

$$M(1, 2) = M(2, 1), \quad \frac{1}{2}[M(1, 1) + M(2, 2)] = M(1, 2), \quad (52)$$

which does not depend on the mentioned permutation symmetry for the components. The symmetry condition leads to definite relations of the Cabibbo angle with the physical parameters of model. The natural unification of condition to the case of three generations is given by the following equations:

$$M(1, 2) = M(2, 1), \quad M(1, 3) = M(3, 1), \quad M(2, 3) = M(3, 2), \quad (53)$$

$$\frac{1}{2}[M(1, 2) + M(2, 3)] = M(1, 3), \quad (54)$$

so that the resulting matrix has the form

$$M = \begin{pmatrix} \mu_1 & \bar{\mu} + \Delta & \bar{\mu} \\ \bar{\mu} + \Delta & \mu_2 & \bar{\mu} - \Delta \\ \bar{\mu} & \bar{\mu} - \Delta & \mu_3 \end{pmatrix}. \quad (55)$$

It can be constructed in the following way:

Permute elements v_1 and v_2 in matrix (46) (the action of permutation operator S_4) and come to the ‘‘heavy’’ basis by the transformation

$$U \cdot M_{RL} \cdot U^\dagger = M_U,$$

where

$$U = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \end{pmatrix}, \quad (56)$$

so that

$$M_U = \begin{pmatrix} v + 2v' + i \cdot \bar{v} & 0 & 0 \\ 0 & v' - v - i \cdot \bar{v} & \bar{v}' \\ 0 & \bar{v}' & v - v' + i \cdot \bar{v} \end{pmatrix}. \quad (57)$$

Further, transform this matrix to the form

$$\tilde{M}_U = \begin{pmatrix} v + 2v' + i \cdot \bar{v} & 0 & 0 \\ 0 & -\bar{v} + i \cdot (v - v') & \bar{v}' \\ 0 & \bar{v}' & -\bar{v} + i \cdot (v - v') \end{pmatrix} \quad (58)$$

by the action of operators

$$U_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix}$$

so that $\tilde{M}_U = U_2 \cdot M_U \cdot U_1$. As one can see, matrix (58) has the form similar to one considered above in the case of two generations, so that the third generation is decoupled, and the real part of matrix for two other generations exactly corresponds to the form of matrix constructed above. By the way, an imaginary part of matrix for two generations is proportional to the identical one. Then, we can easily introduce the rotation in an analogous way as in the model with two generations, so that

$$\cos 2\theta_{12} = \frac{\bar{v}}{\bar{v}'}, \quad (59)$$

and the mass matrix is transformed to

$$M_{(12)} = S_{(1)}^\dagger \cdot \tilde{M}_U \cdot S_{(1)},$$

where

$$S_{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{12} & \sin \theta_{12} \\ 0 & -\sin \theta_{12} & \cos \theta_{12} \end{pmatrix}.$$

Further, make the inverse transformation with matrices $U_{1,2}$ to get the previous texture of imaginary parts for the elements of mass matrix: $\tilde{M}_{(12)} = U_2^\dagger \cdot M_{(12)} \cdot U_1^\dagger$. Then, introduce both the angle of mixing with the third (heavy) generation and the relative phase

$$S_{(3)} = \begin{pmatrix} \cos \theta_3 & 0 & \sin \theta_3 \\ 0 & e^{i\delta} & 0 \\ -\sin \theta_3 & 0 & \cos \theta_3 \end{pmatrix},$$

whereas

$$M_{(3)} = S_{(3)}^\dagger \cdot \tilde{M}_{(12)} \cdot S_{(3)}.$$

In what follows we will use the approximation of low θ_3 value in the linear order over this angle. Finally, coming from the “heavy” basis to the “democratic” one, we find the mass matrix M

$$M = U^\dagger \cdot M_{(3)} \cdot U.$$

Making conditions (54) to be valid, we get that the mass matrix takes form (55), if the following equations are satisfied

$$-\frac{2}{3\sqrt{2}} \frac{v - v' + \sqrt{3}\bar{v} \cos \delta}{v' + \frac{1}{\sqrt{3}}\bar{v} \cos \delta} = \theta_3, \quad (60)$$

$$6 - 8\sqrt{3} \sin \delta - \sqrt{6}\theta_3 \sin \delta + 3\sqrt{2}\theta_3 \tan 2\theta_{12} = 0. \quad (61)$$

Constructing a realistic model, we have to use the condition of observed low values of masses for the junior generations with respect to the mass of major generation as well as the fact that the mixing between two junior generations is quite accurately described by the scheme considered in the model of two generations. So, in the realistic model we suppose

$$\begin{aligned} |v - v'| &\ll |\bar{v}|, \\ |\bar{v}| &\ll v, v', \\ |v - v'| &\ll |\bar{v} - \bar{v}'|. \end{aligned}$$

Then we can easily find

$$\begin{aligned} \sin \delta &\approx \frac{\sqrt{3}}{4}, \\ \theta_3 &\approx -\sqrt{\frac{3}{2}} \frac{2\bar{v}}{3v'} \cos \delta \approx \pm \sqrt{\frac{39}{32}} \frac{m_2}{m_3}, \\ \tan 2\theta_{12} &\approx 2\sqrt{m_1 m_2} / (m_2 - m_1), \\ m_3 &\approx 3v, \\ m_2 &\approx |\bar{v}| + |\bar{v}'|, \\ m_1 &\approx ||\bar{v}| - |\bar{v}'||. \end{aligned} \quad (62)$$

First of all, note that at small angles of generation mixing, the introduction of complex phase δ is necessary, whereas the value of $\sin \delta$ in the leading approximation is certainly fixed. However, we have to point out such uncertainties as signs of $\cos \delta$ ($\delta \rightarrow \pi - \delta$), \bar{v} , \bar{v}' , and, hence, signs of some masses and rotation angles. Therefore, the order of arrangement for the first and second generations is not definite in the diagonal representation, since the relative sign of \bar{v} and \bar{v}' is uncertain. The phenomenological analysis shows that the angle θ_3 corresponds to the mixing between the second generation and the third one.

Now consider the picture of mixing for the charged currents of left-handed fermions, $\psi^{(u)}$ and $\psi^{(d)}$. One can easily understand that the unitary matrix of Cabibbo–Kobayashi–Maskawa is given by the expression⁹

$$V_{CKM} = S_{(1)}^{(u)} \cdot S_{(3)}^{(u)} \cdot S_{(3)}^{(d)\dagger} \cdot S_{(1)}^{(d)\dagger}.$$

⁹In the model with two generations the additional transformation by matrix (16) does not influence the Cabibbo matrix because this transformation is the rotation to the angle $\frac{\pi}{4}$, which is compensated by the inverse rotation for the quarks of other charge. In the model with three generations, the presence of mixing with the third heavy generation leads to the fact that to remove the additional rotation to the

The complex phases in the matrices $S_{(3)}$ for the “up” kind and “down” kind fermions do not cancel each other, if $\delta^{(u)} = \pi - \delta^{(d)}$. For the mixing angles of junior generations $\theta^{(u,d)} = \theta_{12}^{(u,d)}$ with the accuracy up to the sign, we approximately get

$$\sin \theta^{(u,d)} \approx \pm \sqrt{\frac{m_1^{(u,d)}}{m_2^{(u,d)}}}.$$

Thus, the mixing matrix V_{CKM} in the representation of Fritzsche–Xing has the form

$$V_{CKM} = \begin{pmatrix} s_u s_d c + c_u c_d e^{-i\tilde{\delta}} & s_u c_d c - c_u s_d e^{-i\tilde{\delta}} & s_u s \\ c_u s_d c - s_u c_d e^{-i\tilde{\delta}} & c_u c_d c + s_u s_d e^{-i\tilde{\delta}} & c_u s \\ -s_d s & -c_d s & c \end{pmatrix},$$

where $\cos \theta^{(u,d)} = c_{u,d}$, and analogous definitions are made for the sines, and

$$\begin{aligned} \tilde{\delta} &\approx \pi \pm 2 \arcsin \frac{\sqrt{3}}{4}, \\ \theta &= \theta_3^{(u)} - \theta_3^{(d)}. \end{aligned}$$

The features of such representation of the mixing matrix for the charged currents are in detail discussed in papers by Fritzsche and Xing. Let us here note only that with the accuracy up to small corrections over the mass ratios for the fermion generations, we have

$$\cos \tilde{\delta} = -\frac{5}{8}, \quad (63)$$

$$|\theta| = \sqrt{\frac{39}{32}} \left| \frac{m_2^{(u)}}{m_3^{(u)}} \pm \frac{m_2^{(d)}}{m_3^{(d)}} \right|. \quad (64)$$

2.2. Numerical estimates

Consider the matrix of charged current mixing for quarks. This matrix is of special interest phenomenologically, since the measurement of its elements is carried out experimentally in the framework of precision study of the Standard Model, but also theoretically because, in contrast to lots of models introducing the various initial mass-matrices of the fermion generations, the model under consideration can make some definite predictions on both the phase of CP-violation (63) and the relation of angle θ with the masses of quarks.

Indeed, with the accuracy up to small corrections linear over the ratios of generation masses, we find

$$|V_{cb}| \approx |\theta| = \sqrt{\frac{39}{32}} \left| \frac{m_s}{m_b} \pm \frac{m_c}{m_t} \right|.$$

angle $\frac{\pi}{4}$, we have correspondingly to transform the mass matrix: to come to the “heavy” basis, to make the inverse rotations by matrix $S_{(3)}$ and operators $U_{1,2}$, to rotate the matrix of two light generations by (16), and, finally, to be back to the initial texture (operators $U_{1,2}$, $S_{(3)}$ and the rotation from the “heavy” basis to the “democratic” one (56)). By the procedure, the mentioned properties of symmetry for the mass matrix naturally survive in a changed form.

For the Cabibbo angle, we get

$$\sin \theta_c = |s_u - s_d e^{-i\tilde{\delta}}| \approx |s_d + s_u \cos \tilde{\delta}| = \left| \sqrt{\frac{m_d}{m_s}} \pm \frac{5}{8} \sqrt{\frac{m_u}{m_c}} \right|,$$

where the upper sign corresponds to the negative value of relative sign between s_u and s_d . Further,

$$\left| \frac{V_{ub}}{V_{cb}} \right| = \tan \theta_u = \sqrt{\frac{m_u}{m_c}}, \quad (65)$$

$$\left| \frac{V_{td}}{V_{ts}} \right| = \tan \theta_d = \sqrt{\frac{m_d}{m_s}} \approx \sin \theta_c. \quad (66)$$

In the B -meson physics, angles of unitary triangle are usually introduced. So, in the given model we can predict, for example, $\alpha = \arg[-V_{td}V_{tb}^*V_{ud}^*V_{ub}]$, which approximately coincides with the angle $\tilde{\delta}$ up to high accuracy,

$$\sin 2\alpha \approx \sin 2\tilde{\delta} = \pm \frac{5}{32} \sqrt{39} \approx \pm 0.976.$$

For $\beta = \arg[-V_{cd}V_{cb}^*V_{td}^*V_{tb}]$, we obtain

$$\sin 2\beta \approx 2 \frac{s_u}{s_d} \sin \tilde{\delta} \left[1 + \frac{s_u}{s_d} \cos \tilde{\delta} \right] \approx \pm \frac{\sqrt{39}}{4} \sqrt{\frac{m_u m_s}{m_d m_c}} \left[1 \mp \frac{5}{8} \sqrt{\frac{m_u m_s}{m_d m_c}} \right].$$

The universal parameter \mathcal{J} [14] independent of the representation for the mixing matrix of charged currents determines the violation of CP-invariance. In the model it is given by the expression

$$\mathcal{J} = s_u c_u s_d c_d s^2 c \sin \tilde{\delta} \approx \pm \frac{39}{32} \left| \frac{m_s}{m_b} \pm \frac{m_c}{m_t} \right|^2 \sqrt{\frac{m_u m_d}{m_s m_c}} \frac{\sqrt{39}}{8}.$$

Choosing the negative sign in the expressions for the Cabibbo angle and element V_{cb} and supposing that¹⁰

$$\begin{aligned} m_d &= 14 \text{ MeV}, & m_u &= 7 \text{ MeV}, \\ m_s &= 200 \text{ MeV}, & m_c &= 1.4 \text{ GeV}, \\ m_b &= 4.6 \text{ GeV}, & m_t &= 175 \text{ GeV}, \end{aligned} \quad (67)$$

¹⁰In this paper we are not concerned with the problem on the values of quark masses: a scale for the determination of “running” masses, a relation of the current masses with the pole ones, uncertainties of theoretical models in QCD for an extraction of the quark masses from the experimental data and so on. Note only that an admissible variation of quark masses for two light generations can reach 50%, because of both effects of QCD evolution to a scale of the order of t -quark mass and some uncertainties in the initial data of the evolution. So, the set supposed in (67) is generally used for the illustrative purpose though it is quite typical.

we find¹¹ that

$$\sin 2\beta = 0.490,$$

and the set of values shown in Table 1 for the elements of Cabibbo–Kobayashi–Maskawa matrix.

Table 1. The model predictions with the mass set of (67) in comparison with the current experimental data.

quantity	model	experiment
$ V_{us} $	0.220	0.2205 ± 0.0026
$ V_{cb} $	0.0392	0.0392 ± 0.003
$ V_{ub}/V_{cb} $	0.071	0.08 ± 0.02
$ V_{td}/V_{ts} $	0.265	0.22 ± 0.07

Thus, we have shown that in the framework of model the numerical estimates of matrix elements for the mixing of charged quark currents agree with the current experimental data within the accuracy of determination, but also they have a predictive power with respect to the parameters of CP-violation.

Conclusion

In this paper we have constructed the models of potentials for two and three higgses, wherein there are the discrete symmetries of vacuum configurations, Z_2 and Z_3 , correspondingly, which provides the introduction of two and three generations of fermions coupled to the higgses by the Yukawa interaction. Thus, the essential reason for the generation replication is in part the symmetric structure of higgs vacuum.

In the framework of such approach, the hierarchy of generation masses (two generations are light and the only one is heavy) is caused by a small splitting of the quantities defined by the products of vacuum expectation values of higgses and Yukawa coupling constants in the higgs-fermion contact terms.

These physical parameters of model determine the matrix of charged current mixing, whose elements are related to the ratios of generation masses. In the leading approximation for the quark currents mixing, the hierarchy of masses results in the certain predictions on the complex phase of CP-violation and several elements of the matrix, though there is the uncertainty in the choice of several signs because of indefinite values for the model parameters: the arrangement of Yukawa constants and vacuum expectation values.

¹¹Note that the admissible variation of quark masses results in valuable uncertainties in the determination of angle β ($\Delta\beta \sim 5\%$) and elements of mixing matrix. We have also to point out that the choice of quark masses close to the values in the paper by Lucas & Raby [8] leads to the quark-currents mixing matrix approximately equal to what is obtained in the model under consideration.

We have shown that the typical set of quark masses leads to the Cabibbo–Kobayashi–Maskawa matrix which is in a good agreement with the current experimental data in the framework of error bars.

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