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**STUDY OF THE  $\pi^0\pi^0$ -SYSTEM WITH THE GAMS-4000  
SPECTROMETER AT 100 GeV/c**

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(Joint CERN–IHEP experiment)

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**Abstract**

Alde D. et al. Study of the  $\pi^0\pi^0$ -system with the GAMS-4000 spectrometer at 100 GeV/c: IHEP Preprint 98-23. – Protvino, 1998. – p. 24, figs. 11, refs.: 25.

The  $\pi^0\pi^0$ -system produced in the charge exchange  $\pi^-p$ -reaction at 100 GeV/c has been studied. The experiment was performed at the CERN SPS accelerator with the multiphoton hodoscope spectrometer GAMS-4000. A partial wave analysis was carried out in the mass range from 0.8 GeV to 3.0 GeV at  $-t < 0.2$  (GeV/c)<sup>2</sup> with the  $S$ ,  $D$ ,  $G$  and  $J$  waves. The  $S$ -wave exhibited rather complicated behaviour with a series of four bumps separated by three dips, at 1 GeV, 1.5 GeV and 2 GeV, which give the evidence for several scalar resonances. Clear peaks corresponding to the  $f_2(1270)$ ,  $f_4(2050)$  and  $f_6(2510)$  mesons were seen in the higher waves. All the three mesons were produced via a dominating one pion  $t$ -channel exchange. The parameters and production cross sections of these mesons were measured.

**Аннотация**

Алди Д. и др. Изучение  $\pi^0\pi^0$ -системы на спектрометре ГАМС-4000 при импульсе 100 ГэВ/с: Препринт ИФВЭ 98-23. – Протвино, 1998. – 24 с., 11 рис., библиогр.: 25.

Изучена  $\pi^0\pi^0$ -система, образующаяся в зарядовообменной  $\pi^-p$ -реакции при импульсе 100 ГэВ/с. Эксперимент выполнен на ускорителе СПС ЦЕРН. Для регистрации  $\gamma$ -квантов от распадов  $\pi^0$ -мезонов использовался годоскопический многофотонный спектрометр ГАМС-4000. Парциально-волновой анализ  $\pi^0\pi^0$ -системы проведен в интервале масс от 0,8 до 3 ГэВ с учетом  $S$ -,  $D$ -,  $G$ - и  $J$ -волн.  $S$ -волна имеет весьма сложную структуру, в ней наблюдается серия из четырех бампов, разделенных тремя провалами, около 1, 1,5 и 2 ГэВ, которые указывают на существование нескольких скалярных резонансов. В высших волнах наблюдаются четкие пики, соответствующие  $f_2(1270)$ -,  $f_4(2050)$ - и  $f_6(2510)$ -мезонам. Все три мезона образуются путем доминирующего однопионного обмена. Измерены их параметры и сечения образования.

## Introduction

In this paper the GAMS Collaboration presents a study of the  $\pi^o\pi^o$ -system produced in the charge exchange reaction

$$\begin{aligned} \pi^- p &\rightarrow M^o n, \\ &\quad \downarrow \rightarrow \pi^o \pi^o \rightarrow 4\gamma \end{aligned} \tag{1}$$

at 100 GeV/c  $\pi^-$  beam momentum.

In the previous analyses of this reaction performed by the GAMS Collaboration at 38 GeV/c [1,2,3,4,5], a series of interesting results was obtained. A complicated bump-dip structure of the  $S$ -wave at low momentum transfer was revealed. A simultaneous analysis of the GAMS data on the  $S$ -waves in the  $\pi^o\pi^o$ ,  $\eta\eta$  and  $\eta\eta'$  systems together with the data of other experiments [6] indicates the existence of five resonances with quantum numbers  $I^G J^{PC} = 0^+0^{++}$  in the mass range below 1.9 GeV while the quark model predicts only four. One of the five resonances is superfluous for the  $q\bar{q}$ -classification. This state is considered as a ground state scalar glueball candidate. The production mechanisms of the  $f_o(980)$ ,  $f_2(1270)$  and  $f_4(2050)$  were studied at different momentum transfer, a strong upper limit on the production cross section of the  $f_2(1810)$  in the  $\pi^o\pi^o$ -system was set.

The investigation of the  $\pi^o\pi^o$ -system at 100 GeV/c provides a way to get on to the high mass region where a spin 6 resonance  $f_6(2510)$  was previously observed by the GAMS Collaboration [7]. Study of the high mass region is also important for another reason. Recently an interest to the scalar isoscalar mesons has increased in connection with a search for a scalar glueball. The data on the  $S$ -wave  $\pi\pi$ -amplitude are very important for the classification of the  $0^+0^{++}$  mesons and for the identification of states with an enhanced gluonic component. Meanwhile the information on the  $S$ -wave above 1.7–1.8 GeV is now rather scanty and contradictory. The  $S$ -wave separation at high masses is impossible without a detailed partial wave analysis (PWA) because the contribution of this wave is hidden by the higher spin waves which dominate in reaction (1) at mass above 1 GeV.

In the present work a PWA is performed in a wide mass range from 0.8 GeV to 3 GeV at values of four momentum transfer squared  $-t < 0.2$  (GeV/c)<sup>2</sup>.

## 1. Event selection

The experimental data were collected at the CERN SPS accelerator during two runs of measurements in 1984. The multiphoton spectrometer GAMS-4000 was used to detect  $\gamma$ -quanta in the reaction (1) final state. The spectrometer comprised a matrix of  $64 \times 64$  lead glass cells of transverse size  $38 \times 38 \text{ mm}^2$ . A hole of  $2 \times 2$  cells was made in the center for the beam particles not interacting with the liquid hydrogen target to pass. The distance between the target and the  $\gamma$ -spectrometer is equal to 15 m. This allows one to efficiently detect photons from  $\pi^0$  decays in the  $\pi^0\pi^0$  mass range up to 3 GeV. The experimental setup, the data acquisition system, the GAMS-4000 calibration procedures were described in detail [8,9].

Multiplicity of photons, their energies and impact point coordinates in GAMS for each event are determined using a geometrical reconstruction program [10]. Only  $4\gamma$  events are retained to separate the  $\pi^0\pi^0$ -system. A series of cuts is applied to reduce the instrumental background and decrease event leakage from a class with photon multiplicity  $k$  to classes with multiplicities  $k - 1$  or  $k + 1$ . The  $\gamma$ -quanta pair is treated as one  $\gamma$ -quantum if the invariant mass of the pair is less than 25 MeV and the distance between  $\gamma$ -quanta is less than 35 mm. The energy of each photon is required to be larger than the threshold which increases exponentially from 0.6 GeV at GAMS edges to 2.5 GeV in the center. This cut allows one to reject the false photons generated by the sensitive reconstruction program due to fluctuations of the energy deposited in  $\gamma$ -spectrometer cells and to electronic noises. The minimum distance between beam axis and photon impact point in GAMS is required to be larger than 60 mm in order to suppress the background associated with a heavy load of the central cells. This cut decreases also the distortions due to the electromagnetic shower leakage into the central hole. The total energy release in GAMS is confined within the 10% range of the beam energy.

The quality of the selected  $4\gamma$  events is demonstrated in fig. 1, where the invariant mass of  $\gamma$ -pair is shown when the second pair is identified as  $\pi^0$  ( $100 \text{ MeV} < m_{2\gamma} < 170 \text{ MeV}$ ). It is clearly seen from the figure that the background under  $\pi^0$  peak does not exceed 1%.

The final separation of the  $\pi^0\pi^0$ -system from other possible  $4\gamma$ -channels ( $\pi^0\eta$ ,  $\eta\eta$ ,  $\pi^0\eta'$  and  $\eta\eta'$ ) is carried out on the basis of kinematical analysis (3C fit, masses of recoil neutron and two mesons being fixed). The events are selected with the best  $\chi^2$  for the  $\pi^0\pi^0$  hypothesis ( $\chi^2 < 8.3$ , 97% confidence level). A total of 644,000  $\pi^0\pi^0$  events is selected. Only events with  $-t < 0.2 \text{ (GeV/c)}^2$  where one pion exchange (OPE) dominates (see, for example, [1]) are retained for further consideration. Outside this range 21% of all  $\pi^0\pi^0$  events lies.

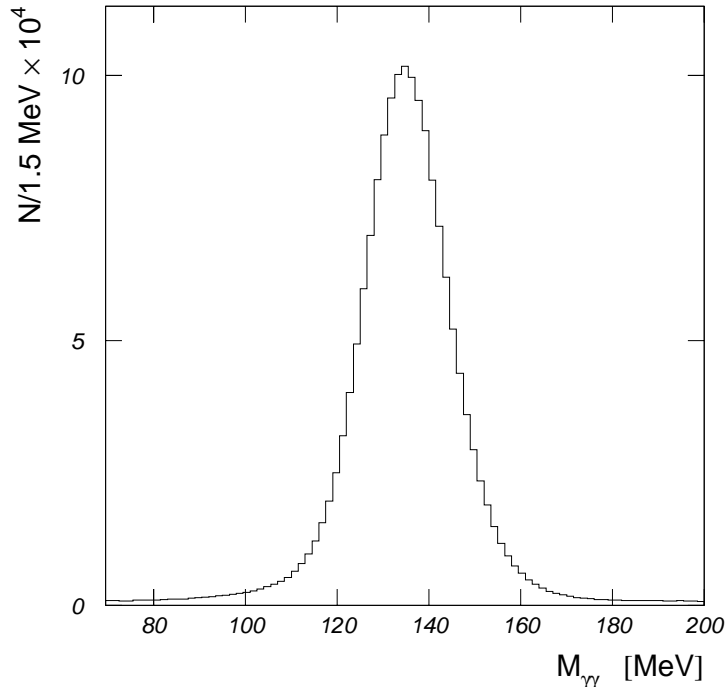


Fig. 1. Invariant mass of  $\gamma$ -pair when the second pair is identified as  $\pi^0$ .

## 2. Mass spectrum

Mass spectrum of the  $\pi^0\pi^0$ -system obtained after kinematical analysis,  $-t < 0.2$  ( $\text{GeV}/c$ )<sup>2</sup>, demonstrates the same characteristic features as that at 38  $\text{GeV}/c$  (fig. 2). It is dominated by the  $f_2(1270)$ , a dip at 1  $\text{GeV}$  is also seen which corresponds to the  $f_0(980)$ . Above the  $f_2(1270)$ , a peak is clearly seen with a mass of about 2  $\text{GeV}$ , it is interpreted with the  $f_4(2050)$ . A peak at 1.7  $\text{GeV}$  cannot be identified with any known resonance. It may be connected with the  $S$ -wave contribution. The PWA is necessary to get an exact answer. A small shoulder is seen above the  $f_4(2050)$  peak at 2.4  $\text{GeV}$ , where the  $f_6(2510)$  was observed previously by the GAMS Collaboration [7].

In the OPE approximation, the reaction (1) angular distribution can be written in the form

$$I(\cos\theta_{GJ}) \sim \left| \sum_l A_l P_l(\cos\theta_{GJ}) \right|^2, \quad (2)$$

where  $A_l$  is the spin  $l$  amplitude,  $P_l(x)$  is the Legendre polynomial,  $\theta_{GJ}$  is the Gottfried-Jackson angle. The angular distributions for the high spin mesons are characterized by a series of minima and maxima, as follows from (2). This allows one to enhance or suppress a signal from the considered resonance by choosing a suitable  $\cos\theta_{GJ}$ -interval. In the interval  $0.75 < \cos\theta_{GJ} < 0.85$  a function  $[P_6(\cos\theta_{GJ})]^2$  reaches its maximum while a polynomial  $P_4(\cos\theta_{GJ})$  squared is close to zero. In the mass spectrum built for the events

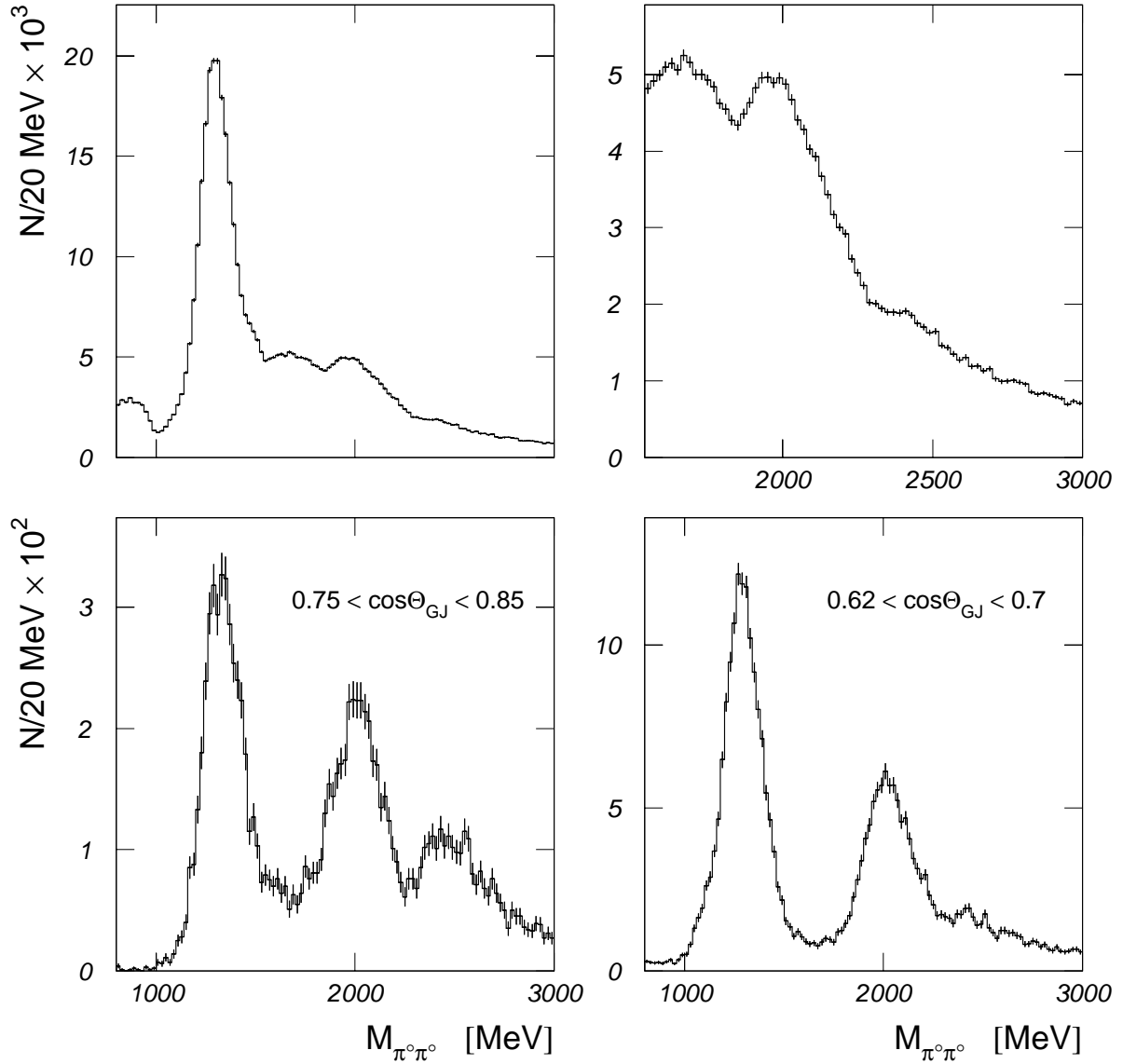


Fig. 2. Two upper plots show mass spectrum of the  $\pi^0\pi^0$ -system after kinematical 3C fit,  $|t| < 0.2$  ( $\text{GeV}/c$ )<sup>2</sup> (high mass region is shown in the right plot). Two lower plots show  $\pi^0\pi^0$  mass spectra in the  $\cos\theta_{GJ}$ -intervals where spin 6 contribution is enhanced (left plot) or suppressed (right plot).

from this interval, a clear  $f_6(2510)$  peak is seen along with the  $f_2(1270)$  and  $f_4(2050)$ . To the contrary, for the events from the  $0.62 < \cos\theta_{GJ} < 0.7$  interval where a polynomial  $[P_6(\cos\theta_{GJ})]^2$  has a minimum, an  $f_4(2050)$  peak is clearly seen, while a signal from the  $f_6(2510)$  is absent (fig. 2).

Further study of the structures observed in the mass spectrum is performed on the basis of PWA.

### 3. Partial wave analysis

#### 3.1. Detection efficiency

Detection efficiency for the reaction (1) events  $\varepsilon$  is computed by the Monte Carlo method as a function of four significant dynamical variables:  $M_{\pi\pi}$ ,  $t$  and two angles in the Gottfried-Jackson frame,  $\theta_{GJ}$  and  $\phi_{TY}$ . The event kinematics is simulated up to the final decay of each  $\pi^0$  to two photons taking into account beam characteristics (dispersions in the energy and the transversal size) and all the geometrical parameters of the setup. A profile of real showers is used to simulate the electromagnetic showers in GAMS [10], fluctuations of energy released in the spectrometer cells covered by the showers are included. The MC events are then processed with the very same programs as in the case of the experimental data. This procedure takes into account in detail the setup parameters, the measurement conditions and all stages of the event selection used in the physical data analysis.

The MC events are generated uniformly distributed over variables  $\cos\theta_{GJ}$ ,  $\phi_{TY}$  and  $e^{\alpha t}$ ,  $\alpha = 5 \text{ (GeV/c)}^{-2}$ , in the mass points equally spaced with 100 MeV step in an interval from 0.6 GeV to 3 GeV. A total of 15 million MC events are generated.

The efficiency is parametrized by the method of [11] and represented in the form of 4-dimensional Fourier series:

$$\varepsilon(\theta_{GJ}, \phi_{TY}, t, M_{\pi\pi}) = \sum_{\substack{l,m \\ k,n}} \varepsilon_{lmkn} \text{Re} \{Y_l^m(\theta_{GJ}, \phi_{TY})\} P_k(a_1 + b_1 e^{\alpha t}) P_n(a_2 + b_2 M_{\pi\pi}), \quad (3)$$

where  $Y_l^m(\theta_{GJ}, \phi_{TY})$  is the spherical harmonic,  $P_k(x)$  is the Legendre polynomial.

Two-dimensional efficiency calculated in two mass points, 1.3 GeV and 2 GeV, integrated over  $t$  in interval  $0 - 0.2 \text{ (GeV/c)}^2$  is presented in fig. 3. The efficiency achieves a maximum at  $\cos\theta_{GJ} \approx 0$  and reduces to zero at  $\cos\theta_{GJ} \rightarrow 1$ . A zone of zero efficiency increases rapidly with the decrease in mass, it occupies about 22% of phase volume in the  $f_2(1270)$  region. At high masses  $\varepsilon$  is equal to zero only in a small part of phase space near  $\cos\theta_{GJ} \approx 1$ . The efficiency integrated over  $\cos\theta_{GJ}$ ,  $\phi_{TY}$  and  $t$  reaches a maximum at  $M_{\pi\pi} \approx 2.2 \text{ MeV}$  and then decreases only slightly with the increase in mass up to 3 GeV. This provides favorable conditions for PWA at high masses.

#### 3.2. Angular distributions

The angular distribution of the reaction (1) events in the Gottfried-Jackson frame is given by a sum of two non-interfering terms

$$I(\theta_{GJ}, \phi_{TY}) = |h_o(\theta_{GJ}) + h_-(\theta_{GJ}) \cos \phi_{TY}|^2 + |h_+(\theta_{GJ}) \sin \phi_{TY}|^2. \quad (4)$$

The first term in (4) corresponds to the exchange with unnatural-parity in the reaction (1)  $t$ -channel, the second one describes the natural-parity exchange. Only the waves with spin  $l \leq l_m = 6$  are taken into account in the PWA. The higher spin contribution is negligible

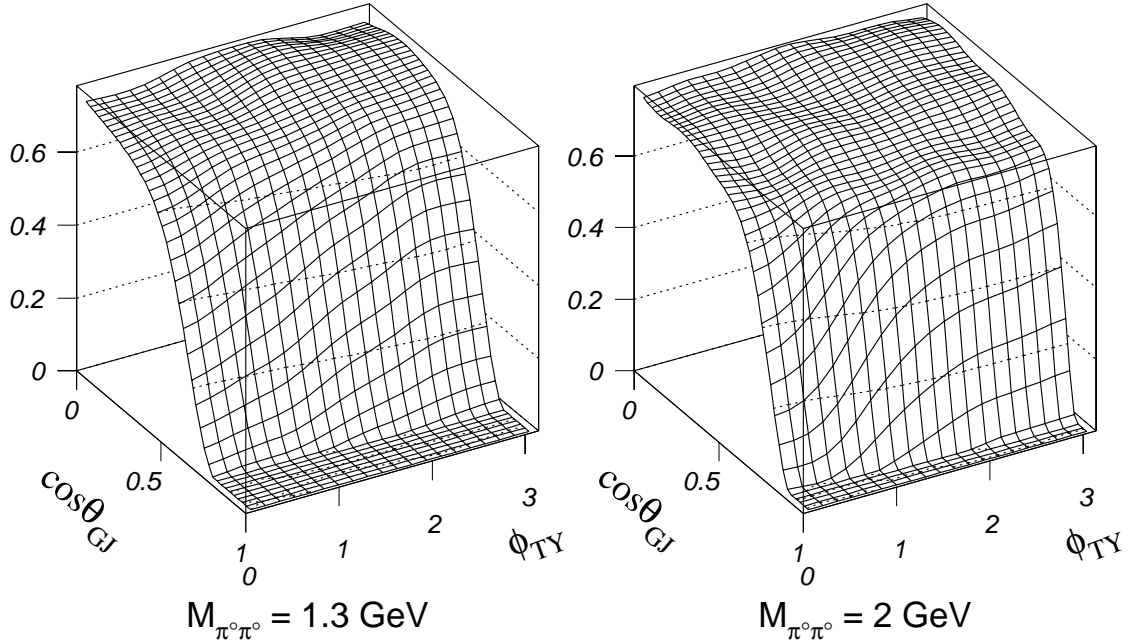


Fig. 3. Detection efficiency of the reaction (1) events integrated over  $t$  from 0 to 0.2 (GeV/c)<sup>2</sup> at  $M_{\pi\pi} = 1.3$  GeV and 2 GeV.

in the mass range under study. The function  $h_o$  includes the amplitudes with spin  $z$ -projections  $m = 0$ , the amplitudes with  $|m| = 1$  form the functions  $h_-$  and  $h_+$ . The waves with  $|m| > 1$  are negligibly small in the whole mass range. The functions  $h_o$ ,  $h_-$  and  $h_+$  can be written in terms of amplitudes of the  $S$ ,  $D$ ,  $G$  and  $J$  waves, corresponding to spin values 0, 2, 4 and 6, as follows

$$\sqrt{4\pi}h_o(\theta_{GJ}) = SP_0^0(\cos\theta_{GJ}) + \sqrt{5}D_oP_2^0(\cos\theta_{GJ}) + \sqrt{9}G_oP_4^0(\cos\theta_{GJ}) + \sqrt{13}J_oP_6^0(\cos\theta_{GJ}), \quad (5)$$

$$\sqrt{4\pi}h_{\pm}(\theta_{GJ}) = \sqrt{3/5}D_{\pm}P_2^1(\cos\theta_{GJ}) + \sqrt{9/10}G_{\pm}P_4^1(\cos\theta_{GJ}) + \sqrt{13/21}J_{\pm}P_6^1(\cos\theta_{GJ}), \quad (6)$$

where  $P_l^m(\cos\theta_{GJ})$  is the associated Legendre function.

Angular distribution (4) can be expanded in the spherical harmonic  $Y_L^M(\Omega)$  series as follows

$$I(\Omega_{GJ}) = \sum_{L=0}^{2l_m} \left[ t_L^0 Y_L^0(\Omega_{GJ}) + 2 \sum_{M=1}^2 t_L^M \text{Re} \{ Y_L^M(\Omega_{GJ}) \} \right], \quad (7)$$

where  $\Omega_{GJ} \equiv [\cos\theta_{GJ}, \phi_{TY}]$ .

The PWA is carried out in 20 MeV mass bins (bin size is doubled at  $M_{\pi\pi} > 1.5$  GeV). Events with  $\varepsilon < 0.02$  are excluded from consideration. The experimental angular distributions are fitted independently in each mass bin  $\Delta M_{\pi\pi}$  with the event-by-event maximum



likelihood method. The MINUIT program [12] is used to minimize functional

$$F = - \sum_{i=1}^N \ln I(\Omega_i) + \sum_{LM} t_L^M \varepsilon_L^M, \quad (8)$$

where  $N$  is the number of experimental events in  $\Delta M_{\pi\pi}$  bin,  $\varepsilon_L^M$  are expansion (3) coefficients, calculated in the center of  $\Delta M_{\pi\pi}$  bin and integrated over  $t$  taking into account the experimental  $t$ -distribution,  $t_L^M$  are spherical harmonic moments (7), expressed in terms of the partial amplitudes [13].

In the case of the  $S$ ,  $D$ ,  $G$  and  $J$  waves, angular distribution (4) is expressed via ten complex amplitudes (three amplitudes for each  $l$ , except for the  $S$ -wave which comes with only one amplitude). One amplitude, each for natural and unnatural spin-parities, can be set real, therefore, the angular distribution in each mass bin is characterized by 18 real parameters. Amplitude modules and relative phases of the partial waves are taken as parameters to be determined in PWA.

### 3.3. PWA ambiguity

A system of equations which expresses the  $t_L^M$  moments via the partial amplitudes [13] is bilinear and has, therefore, multiple solutions. Fit to the angular distributions gives only one solution in each mass bin, i.e. one set of parameters (amplitude modules and relative phases). All other solutions for the system of two pseudoscalar particles can be found with the well-known method [14,15]. First, the ambiguity problem is investigated for the partial waves with unnatural-parity exchange. For this purpose, continuous and continuously differentiated function is defined

$$\begin{aligned} g(\theta_{GJ}) &= h_o(\theta_{GJ}) + h_-(\theta_{GJ}), \\ g(-\theta_{GJ}) &= h_o(\theta_{GJ}) - h_-(\theta_{GJ}). \end{aligned} \quad (9)$$

Introducing a variable  $t = \tan(\theta_{GJ}/2)$  and then  $u = 1/t - t$ , one can define a new function

$$G(u) = t^{-l_m} (1 + t^2)^{l_m} g(t) = a_{l_m} \prod_{k=1}^{l_m} (u - u_k), \quad (10)$$

which is a polynomial of order  $l_m$  and therefore can be expressed through its complex roots  $u_k$  ( $a_{l_m}$  is a complex constant).

One can replace any of the  $u_k$  roots by complex-conjugated one and calculate the corresponding set of angular distribution parameters. These parameters are an equally valid solution because the reaction (1) angular distribution does not change under complex conjugation of the  $G(u)$  roots (see [15]). Hence, there are, in general,  $2^{l_m-1}$  non-trivial solutions for the waves with unnatural-parity exchange, after eliminating those which may be obtained by complex conjugation of the entire  $G(u)$  function.

There are therefore 8 non-trivial solutions for unnatural-parity amplitudes in case of  $S$ ,  $D_o$ ,  $D_{\pm}$ ,  $G_o$  and  $G_{\pm}$  waves. The number of solutions increases to 32 if  $J_o$ ,  $J_-$  and  $J_+$  waves are added.

When the ambiguity problem is resolved for the unnatural-parity amplitudes, natural-parity amplitudes can be found using the moments with  $M = 2$ , which can be expressed through the amplitudes with  $|m| = 1$ . An extra ambiguity for the waves with natural-parity exchange appears, if  $l_m \geq 6$ . This ambiguity may be eliminated by using the Ochs-Wagner model [17], which predicts the equality of the amplitudes with  $|m| = 1$  for each  $l$ .

### 3.4. Bootstrapping procedure

The procedure described above allows one to solve the ambiguity problem in each mass bin. To link solutions in the adjacent mass bins, the requirements are introduced that real and imaginary parts of the  $G(u)$  roots  $u_k$  be continuous and continuously differentiated functions of  $M_{\pi\pi}$ . For this purpose, the following functional is introduced

$$\Phi = \sum_{k=1}^{l_m} \frac{(a_k^{n-1} - a_{\lambda_k}^n)^2}{(\Delta a_k^{n-1})^2 + (\Delta a_{\lambda_k}^n)^2} + \sum_{k=1}^{l_m} \frac{(a_k^{n-2} - 2a_k^{n-1} + a_{\lambda_k}^n)^2}{(\Delta a_k^{n-2})^2 + 4(\Delta a_k^{n-1})^2 + (\Delta a_{\lambda_k}^n)^2}, \quad (11)$$

where

$$a_k^n = \begin{cases} \operatorname{Re} u_k^n & \text{if } 1 \leq k \leq l_m, \\ |\operatorname{Im} u_k^n| & \text{if } l_m + 1 \leq k \leq 2l_m, \end{cases}$$

superscripts indicate the ordinal number of mass bin. Weight of each term in (11) depends on statistical errors of the real and imaginary parts of the roots entering this term.

All possible permutations of the roots  $\{\lambda_k\}$  (in all  $l_m!$ ) are sorted in  $n$ -th bin and the  $\Phi$  value is calculated for each permutation (only the first term in (11) is taken into account for the first two bins). Finally, the roots in  $n$ -th bin are ordered in such a way that functional  $\Phi$  has a minimal value.

The statistical errors of the roots in (11) are estimated by the Monte Carlo method. A total of 10,000 sets of the angular distribution parameters (amplitude modules and relative phases) are generated in each mass bin according to the Gaussian distributions with the mean values and the dispersions determined by MINUIT for one of the solutions. Then, roots of the  $F(u)$  are calculated for each set, their real and imaginary parts are put in histograms. Dispersions of the obtained distributions are used as estimates of the statistical errors  $\Delta \operatorname{Re} u_k^n$  and  $\Delta \operatorname{Im} u_k^n$ .

Figures 4 and 5 show the real and imaginary parts of the  $F(u)$  roots as functions of mass (for the cases  $l_m = 4$  and  $l_m = 6$ ) after imposing the bootstrapping procedure. A behaviour of each root is clearly traced, the imaginary parts are well separated in those points where the real parts cross each other and vice versa, if the imaginary parts of two roots are close to each other, their real parts are separated quite well. It shows that the bootstrapping procedure based on functional (11) allows one to identify unambiguously all the PWA solutions in the whole mass range from 0.8 GeV to 3 GeV.

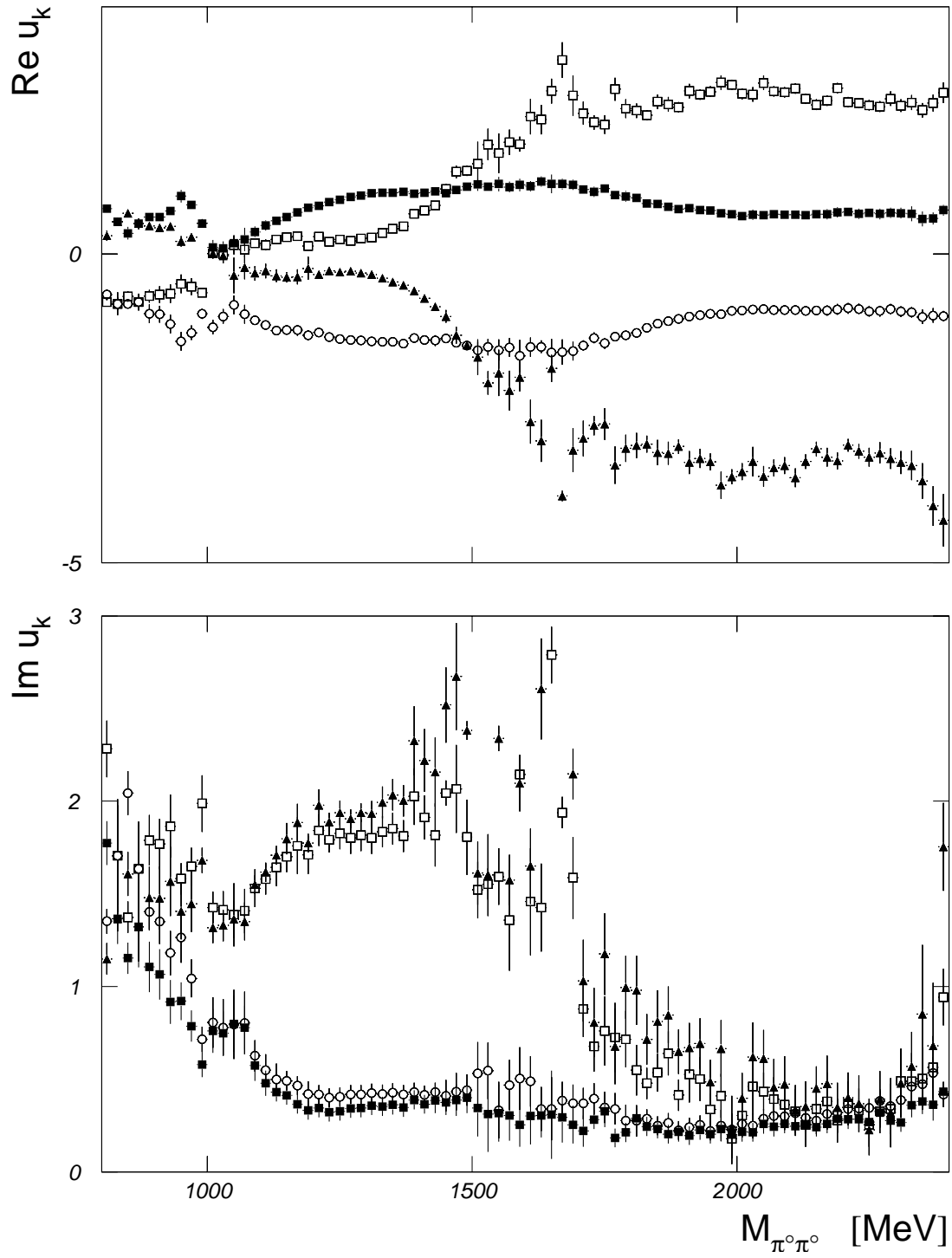


Fig. 4. Real and imaginary parts of  $F(u)$  roots (see text) as functions of mass obtained in PWA with  $S$ ,  $D$  and  $G$  waves.

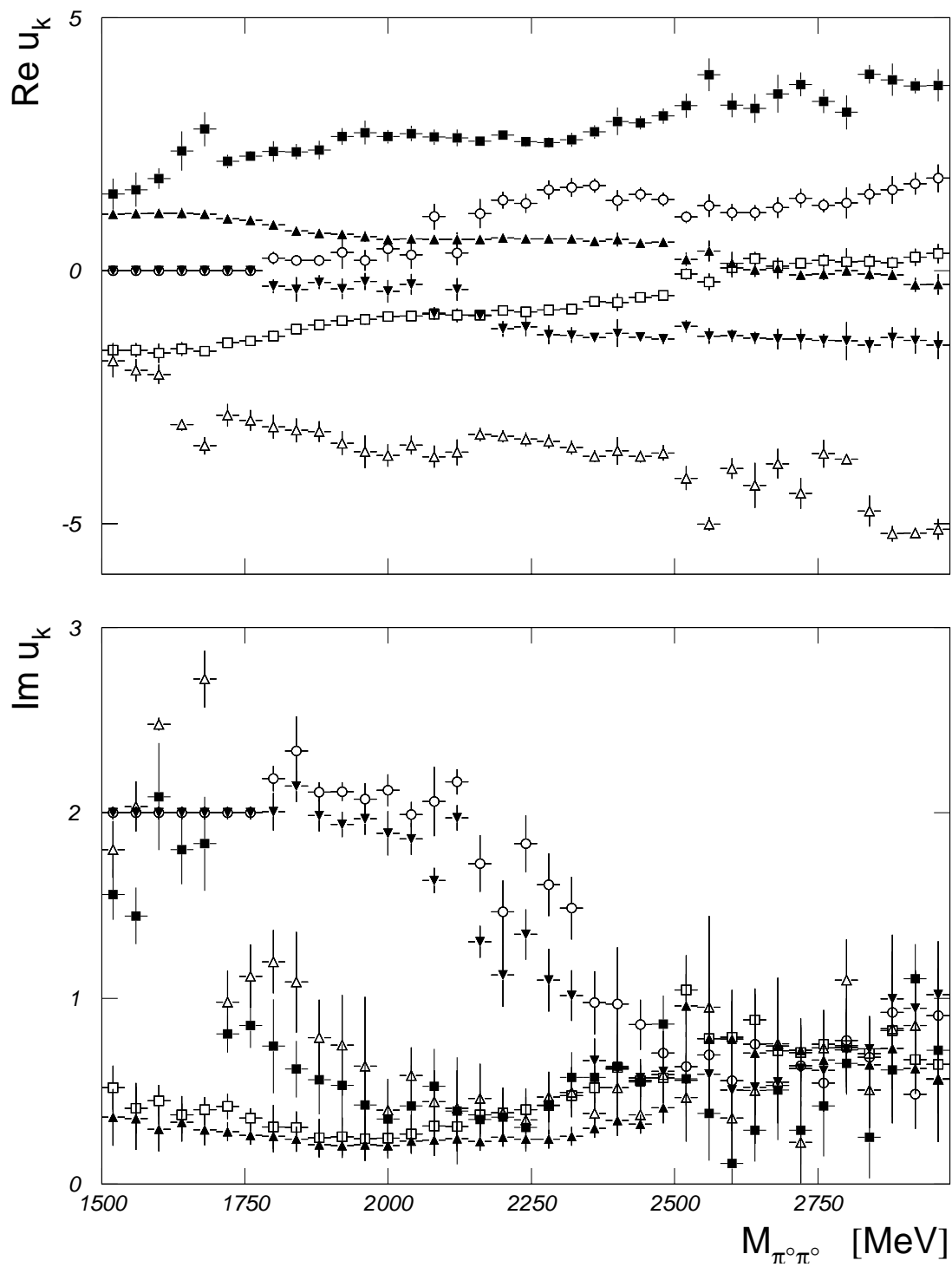


Fig. 5. The same for the case of the  $S$ ,  $D$ ,  $G$  and  $J$  waves.

## 4. Partial wave analysis results

### 4.1. PWA below 2.4 GeV

At the first stage, a PWA is carried out in the mass range from 0.8 GeV to 2.4 GeV taking into account  $S$ ,  $D_o$ ,  $D_-$ ,  $D_+$ ,  $G_o$ ,  $G_-$  and  $G_+$  waves. The detection efficiency drops sharply at low masses (in the  $f_2(1270)$  region and below, see above) which makes difficult the PWA here. Because of this, some extra conditions are imposed at low masses. Modules of the  $G_o$ ,  $G_-$  and  $G_+$  amplitudes squared below 1.5 GeV are described by the exponential curves dropping fast and smoothly with decrease in mass and fixed. Similar constraints are applied to the  $D_o$ ,  $D_-$  and  $D_+$  waves below 1 GeV. The Ochs-Wagner model [17] predicts an equality of the amplitudes with natural and unnatural spin-parity exchange ( $|m| = 1$ ) in the resonance region. This model works well for the  $D_-$  and  $D_+$  waves at 38 GeV/c [1], amplitude modules of these waves are equal to each other not only in the  $f_2(1270)$  mass region but at higher masses also. In our analysis two constraints,  $|D_+| = |D_-|$  and  $|G_+| = |G_-|$ , predicted by the Ochs-Wagner model are applied. A quality of the angular distribution fit for one of the mass bins at the  $f_2(1270)$  peak is shown in fig. 6. In other bins the quality is similar.

Eight non-trivial PWA solutions are found by the method described above. One can reduce significantly the number of solutions by using some physical requirements. Of all solutions four are characterized by the unphysical behaviour of the  $G$ -waves at low masses (clear peaks in the  $f_2(1270)$  region). Furthermore, some solutions do not comply with phase coherence for the  $D_o$  and  $D_-$  waves in the  $f_2(1270)$  mass region and for the  $G_o$  and  $G_-$  waves in the  $f_4(2050)$  mass region. All these solutions are excluded from further consideration. The only solution complying with the physical conditions is shown in fig. 7. It should be noted that the behaviour of the relative phases of the  $D_o$  and  $D_-$  waves as well as the  $G_o$  and  $G_-$  waves agrees well with phase coherence. In a PWA with  $S$ ,  $D$ ,  $G$  and  $J$  waves phase coherence is applied to decrease the number of angular distribution parameters.

### 4.2. PWA in the mass region up to 3 GeV

To study a high mass region a PWA is carried out from 1.8 GeV to 3 GeV in 40 MeV mass bins with  $S$ ,  $D_o$ ,  $D_-$ ,  $D_+$ ,  $G_o$ ,  $G_-$ ,  $G_+$ ,  $J_o$ ,  $J_-$  and  $J_+$  waves taken into account. Contribution of the  $J_-$  and  $J_+$  waves is found to be equal to zero within the statistical errors. These waves are not taken into account in further analysis. In order to reduce the number of parameters to be determined from the angular distribution fit, a series of constraints is applied. First, it is required  $|D_+| = |D_-|$  and  $|G_+| = |G_-|$  in accordance with the Ochs-Wagner model. Then, phase coherence is applied:  $\phi_{D_o} = \phi_{D_-}$  and  $\phi_{G_o} = \phi_{G_-}$ . At last, it is found that phase differences of the waves with natural and unnatural spin-parities ( $|m| = 1$ ) corresponding to the same  $l$  are identical within the errors bars. This allows one to put these phase differences to be equal to each other:  $\phi_{G_+} - \phi_{D_+} = \phi_{G_-} - \phi_{D_-}$ . In our PWA there are nine real parameters to be determined in each mass bin: six amplitude modules  $|S|$ ,  $|D_o|$ ,  $|D_-|$ ,  $|G_o|$ ,  $|G_-|$ ,  $|J_o|$ , and three relative phases  $\phi_{D_o} - \phi_S$ ,

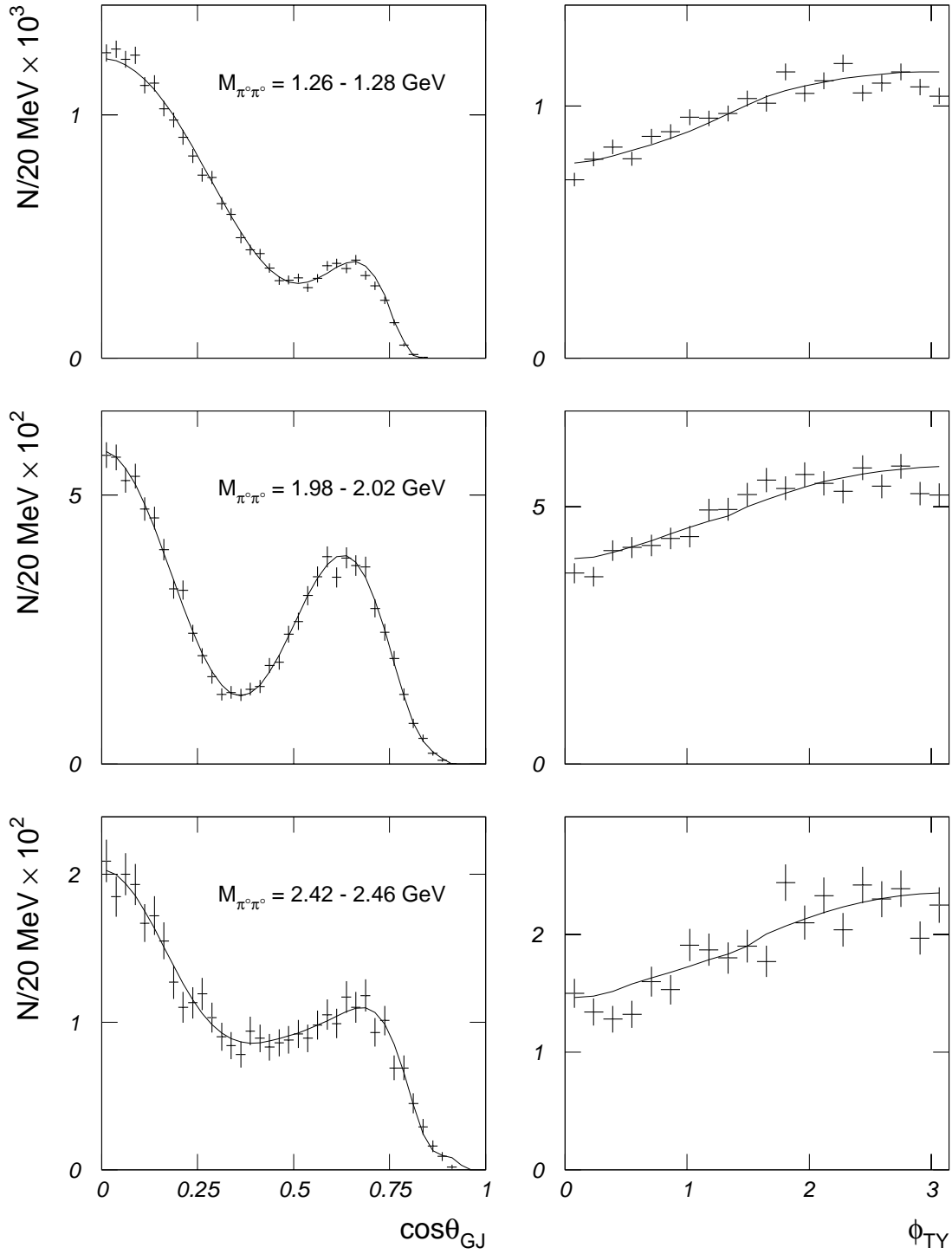


Fig. 6. Angular distributions of reaction (1) in the Gottfried-Jackson frame at the maxima of the  $f_2(1270)$  (two upper plots),  $f_4(2050)$  (two middle plots) and  $f_6(2510)$  peaks. Curves show the theoretical distributions multiplied by the detection efficiency.

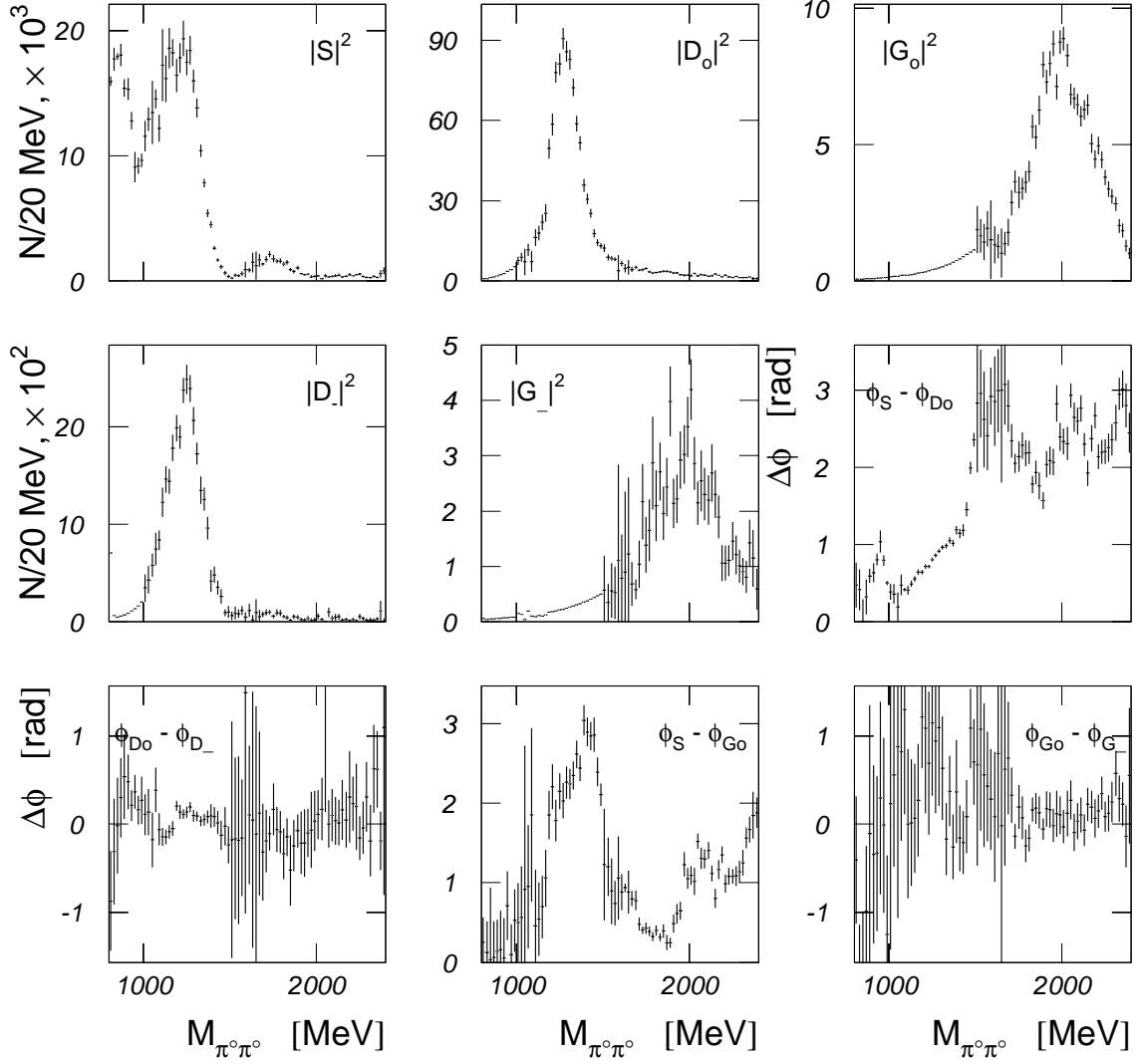


Fig. 7. Physical solution obtained in PWA with  $S$ ,  $D$  and  $G$  waves.

$\phi_{D_0} - \phi_{G_0}$  and  $\phi_{D_0} - \phi_{J_0}$ . All phases are measured relative to the  $D_0$ -wave phase because there are no resonance structures in this wave above the  $f_2(1270)$  (see [5]), therefore the  $D_0$ -wave phase is nearly constant at high masses. A quality of the angular distribution fit for two mass bins in the maxima of the  $f_4(2050)$  and  $f_6(2510)$  peaks is shown in fig. 6.

32 non-trivial PWA solutions found with the method described above may be classified into two equal sets. The solutions from one of these sets are rejected because of the unphysical behaviour of the  $J$ -wave. A clear peak is seen in the  $J_0$ -wave at  $f_4(2050)$  mass. Also a significant  $J_-$ -wave with unphysical structure around 2 GeV appears in these solutions.

Each of the other 16 solutions exhibits a clear  $f_6(2510)$  peak in the  $J_o$ -wave, below this peak the  $J_o$ -wave intensity decreases smoothly to zero. The  $J_-$ -wave does not exhibit any statistically significant structure, its intensity is equal to zero within the errors bars for each solution. The  $G_o$ -wave exhibits a clear  $f_4(2050)$  peak for all 16 solutions while the  $G_-$ -wave demonstrates various behaviours for different solutions. The  $f_4(2050)$  is seen as a prominent peak in eight solutions. In other eight solutions the  $G_-$ -wave intensity decreases monotonously with mass decrease, a shoulder is seen at 2 GeV. The  $G_o$ -wave for these solutions has a noticeable background but the  $f_4(2050)$  signal is an order of magnitude higher in this wave as compared to the  $G_-$ -wave, and  $f_4(2050)$  is observed as a clear peak against a background of about 20%. Such a structure of the  $G$ -waves may be explained by their unphysical behaviour at low masses. The  $G$ -waves exhibit peaks in the  $f_2(1270)$  region (see sect. 4.1), the tails of these peaks are extended to the higher masses. These eight solutions are eliminated also as unphysical ones.

In each of the retained eight solutions, the  $f_4(2050)$  is clearly seen both in the  $G_o$  and  $G_-$  waves, while the  $S$ ,  $D_o$  and  $D_-$  waves have different behaviour. There are statistically significant structures in the  $D_o$  and  $D_-$  waves in the mass range of 1.8–2.5 GeV in all but one solutions. This is in contradiction with the results of the PWA with  $S$ ,  $D$  and  $G$  waves obtained in this work (see also [4]). According to these analyses, the intensities of the  $D_o$  and  $D_-$  waves decrease smoothly above the  $f_2(1270)$  peak. The only solution complied with these results is shown in fig. 8. Below 1.8 GeV the physical solution obtained in PWA with  $S$ ,  $D$  and  $G$  waves is shown (see sect. 4.1).

### 4.3. Spherical harmonic moments

Spherical harmonic moments  $\sqrt{4\pi}t_L^M$  restored from the amplitudes are shown in fig. 9 (the moments with  $M \geq 2$  are equal to zero within the errors bars). The moments demonstrate features typical for all PWA solutions because they are determined unambiguously from the angular distributions. Clear  $f_2(1270)$  peaks are seen in the moments with  $L \leq 4$ . Ratios of the  $t_M^0$  and  $t_M^1$  at mass of the  $f_2(1270)$  allow one to estimate the ratio of the  $D_-$  and  $D_o$  waves intensities (on condition that one can neglect the  $S$  and  $G$  wave contributions at this mass):

$$\frac{|D_-|^2}{|D_o|^2} \approx \frac{1}{2} \left( \frac{t_2^1}{t_2^0} \right)^2 \approx 0.035, \quad \frac{|D_-|^2}{|D_o|^2} \approx \frac{5}{3} \left( \frac{t_4^1}{t_4^0} \right)^2 \approx 0.037. \quad (12)$$

This estimations appear to be somewhat larger than the value obtained from the analysis in terms of amplitudes, which can be explained by the approximate character of (12).

The  $f_4(2050)$  is clearly seen in the  $t_6^0$  and  $t_6^1$ . A signal from this meson becomes more prominent in the moments with  $L = 8$  because these moments ( $t_8^0$  and  $t_8^1$ ) are expressed via  $G$ -amplitudes only (the  $J_o$ -wave gives negligibly small contribution at 2 GeV):

$$\begin{aligned} \sqrt{4\pi}t_8^0 &= \frac{490\sqrt{17}}{2431}|G_o|^2 - \frac{392\sqrt{17}}{2431}(|G_-|^2 + |G_+|^2), \\ \sqrt{4\pi}t_8^1 &= \frac{294\sqrt{85}}{2431}|G_o||G_-|\cos(\phi_{G_o} - \phi_{G_-}). \end{aligned} \quad (13)$$



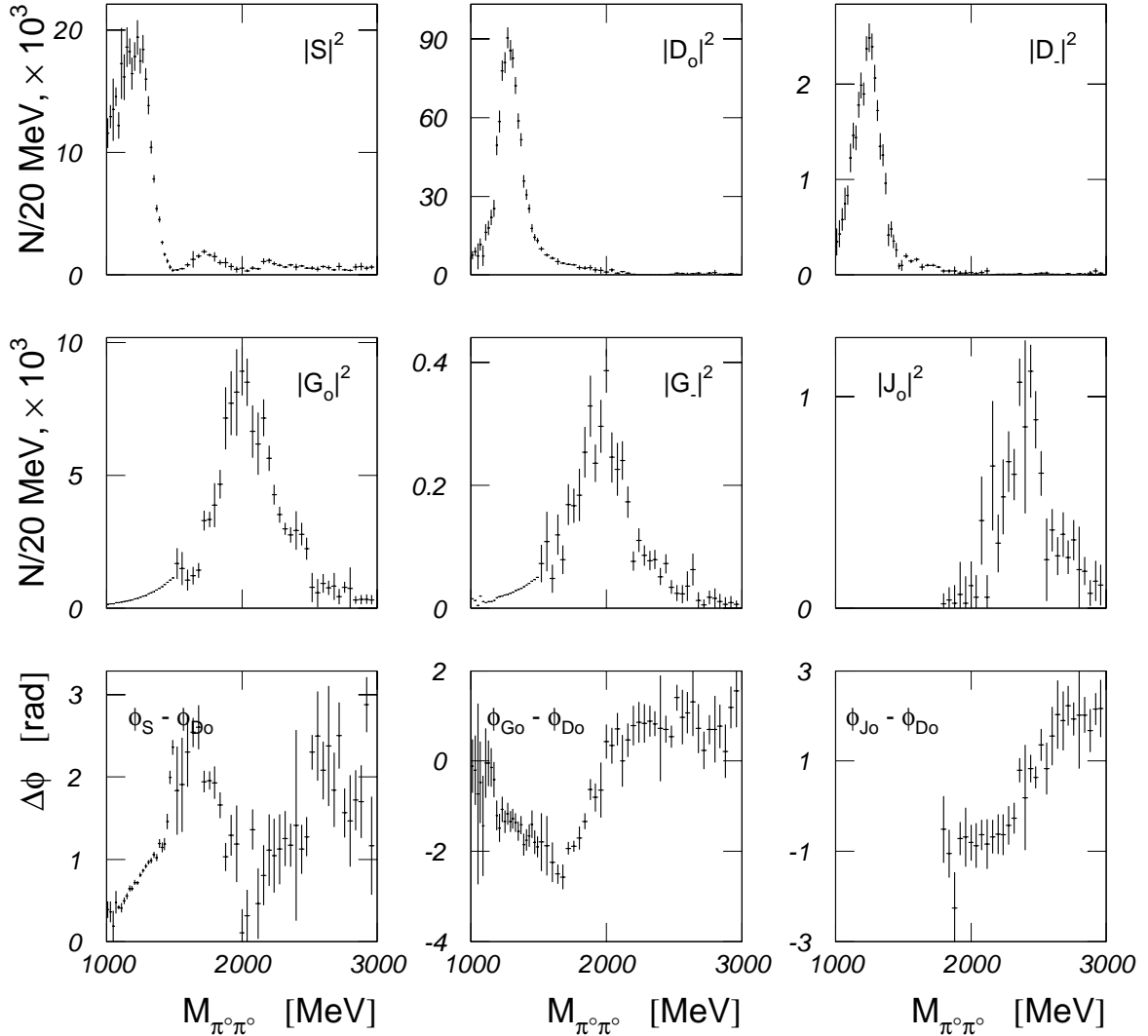


Fig. 8. Physical solution obtained in PWA with  $S$ ,  $D$ ,  $G$  and  $J$  waves.

If one puts  $\phi_{G_0} = \phi_{G_-}$  and  $|G_+| = |G_-|$ , the values of  $|G_-|$  and  $|G_0|$  can be easily determined from (13). In the  $f_4(2050)$  mass region a  $|G_-|^2/|G_0|^2$  ratio is equal to 0.026 in a good agreement with the PWA results in terms of partial waves.

The  $t_{12}^0$  is proportional to the  $|J_0|^2$ . A clear  $f_6(2510)$  peak is seen in this moment.

#### 4.4. Discussion

The behaviour of the physical solution found in this work is in good agreement with the results obtained at 38 GeV/c [1,4,5]. The  $S$ -wave has a rather complicated structure (fig.10). It demonstrates a series of four bumps separated with three dips, at 1, 1.5 and 2 GeV. The first two dips were observed earlier at 38 GeV/c. The former is associated with

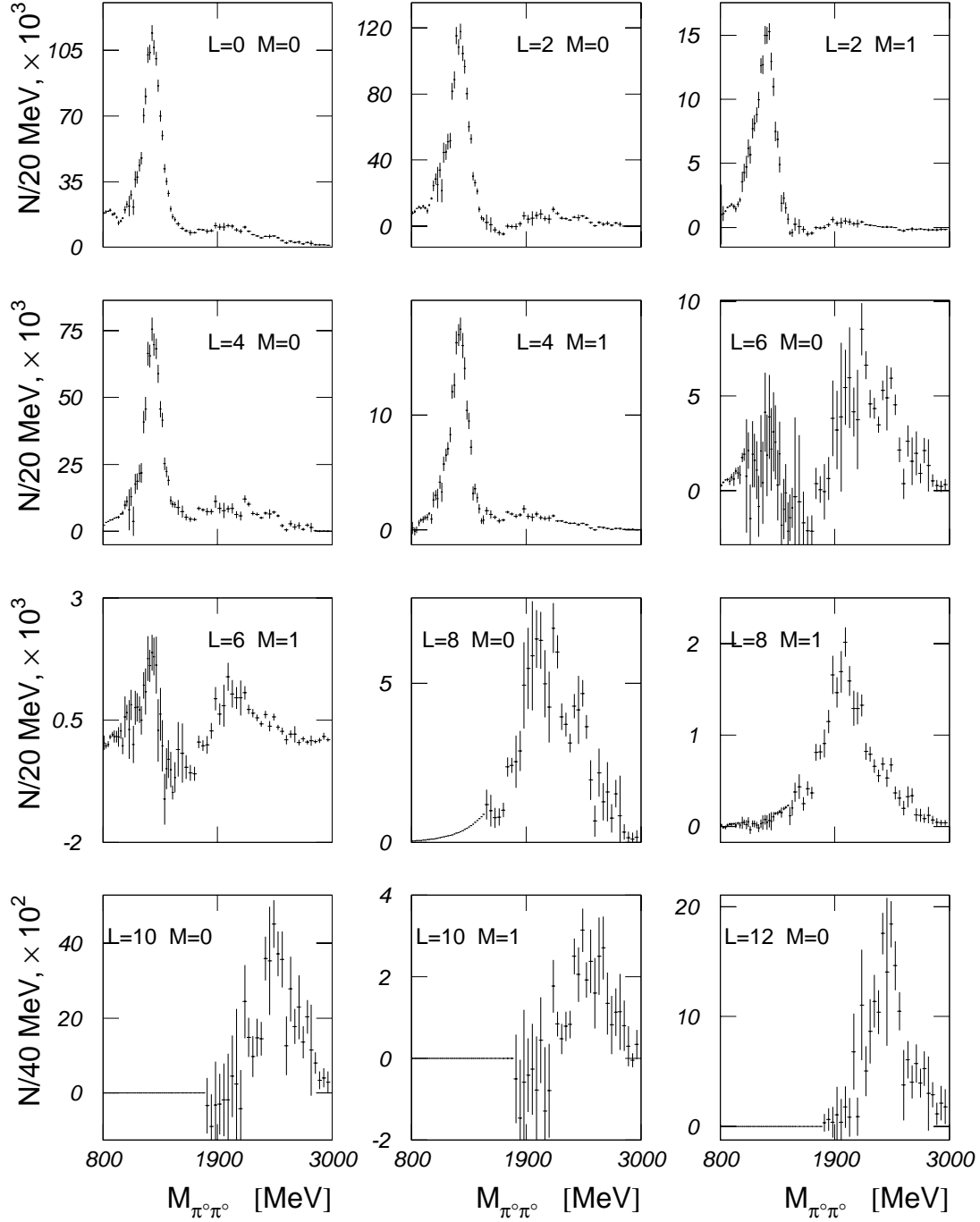


Fig. 9. Spherical harmonic moments restored from the partial amplitudes obtained in PWA with  $S$ ,  $D$ ,  $G$  and  $J$  waves.

the  $f_0(980)$ , this scalar meson was studied in detail in [2,3,18]. The bump at 1.5 GeV seen previously in [4,5] is attributed to another scalar resonance,  $f_0(1500)$ , a ground state scalar glueball candidate. The  $S$ -wave structure above 2 GeV is less prominent at 38 GeV/c

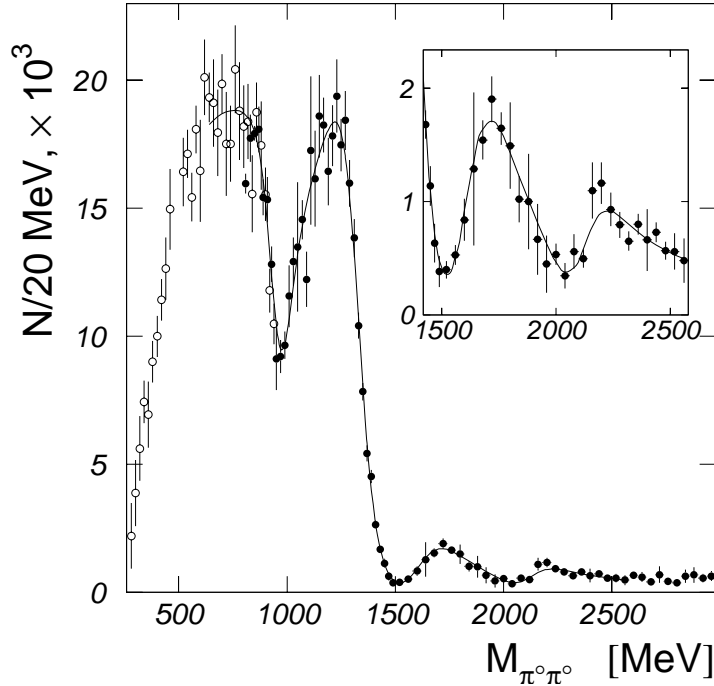


Fig. 10.  $S$ -wave amplitude module squared for the physical solution. Below 1 GeV the values of  $|S|^2$  obtained at 38 GeV/c [25] are shown by open circles normalized to the present data. High mass region is shown in insertion. A curve shows the fit with the dependence described in text.

due to insufficient detection efficiency [5]. At 100 GeV/c the efficiency at high masses is much better, because of this a dip is clearly seen in the  $S$ -wave at 2 GeV followed by one more bump. It is important to note that the last bump appeared when the  $J_0$ -wave was added in PWA model (sect. 4.2). The analysis with  $S$ ,  $D$  and  $G$  waves only did not reveal any structure in the  $S$ -wave above 2 GeV (fig.7). A similar situation took place with the bump at 1.7 GeV [4]. It appeared in the  $S$ -wave after the  $S$ ,  $D$  and  $G$  waves have been taken into account correctly. In the first analysis [1], where apart from  $S$ ,  $D_0$ ,  $D_-$  and  $D_+$  waves only  $G_0$ -wave was included with the  $f_4(2050)$  tabulated parameters, the bump in the  $S$ -wave at 1.7 GeV was absent. It shows once more the importance of high wave account for correct determination of the  $S$ -wave at high masses.

The  $S$ -wave phase measured relative to the  $D_0$ -wave phase has rather a complicated behaviour. It increases sharply around 1.5 GeV, confirming the presence of the  $f_0(1500)$ . The phase changes rapidly at 2 GeV as well. Along with the dip in the  $S$ -wave, this indicates the existence of a scalar resonance in this mass range. Such conclusion is confirmed by the results of  $\eta\eta$ -system study at 100 GeV/c [19]. The  $S$ -wave for one of two solutions found in [19] demonstrates above the  $\eta\eta$ -threshold a behaviour, similar to the behaviour of the  $S$ -wave, obtained in the present work. It shows three bumps, separated by dips at 1.45 GeV and 1.9 GeV.

The behaviour of the  $D$  and  $G$  waves are in good agreement with the Ochs-Wagner model prediction. A ratio of  $D_o$  and  $D_-$  intensities at  $f_2(1270)$  mass is about 3%. The same ratio obeys for  $G_o$  and  $G_-$  intensities at mass of the  $f_4(2050)$ .

The  $G_o$ -wave phase measured relative to the  $D_o$ -wave phase changes by  $\pi$  at 2 GeV confirming the presence of the  $f_4(2050)$ . The relative phase of the  $J_o$  and  $D_o$  waves demonstrates a similar behaviour in the  $f_6(2510)$  mass region.

## 5. Resonance parameters and production cross sections

### 5.1. Scalar resonances

To estimate the parameters and production cross sections of the scalar resonances produced in reaction (1), a fit to the  $S$ -wave amplitude module squared has been carried out. The following parametrization is used

$$A(M_{\pi\pi}) = G(M_{\pi\pi}) + \sum_{n=1}^{N_{res}} a_n e^{i\theta_n} B_n(M_{\pi\pi}), \quad (14)$$

$$G(M_{\pi\pi}) = (M_{\pi\pi} - 2m_{\pi^0})^\alpha e^{-\beta M_{\pi\pi} - \gamma M_{\pi\pi}^2}, \quad (15)$$

where  $a_n$  and  $\theta_n$  are the amplitude and phase of the  $n$ -th resonance, respectively,  $\alpha$ ,  $\beta$  and  $\gamma$  are real parameters. A relativistic formula [20] is used for the Breit-Wigner function  $B(M_{\pi\pi})$ :

$$B(M_{\pi\pi}) = \left( \frac{M_{\pi\pi}}{\sqrt{q}} \right) \sqrt{2l+1} \frac{M_R \Gamma}{M_R^2 - M_{\pi\pi}^2 - i M_R \Gamma}, \quad (16)$$

$$\Gamma = \Gamma_R \left( \frac{q}{q_R} \right)^{2l+1} \frac{D_l(qRr)}{D_l(qr)}, \quad (17)$$

where  $q$  is a  $\pi^0$ 's momentum in c.m.s. of dipion,  $l$ ,  $M_R$  and  $\Gamma_R$  are spin, mass and width of the resonance, respectively,  $q_R$  is a  $\pi^0$ 's momentum at  $M_{\pi\pi} = M_R$ ,  $r$  is interaction radius being equal to 1 Fm (fit results depend only slightly on this parameter),  $D_l(x)$  is a Blatt-Weiskopf barrier factor [21].

Function (14) is convoluted with a Gaussian distribution

$$\zeta(M, M_{\pi\pi}) = C \exp \left\{ -\frac{(M - M_{\pi\pi})^2}{2\sigma^2(M)} \right\}, \quad (18)$$

$\sigma(M) = 0.009 + 0.021 \cdot M$  GeV, in order to account for the experimental mass resolution:

$$|S(M_{\pi\pi})|^2 = \int dM \zeta(M, M_{\pi\pi}) |A(M)|^2. \quad (19)$$

Apart from three resonances discussed in sect. 4.4, one more scalar resonance around 1.3 GeV is needed to describe the  $S$ -wave amplitude module squared in the whole mass range under study. The fit quality deteriorates significantly without the  $f_o(1300)$  especially for the bump at 1.3 GeV. The dip at 1.5 GeV appears due to destructive interference

of the  $f_o(1300)$  and  $f_o(1500)$  with a non-resonant part of the S-wave (fig. 10). As a result of the interference the  $f_o(1500)$  mass is shifted to higher values as compared to the dip position.

To check the stability of the obtained results, we varied the low and the high edges of the mass interval. We tried also to use the Breit-Wigner function or polynomial instead of  $G(M)$  in (14). Variations of the  $f_o(980)$  and  $f_o(1300)$  parameters lie within the statistical errors:  $M = 960 \pm 10$  MeV and  $\Gamma = 70 \pm 20$  MeV for the  $f_o(980)$  and  $M = 1315 \pm 30$  MeV and  $\Gamma = 190 \pm 50$  MeV for the  $f_o(1300)$ . Mass and width of the  $f_o(1500)$  are determined with larger errors:  $M = 1580 \pm 80$  MeV,  $\Gamma = 280 \pm 100$  MeV. As for the resonance corresponding to the dip at 2 GeV, for its mass and width, the following values are obtained

$$\begin{aligned} M &= 2010 \pm 60 \text{ MeV}, \\ \Gamma &= 240 \pm 100 \text{ MeV}. \end{aligned} \tag{20}$$

The errors shown in (20) include both systematical and statistical errors. In what follows this resonance will be called  $f_o(2010)$ . A scalar state with the same mass  $2020 \pm 35$  MeV and slightly larger width  $410 \pm 50$  MeV was observed in the  $\pi^+\pi^-\pi^+\pi^-$  system produced in  $pp$  central collisions at 450 GeV/c [22].

The production cross sections of the scalar resonances are estimated using the number of events under the Breit-Wigner curves and normalized to the  $f_2(1270)$  cross section (see below):

$$\begin{aligned} \sigma(\pi^-p \rightarrow f_o(980)n) \times \text{BR}(f_o(980) \rightarrow \pi^o\pi^o) &= 5,4 \pm 1,2 \text{ nb}, \\ \sigma(\pi^-p \rightarrow f_o(1300)n) \times \text{BR}(f_o(1300) \rightarrow \pi^o\pi^o) &= 70 \pm 15 \text{ nb}, \\ \sigma(\pi^-p \rightarrow f_o(1500)n) \times \text{BR}(f_o(1500) \rightarrow \pi^o\pi^o) &= 12 \pm 3 \text{ nb}, \\ \sigma(\pi^-p \rightarrow f_o(2010)n) \times \text{BR}(f_o(2010) \rightarrow \pi^o\pi^o) &= 3 \pm 1 \text{ nb}. \end{aligned} \tag{21}$$

Data on the S-wave in the  $\pi^o\pi^o$ -system produced at 100 GeV/c obtained in the present work does not contradict the existence of the fifth resonance [6]. This state can be identified with a wide bump in the S-wave extending from the threshold up to  $\sim 2.5$  GeV with the dips against its background (fig. 10). Use of the Breit-Wigner function instead of  $G(M)$  (see (14)) to describe a non-resonant part of the S-wave gives an equally good fit ( $\chi^2$  divided by the number of freedom increase only slightly, from 0.98 to 1.07). According to [6] the broad resonance is crucial for large interference effects seen in the S-wave.

## 5.2. Resonances with higher spins

In order to determine the masses, widths and production cross sections of the resonances observed in the  $D$ ,  $G$  and  $J$  waves, amplitude modules squared of these waves are fitted with the sums of the Breit-Wigner curves (16) and backgrounds.

The  $D_o$ -amplitude module squared is well described by the Breit-Wigner resonance with a mass of  $1283 \pm 5$  MeV and a width of  $171 \pm 10$  MeV, which are in good agreement with the  $f_2(1270)$  tabulated parameters [23] (fig.11). A peak in the  $D_-$ -wave is not fitted with the Breit-Wigner function, which may be explained by distortions due to insufficient efficiency at low masses.

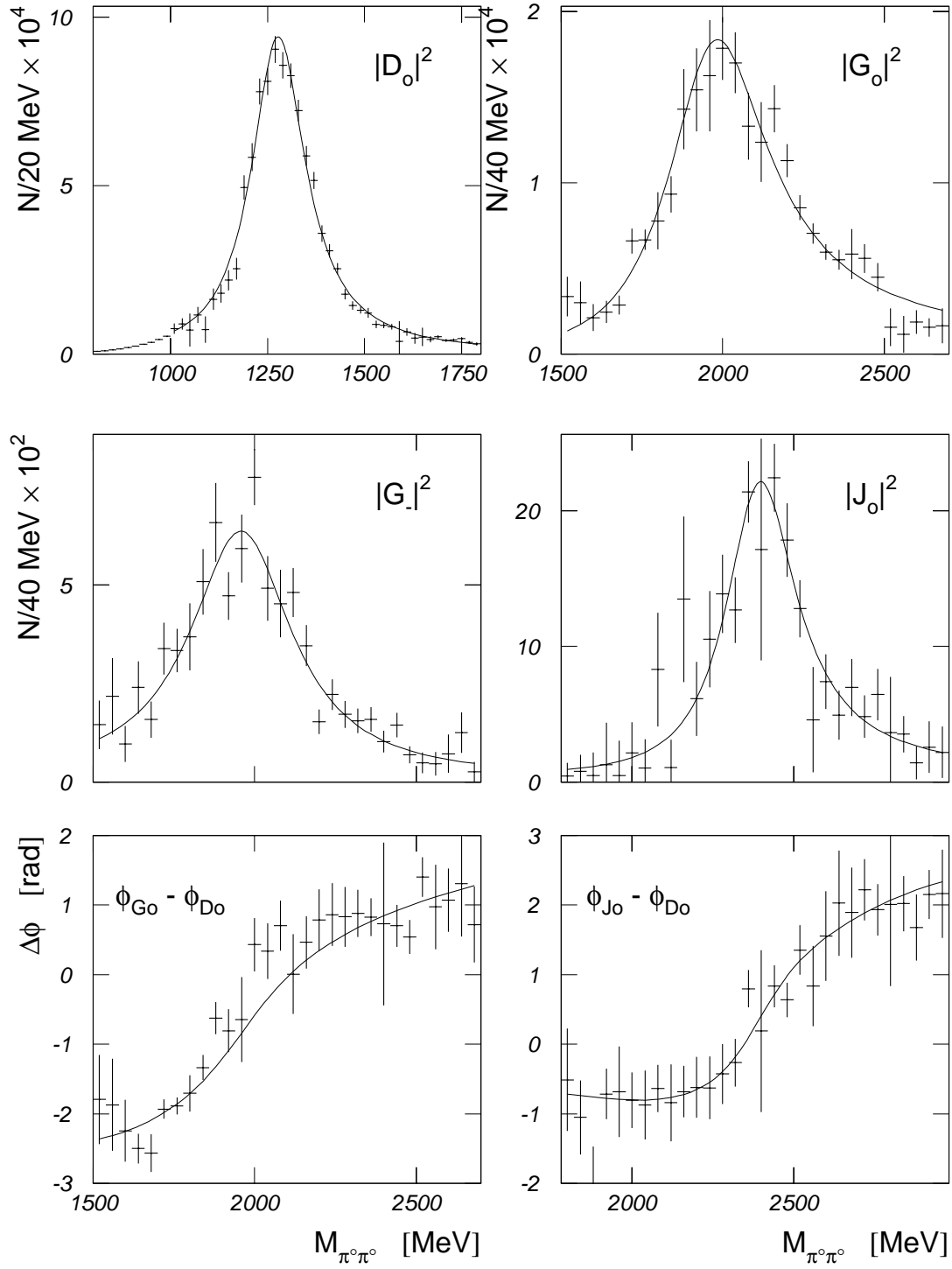


Fig. 11. Simultaneous fit to the  $G_0$ ,  $G_-$  and  $J_0$  amplitudes and relative phases of the  $G_0$  and  $D_0$ ,  $J_0$  and  $D_0$  waves (see details in the text).

The  $f_4(2050)$  parameters are found on the basis of the simultaneous fit to the  $D_o$ ,  $G_o$ ,  $G_-$  amplitude modules squared and the relative phase of the  $D_o$  and  $G_o$  waves (fig. 11). A coherent sum of resonance (16) for spin four and a background (complex constant) is used to describe the  $G_o$  and  $G_-$  amplitudes. The  $D_o$ -amplitude is parametrized in a similar way, the  $f_2(1270)$  parameters being fixed to their tabulated values. The fit gives the following values for the  $f_4(2050)$  mass and width (taking into account a mass resolution,  $\sigma_M = 50$  MeV at  $M_{\pi\pi} = 2$  GeV):

$$\begin{aligned} M &= 1998 \pm 15 \text{ MeV}, \\ \Gamma &= 395 \pm 40 \text{ MeV}. \end{aligned} \quad (22)$$

The  $f_6(2510)$  parameters are determined from a simultaneous fit to the  $D_o$  and  $J_o$  amplitude modules squared and relative phase of these waves (fig. 11). The  $J_o$ -amplitude is parametrized as a coherent sum of resonance (16) for spin six and a background (complex constant). The  $D_o$ -amplitude is described with a complex polynomial of first order ( $D_o$ -amplitude parametrization in a form of the tail of the Breit-Wigner function gives unsatisfactory description). The following values are obtained for the  $f_6(2510)$  mass and width:

$$\begin{aligned} M &= 2420 \pm 30 \text{ MeV}, \\ \Gamma &= 270 \pm 60 \text{ MeV}. \end{aligned} \quad (23)$$

The width is determined taking into account a mass resolution ( $\sigma_M = 60$  MeV at  $M_{\pi\pi} = 2.4$  GeV).

Production cross sections of the  $f_4(2050)$  and  $f_6(2510)$  are normalized to the cross section of the  $f_2(1270)$  measured in one of our previous works with an accuracy of 8% [1]. The production cross section of the  $f_2(1270)$  is equal to  $326 \pm 30$  nb at 100 GeV/c. This value is obtained using the following energy dependence of the cross section [24]

$$\begin{aligned} \sigma(\pi^- p \rightarrow f_2(1270)n) \times \text{BR}(f_2(1270) \rightarrow \pi^o \pi^o) = \\ (2.61 \pm 0.20) \mu\text{b} (p_{lab}/38 \text{ GeV}/c)^{-2.15 \pm 0.05}. \end{aligned} \quad (24)$$

A total number of the  $f_2(1270)$  mesons detected in the  $D_o$ ,  $D_-$  and  $D_+$  waves is equal to  $(1.25 \pm 0.02) \times 10^6$ , a corresponding sensitivity of the experiment is  $0.261 \pm 0.024$  pb per one event of reaction (1). A ratio of the production cross sections of the  $f_2(1270)$  in the  $D_-$  and  $D_o$  waves is found to be  $0.026 \pm 0.002$ , being two times smaller as compared to the value of  $0.059 \pm 0.006$  obtained at 38 GeV/c [1].

The production cross sections of the  $f_4(2050)$  in the  $G_o$  and  $G_{\pm}$  waves at 100 GeV/c are found to be

$$\begin{aligned} \sigma_{G_o}(\pi^- p \rightarrow f_4(2050)n) \times \text{BR}(f_4(2050) \rightarrow \pi^o \pi^o) &= 74, 8 \pm 7, 2 \text{ nb}; \\ \sigma_{G_{\pm}}(\pi^- p \rightarrow f_4(2050)n) \times \text{BR}(f_4(2050) \rightarrow \pi^o \pi^o) &= 2, 5 \pm 0, 3 \text{ nb}. \end{aligned} \quad (25)$$

A total  $f_4(2050)$  production cross section in three  $G$ -waves (on condition that  $|G_+| = |G_-|$ ) is equal to

$$\sigma_{tot}(\pi^- p \rightarrow f_4(2050)n) \times \text{BR}(f_4(2050) \rightarrow \pi^o \pi^o) = \sigma_{G_o} + 2\sigma_{G_-} = 79.8 \pm 7.8 \text{ nb}. \quad (26)$$

A ratio of the  $f_4(2050)$  production cross sections in the  $G_-$  and  $G_o$  waves is  $0.033 \pm 0.003$ . A similar ratio measured at 38 GeV/c is more than two times larger  $0.075 \pm 0.007$  [4].

The  $f_6(2510)$  production cross section in the  $J_o$ -wave at 100 GeV/c is found to be

$$\sigma_{J_o}(\pi^- p \rightarrow f_6(2510)n) \times \text{BR}(f_6(2510) \rightarrow \pi^o \pi^o) = 9.9 \pm 1.4 \text{ nb}. \quad (27)$$

An upper limit is set for the  $f_6(2510)$  production cross section in the  $J_{\pm}$ -waves at 95% confidence level

$$\frac{\sigma_{J_{\pm}}(\pi^- p \rightarrow f_6(2510)n) \times \text{BR}(f_6(2510) \rightarrow \pi^o \pi^o)}{\sigma_{J_o}(\pi^- p \rightarrow f_6(2510)n) \times \text{BR}(f_6(2510) \rightarrow \pi^o \pi^o)} < \frac{1}{10}. \quad (28)$$

It is interesting to compare the results obtained in this work with the Ochs-Wagner model predictions. According to the model

$$\frac{|A_-^l|^2}{|A_o^l|^2} = \frac{c_A}{M_{\pi\pi}^2} l(l+1), \quad (29)$$

where  $A_o^l$  and  $A_-^l$  are the amplitudes with unnatural-parity exchange corresponding to  $|m| = 0$  and 1, respectively.

By using (29), one can calculate a  $|G_-|^2/|G_o|^2$  ratio at mass of the  $f_4(2050)$  from the  $D_-$  and  $D_o$  wave intensities in the  $f_2(1270)$  mass region. This ratio is equal to 0.034. A similar ratio for the  $J_-$  and  $J_o$  waves in the  $f_6(2510)$  mass region is equal to 0.051. Both values are in good agreement with the results obtained in the present work.

Ratios of the production cross sections of the  $f_2(1270)$  in the  $D_o$  and  $D_{\pm}$  waves and of the  $f_4(2050)$  in the  $G_o$  and  $G_{\pm}$  waves at 38 GeV/c and 100 GeV/c are found to be

$$\begin{aligned} f_2(1270), D_o\text{-wave} : \sigma_{100 \text{ GeV/c}}/\sigma_{38 \text{ GeV/c}} &= 0,134 \pm 0,016; \\ f_2(1270), D_{\pm}\text{-wave} : \sigma_{100 \text{ GeV/c}}/\sigma_{38 \text{ GeV/c}} &= 0,054 \pm 0,007; \\ f_4(2050), G_o\text{-wave} : \sigma_{100 \text{ GeV/c}}/\sigma_{38 \text{ GeV/c}} &= 0,106 \pm 0,016; \\ f_4(2050), G_{\pm}\text{-wave} : \sigma_{100 \text{ GeV/c}}/\sigma_{38 \text{ GeV/c}} &= 0,046 \pm 0,007. \end{aligned} \quad (30)$$

It is seen from (30) that energy dependence of the production cross sections is essentially the same both for the  $f_2(1270)$  and  $f_4(2050)$ .

## Conclusions

A partial wave analysis of the  $\pi^o \pi^o$ -system produced in the  $\pi^- p \rightarrow \pi^o \pi^o n$  reaction at 100 GeV/c has been carried out taking into account  $S$ ,  $D$ ,  $G$  and  $J$  waves. The unique physical solution is found in the whole mass range under study from 0.8 GeV to 3 GeV, its behaviour agrees well with the behaviour of the solutions obtained earlier at 38 GeV/c. The  $S$ -wave for the physical solution has rather a complicated structure with three dips at 1, 1.5 and 2 GeV. These dips appear due to destructive interference of the  $f_o(980)$ ,  $f_o(1300)$ ,  $f_o(1500)$  and  $f_o(2010)$  with the wide bump extending from the threshold up to  $\sim 2.5$  GeV. Parameters and production cross sections of these resonances have been measured.



Mesons with high spins,  $f_2(1270)$ ,  $f_4(2050)$  and  $f_6(2510)$ , are produced in the  $\pi^- p \rightarrow \pi^0 \pi^0 n$  reaction via dominating one pion exchange with a small absorption (about 3% for the  $f_2(1270)$  and  $f_4(2050)$  and less when 10% for the  $f_6(2510)$ ), which is in agreement with the Ochs-Wagner model prediction. Parameters and production cross sections of the  $f_2(1270)$ ,  $f_4(2050)$  and  $f_6(2510)$  have been determined.

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