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## THEORETICAL STATUS OF THE $B_c$ MESON

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**Abstract**

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Theoretical predictions on the  $B_c$  meson properties are reviewed.

**Аннотация**

Герштейн С.С., Киселев В.В., Лиходед А.К., Ткабладзе А.В., Бережной А.В., Онищенко А.И. Теоретический статус  $B_c$ -мезона: Препринт ИФВЭ 98-22. – Протвино, 1998. – 26 с., 12 рис., 8 табл., библиогр.: 29.

Сделан обзор теоретических предсказаний для свойств  $B_c$ -мезонов.

## Introduction

Theoretical and experimental studies of the heavy quark sector in the Standard Model are of great interest to complete the whole quantitative picture of fundamental interactions.

In the bottom quark physics experimentalists step from the  $10^6$  yield of hadrons containing  $b$ -quark at present facilities up to the  $10^9$  yield in the foreseeable future to measure rare processes like the CP-violation and possible effects beyond the SM. To distinguish the hadronic dynamics from the latter effects at the quark level one needs a perfect understanding of QCD interactions binding the quarks into hadrons.

An accompanying problem is the observation and study of the  $(\bar{b}c)$  state, yielding a  $10^{-3}$  fraction of beauty hadrons at high energies.

The  $B_c$  meson allows one:

1. To accomplish the QCD-based models of hadrons with the bottom quarks.
2. To study the specific production and decay mechanisms.
3. To measure the SM parameters.

The basic state of  $B_c$  is the long-lived heavy quarkonium, which can be searched for in a way analogous to the observation of beauty mesons with a light quark [1].

At CDF a background is still strong to isolate the  $B_c$  event at low statistics available [2]. At present, the LEP Collaborations have reported on several candidates for the  $B_c$  decays [3].

The mean values averaged over the  $\psi\pi$  mode are equal to

$$m_{B_c} = 6.33 \pm 0.05 \text{ GeV}, \quad (1)$$

$$\tau_{B_c} = 0.28_{-0.20}^{+0.10} \text{ ps}. \quad (2)$$

OPAL has reported also

$$f(\bar{b} \rightarrow B_c^+) \cdot \text{BR}(B_c^+ \rightarrow \psi\pi^+) = (3.8_{-2.4}^{+5.0} \pm 0.5) \cdot 10^{-5}.$$

The key question of the report is

What are the theoretical expectations of the  $B_c$ -meson properties?

Table 1. The basic characteristics of events with the  $B_c$ -meson candidates at LEP.

Collab.	Mode	mass of $B_c$ , GeV	lifetime, ps
OPAL <sub>old</sub>	$\psi\pi^\pm$	$6.31 \pm 0.17$	—
OPAL <sub>new</sub>	$\psi\pi^\pm$	$6.29 \pm 0.17$	$-0.06 \pm 0.19$
OPAL <sub>new</sub>	$\psi\pi^\pm$	$6.33 \pm 0.06$	$0.09 \pm 0.10$
ALEPH	$\psi l^\pm \nu$	$5.96 \pm 0.24$	$1.77 \pm 0.17$
DELPHI	$\psi\pi^\pm$	$6.35 \pm 0.09$	$0.38 \pm 0.06$
DELPHI	$\psi(3\pi)^\pm$	$6.12 \pm 0.02$	$0.41 \pm 0.07$

## 1. The mass spectrum of the $(\bar{b}c)$ family

The most accurate estimates of  $(\bar{b}c)$  masses [4,5] can be obtained in the framework of nonrelativistic potential models based on the NRQCD expansion over both  $1/m_Q$  and  $v_{rel} \rightarrow 0$  [6].

The uncertainty of evaluation is about 30 MeV. The reason is the following. The potential models [7] were justified for the well measured masses of charmonium and bottomonium. So, the potentials with different global behaviours, i.e. with the different  $r \rightarrow \infty$  and  $r \rightarrow 0$  asymptotics, have the same form in the range of mean distances between the quarks in the heavy quarkonia at  $0.2 < r < 1$  fm [8]. The observed **regularity** is the distances between the excitation levels are approximately flavor-independent. This fact is exact for the logarithmic potential (the Feynman–Hell-Mann theorem), where the average kinetic energy of quarks  $T$  is a constant value independent of the excitation numbers (the virial theorem) [9]. A slow dependence of the level distances on the reduced mass can be taken into account by the use of the Martin potential (power law:  $V(r) = A(r/r_0)^a + C$ ,  $a \ll 1$ ) [10], wherein the predictions are in agreement with the QCD-motivated Buchmüller-Tye potential with the account for the two-loop evolution of the coupling constant at short distances [11].

So, one gets the picture of  $(\bar{b}c)$  levels which is very close to the texture of charmonium and bottomonium. The difference is the jj-binding instead of the LS one.

The spin-dependent perturbation of the potential includes the contribution of the effective one-gluon exchange (the vector part) as well as the scalar confining term [12].

$$\begin{aligned}
 V_{SD}(\vec{r}) = & \left( \frac{\vec{L} \cdot \vec{S}_c}{2m_c^2} + \frac{\vec{L} \cdot \vec{S}_b}{2m_b^2} \right) \left( -\frac{dV(r)}{rdr} + \frac{8}{3} \alpha_s \frac{1}{r^3} \right) + \\
 & + \frac{4}{3} \alpha_s \frac{1}{m_c m_b} \frac{\vec{L} \cdot \vec{S}}{r^3} + \frac{4}{3} \alpha_s \frac{2}{3m_c m_b} \vec{S}_c \cdot \vec{S}_b 4\pi \delta(\vec{r}) \\
 & + \frac{4}{3} \alpha_s \frac{1}{m_c m_b} (3(\vec{S}_c \cdot \vec{n}) (\vec{S}_b \cdot \vec{n}) - \vec{S}_c \cdot \vec{S}_b) \frac{1}{r^3}, \quad \vec{n} = \frac{\vec{r}}{r}.
 \end{aligned} \tag{3}$$

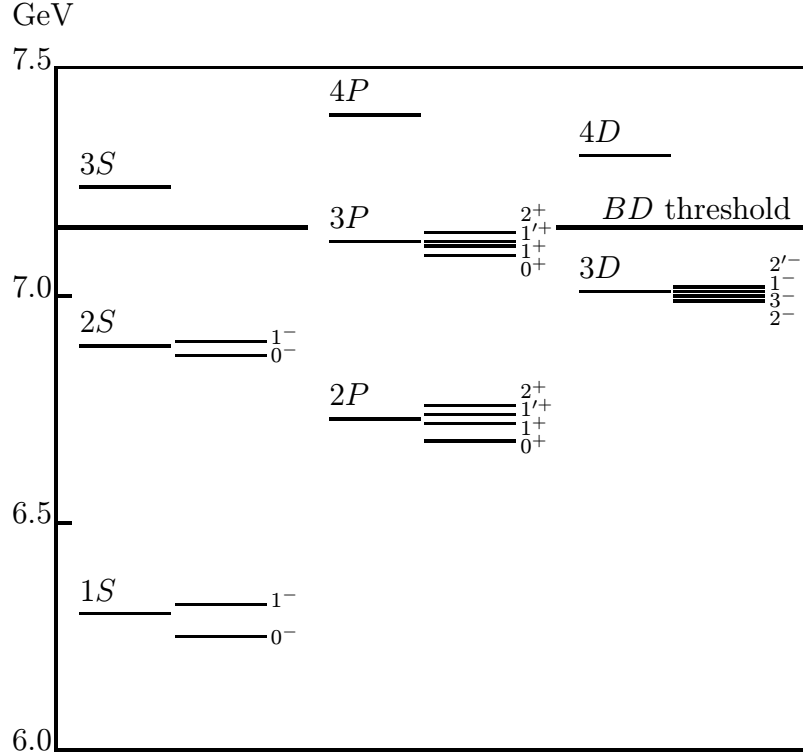


Fig. 1. The mass spectrum of  $(\bar{b}c)$  with account for the spin-dependent splittings.

The model-dependent value of effective  $\alpha_s$  [5] can be extracted from the data on the splitting in the charmonium

$$M(\psi) - M(\eta_c) = \alpha_s \frac{8}{9m_c^2} |R(0)|^2 \approx 117 \text{ MeV}.$$

We take into account the renormalization-group dependence of  $\alpha_s$  at the one-loop accuracy by means of introduction of the quarkonium scale [4]

$$\mu^2 = \langle \mathbf{p}^2 \rangle = 2 \langle T \rangle m_{red}.$$

The estimated difference between the masses of basic pseudoscalar state and its vector excitation [4] is equal to

$$M(B_c^{*+}) - M(B_c^+) = 65 \pm 15 \text{ MeV}.$$

The mass of the ground state [4] equals

$$M(B_c^+) = 6.25 \pm 0.03 \text{ GeV}, \quad (4)$$

which is inside the quoted region of the  $B_c$  candidates at LEP.

**Table 2.** The masses of bound ( $\bar{b}c$ )-states below the threshold of decay into the pair of heavy mesons  $BD$ , in GeV, in the models with the Martin and BT potentials. The spectroscopic notations of states are  $n^{2j_c}L_J$ , where  $j_c$  is the total angular momentum of  $c$ -quark,  $n$  is the principal quantum number,  $L$  is the orbital angular momentum,  $J$  is the total spin of meson.

state	Martin	BT
$1^1S_0$	6.253	6.264
$1^1S_1$	6.317	6.337
$2^1S_0$	6.867	6.856
$2^1S_1$	6.902	6.899
$2^1P_0$	6.683	6.700
$2P\ 1^+$	6.717	6.730
$2P\ 1'^+$	6.729	6.736
$2^3P_2$	6.743	6.747
$3^1P_0$	7.088	7.108
$3P\ 1^+$	7.113	7.135
$3P\ 1'^+$	7.124	7.142
$3^3P_2$	7.134	7.153
$3D\ 2^-$	7.001	7.009
$3^5D_3$	7.007	7.005
$3^3D_1$	7.008	7.012
$3D\ 2'^-$	7.016	7.012

#### RADIATIVE TRANSITIONS

The bright feature of the ( $\bar{b}c$ ) family is that there are no annihilation decay modes due to the strong interaction. So, the excitations, in a cascade way, decay into the ground state with the emission of photons and pion-pion pairs.

The formulae for the E1-transitions are slightly modified.

$$\begin{aligned}
\Gamma(\bar{n}P_J \rightarrow n^1S_1 + \gamma) &= \frac{4}{9} \alpha_{\text{em}} Q_{\text{eff}}^2 \omega^3 I^2(\bar{n}P; nS) w_J(\bar{n}P) , \\
\Gamma(\bar{n}P_J \rightarrow n^1S_0 + \gamma) &= \frac{4}{9} \alpha_{\text{em}} Q_{\text{eff}}^2 \omega^3 I^2(\bar{n}P; nS) (1 - w_J(\bar{n}P)) , \\
\Gamma(n^1S_1 \rightarrow \bar{n}P_J + \gamma) &= \frac{4}{27} \alpha_{\text{em}} Q_{\text{eff}}^2 \omega^3 I^2(nS; \bar{n}P) (2J + 1) w_J(\bar{n}P) , \\
\Gamma(n^1S_0 \rightarrow \bar{n}P_J + \gamma) &= \frac{4}{9} \alpha_{\text{em}} Q_{\text{eff}}^2 \omega^3 I^2(nS; \bar{n}P) (2J + 1) (1 - w_J(\bar{n}P)) , \\
\Gamma(\bar{n}P_J \rightarrow nD_{J'} + \gamma) &= \frac{4}{27} \alpha_{\text{em}} Q_{\text{eff}}^2 \omega^3 I^2(nD; \bar{n}P) (2J' + 1) w_J(\bar{n}P) w_{J'}(nD) S_{JJ'} , \\
\Gamma(nD_J \rightarrow \bar{n}P_{J'} + \gamma) &= \frac{4}{27} \alpha_{\text{em}} Q_{\text{eff}}^2 \omega^3 I^2(nD; \bar{n}P) (2J' + 1) w_{J'}(\bar{n}P) w_J(nD) S_{J'J} ,
\end{aligned} \tag{5}$$

where  $\omega$  is the photon energy,  $\alpha_{\text{em}}$  is the electromagnetic fine structure constant. In eq.(5)

one uses

$$Q_{\text{eff}} = \frac{m_c Q_{\bar{b}} - m_b Q_c}{m_c + m_b}, \quad (6)$$

where  $Q_{c,b}$  are the electric charges of the quarks. For the  $B_c$  meson with the parameters from the Martin potential, one gets  $Q_{\text{eff}} = 0.41$ .  $w_J(nL)$  is the probability that the spin  $S = 1$  in the  $nL$  state.  $S_{JJ'}$  are the statistical factors. The  $I(\bar{n}L; nL')$  value is expressed through the radial wave functions

$$I(\bar{n}L; nL') = \left| \int R_{\bar{n}L}(r) R_{nL'}(r) r^3 dr \right|. \quad (7)$$

**Table 3.** The total widths of excited bound ( $\bar{b}c$ )-states below the threshold of decay into the  $BD$ -pair in the model with Martin potential and the branching ratios of the dominant decay modes.

state	$\Gamma_{\text{tot}}$ , KeV	dominant decay mode	BR, %
$1^1 S_1$	0.06	$1^1 S_0 + \gamma$	100
$2^1 S_0$	67.8	$1^1 S_0 + \pi\pi$	74
$2^1 S_1$	86.3	$1^1 S_1 + \pi\pi$	58
$2^1 P_0$	65.3	$1^1 S_1 + \gamma$	100
$2P 1^+$	89.4	$1^1 S_1 + \gamma$	87
$2P 1'^+$	139.2	$1^1 S_0 + \gamma$	94
$2^3 P_2$	102.9	$1^1 S_1 + \gamma$	100
$3^1 P_0$	44.8	$2^1 S_1 + \gamma$	57
$3P 1^+$	65.3	$2^1 S_1 + \gamma$	49
$3P 1'^+$	92.8	$2^1 S_0 + \gamma$	63
$3^3 P_2$	71.6	$2^1 S_1 + \gamma$	69
$3D 2^-$	95.0	$2P 1^+ + \gamma$	47
$3^5 D_3$	107.9	$2^3 P_2 + \gamma$	71
$3^3 D_1$	155.4	$2^1 P_0 + \gamma$	51
$3D 2'^-$	122.0	$2P 1'^+ + \gamma$	38

For the dipole magnetic M1-transitions, one has

$$\Gamma(\bar{n}^1 S_i \rightarrow n^1 S_f + \gamma) = \frac{16}{3} \mu_{\text{eff}}^2 \omega^3 (2f + 1) A_{if}^2, \quad (8)$$

where

$$A_{if} = \int R_{\bar{n}S}(r) R_{nS}(r) j_0(\omega r/2) r^2 dr,$$

and

$$\mu_{\text{eff}} = \frac{1}{2} \frac{\sqrt{\alpha_{\text{em}}}}{2m_c m_b} (Q_c m_b - Q_{\bar{b}} m_c). \quad (9)$$

Note, in contrast to the  $\psi$  and  $\Upsilon$  particles, the total width of the  $B_c^*$  meson is equal to the width of its radiative decay into the  $B_c(0^-)$  state.

Thus, below the threshold of decay into the  $BD$ -pair, the theory predicts the existence of 16 narrow ( $\bar{b}c$ ) states, which do not annihilate due to the strong interactions, but they have the cascade radiative transitions into the ground long-lived pseudoscalar state, the  $B_c^+$  meson.

As for the states lying above the threshold of decay into the heavy meson  $BD$  pair, the width of  $B_c^{*+}(3S) \rightarrow B^+D^0$ , for example, can be calculated in the framework of sum rules for the meson currents, where the scaling relation [13] takes place for the  $g$  constants of similar decays of  $\Upsilon(4S) \rightarrow B^+B^-$  and  $\psi(3770) \rightarrow D^+D^-$ ,

$$\frac{g^2}{M} \left( \frac{4m_{12}}{M} \right) = const.$$

The relation is caused by the dependence of energy gap between the vector and pseudoscalar heavy meson states:  $\Delta M_{1,2} \cdot M_{1,2} = const.$ , where  $M_{1,2}$  are the meson masses in the final state,  $m_{12}$  is their reduced mass. The width of this decay has a strong dependence on the  $B_c^{*+}(3S)$  mass, and at  $M = 7.25$  GeV it is equal to  $\Gamma = 90 \pm 35$  MeV, where the uncertainty is determined by the accuracy of method used.

Table 4. The predictions of scaling relation in comparison with the current experimental data.

value	exp.	scaling rel.
$g_{\Upsilon(4S) \rightarrow B^+B^-}$	52	input
$g_{\psi(3770) \rightarrow D^+D^-}$	31	31
$g_{B_c^{*+}(3S) \rightarrow B^+D^0}$	–	49

## 2. Leptonic constants

In the framework of potential models the asymptotic behaviour at  $r \rightarrow \infty$ ,  $r \rightarrow 0$  is significant for the determination of the leptonic coupling constants  $f$  for the  $nS$ -levels. In the leading approximation, the  $f$  value does not depend on the spin state of the level and it is determined by the value of the radial wave function at the origin,  $R(0)$ , being model-dependent,

$$\tilde{f}_n = \sqrt{\frac{3}{\pi M_n}} R_{nS}(0).$$

Taking into account the hard gluon corrections, the constants of vector and pseudoscalar states equal

$$f_n^{V,P} = \tilde{f}_n \left( 1 + \frac{\alpha_s}{\pi} \left( \frac{m_1 - m_2}{m_1 + m_2} \ln \frac{m_1}{m_2} - \delta^{V,P} \right) \right), \quad (10)$$



where  $m_{1,2}$  are the quark masses,  $\delta^V = 8/3$ ,  $\delta^P = 2$  [14,15,16], and the QCD coupling constant is determined at the scale of the quark masses.

The corresponding uncertainty due to the model dependence is expressed by a factor of two.

**Table 5.** The radial wave functions at the origin,  $R_{nS}(0)$  (in  $\text{GeV}^{3/2}$ ) and  $R'_{nP}(0)$  (in  $\text{GeV}^{5/2}$ ), obtained in the Schrödinger equation with the Martin and BT potentials as well as in the sum rules.

n	Martin	BT	SR
$R_{1S}(0)$	1.31	1.28	1.20
$R_{2S}(0)$	0.97	0.99	0.85
$R'_{2P}(0)$	0.55	0.45	–
$R'_{3P}(0)$	0.57	0.51	–

The QCD sum rules [17] allow one to determine the leptonic constants for the heavy quarkonium states with a much better accuracy.

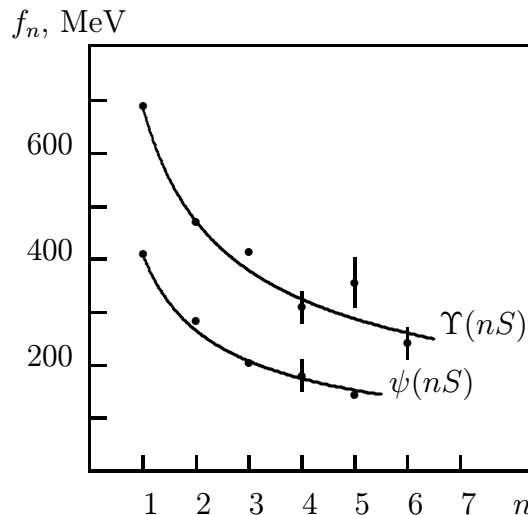
Standard schemes of the sum rules give an opportunity to calculate the ground state constants for vector and pseudoscalar currents with the account for corrections from the quark-gluon condensates, which have the power form over the inverse heavy quark mass.

#### QUASILOCAL SUM RULES

There is a region of the momentum numbers for the spectral density of the two-point current correlator, where the condensate contributions are not significant. In this region, the integral representation for the contribution of resonances lying below the threshold of decay into the pair of heavy mesons, allows one to use the regularity of the quarkonium state density mentioned above, and to derive the scaling relations [15] for the leptonic constants of the ground state quarkonia with different quark contents and for the excited states. Thus, for vector states we have

$$\frac{f_n^2}{M_n} \left( \frac{M_n}{M_1} \right)^2 \left( \frac{m_1 + m_2}{4m_{12}} \right)^2 = \frac{c}{n}, \quad (11)$$

where  $m_{12} = m_1 m_2 / (m_1 + m_2)$  is the reduced mass of quarks, and the constant  $c$  is determined by the average kinetic energy of quarks, the QCD coupling constant at the



**Fig. 2.** The dependence of leptonic constants for the  $nS$ -levels of bottomonium and charmonium,  $f_n$ , as it is calculated in the Quasiloal Sum Rules.

scale of average momentum transfers in the system and the hard gluon correction factor to the vector current. Numerically, in the method accuracy the  $c$  value turns out to depend on no quark flavour and the excitation number in the system. Relation (11) is in a good agreement with the data on the coupling constants for the families of  $\psi$  and  $\Upsilon$  particles and, thus, it can be a reliable basis for the prediction of leptonic constants for the  $B_c$ -meson family. The constants for the pseudoscalar states of  $nS$ -levels are determined by the relation

$$f_n^P = f_n \left( 1 + \frac{2\alpha_s}{3\pi} \right) \frac{m_1 + m_2}{M_n} .$$

**Table 6.** The leptonic constants for the vector and pseudoscalar states of  $nS$ -levels in the  $(\bar{b}c)$ -system,  $f_n$  and  $f_n^P$ , calculated in the sum rules resulting in the scaling relation. The accuracy is equal to 6%.

n	$f_n$ , MeV	$f_n^P$ , MeV
1	385	397
2	260	245

### 3. $B_c$ decays

#### Lifetime.

The  $B_c$ -meson decay processes can be subdivided into three classes:

- 1) the  $\bar{b}$ -quark decay with the spectator  $c$ -quark,
- 2) the  $c$ -quark decay with the spectator  $\bar{b}$ -quark and
- 3) the annihilation channel  $B_c^+ \rightarrow l^+ \nu_l (c\bar{s}, u\bar{s})$ , where  $l = e, \mu, \tau$ .

In the  $\bar{b} \rightarrow \bar{c}c\bar{s}$  decays one separates also the Pauli interference with the  $c$ -quark from the initial state. In accordance with the given classification, the total width is the sum over the partial widths

$$\Gamma(B_c \rightarrow X) = \Gamma(b \rightarrow X) + \Gamma(c \rightarrow X) + \Gamma(\text{ann.}) + \Gamma(\text{PI}) . \quad (12)$$

For the annihilation channel the  $\Gamma(\text{ann.})$  width can be reliably estimated in the framework of inclusive approach, where one takes the sum of the leptonic and quark decay modes with account for the hard gluon corrections to the effective four-quark interaction of weak currents. These corrections result in the factor of  $a_1 = 1.22 \pm 0.04$ . The width is expressed through the leptonic constant of  $f_{B_c} = f_1^P \approx 400$  MeV. This estimate of the quark-contribution does not depend on a hadronization model, since a large energy release of the order of the meson mass takes place. From the following expression, one can see that the contribution by light leptons and quarks can be neglected,

$$\Gamma(\text{ann.}) = \sum_{i=\tau,c} \frac{G_F^2}{8\pi} |V_{bc}|^2 f_{B_c}^2 M m_i^2 (1 - m_i^2/m_{B_c}^2)^2 \cdot C_i , \quad (13)$$

where  $C_\tau = 1$  for the  $\tau^+ \nu_\tau$ -channel and  $C_c = 3|V_{cs}|^2 a_1^2$  for the  $c\bar{s}$ -channel.

As for the non-annihilation decays, in the approach of the operator product expansion for the quark currents of weak decays [18], one takes into account the  $\alpha_s$ -corrections to the free quark decays and uses the quark-hadron duality for the final states. Then one considers the matrix element for the transition operator over the bound meson state. The latter allows one also to take into account the effects caused by the motion and virtuality of decaying quark inside the meson because of the interaction with the spectator. In this way the  $\bar{b} \rightarrow \bar{c}l\bar{s}$  decay mode turns out to be suppressed almost completely due to the Pauli interference with the charm quark from the initial state. Besides, the  $c$ -quark decays with the spectator  $\bar{b}$ -quark are essentially suppressed in comparison with the free quark decays because of a large bound energy in the initial state.

**Table 7.** The branching ratios of the  $B_c$  decay modes calculated in the framework of inclusive approach and in the exclusive quark model with the parameters  $|V_{bc}| = 0.040$ ,  $\tilde{f}_{B_c} = 0.47$  GeV,  $\tilde{f}_\psi = 0.54$  GeV,  $\tilde{f}_{B_s} = 0.3$  GeV,  $m_b = 4.8 - 4.9$  GeV,  $m_c = 1.5 - 1.6$  GeV,  $m_s = 0.55$  GeV. The accuracy is about 10%.

$B_c$ decay mode	Inclus., %	Exclus., % [21]
$\bar{b} \rightarrow \bar{c}l^+\nu_l$	3.9	3.7
$\bar{b} \rightarrow \bar{c}u\bar{d}$	16.2	16.7
$\sum \bar{b} \rightarrow \bar{c}$	25.0	25.0
$c \rightarrow sl^+\nu_l$	8.5	10.1
$c \rightarrow su\bar{d}$	47.3	45.4
$\sum c \rightarrow s$	64.3	65.6
$B_c^+ \rightarrow \tau^+\nu_\tau$	2.9	2.0
$B_c^+ \rightarrow c\bar{s}$	7.2	7.2

In the framework of exclusive approach, it is necessary to sum widths of different decay modes calculated in the potential models [19,20]. While considering the semileptonic decays due to the  $\bar{b} \rightarrow \bar{c}l^+\nu_l$  and  $c \rightarrow sl^+\nu_l$  transitions, one finds that in the former decays the hadronic final state is practically saturated by the lightest bound  $1S$ -state in the  $(\bar{c}c)$ -system, i.e. by the  $\eta_c$  and  $J/\psi$  particles, and in the latter decays, the  $1S$ -states in the  $(\bar{b}s)$ -system, i.e.  $B_s$  and  $B_s^*$ , can only enter the accessible energetic gap. The energy release in the latter transition is low in comparison with the meson masses, and, therefore, a visible deviation from the picture of quark-hadron duality is possible. Numerical estimates show that the value of  $B_c \rightarrow (\bar{b}s)l^+\nu_l$  decay width is two times less in the exclusive approach than in the inclusive method, though this fact can be caused by the choice of narrow wave package for the  $B_s^{(*)}$  mesons in the quark model, so that  $\tilde{f}_{B_s} \approx 150$  MeV, while in the limit of static heavy quark, one should expect a larger value for the leptonic constant. This increase will lead to the widening of the wave package and, hence, to the increase of the overlapping integral for the wave functions of  $B_c$  and  $B_s^{(*)}$ .

Further, the  $\bar{b} \rightarrow \bar{c}u\bar{d}$  channel, for example, can be calculated through the given decay width of  $\bar{b} \rightarrow \bar{c}l^+\nu_l$  with account for the color factor and hard gluon corrections to the

four-quark interaction. It can be also obtained as a sum over the widths of decays with the  $(u\bar{d})$ -system bound states.

The results of calculation for the total  $B_c$  width in the inclusive and exclusive approaches give the values consistent with each other, if one takes into account the most significant uncertainty related to the choice of quark masses (especially for the charm quark), so that finally, we have

$$\tau(B_c^+) = 0.55 \pm 0.15 \text{ ps}, \quad (14)$$

and the observed  $B_c$  candidates in the  $\psi\pi$  mode at LEP have quite a close value of the lifetime.

#### Exclusive decays.

The consideration of exclusive  $B_c$ -decay modes supposes an introduction of model for the hadronization of quarks into the mesons with the given quantum numbers. The QCD sum rules for the three-point correlators of quark currents and potential models are among those hadronization models.

A feature of the sum rule application to the mesons containing two heavy quarks, is the account for a significant role of the coulumb-like  $\alpha_s/v$ -corrections due to the gluon exchange between the quarks composing the meson and moving with a relative velocity  $v$ . So, in the semileptonic decays of  $B_c^+ \rightarrow \psi(\eta_c)l^+\nu_l$ , the heavy  $(\bar{Q}_1Q_2)$  quarkonium is present in both the initial and final states, and, therefore, the contribution of coulumb-like corrections exhibits in a particularly strong form. The use of tree approximation for the perturbative contribution into the three-point correlator of quark currents leads to a large deviation between the values of transition form-factors, calculated in the sum rules and potential models, respectively [22]. The account for the  $\alpha_s/v$ -corrections removes this contradiction [23].

Thus, the meson potential models, based on the covariant expression for the form-factors of weak  $B_c$  decays through the overlapping of quarkonium wave functions in the initial and final states, and the QCD sum rules give the consistent description of semileptonic  $B_c$ -meson decays.

Further, the hadronic decay widths can be obtained on the basis of assumption on the factorization of the weak transition between the quarkonia and the hadronization of products of the virtual  $W^{*+}$ -boson decay [24]. The accuracy of factorization has to raise with the increase of  $W$ -boson virtuality. This fact is caused by the suppression of interaction in the final state. In this way, the hadronic decays can be calculated due to the use of form-factors for the semileptonic transitions with the relevant description of  $W^*$  transition into the hadronic state.

More generally, the effective four-fermion hamiltonian for the nonleptonic decays of the  $c$ - and  $b$ -quarks has the form

$$\mathcal{H}_{\text{eff}}^c = \frac{G}{2\sqrt{2}} V_{uq_1} V_{cq_1}^* [C_+(\mu)O_+^c + C_-(\mu)O_-^c] + h.c. , \quad (15)$$

$$\mathcal{H}_{\text{eff}}^b = \frac{G}{2\sqrt{2}} V_{q_1b} V_{q_2q_3}^* [C_+(\mu)O_+^b + C_-(\mu)O_-^b] + h.c. , \quad (16)$$

**Table 8.** The branching ratios of exclusive  $B_c$  decay modes, calculated in the framework of covariant quark model with the parameters  $|V_{bc}| = 0.040$ ,  $\tilde{f}_{B_c} = 0.47$  GeV,  $\tilde{f}_\psi = 0.54$  GeV,  $\tilde{f}_{B_s} = 0.3$  GeV,  $m_b = 4.8 - 4.9$  GeV,  $m_c = 1.5 - 1.6$  GeV,  $m_s = 0.55$  GeV. The accuracy equals 10%.

$B_c$ decay mode	BR, %	$B_c$ decay mode	BR, %
$\psi l^+ \nu_l$	2.5	$\eta_c l^+ \nu_l$	1.2
$B_s^* l^+ \nu_l$	6.2	$B_s l^+ \nu_l$	3.9
$\psi \pi^+$	0.2	$\eta_c \pi^+$	0.2
$B_s^* \pi^+$	5.2	$B_s \pi^+$	5.5
$\psi \rho^+$	0.6	$\eta_c \rho^+$	0.5
$B_s^* \rho^+$	22.9	$B_s \rho^+$	11.8

where

$$O_{\pm}^c = (\bar{q}_{1\alpha} \gamma_\nu (1 - \gamma_5) c_\beta) (\bar{u}_\gamma \gamma^\nu (1 - \gamma_5) q_{2\delta}) (\delta_{\alpha\beta} \delta_{\gamma\delta} \pm \delta_{\alpha\delta} \delta_{\gamma\beta}),$$

$$O_{\pm}^b = (\bar{q}_{1\alpha} \gamma_\nu (1 - \gamma_5) b_\beta) (\bar{q}_{3\gamma} \gamma^\nu (1 - \gamma_5) q_{2\delta}) (\delta_{\alpha\beta} \delta_{\gamma\delta} \pm \delta_{\alpha\delta} \delta_{\gamma\beta}).$$

The factors  $C_{\pm}^{c,b}(\mu)$  account for the strong corrections to the corresponding four-fermion operators because of hard gluons.

In the leading logarithm approximation at  $\mu > m_c$ , one has

$$C_+(\mu) = \left( \frac{\alpha_s(M_W^2)}{\alpha_s(m_b^2)} \right)^{6/23} \left( \frac{\alpha_s(m_b^2)}{\alpha_s(\mu^2)} \right)^{6/25},$$

$$C_-(\mu) = [C_+(\mu)]^{-2}. \quad (17)$$

At  $\mu > m_b$ , one finds

$$C_+(\mu) = \left( \frac{\alpha_s(M_W^2)}{\alpha_s(m_b^2)} \right)^{6/23} \left( \frac{\alpha_s(m_b^2)}{\alpha_s(\mu^2)} \right)^{-3/25}, \quad (18)$$

$$C_-(\mu) = \left( \frac{\alpha_s(M_W^2)}{\alpha_s(m_b^2)} \right)^{-12/23} \left( \frac{\alpha_s(m_b^2)}{\alpha_s(\mu^2)} \right)^{-12/25}. \quad (19)$$

The  $a_1$  and  $a_2$  coefficients, accounting for the renormalization of the four-fermion operators, are defined in the following way:

$$a_1 = C_+ \frac{N_c + 1}{2N_c} + C_- \frac{N_c - 1}{2N_c}, \quad (20)$$

$$a_2 = C_+ \frac{N_c + 1}{2N_c} - C_- \frac{N_c - 1}{2N_c}. \quad (21)$$

In the limit  $N_c \rightarrow \infty$ , one has

$$a_1 \approx 0.5 \cdot (C_+ + C_-),$$

$$a_2 \approx 0.5 \cdot (C_+ - C_-). \quad (22)$$

In the theory  $a_2/a_1 \sim 0.2$ , which is in agreement with the experimental data, so that the  $B_c$  decay modes proportional to  $a_2^2$  are essentially suppressed.

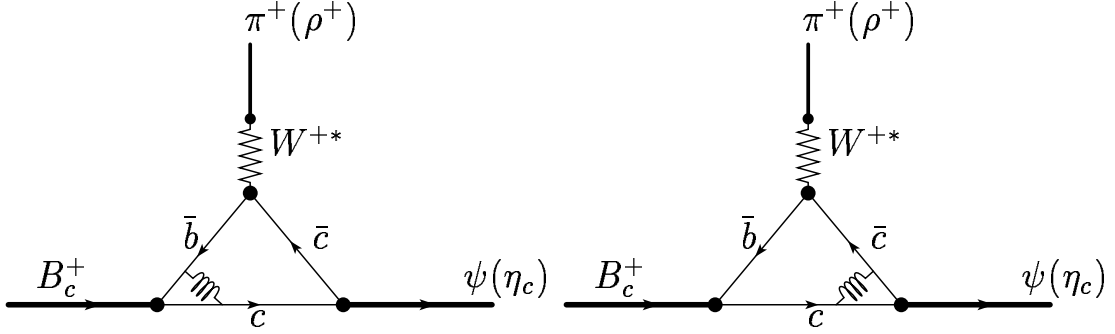


Fig. 3. The diagrams of  $B_c^+ \rightarrow \psi(\eta_c)\pi^+(\rho^+)$  decays with account for the hard gluon exchange between the constituents.

A decrease of the invariant mass for the hadron system results in an increase of the recoil meson momentum. This causes the problem of applicability for the formalism of overlapping for the quarkonium wave functions, because, in this kinematics, the narrow wave packages are displaced relative to each other in the momentum space into the range of distribution tails. In this situation one has to take into account a hard gluon exchange between the quarkonium constituents, which destroys the spectator picture of weak transition in the potential approach.

The widths of  $B_c^+ \rightarrow \psi\pi^+$  and  $B_c^+ \rightarrow \eta_c\pi^+$  decays [25] have the following forms:

$$\Gamma(B_c^+ \rightarrow \psi\pi^+) = G_F^2 |V_{bc}|^2 \frac{128\pi\alpha_s^2}{81} \left(\frac{M+m}{M-m}\right)^3 \frac{f_\pi^2 \tilde{f}_{B_c}^2 \tilde{f}_\psi^2 M^3}{(M-m)^2 m^2} a_1^2, \quad (23)$$

$$\frac{\Gamma(B_c^+ \rightarrow \eta_c\pi^+)}{\Gamma(B_c^+ \rightarrow \psi\pi^+)} = \frac{[5(M-m)^2(M+m) + (M-m)^3 + 8m^3]^2}{16(M+m)^2 M^4}, \quad (24)$$

where  $m = m_\psi$  or  $m = m_{\eta_c}$ , respectively.

Numerically, one finds

$$\text{BR}^{\text{HS}}(B_c^+ \rightarrow \psi\pi^+) = 0.77 \pm 0.19\%, \quad (25)$$

$$\text{BR}^{\text{HS}}(B_c^+ \rightarrow \eta_c\pi^+) = 1.00 \pm 0.25\%, \quad (26)$$

$$\text{BR}^{\text{HS}}(B_c^+ \rightarrow \psi\rho^+) = 2.25 \pm 0.56\%, \quad (27)$$

$$\text{BR}^{\text{HS}}(B_c^+ \rightarrow \eta_c\rho^+) = 2.78 \pm 0.70\%, \quad (28)$$

so that the matrix element is twice enhanced in comparison with that of the potential model value due to the contribution of second t-channel diagram.

The relative yield of excited charmonium states can be also evaluated

$$\frac{\text{BR}^{\text{HS}}(B_c^+ \rightarrow \psi(2S)\pi^+)}{\text{BR}^{\text{HS}}(B_c^+ \rightarrow \psi\pi^+)} = \frac{\text{BR}^{\text{HS}}(B_c^+ \rightarrow \eta_c(2S)\pi^+)}{\text{BR}^{\text{HS}}(B_c^+ \rightarrow \eta_c\pi^+)} \approx 0.36. \quad (29)$$

As for the extraction of  $B_c$  signal in the hadronic background, the decay modes with  $\psi$  in the final state are the most preferable, because the latter particle can be easily identified by its leptonic decay mode. This advantage is absent in the  $B_c$  decay modes with the final state containing the  $\eta_c$  or  $B_s^{(*)}$  mesons, which are the objects, whose experimental registration is impeded by a large hadron background.

From the values of branching ratios shown above, one can easily obtain, that the total probability of  $\psi$  yield in the  $B_c$  decays equals  $\text{BR}(B_c^+ \rightarrow \psi X) = 0.17$ .

It is worth to note that the key role in the  $B_c$  signal observation plays the presence of the **vertex detector**, which allows one to extract events with the weak decays of long-lived particles containing heavy quarks. In the case under consideration it gives the possibility to suppress the background from the direct  $\psi$  production. In the semileptonic  $B_c^+ \rightarrow \psi l^+ \nu_l$  decays, the presence of vertex detector and large statistics of events allows one to determine the  $B_c$ -meson mass and to separate the events with its decays from the ordinary  $B_u^+$ -meson decays, which have no  $\psi l^+$  mode. In the  $\psi \pi^+$  decay, the direct measurement of  $B_c$  mass is possible. The detector efficiency in the reconstruction of three-particle secondary vertex ( $l^+ l^-$  from decays of  $\psi$  and  $\pi^+$  or  $l^+$ ) becomes the most important characteristics here. The low efficiency of the LEP detectors ( $\epsilon \approx 0.15$ ), for example, makes the  $B_c$  observation be hardly reachable in the experiments at the electron-positron collider.

## 4. $B_c$ production

The  $(\bar{b}c)$  system is a heavy quarkonium, i.e. it contains two heavy quarks. This determines the general features for the  $B_c$  meson production in various interactions:

1. Perturbative calculations for the hard associative production of two heavy pairs of  $\bar{c}c$  and  $\bar{b}b$  cause the suppressed  $B_c$  yield as a  $10^{-3}$  fraction of beauty hadrons.
2. A soft nonperturbative binding of nonrelativistic quarks in the color-singlet state can be described in the framework of potential models.

The latter means the factorization of hard and soft amplitudes, so that the soft factor is determined by the radial wave function at the origin for the S-wave quarkonium and its first derivative for the P-wave one. Thus, the analysis of  $B_c$  production mechanisms is reduced to the consideration of perturbative amplitudes in high orders over  $\alpha_s$ .

### $e^+e^-$ annihilation

In  $e^+e^-$ -annihilation at large energies ( $M_{B_c}^2/s \ll 1$ ), the consideration of leading diagrams for the  $B_c$ -meson production gives the factorized scaling result for the differential cross-section over the energy fraction carried out by meson,  $d\sigma/dz = \sigma(\bar{b}b) \cdot D(z)$ , where  $z = 2E_{B_c}/\sqrt{s}$ . This distribution allows the interpretation as the hard production of heavier  $\bar{b}$ -quark with the subsequent fragmentation into  $B_c$ , so that  $D(z)$  is the fragmentation function. In this approach one can obtain analytic expressions for the fragmentation functions into the different spin states of  $S$ ,  $P$ , and  $D$ -wave levels [26].

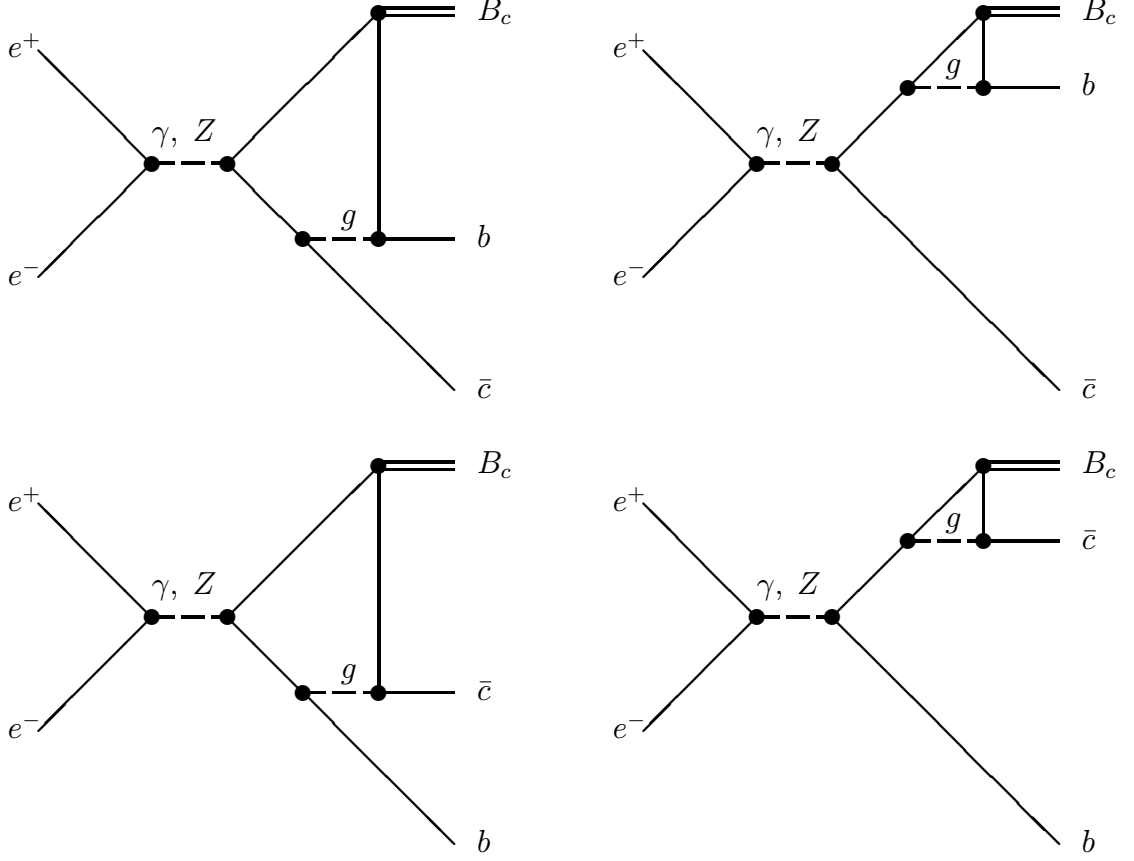


Fig. 4. The diagrams of the  $B_c$ -meson production in  $e^+e^-$ -annihilation.

Denoting  $r = m_c/(m_b + m_c)$ , for the pseudoscalar state, one finds

$$D(z)_{\bar{b} \rightarrow B_c} = \frac{8\alpha_s^2 |\Psi(0)|^2}{81m_c^3} \frac{rz(1-z)^2}{(1-(1-r)z)^6} (6 - 18(1-2r)z + (21 - 74r + 68r^2)z^2 - 2(1-r)(6 - 19r + 18r^2)z^3 + 3(1-r)^2(1-2r+2r^2)z^4), \quad (30)$$

for the vector meson, the perturbative fragmentation function is equal to

$$D(z)_{\bar{b} \rightarrow B_c^*} = \frac{8\alpha_s^2 |\Psi(0)|^2}{27m_c^3} \frac{rz(1-z)^2}{(1-(1-r)z)^6} (2 - 2(3-2r)z + 3(3-2r+4r^2)z^2 - 2(1-r)(4-r+2r^2)z^3 + (1-r)^2(3-2r+2r^2)z^4), \quad (31)$$

These expressions take into account the spin structure of both interactions and bound states, but they are very close to the Peterson et al. model [27], where

$$D(z)_{\bar{b} \rightarrow B_c^{(*)}} \sim \frac{1}{z} \frac{1}{(m_b^2 - \frac{M^2}{z} - \frac{m_c^2}{1-z})^2} \sim \frac{rz(1-z)^2}{(1-(1-r)z)^4},$$



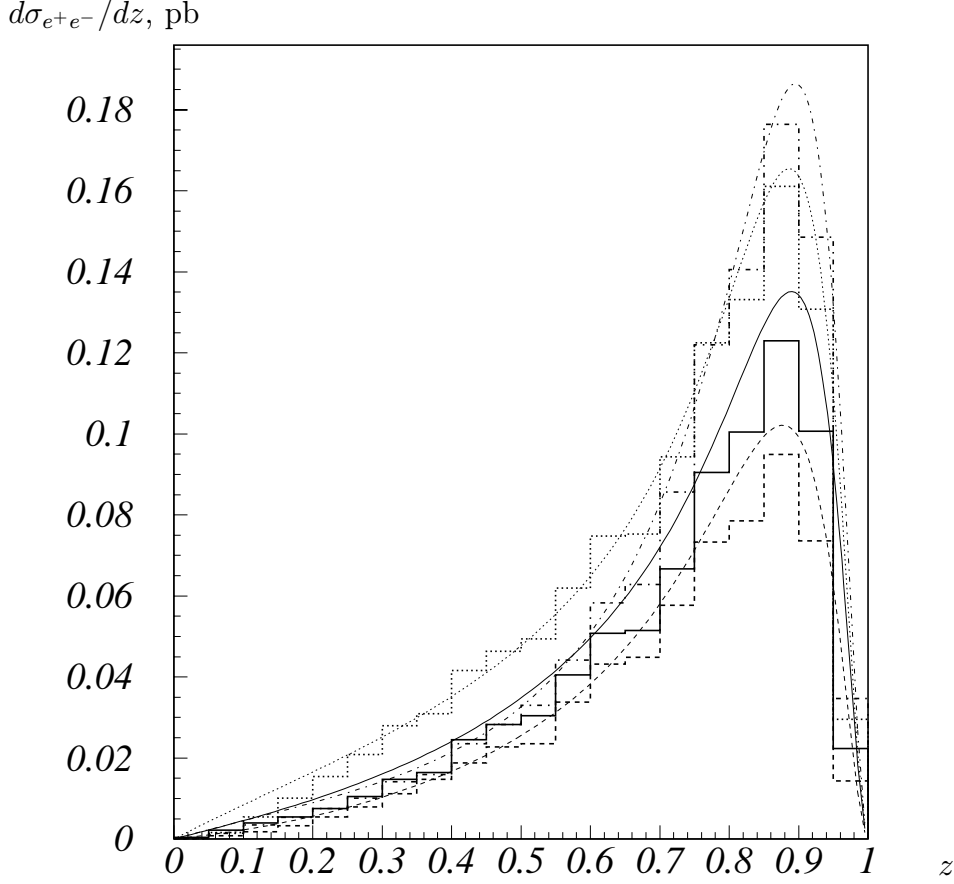


Fig. 5. The differential cross-sections over  $z$  for the  $B_c$  production in  $e^+e^- \rightarrow \gamma^* \rightarrow B_c + X$  (histograms) in comparison with the fragmentation contribution calculated analytically (smooth curves) at 100 GeV for the following states of  $B_c$ :  $^1P_1$  (solid line),  $^3P_0$  (dashed line),  $^3P_1$  (dotted line),  $^3P_2$  (dash-dotted line).

so that the calculated functions are slightly harder. Numerically, one finds  $\tilde{f}(\bar{b} \rightarrow B_c^+) = \sigma(B_c^+)/\sigma(b\bar{b}) = (1.3 \pm 0.4) \cdot 10^{-3}$ .

Combining the latter with the prediction of branching ratios, one gets

$$[f(\bar{b} \rightarrow B_c^+) \cdot \text{BR}(B_c^+ \rightarrow \psi\pi^+)]_{\text{TH}} = (0.22 \pm 0.09) \cdot 10^{-5},$$

where  $f(\bar{b} \rightarrow B_c^+) = \sigma(Z \rightarrow B_c^+)/\sigma(Z \rightarrow q\bar{q})$ , which may be compared with the OPAL estimate

$$[f(\bar{b} \rightarrow B_c^+) \cdot \text{BR}(B_c^+ \rightarrow \psi\pi^+)]_{\text{OPAL}} = (3.8_{-2.4}^{+5.0} \pm 0.5) \cdot 10^{-5}.$$

If one takes into account both the single DELPHI candidate in the  $\psi\pi$  mode and the absence of the event at ALEPH, then assuming the equal detector-efficiencies, one estimates

$$[f(\bar{b} \rightarrow B_c^+) \cdot \text{BR}(B_c^+ \rightarrow \psi\pi^+)]_{\text{LEP}} = (1.9_{-1.2}^{+2.5} \pm 0.3) \cdot 10^{-5}.$$

## Photon-photon collisions

In photon-photon interactions for the  $B_c$  production in the leading approximation of perturbation theory, one can isolate three gauge-invariant groups of diagrams, which can be interpreted as:

- 1) the hard photon-photon production of  $b\bar{b}$  with the subsequent fragmentation of  $\bar{b} \rightarrow B_c^+(nL)$ , where  $n$  is the principal quantum number of the  $(\bar{b}c)$ -quarkonium,  $L = 0, 1 \dots$  is the orbital angular momentum,
- 2) the corresponding production and fragmentation for the  $c$ -quarks, and
- 3) the recombination diagrams of  $(\bar{b}c)$ -pair into  $B_c^+$ , wherein the quarks of different flavours are connected to the different photon lines.

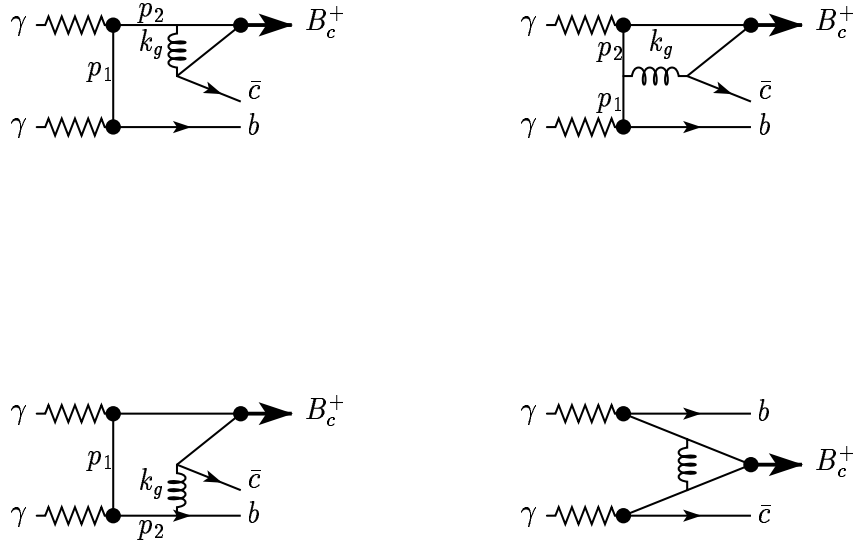


Fig. 6. The contribution of fragmentation for  $\bar{b} \rightarrow B_c^+$  in the photonic production (three initial diagrams) and the recombination (the last diagram).

In this case, the results of calculation for the complete set of diagrams in the leading order of perturbation theory show that the group of  $b$ -fragmentation diagrams at high transverse momenta  $p_T(B_c) \gg M_{B_c}$  can be described by the fragmentation model with the fragmentation function  $D_{\bar{b} \rightarrow B_c^+}(z)$ , calculated in the  $e^+e^-$ -annihilation. The set of  $c$ -fragmentation diagrams does not allow the description in the framework of fragmentation model. The recombination diagrams give the dominant contribution to the total cross-section for the photon-photon production of  $B_c$  [28].

The reason is that the photon coupled to the charm quark can be considered as the "resolved" one, so that the bottom quark is hardly produced off the charm quark appearing in the structure function of the photon, as it is evaluated in the lowest order of perturbative theory.

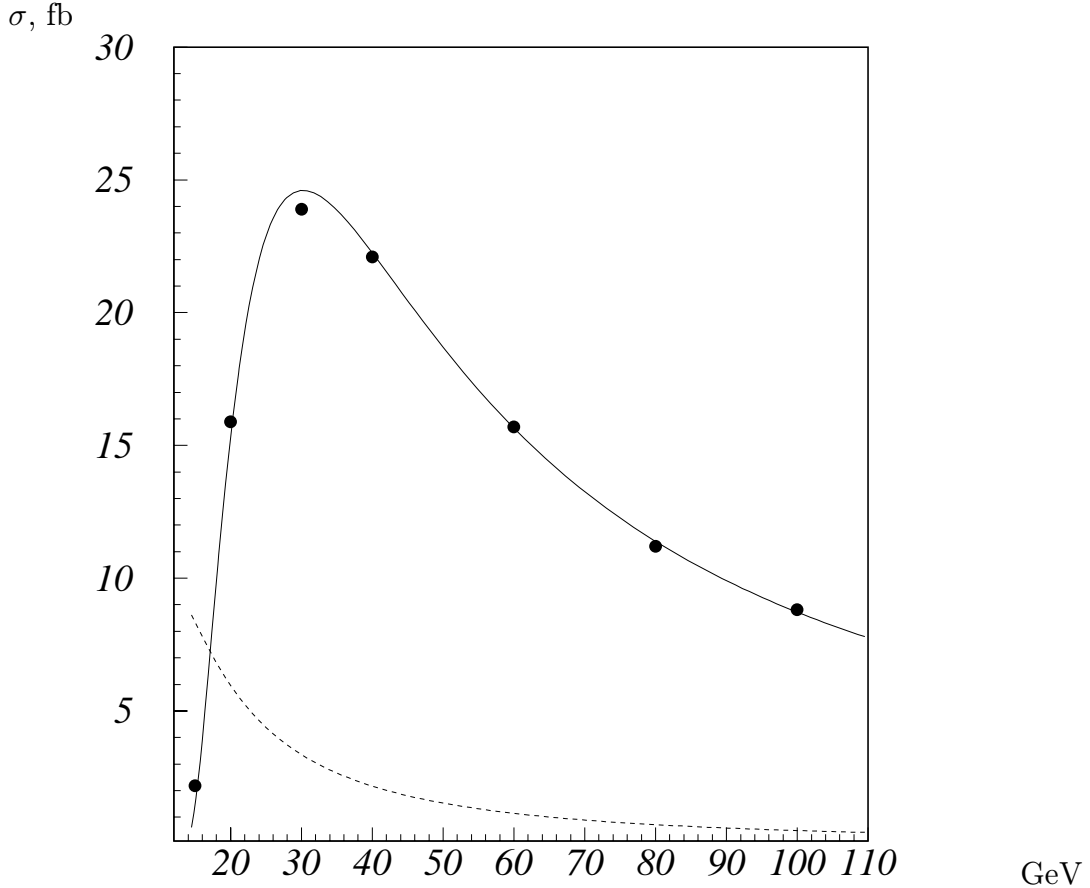


Fig. 7. The summed total cross-section versus the energy of photonic production for the P-wave states of  $B_c$  ( $\bullet$ ) in comparison with the fragmentation contribution (dashed line). The solid-line curve is the fit of points.

### Hadronic production of $B_c$

The parton subprocess of gluon-gluon fusion  $gg \rightarrow B_c^+ + b + \bar{c}$  dominates in the hadron-hadron production of  $B_c$  mesons. In the leading approximation of QCD perturbation theory, it requires the calculation of 36 diagrams in the fourth order over the  $\alpha_s$  coupling constant. In this case, there are no isolated gauge-invariant groups of diagrams, which would allow the interpretation similar to the consideration of  $B_c$  production in  $e^+e^-$  annihilation and photon-photon collisions.

By the general theorem on factorization, it is clear that at high transverse momenta, the fragmentation of the heavier quark  $Q \rightarrow (Q\bar{q}) + q$  must dominate. It is described by the factorized formula

$$\frac{d\sigma}{dp_T} = \int \frac{d\hat{\sigma}(\mu; gg \rightarrow Q\bar{Q})}{dk_T} \Big|_{k_T=p_T/x} \cdot D^{Q \rightarrow (Q\bar{q})}(x; \mu) \frac{dx}{x}, \quad (32)$$

where  $\mu$  is the factorization scale,  $d\hat{\sigma}/dk_T$  is the cross-section for the gluon-gluon production of quarks  $Q + \bar{Q}$ ,  $D$  is the fragmentation function.

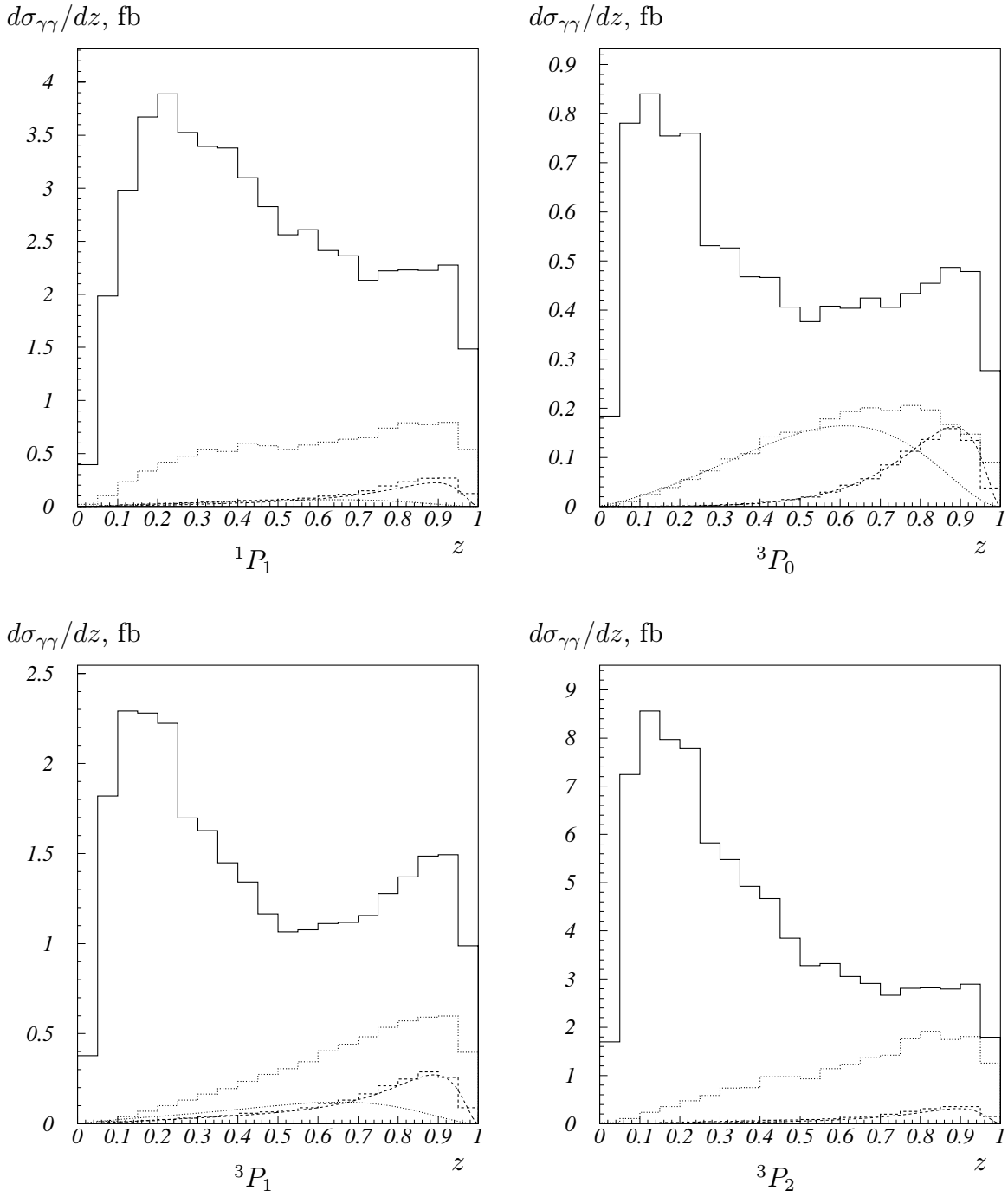


Fig. 8. The differential distributions over  $z$  for the photonic production of P-wave states of  $B_c$  at 100 GeV: a complete set of diagrams (solid histogram), the  $\bar{b}$ -fragmentation diagrams in comparison with the prediction of fragmentation model (dashed histogram and curve), the  $c$ -fragmentation diagrams and the fragmentation model (dotted histogram and curve), correspondingly.

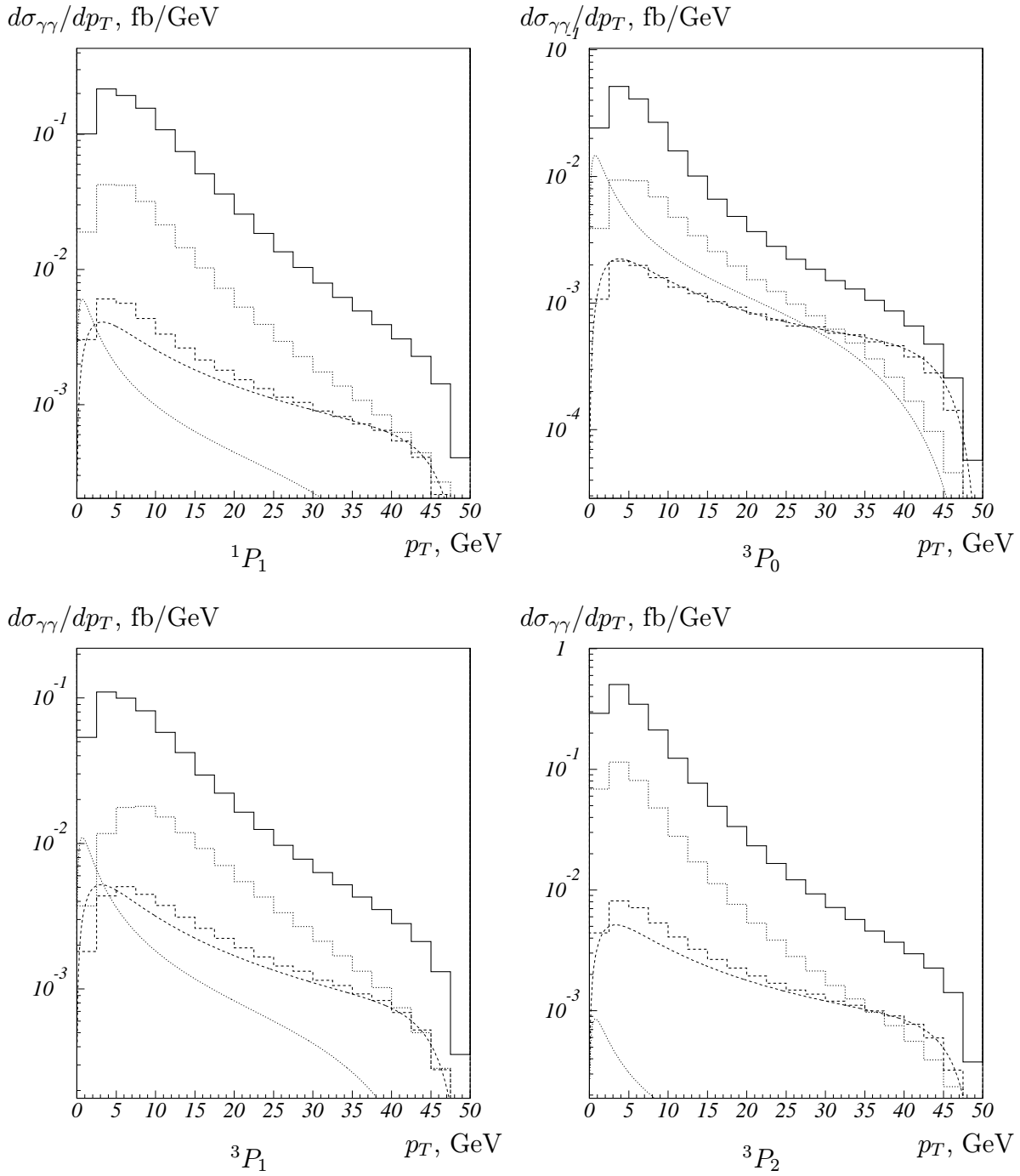


Fig. 9. The distributions over the transverse momentum.

$d\hat{\sigma}/dp_T, \text{ nb/GeV}$

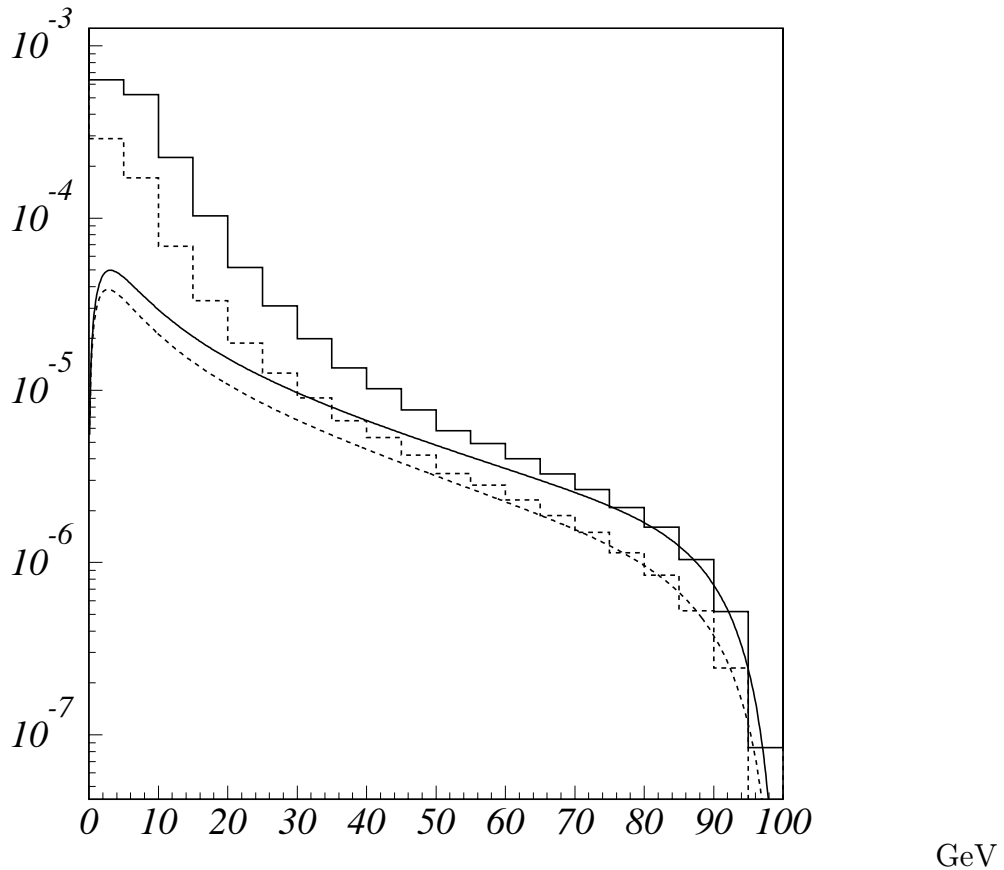


Fig. 10. The differential cross-section for the  $B_c^{(*)}$  meson production in gluon-gluon collisions as calculated in the perturbative QCD over the complete set of diagrams in the  $O(\alpha_s^4)$  order at 200 GeV. The dashed and solid histograms present the pseudoscalar and vector states, respectively, in comparison with the results of fragmentation model shown by the corresponding smooth curves.

The calculation for the complete set of diagrams of the  $O(\alpha_s^4)$ -contribution [29] allows one to determine a value of the transverse momentum  $p_T^{min}$ , being the low boundary of the region, where the subprocess of gluon-gluon  $B_c$ -meson production enters the regime of factorization for the hard production of  $b\bar{b}$ -pair and the subsequent fragmentation of  $\bar{b}$ -quark into the bound  $(\bar{b}c)$ -state, as it follows from the theorem on the factorization of the hard processes in the perturbative QCD.

The  $p_T^{min}$  value turns out to be much greater than the  $M_{B_c}$  mass, so that the dominant contribution into the total cross-section of gluon-gluon  $B_c$ -production is given by the diagrams of nonfragmentational type, i.e. by the recombination of heavy quarks. Furthermore, the convolution of the parton cross-section with the gluon distributions inside the initial hadrons leads to the suppression of contributions at large transverse momenta as well as the subprocesses with large energy in the system of parton mass centre, so that

the main contribution into the total cross-section of hadronic  $B_c$ -production is given by the region of energies less or comparable to the  $B_c$ -meson mass, where the fragmentation model cannot be applied by its construction. Therefore, one must perform the calculations with the account for all contributions in the given order under consideration in the region near the threshold.

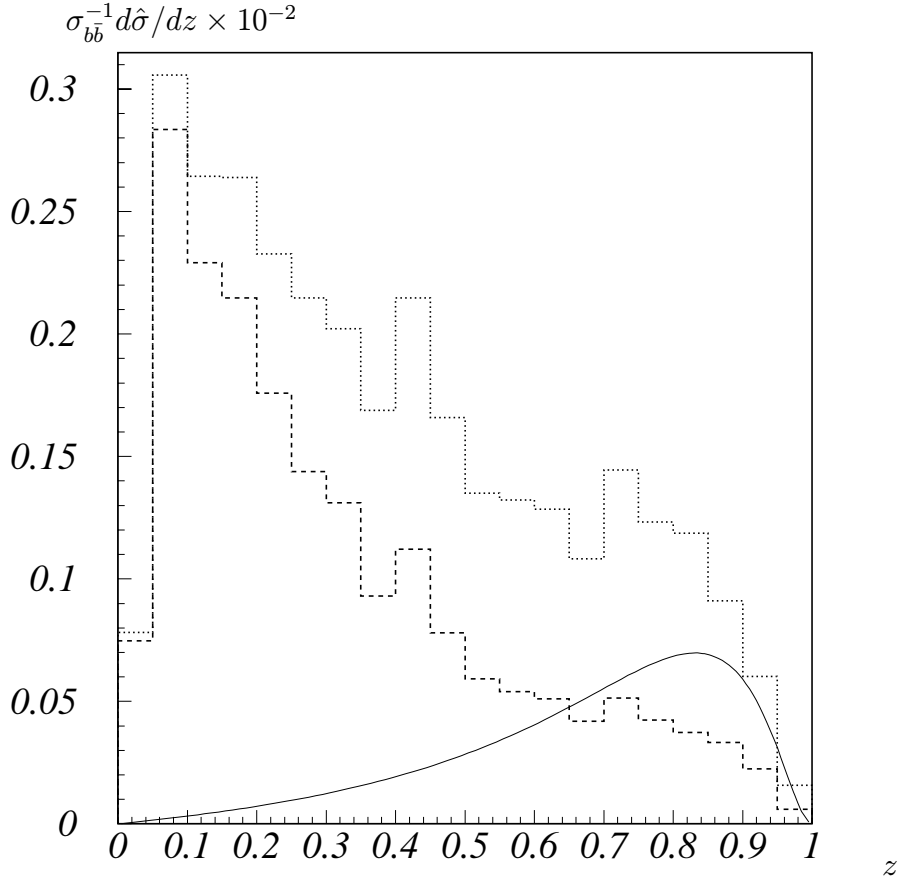


Fig. 11.  $d\hat{\sigma}/dz$  for the  $B_c$ -mesons at the gluon interaction energy of 100 GeV. The dashed histogram presents the  $gg \rightarrow B_c + \bar{b} + c$  process, the dotted one is the abelian case. The curve shows the result of the fragmentation model. The cross-sections are normalized over the cross-section of the  $b\bar{b}$  pair production.

The large numeric value of  $p_T^{\min}$  points to the fact that the basic amount of events of the hadronic  $B_c^{(*)}$ -production does not certainly allow the description in the framework of the fragmentation model. This conclusion looks more evident, if one considers the  $B_c$ -meson spectrum over the energy.

The basic part of events for the gluon-gluon production of  $B_c$  is accumulated in the region of low  $z$  close to 0, where the recombination being essentially greater than the fragmentation, dominates. One can draw the conclusion on the essential destructive interference in the region of  $z$  close to 1, for the pseudoscalar state.

We have in detail considered the contributions of each diagram in the region of  $z \rightarrow 1$ . In the covariant Feynman gauge, the diagrams of the gluon-gluon production of  $Q + \bar{Q}$

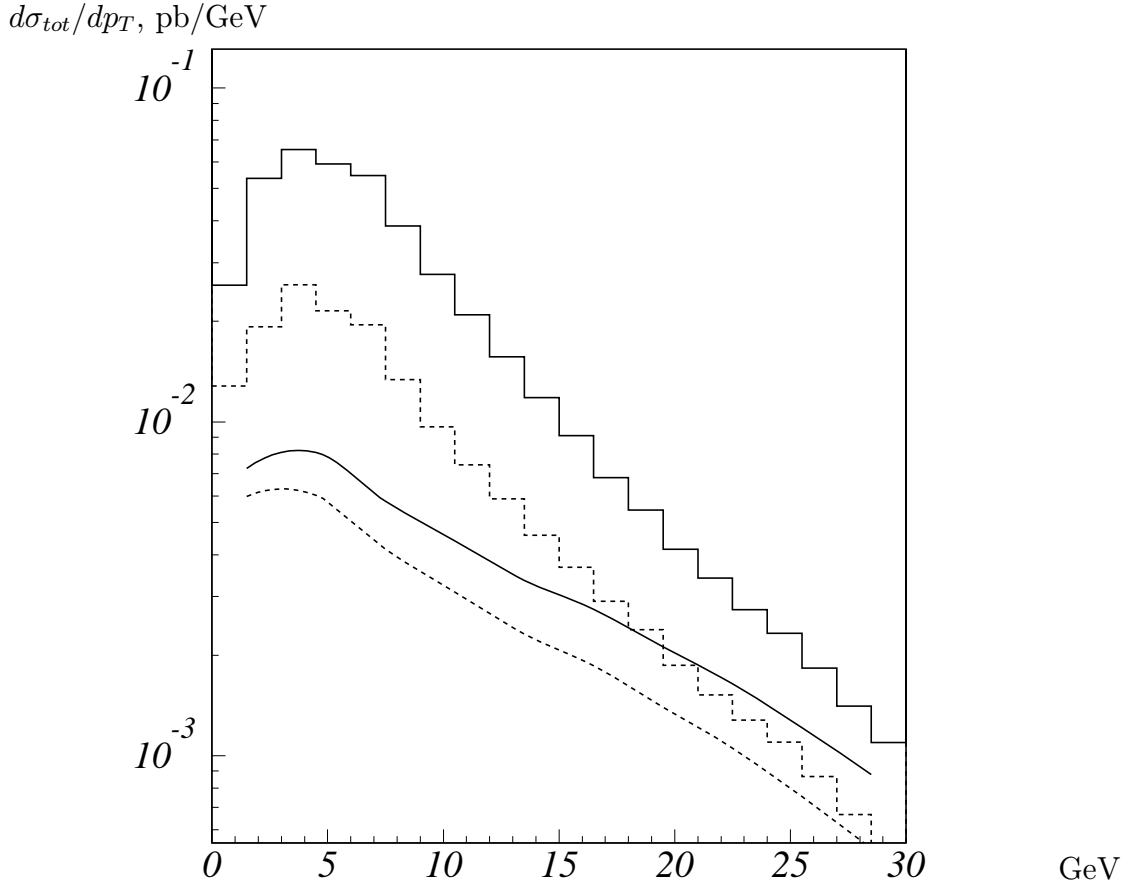


Fig. 12. The differential cross-sections for the  $B_c(B_c^*)$ -meson production in  $p\bar{p}$ -collisions versus the transverse momentum at the interaction energy 1.8 TeV with the cut off the gluon-gluon energy  $E > 60$  GeV.

with the subsequent  $Q \rightarrow (Q\bar{q})$  fragmentation dominate as well as the diagrams, when the  $q\bar{q}$  pair is produced in the region of the initial gluon splitting. However, the contribution of the latter diagrams leads to the destructive interference with the fragmentation amplitude, and this results in the "reduction" of the production cross-section in the region of  $z$  close to 1. In the axial gauge with the vector  $n^\mu = p_Q^\mu$ , this effect of the interference still manifests itself brighter, since the diagrams like the splitting of gluons dominate by several orders of magnitude over the fragmentation, but the destructive interference results in the cancellation of such extremely large contributions. This interference is caused by the nonabelian nature of QCD, i.e., by the presence of the gluon self-action vertices.

To stress the role of the interference diagrams related to the nonabelian self-action of gluons, we have considered the process with abelian currents. In the abelian case the effect of the destructive interference due to the additional contribution of the self-action of gauge quanta, is absent. So, the agreement between the factorized model of fragmentation and the exact perturbative calculation is quite good at  $z$  close to 1.



The direct verification of the given mechanism for the  $B_c$ -meson production could be the comparison of the  $B_c$ -meson spectra in two hemispheres in the region of the gluon fragmentation and in the photon one, in the photonic production of  $B_c$  on nucleons. At LHC with the luminosity  $\mathcal{L} = 10^{34} \text{ cm}^{-2}\text{s}^{-1}$  and  $\sqrt{s} = 14 \text{ TeV}$ , one could expect [21]

$$4.5 \cdot 10^{10} \text{ } B_c^+ \text{ events per year,}$$

which leads, at the efficiency of the decay reconstruction  $\epsilon = 0.1$ , to

$$3 \cdot 10^4 \text{ } B_c^+ \rightarrow \psi\pi^+ \text{ events per year,}$$

with no taking into account some detector-design cuts off angles and momenta.

## Conclusions

- The family of  $(\bar{b}c)$  mesons contains 16 narrow states, the excited ones decay into the ground pseudoscalar state due to the radiative cascades.
- The mass of ground state is expected to have the value

$$m_{B_c} = 6.25 \pm 0.03 \text{ GeV,}$$

which is quite close to the value of  $B_c \rightarrow \psi\pi$  candidates at LEP

$$[m_{B_c}]_{\text{LEP}} = 6.33 \pm 0.05 \text{ GeV.}$$

- The  $B_c^+$  meson is the long-lived particle with the predicted lifetime equal to

$$\tau_{B_c} = 0.55 \pm 0.15 \text{ ps,}$$

which must be compared with the LEP measurements of the candidates

$$[\tau_{B_c}]_{\text{LEP}} = 0.28_{-0.20}^{+0.10} \text{ ps.}$$

- In  $e^+e^-$ -annihilation, the fragmentation of  $\bar{b} \rightarrow B_c^{(*)+}$  dominates, so that one expects

$$[f(\bar{b} \rightarrow B_c^+) \cdot \text{BR}(B_c^+ \rightarrow \psi\pi^+)]_{\text{TH}} = (0.22 \pm 0.09) \cdot 10^{-5},$$

which is less than the  $B_c^+ \rightarrow \psi\pi^+$  candidates rate at LEP

$$[f(\bar{b} \rightarrow B_c^+) \cdot \text{BR}(B_c^+ \rightarrow \psi\pi^+)]_{\text{LEP}} = (1.9_{-1.2}^{+2.5} \pm 0.3) \cdot 10^{-5}.$$

- The fragmentation regime works in the hadronic production of  $B_c$  at high transverse momenta  $p_T > 35 \text{ GeV}$  only. The recombination is essential at lower momenta. In the forward production of  $B_c$  along the hadron beam, the destructive interference of fragmentation diagrams with the gluon-splitting ones takes place.
- The most promising production rate of  $B_c$  for its observation is expected in hadron-hadron collisions, especially at LHC and upgraded Tevatron.

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