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S.M.Klishevich

MASSIVE FIELDS OF ARBITRARY INTEGER SPIN IN HOMOGENEOUS ELECTROMAGNETIC FIELD

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Abstract

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We study the interaction of gauge fields of arbitrary integer spins with the constant electromagnetic field. We reduce the problem of obtaining the gauge-invariant Lagrangian of integer spin fields in the external field to purely algebraic problem of finding a set of operators with certain features using the representation of the high-spin fields in the form of some vectors on a pseudo-Hilbert space. We consider such a construction up to the second order in the electromagnetic field strength and also present an explicit form of interaction Lagrangian for a massive particle with spin s in terms of symmetrical tensor fields in the linear approximation. The obtained result does not depend on dimensionality of space-time.

Аннотация

Клишевич С.М. Массивное поле произвольного целого спина в однородном электромагнитном поле: Препринт ИФВЭ 98-24. – Протвино, 1998. – 14 с., библиогр.: 26.

В данной работе мы изучаем взаимодействие массивных калибровочных полей произвольных целых спинов с постоянным электромагнитным полем. Основываясь на представлении полей высоких спинов в виде векторов некоторого псевдогильбертового пространства, мы сводим проблему получения калибровочно-инвариантного лагранжиана полей целых спинов во внешнем поле к чисто алгебраической задаче отыскания некоторого набора операторов с определенными свойствами. Мы рассматриваем такое построение вплоть до второго порядка по напряженности электромагнитного поля и приводим явный вид лагранжиана взаимодействия для массивной частицы со спином s в терминах симметричных тензорных полей в линейном приближении. Полученный результат не зависит от размерности пространства-времени.

1. Introduction

In spite of the long history [1,2,3] the problem of obtaining the consistent interaction of high-spin fields is far from its completion till now.

The problem of obtaining a consistent description of "minimal" interactions of high-spin fields with the Abelian vector field has a particular place as well as their gravitational interactions. In some sense these interactions are the test ones since they allow one to connect the fields of high spins with the observable world.

It has been realized [4] that for the massless fields of spins $s \geq \frac{3}{2}$ in an asymptotically flat space-time one could not build the consistent "minimal" interaction with an Abelian vector field. The same is valid for the gravity interaction of fields with spins $s \geq 2$ [5] as well. It is possible to argue the given statement as follows [4]: A free gauge-invariant Lagrangian for integer spin fields in the flat space has structure $\mathcal{L}_0 = \partial\Phi\partial\Phi$ with transformations $\delta\Phi = \partial\xi$. The introduction of the "minimal" interaction means a replacement of the usual derivative by the covariant one $\partial \to \mathcal{D}$. In this, the gauge invariance fails and a residual of type $[\mathcal{D}, \mathcal{D}]\,\mathcal{D}\Phi\xi = \mathcal{R}\mathcal{D}\Phi\xi$ appears, where $\mathcal{R}$ is the strength tensor of the electromagnetic field or the Riemann tensor. In the case of the electromagnetic interaction for the fields with spins $s \geq \frac{3}{2}$, one cannot cancel the residual by any changes of the Lagrangian and the transformations in the linear approximation. Therefore, in such a case this approximation is absent, but since linear approximation does not depend on the presence in the system of any other fields, this means that the whole theory of interaction does not exist either. The same is valid for the fermionic fields. In the case of the gravity interaction the residual for the field with spin $\frac{3}{2}$ is proportional to the gravity equations of motion: $\delta\mathcal{L}_0 \sim i(\bar{\psi}^\mu\gamma^\nu\eta)(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)$. One can compensate such a residual by modifying the Lagrangian and the transformations. As a result, the theory of the supergravity appears. For fields with spins $s > 2$ the residual contains terms proportional to Riemann tensor $R_{\mu\nu\,\alpha\beta}$. It is impossible to cancel such terms in an asymptotically flat space. Hence, the gravity interaction does not exist for any massless fields with spin $s > \frac{3}{2}$. So, even if the massless high-spin fields possess a nontrivial self-action, in any case they are a "thing in itself".

These difficulties can be overcome in several ways. In case of the gravity interaction, one can consider the fields in a constant curvature space. Then, the Lagrangian for gravity would have an additional term $\Delta\mathcal{L} \sim \sqrt{-g}\lambda$, where λ is the cosmological constant. Modification of the Lagrangian and the transformations leads to a mixing of terms with different numbers of derivatives. This allows one to compensate the residual with terms proportional to $R_{\mu\nu\alpha\beta}$. The complete theory will be represented as series in inverse value of the cosmological constant [6,7]. This means non-analyticity of the theory with respect to λ at zero, i.e. the impossibility of a smooth transition to the flat space. Such a theory was considered in Refs. [6,7,8].

It is also possible to try to avoid these difficulties if one considers massive high-spin fields [13,9].

In the literature the electromagnetic [10] and gravitational [11,12] interactions of arbitrary spin fields were considered at the lowest order. Under consideration of the interactions, the authors start from the free theory of the massive fields in the classical form [13]. The "minimal" introduction of the interaction leads to contradictions, therefore, it is necessary to consider non-minimal terms in the interaction Lagrangian. Since the massive Lagrangian for spin s fields [13] is not gauge invariant, in such an approach there are not any restrictions on the form of a non-minimal interaction and to build a consistent theory it is necessary to introduce additional restrictions. So, for instance, when studying the electromagnetic interaction [10], the authors used the requirement that tree level amplitudes of any theory of massive particles interacting with the photon had to possess a smooth limit $M \to 0$ at the fixed electric charge. Under such a requirement, the unitary limit is achieved at energy $E \gg M/e$ only. This would mean unitarity of the theory at the tree level. Such restriction provides the gyromagnetic ratio $g = 2$ for the massive particles of any spins rather than $g = \frac{1}{s}$ as it would be obtained if one considered the "minimal" interaction [13] only. When investigating the gravitational interactions[11,12], the authors required for the theory to have tree-level unitarity up to Planck scale.

It seems to us it is more suitable to use the gauge-invariant approach when one analyzes an interaction of the massive fields [14,4,9], [15] or [16]. Under such approach the interaction is considered as a deformation of an initial gauge algebra and Lagrangian[1] [17,9]. Although, generally speaking, the gauge invariance does not guarantee consistency of the massive theories but, in any case, it allows one to narrow the search and conserves the right number of the physical degree of freedom. Besides, such an approach is quite convenient and practical.

At present only the superstring theory claims to have the consistent description of the interaction of the high-spin fields. But the interacting strings describe an infinite set of the fields and the question about an interaction of the finite number of the fields is still open. In the case of the constant Abelian field one can obtain a gauge-invariant Lagrangian that describes the interaction consistently for the fields of each string level. So, in Refs. [16,18] the first and the second massive levels of the open boson string were explored. These levels contain the massive fields with spins 2 and 3. However, as was shown in [18],

[1] Of course, one must consider only a non-trivial deformation of the free algebra and the Lagrangian which cannot be completely gauged away or removed by a redefinition of the fields.

the presence of the constant electromagnetic field leads to the mixing of states on each string level. Therefore, it is impossible to obtain the electromagnetic interaction for a single field of spin s in such an approach.

In this paper we consider the interaction of an arbitrary massive field of spin s with the homogeneous electromagnetic field up to the second order in the strength.

We represent a free state with the arbitrary integer spin s as some state $|\Phi^s\rangle$ of a Pseudo-Hilbert space[2] The tensor fields corresponding to the particle of spin s are coefficient functions of the state $|\Phi^s\rangle$. In the considered Fock space we introduce a set of some operators by means of which we define the gauge transformations and the necessary constraints for the state $|\Phi^s\rangle$. The gauge-invariant Lagrangian has the form of expectation value of a Hermitian operator taken over state $|\Phi^s\rangle$ which consists of the operators. Using such a construction, in section 2 we obtain the gauge-invariant Lagrangian describing the particles with an arbitrary integer spin both in the massless and massive case in terms of the coefficient functions.

In the considered approach the gauge invariance is a consequence of commutation relations of the introduced operators. The introduction of the interaction by replacing the usual derivatives with the covariant ones leads to a change of algebraic features of the operators and, as consequence, to the loss of the gauge invariance. The problem of recovering the invariance is reduced to an algebraic problem of finding such modified operators which depend on the electromagnetic field strength which satisfy the same commutation relations as initial operators in the absence of the external field. It is possible to argue the hope for the existence of such operators from the consideration that the gauge transformation algebra in the free case and in the presence of an external field is the same (trivial)[3] However, we should note that in the massless case one cannot realize such a construction. For the massive theory (section 3) we construct the set of the operators having the algebraic features of the free ones up to the second order in strength. Besides we give the explicit form of the interaction Lagrangian in terms of the tensor field for a special case of the constructed linear approximation.

2. Free Field with Spin s

Massless fields. Let us consider the Fock space generated by creation and annihilation operators $\bar{a}_\mu$ and a_μ which are vectors on the D-dimensional Minkowski space $\mathcal{M}_D$ and which satisfy the following algebra

$$[a_\mu, \bar{a}_\nu] = g_{\mu\nu}, \quad a_\mu^\dagger = \bar{a}_\mu, \tag{1}$$

where $g_{\mu\nu}$ is the metric tensor with signature $\|g_{\mu\nu}\| = \mathrm{diag}(-1, 1, 1, ..., 1)$. Since the metric is indefinite, the Fock space which realizes the representation of the Heisenberg algebra (1) is Pseudo-Hilbert.

[2]The representation of the free fields of arbitrary integer spin in such a form was considered in Refs. [15,22]

[3]But transformations for the massive fields are not trivial.

Let us consider the state of the introduceded space of the following type:

$$|\Phi^s\rangle = \frac{1}{\sqrt{s}}\Phi_{\mu_1\ldots\mu_s}(x)\prod_{i=1}^{s}\bar{a}_{\mu_i}|0\rangle. \tag{2}$$

Coefficient function $\Phi_{\mu_1\ldots\mu_s}(x)$ is a symmetrical tensor of rank s on space $\mathcal{M}_D$. One can describe the state with spin[4] s by means of this tensor if one imposes the following condition on it:

$$\Phi_{\mu\mu\nu\nu\mu_4\ldots\mu_s} = 0. \tag{3}$$

In terms of such fields Lagrangian [19,20]

$$\begin{aligned}\mathcal{L}_s &= \frac{1}{2}(\partial_\mu\Phi^s)\cdot(\partial_\mu\Phi^s) - \frac{s}{2}(\partial\cdot\Phi^s)\cdot(\partial\cdot\Phi^s) - \frac{s(s-1)}{4}(\partial_\mu\Phi'^s)\cdot(\partial_\mu\Phi'^s)\\ &- \frac{s(s-1)}{2}(\partial\cdot\partial\cdot\Phi^s)\cdot\Phi'^s - \frac{1}{8}s(s-1)(s-2)(\partial\cdot\Phi'^s)\cdot(\partial\cdot\Phi'^s)\end{aligned} \tag{4}$$

is invariant under transformations

$$\delta\Phi_{\mu_1\ldots\mu_s} = \partial_{(\mu_1}\Lambda_{\mu_2\ldots\mu_{s-1})}, \tag{5}$$

$$\Lambda_{\mu\mu\mu_3\ldots\mu_{s-1}} = 0. \tag{6}$$

The following notation $\Phi' = \Phi_{\mu\mu\ldots}$ is used here while the point means the contraction of all indexes.

Let us introduce the following operators in our pseudo-Hilbert space

$$L_1 = p\cdot a, \quad L_{-1} = L_1^\dagger, \quad L_2 = \frac{1}{2}a\cdot a, \quad L_{-2} = L_2^\dagger, \quad L_0 = p^2. \tag{7}$$

Here $p_\mu = i\partial_\mu$ is the momentum operator that acts on the space of the coefficient function.

Operators of such a type appear as constraints of a two-particle system when one quantizes[5] of the one [21] that. Operators (7) satisfy the following commutation relations:

$$\begin{array}{rclrcl} [L_1,L_{-2}] &=& L_{-1}, & [L_1,L_2] &=& 0,\\ [L_2,L_{-2}] &=& N+\frac{D}{2}, & [L_0,L_n] &=& 0,\\ [L_1,L_{-1}] &=& L_0, & [N,L_n] &=& -nL_n, \quad n=0,\pm1,\pm2. \end{array} \tag{8}$$

Here $N = \bar{a}\cdot a$ is a level operator that defines the spin of the states. So, for instance, for state (2)

$$N|\Phi^s\rangle = s|\Phi^s\rangle.$$

In terms of operators (8) condition (3) can be written as

$$(L_2)^2|\Phi^s\rangle = 0, \tag{9}$$

[4] We consider representations of type $D(\frac{s}{2},\frac{s}{2})$ only.

[5] It is also possible to regard operators (7) as some truncation of the Virasoro algebra.

while gauge transformations (5) take the form

$$\delta|\Phi^s\rangle = L_{-1}|\Lambda^{s-1}\rangle. \tag{10}$$

Here, the gauge state

$$|\Lambda^{s-1}\rangle = \Lambda_{\mu_1\ldots\mu_{s-1}} \prod_{i=1}^{s-1} \bar{a}_{\mu_i}|0\rangle$$

satisfies the condition

$$L_2|\Lambda\rangle = 0. \tag{11}$$

For the coefficient functions this condition is equivalent to (6).

Lagrangian (4) can be written as an expectation value of some Hermitian operator taken over state (2)

$$\mathcal{L}_s = \langle\Phi^s|\mathcal{L}(L)|\Phi^s\rangle, \quad \langle\Phi^s| = |\Phi^s\rangle^\dagger, \tag{12}$$

where

$$\begin{aligned} \mathcal{L}(L) &= L_0 - L_{-1}L_1 - 2L_{-2}L_0L_2 - L_{-2}L_{-1}L_1L_2 \\ &\quad + \{L_{-2}L_1L_1 + \text{h.c.}\}\,. \end{aligned} \tag{13}$$

Lagrangian (12) is invariant under transformations (10) as consequence of relation

$$[\mathcal{L}(L), L_{-1}] = (...)L_{-2}.$$

Massive fields Let us consider the massive states of arbitrary spin s in similar manner. For this we have to extend our Fock space by the introduction of the scalar creation and annihilation operators $\bar{b}$ and b that satisfy the usual commutation relations

$$\left[b, \bar{b}\right] = 1, \quad b^\dagger = \bar{b}. \tag{14}$$

Operators (7) are modified as follows:

$$L_1 = p\cdot a + mb, \quad L_2 = \frac{1}{2}\left(a\cdot a + b^2\right), \quad L_0 = p^2 + m^2. \tag{15}$$

Here m is an arbitrary parameter having the dimensionality of mass. When an interaction is absent, one can consider such a transition as the dimensional reduction $\mathcal{M}_{D+1} \to \mathcal{M}_D \otimes S^1$ with the radius of sphere $R \sim 1/m$ (refer also to [15,22]).

We shall describe the massive field with spin s as the following vector in the extended Fock space:

$$|\Phi^s\rangle = \sum_{n=0}^{s} \frac{1}{\sqrt{n!(s-n)!}} \Phi_{\mu_1\ldots\mu_n}(x)\bar{b}^{s-n} \prod_{i=1}^{n} \bar{a}_{\mu_i}|0\rangle. \tag{16}$$

As in the massless field case, this state satisfies the same condition (9) but in terms of operators (15). The algebra of operators (8) changes weakly, it is only the commutator that modified

$$[L_2, L_{-2}] = N + \frac{D+1}{2}. \tag{17}$$

Here, as in the massless case, operator $N = \bar{a}\cdot a + \bar{b}b$ defines a spin of the massive states. The Lagrangian describing the massive field of spin s has the form of (13) also, where the expectation value is taken over state (16). Such a Lagrangian is invariant under transformations (10) with the gauge Fock vector

$$|\Lambda^{s-1}\rangle = \sum_{n=0}^{s-1} \frac{1}{\sqrt{(n+1)!(s-n-1)!}} \Lambda_{\mu_1\ldots\mu_n} \bar{b}^{s-n-1} \prod_{i=1}^{n} \bar{a}_{\mu_i}|0\rangle,$$

that satisfies condition (6).

Having calculated expectation (13) we obtain the explicit expression for the Lagrangian describing the massive state with arbitrary spin s in terms of the coefficient functions

$$\begin{aligned}
\mathcal{L}_0 \;=\; & \sum_{n=0}^{s} \bar{\Phi}^n \cdot p^2 \Phi^n \left(1 - C^2_{s-n}\right) - \sum_{n=2}^{s} \bar{\Phi}'^n \cdot p^2 \Phi'^n C^2_n \\
& - \frac{1}{2}\sum_{n=1}^{s} \left(\bar{\Phi}^n \cdot p\right)\cdot\left(p\cdot\Phi^n\right) n \left(2 + C^2_{s-n}\right) - \frac{3}{2}\sum_{n=3}^{s}\left(\bar{\Phi}'^n\cdot p\right)\cdot\left(p\cdot\Phi'^n\right) C^3_n \\
& - \Bigg\{ \frac{1}{2}\sum_{n=3}^{s}\left(\bar{\Phi}'^n\cdot p\right)\cdot\left(p\cdot\Phi^{n-2}\right)(n-2)\sqrt{C^2_n C^2_{s-n+2}} \\
& + \sum_{n=2}^{s} \bar{\Phi}'^n \cdot p^2 \Phi^{n-2}\sqrt{C^2_n C^2_{s-n+2}} - \sum_{n=2}^{s}\bar{\Phi}'^n\cdot\left(p\cdot p\cdot\Phi^n\right) C^2_n \\
& - \sum_{n=2}^{s}\bar{\Phi}^{n-2}\cdot\left(p\cdot p\cdot\Phi^n\right)\sqrt{C^2_n C^2_{s-n+2}} + h.c \Bigg\} \\
& - m\Bigg\{\frac{1}{2}\sum_{n=1}^{s}\left(\bar{\Phi}^n\cdot p\right)\cdot\Phi^{n-1}\left(2 - C^1_{s-n} + C^2_{s-n}\right)\sqrt{nC^1_{s-n+1}} \\
& - \frac{1}{4}\sum_{n=2}^{s}\bar{\Phi}'^n\cdot\left(p\cdot\Phi^{n-1}\right)(n-1)(4 - C^1_{s-n})\sqrt{nC^1_{s-n+1}} \\
& + \frac{3}{2}\sum_{n=3}^{s}\left(\bar{\Phi}'^n\cdot p\right)\cdot\Phi^{n-3}\sqrt{C^3_n C^3_{s-n+3}} \\
& + \frac{1}{2}\sum_{n=3}^{s}\left(\bar{\Phi}'^n\cdot p\right)\cdot\Phi'^{n-1} C^2_n\sqrt{nC^1_{s-n+1}} + h.c.\Bigg\} \\
& + \frac{m^2}{2}\Bigg\{\sum_{n=0}^{s}\bar{\Phi}^n\cdot\Phi^n\left(2 - 2C^1_{s-n} + 2C^2_{s-n} - 3C^3_{s-n}\right) \\
& - 2\sum_{n=2}^{s}\left\{\bar{\Phi}'^n\cdot\Phi^{n-2}C^1_{s-n}\sqrt{C^2_n C^2_{s-m+2}} + h.c.\right\} \\
& - 2\sum_{n=2}^{s}\bar{\Phi}'^n\cdot\Phi'^n C^2_n(2 + C^1_{s-n})\Bigg\}.
\end{aligned}$$

Here Φ^n denotes a symmetrical tensor field of rank n and the notation $C^m_n = \frac{n(n-1)\ldots(n-m+1)}{m!}$. Condition (9) for the fields has the form in this case

$$\sqrt{C^n_s C^2_n} C^2_{n-2}\Phi''^n + 2\sqrt{C^{n-2}_s} C^2_{n-2} C^2_{s-n+2}\Phi'^{n-2} + \sqrt{C^{n-4}_s} C^2_{s-n+4} C^2_{s-n+2}\Phi^{n-4} = 0, \quad (18)$$

where $n = 4, ..., s$. Correspondingly, the gauge transformations have form

$$\delta_0 \Phi^n = \left\{ p \Lambda^{n-1} \right\}_{s.} + m \sqrt{\frac{s-n}{n+1}} \Lambda^n. \tag{19}$$

Here $\{...\}_{s.}$ denotes the symmetrization over all indexes. Condition (11) can be written as

$$C_n^2 \Lambda'^n + C_{s-n+1}^2 \Lambda^{n-2} = 0, \qquad n = 2, ..., s-1. \tag{20}$$

Obviously, the dimensioned parameter m has the sense of mass of the state. For convenience, hereinafter we assume $m = 1$.

It is possible to derive the same result from (4) if one uses the dimensional reduction [23,24] of the massless theory.

3. Electromagnetic Interaction of Massive Spin s Field

In this section we shall consider the interaction of the gauge massive field of arbitrary integer spin s with the constant electromagnetic field.

We introduce the interaction by means of the "minimal coupling", i.e. we replace the usual momentum operators with the $U(1)$-covariant ones $p_\mu \to \mathcal{P}_\mu$. The commutator of the covariant momenta defines the electromagnetic field strength

$$[\mathcal{P}_\mu, \mathcal{P}_\nu] = F_{\mu\nu}. \tag{21}$$

For convenience we included the imaginary unit and the coupling constant into the defenition of the strength tensor.

In the definition of operators (15), we replace the usual momenta by the covariant ones as well. As a result the operators cease to obey algebra (8). Therefore, Lagrangian (13) loses the invariance under transformations (10).

To restore algebra (8), (17) let us represent operators (15) as some normal ordered functions of the creation and annihilation operators as well as of the electromagnetic field strength i.e.

$$L_i = L_i \left(\bar{a}_\mu, \bar{b}, a_\mu, b, F_{\mu\nu} \right).$$

As a matter of fact, this means that these operators will belong to the extended universal enveloping Heisenberg algebra (1), (14). The particular form of operators L_i will be defined from the condition recovering commutation relations (8) and (17) by these operators. It is worth noting that it is enough to define the form of operators L_1 and L_2, since one can take the following expressions:

$$L_0 \stackrel{\text{def}}{=} [L_1, L_{-1}], \quad N \stackrel{\text{def}}{=} [L_2, L_{-2}] - \frac{D+1}{2}. \tag{22}$$

as definitions of operators L_0 and N.

Since we have turned to the extended universal enveloping Heisenberg algebra, the arbitrariness in the definition of operators[6] a and b appears. Besides, in the right-hand side of (1), (14), we can admit the presence of the arbitrary operator functions depending on a, b, and $F_{\mu\nu}$. In this, such a modification of the operators must not lead to breaking of the Jacobi identity and in limit $F_{\mu\nu} \to 0$ they must restore the initial algebra. However, it is easy to make sure that using the arbitrariness in the definition of the creation and annihilation operators, we can restore algebra (1), (14) at least up to the second order in the strength.

We shall search for operators L_1 and L_2 in the form of expansion in the strength tensor which is equivalent to that in the coupling constant.

Let us consider the linear approximation.

Operator L_1 should be no higher than linear on operator $\mathcal{P}_\mu$, since the presence of the higher number of that changes the type of the gauge transformations and the number of physical degrees of freedom. Therefore, in this approximation we shall search for them in the form

$$L_1^{(1)} = (\bar{a}Fa)\, h_0(\bar{b}, b)b + (\mathcal{P}Fa)\, h_1(\bar{b}, b) + (\bar{a}F\mathcal{P})\, h_2(\bar{b}, b)b^2. \tag{23}$$

At the same time operator L_2 cannot depend on the momentum operators at all, since condition (9) defines purely algebraic constraints on the coefficient functions. Therefore, in this order we choose operator L_2 in the following form:

$$L_2^{(1)} = (\bar{a}Fa)\, h_3(\bar{b}, b)b^2, \tag{24}$$

Here $h_i(\bar{b}, b)$ are the normal ordered operator functions of the type

$$h_i(\bar{b}, b) = \sum_{n=0}^{\infty} H_n^i \bar{b}^n b^n,$$

where H_n^i are the arbitrary real coefficients. We consider only the real coefficients since the operators with purely imaginary coefficients do not give any contribution to the "minimal" interaction.

Let us define a particular form of functions h_i from the condition of recovering commutation relations (8) by operators (23) and (24). This algebra is entirely defined by (22) and by the following commutators

$$[L_2, L_1] = 0, \tag{25}$$
$$[L_2, L_{-1}] = L_1, \tag{26}$$
$$[L_0, L_1] = 0. \tag{27}$$

Having calculated (25) and passed to normal symbols of the creation and annihilation operators, we obtain a system of differential equations for the normal symbols of operator

[6] Such an arbitrariness has been also presented earlier as an internal automorphism of the Heisenberg-Weil algebra defining the Fock space [25]. But exactly the transformations depending on $F_{\mu\nu}$ are important for us.

functions h_i. For the normal symbols of the operator functions we shall use the same notations. This does not lead to the mess since we consider the operator functions as the functions of two variables while their normal symbols as the functions of one variable. From (25) we have the equations

$$\begin{aligned}
&\frac{1}{2}h_2''(x) + h_2'(x) = 0, \\
&\frac{1}{2}h_0''(x) + h_0'(x) - h_3'(x) = 0, \\
&\frac{1}{2}h_1''(x) + h_1'(x) - h_2(x) - h_3(x) = 0.
\end{aligned} \tag{28}$$

Here the prime denotes the derivative with respect to x, while $x = \bar{\beta}\beta$, where $\bar{\beta}$ and β are the normal symbols of operators $\bar{b}$ and b, correspondingly.

Similarly, from (26) we derive one more system of equations:

$$\begin{aligned}
&x^2\left(\frac{1}{2}h_2''(x) + h_2'(x)\right) + 2x\left(h_2'(x) + h_2(x)\right) + h_2(x) - 2h_1(x) = 0, \\
&\frac{1}{2}h_1''(x) + h_1'(x) - h_2(x) + h_3(x) = 0, \\
&x\left(\frac{1}{2}h_0''(x) + h_0'(x) + h_3'(x)\right) + h_0'(x) + 2h_3(x) = 0.
\end{aligned} \tag{29}$$

Having solved the systems of equations (28) and (29), we obtain the particular form of functions h_1:

$$\begin{aligned}
h_0(x) &= \text{const}, \\
h_1(x) &= d_1\left(\frac{1}{2} - x\right)e^{-2x} + d_2\left(\frac{1}{2} + x\right), \\
h_2(x) &= d_1 e^{-2x} + d_2, \\
h_3(x) &= 0.
\end{aligned} \tag{30}$$

Here d_1 and d_2 are the arbitrary real parameters. Using (30) we obtain from (27)

$$h_0(x) = 1 - d_2.$$

The transition to the operator functions is realized in the conventional manner:

$$h_i(\bar{b}, b) = \exp\left(\bar{b}\frac{\partial}{\partial\bar{\beta}}\right)\exp\left(b\frac{\partial}{\partial\beta}\right)h_i(\bar{\beta}\beta)\Big|_{\substack{\bar{\beta}\to 0\\ \beta\to 0}} = \,:h_i(\bar{b}b):\,.$$

Normal symobls of operators L_n has the following form in this approximation:

$$\begin{aligned}
L_1^{(1)} &= (1 - d_2)(\bar{\alpha}F\alpha)\beta + \left(e^{-2\bar{\beta}\beta}d_1\left(\frac{1}{2} - \bar{\beta}\beta\right) + d_2\left(\frac{1}{2} + \bar{\beta}\beta\right)\right)(\mathcal{P}F\alpha) \\
&\quad + \left(e^{-2\bar{\beta}\beta}d_1 + d_2\right)(\bar{\alpha}F\mathcal{P})\beta^2, \\
L_0^{(1)} &= (1 - 2d_2)(\bar{\alpha}F\alpha) + \left\{(1 + 2d_2)(\mathcal{P}F\alpha)\bar{\beta} + h.c.\right\}, \\
L_2^{(1)} &= 0,
\end{aligned}$$

where $\bar{\alpha}_\mu$ and α_μ are the normal symbols of operators $\bar{a}_\mu$ and a_μ. Since operator L_2 has not changed in the linear approximation, hence from (22) it follows that operator N and constraints (18), (20) have not changed either.

Thus, we have obtained the general form of operators L_n which obey algebra (8) in the linear approximation. This means that in this order Lagrangian (13) is invariant under transformations (10).

From (31) it is clear that there is the two-parametric arbitrariness in the linear approximation. But one of the arbitrary parameters d_1 and d_2 is determined in the second approximation In this, there are two solutions: when d_1 vanishes and d_2 is arbitrary, and vice versa, when d_1 is a free parameter and d_2 is equal to $\frac{1}{2}$. One can verify that in the second case the gyromagnetic ratio vanishes. Below we will consider the first solution only. Having set $d_1 = 0$ we calculate expectation value (13) and obtain the linear in strength expression of the Lagrangian describing e/m interaction of the massive field with spin s in terms of the coefficient functions:

$$
\begin{aligned}
\mathcal{L}^{(1)} = & \sum_{n=1}^{s} \bar{\Phi}^n \cdot F \cdot \Phi^n n \left(1 - 2d_2 + 2\left(d_2 - 1\right) C^1_{s-n} + \left(3 - 2d_2\right) C^2_{s-n}\right. \\
& \left. + 3\left(d_2 - 1\right) C^3_{s-n}\right) + 3 \sum_{n=3}^{s} \bar{\Phi}'^n \cdot F \cdot \Phi'^n C^3_n \left(2d_2 - 1 + \left(d_2 - 1\right) C^1_{s-n}\right) \\
& + \left\{ \sum_{n=3}^{s} \bar{\Phi}'^n \cdot F \cdot \Phi^{n-2} \sqrt{C^2_n C^2_{s-n+2}} (n-2) \left(1 + \left(d_2 - 1\right) C^1_{s-n}\right)\right. \\
& + \frac{1}{2} \sum_{n=2}^{s} \left(\mathcal{P} \cdot \bar{\Phi}^n\right) \cdot F \cdot \Phi^{n-1} \sqrt{n C^1_{s-n+1}} (n-1)(d_2 - 1)\left(2 - 2C^1_{s-n} + C^2_{s-n}\right) \\
& + \sum_{n=1}^{s} \left(\bar{\Phi}^n \cdot F \cdot \mathcal{P}\right) \cdot \Phi^{n-1} \sqrt{n C^1_{s-n+1}} \left(2 + 3d_2 - 3d_2 C^1_{s-n} - \left(2 - \frac{7}{2} d_2\right) C^2_{s-n}\right. \\
& \left. - 6 d_2 C^3_{s-n}\right) + \frac{3}{2} \sum_{n=4}^{s} \left(\mathcal{P} \cdot \bar{\Phi}'^n\right) \cdot F \cdot \Phi'^{n-1} \sqrt{n C^1_{s-n+1}} C^3_{n-1} (d_2 - 1) \\
& + \frac{1}{2} \sum_{n=2}^{s} \bar{\Phi}'^n \cdot \left(\mathcal{P} \cdot F \cdot \Phi^{n-1}\right) \sqrt{n C^1_{s-n+1}} (n-1) \left(d_2 - \left(1 + \frac{d_2}{4}\right) C^1_{s-n} - 8 C^2_{s-n}\right) \\
& - \frac{1}{2} \sum_{n=3}^{s} \bar{\Phi}'^n \cdot F \cdot \left(\mathcal{P} \cdot \Phi^{n-1}\right) \sqrt{n C^1_{s-n+1}} C^2_{n-1} (d_2 - 1) \left(4 - C^1_{s-n}\right) \\
& - \sum_{n=3}^{s} \left(\bar{\Phi}'^n \cdot F \cdot \mathcal{P}\right) \cdot \Phi'^{n-1} \sqrt{n C^1_{s-n+1}} C^2_{n-1} \left(1 + \frac{9}{4} d_2 - d_2 C^1_{s-n}\right) \\
& - 3 \sum_{n=3}^{s} \left(\bar{\Phi}'^n \cdot F \cdot \mathcal{P}\right) \cdot \Phi^{n-3} \sqrt{C^3_n C^3_{s-n+3}} \left(1 + \frac{d_2}{4} C^1_{s-n}\right) \\
& + \frac{3}{2} \sum_{n=4}^{s} \bar{\Phi}^{n-3} \cdot F \cdot \left(\mathcal{P} \cdot \Phi'^n\right) \sqrt{C^3_n C^3_{s-n+3}} (n-3)(d_2 - 1) \\
& - \frac{d_2}{4} \sum_{n=1}^{s} \left(\bar{\Phi}^n \cdot F \cdot \mathcal{P}\right) \cdot \left(\mathcal{P} \cdot \Phi^n\right) n \left(2 + 4C^1_{s-n} - 7C^2_{s-n} + 6C^3_{s-n}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+ d_2 \sum_{n=2}^{s} \bar{\Phi}'^n \cdot (\mathcal{P} \cdot F \cdot (\mathcal{P} \cdot \Phi^n))\, C_n^2 \left(1 + 2C_{s-n}^1 - C_{s-n}^2\right) \\
&- \frac{3}{4} d_2 \sum_{n=3}^{s} \left(\bar{\Phi}'^n \cdot F \cdot \mathcal{P}\right) \cdot (\mathcal{P} \cdot \Phi'^n)\, C_n^3 \left(1 + 2C_{s-n}^1\right) \\
&- d_2 \sum_{n=2}^{s} \left(\left(\bar{\Phi}^n \cdot \mathcal{P}\right) \cdot F \cdot \mathcal{P}\right) \cdot \Phi^{n-2} \sqrt{C_n^2 C_{s-n+2}^2} \left(1 - 2C_{s-n}^1 + C_{s-n}^2\right) \\
&- \frac{d_2}{2} \sum_{n=4}^{s} \left(\left(\bar{\Phi}'^n \cdot \mathcal{P}\right) \cdot F \cdot \mathcal{P}\right) \cdot \Phi'^{n-2} C_{n-2}^2 \sqrt{C_n^2 C_{s-n+2}^2} \\
&+ \frac{d_2}{4} \sum_{n=3}^{s} \left(\bar{\Phi}'^n \cdot F \cdot \mathcal{P}\right) \cdot \left(\mathcal{P} \cdot \Phi^{n-2}\right) (n-2) \sqrt{C_n^2 C_{s-n+2}^2} \left(7 - 2C_{s-n}^1\right) \\
&- \frac{d_2}{4} \sum_{n=3}^{s} \left(\bar{\Phi}'^n \cdot \mathcal{P}\right) \cdot \left(\mathcal{P} \cdot F \cdot \Phi^{n-2}\right) (n-2) \sqrt{C_n^2 C_{s-n+2}^2} \left(1 + 2C_{s-n}^1\right) \\
&-3 d_2 \sum_{n=4}^{s} \left(\left(\bar{\Phi}'^n \cdot \mathcal{P}\right) \cdot F \cdot \mathcal{P}\right) \cdot \Phi^{n-4} \sqrt{C_n^4 C_{s-n+4}^4} + h.c. \Bigg\} .
\end{aligned}
$$

Correspondingly, in this approximation the gauge transformations have the form:

$$
\begin{aligned}
\delta_1 \Phi^n &= \frac{d_2}{2} (1 + 2(s-n)) \left\{ (F \cdot \mathcal{P})\, \Lambda^{n-1} \right\}_{s.} \\
&\quad + 2 d_2 (n+1)\, C_{s-n}^2 \sqrt{\frac{C_{s-n}^2}{C_{n+2}^2}} \left(\mathcal{P} \cdot F \cdot \Lambda^{n+1}\right) \\
&\quad + (1 - d_2)\, n\, (s-n-1) \sqrt{\frac{s-n}{n+1}} \left\{ F \cdot \Lambda^n \right\}_{s.} .
\end{aligned}
$$

We should note that the construction obtained is free from pathologies. Indeed, the gauge invariance ensures the right number of physical degree of freedom. In this, the model is causal in the linear approximation since by virtue of the antisymmetry and homogeneity of $F_{\mu\nu}$ the characteristic determinant[7] for equations of motion of any massive state has the form $D(n) = (n^2)^p + \mathcal{O}(F^2)$, where n_μ is a normal vector to the characteristic surface and the integer constant p depends on the spin of massive state. The equations of motion will be causal (hyperbolic) if the solutions n^0 to $D(n) = 0$ are real for any $\vec{n}$. In our case condition $D(n) = 0$ corresponds to the ordinary light cone in this order.

Let us consider the quadratic approximation in the strength. As in preceding order taking the general ansatz for operators L_1, L_2 and having required the recovering of relations (8), we obtain a system of inhomogeneous differential equations for the second order. As was mentioned this system has the two solutions for parameters d_1 and d_2 and we choose the solution when d_1 vanishes and d_2 is arbitrary. According to our choice operators L_1 and L_2 have the following form in this approximation:

$$
L_1^{(2)} = (\bar{\alpha} F \mathcal{P})\, (\bar{\alpha} F \alpha) \beta^2 \left(\frac{1}{4} - d_2^2 - d_2\right) - (\bar{\alpha} F F \bar{\alpha}) \beta^3 \left(\frac{1}{3} e^{-2\bar{\beta}\beta} c_2 + \frac{1}{2} d_2 + \frac{1}{8}\right)
$$

[7] The determinant is entirely determinated by the coefficients of the highest derivatives in equations of motion after gauge fixing and resolving all the constaraints [26].

$$
\begin{aligned}
&+ (\bar\alpha F F\mathcal{P})\,\beta^2\left(\frac{1}{3}e^{-2\bar\beta\beta}c_2+2c_1-\frac{1}{2}d_2^2-\frac{1}{2}d_2+\frac{1}{8}\right)\\
&+ (\bar\alpha F\alpha)^2\,\beta\left(d_2^2+\frac{1}{2}d_2-\frac{5}{8}\right)+F^2\beta\left(c_3-\frac{1}{8}\,(1+4d_2)\,\bar\beta\beta\right)\\
&+ (\bar\alpha F F\alpha)\,\beta\left(\left(d_2+\frac{1}{4}\right)\bar\beta\beta+d_2^2+\frac{3}{2}d_2-\frac{3}{8}\right)\\
&- (\alpha F F\alpha)\,\bar\beta\left(\frac{1}{3}c_2e^{-2\bar\beta\beta}\left(1-\bar\beta\beta\right)+\frac{1}{8}\,(1+4d_2)\left(1+\bar\beta\beta\right)+2c_1\right)\\
&+ (\bar\alpha F\alpha)\,(\mathcal{P}F\alpha)\left(\bar\beta\beta+\frac{1}{2}\right)\left(\frac{1}{4}-d_2\,(d_2+1)\right)\\
&- (\mathcal{P}F F\alpha)\left(c_4+\frac{1}{8}\left(4d_2^2+4d_2-1\right)\bar\beta\beta\right),
\end{aligned}
$$

$$
\begin{aligned}
L_2^{(2)} \;=\;& - (\alpha F F\alpha)\left(\frac{1}{6}c_2e^{-2\bar\beta\beta}\left(1-2\bar\beta\beta\right)+c_1\left(1+2\bar\beta\beta\right)\right)\\
&+ \left(2\,(\bar\alpha F F\alpha)-F^2\right)\beta^2\left(c_1+\frac{1}{6}e^{-2\bar\beta\beta}c_2\right).
\end{aligned}
$$

Here c_1, c_2, c_3, and c_4 are the arbitrary real parameters.

In contrast to the preceding order, operator L_2 and, correspondingly, N and constraints (18), (20) depend on $F_{\mu\nu}$ in this approximation.

Operator L_0 is defined via the commutator of operators L_1, L_{-1} and the part which is proportional to F^2 has the form:

$$
\begin{aligned}
L_0^{(2)} \;=\;& (\mathcal{P}F F\mathcal{P})\left(\frac{1}{4}d_2^2-2c_4+\left(d_2^2-d_2+\frac{1}{4}\right)\bar\beta\beta\right)\\
&+ (\bar\alpha F\mathcal{P})\,(\mathcal{P}F\alpha)\left(d_2^2-d_2+\frac{1}{4}\right)\left(1+2\bar\beta\beta\right)\\
&+ (\bar\alpha F\alpha)^2\left(3d_2^2-d_2-\frac{1}{4}\right)+(\bar\alpha F F\alpha)\left((2d_2+1)\,\bar\beta\beta+3d_2^2+\frac{1}{4}\right)\\
&- F^2\left(\frac{1}{2}\bar\beta\beta-2c_3-\frac{1}{2}d_2\right)+\left\{(\bar\alpha F\mathcal{P})^2\,\beta^2\left(d_2^2-d_2+\frac{1}{4}\right)\right.\\
&- (\alpha F F\alpha)\,\bar\beta^2\left(d_2+\frac{1}{2}\right)-(\bar\alpha F\mathcal{P})\,(\bar\alpha F\alpha)\,\beta\left(4d_2^2-d_2+\frac{1}{2}\right)\\
&\left.- (\bar\alpha F F\mathcal{P})\,\beta\left(2d_2^2-\frac{1}{2}d_2+\frac{1}{4}\right)+h.c.\right\}.
\end{aligned}
$$

Thereby, we have restored algebra (8) up to the second order in the electromagnetic field strength. It means that we restored the gauge invariance of Lagrangian (13) in the same order as well. In this we have not used the dimensionality of space-time anywhere explicitly, i.e. the obtained expressions do not depend on it.

4. Conclusion

In this paper we have constructed the Lagrangian describing the interaction of the massive fields of arbitrary integer spins with the homogeneous electromagnetic field up to the second order in the strength. It is noteworthy that unlike the string approach [16] our consideration does not depend on the space-time dimensionality, and, moreover, we have described the interaction of the single field with spin s while in the string approach the presence of the constant electromagnetic field leads to the mixing of states with different spins [18] and one cannot consider any states separately.

We should note that the case of the constant Abelian field can be easily extended to a non-Abelian case. In this case we have to consider the external field as the covariantly constant one. In this, one should take the whole vacuum as $|0\rangle \otimes e^i$, where e^i are a basis vector of space of linear representation of a non-Abelian group. The covariant derivative has the form $\partial_\mu + A_\mu^a T^a$, where T^a are the operators realizing the representation. Such a modification does not change the algebraic features of our scheme and, therefore, all the derived results are valid in this case as well.

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С.М.Клишевич
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