V.E. Rochev

ON HIGGS MECHANISM
IN NON-PERTURBATIVE REGION

Abstract

Rochev V.E. On Higgs Mechanism in Non-Perturbative Region: IHEP Preprint 98–4. – Protvino, 1998. – p. 12, refs.: 10.

Generalization of the Higgs mechanism which takes into account the contributions of gauge field vacuum configuration into the formation of the physical vacuum is considered. For the Abelian Higgs model the triviality bound $m_H \leq 1.15 m_A$ is found.

Аннотация

Рочев В.Е. О механизме Хиггса в непертурбативной области: Препринт ИФВЭ 98–4. – Протвино, 1998. – 12 с., библиогр.: 10.

Рассмотрено обобщение механизма Хиггса, учитывающее вклад вакуумных конфигураций калибровочного поля в формирование физического вакуума теории. В абелевой модели Хиггса получено ограничение тривиальности $m_H \leq 1.15 m_A$.

1. Introduction

Higgs mechanism is one of the crucial points of Standard Model and simultaneously one of the most mysterious its properties. Considerable efforts on the experimental search for Higgs particles have not still lead to success. Due to the recent data [1] the lower bound of the Higgs particle mass is $M_H > 69.5$ GeV. Theoretical investigation of the scalar sector of the standard model is also far from completeness. In attempting to go beyond the framework of the quasiclassical approximation and the perturbation theory, one encounters a number of difficulties and the principal problem of them is the well-known triviality of quadric scalar self-interaction (see [2] and refs. therein): the renormalized coupling constant of ϕ_4^4-interaction tends to zero at the cutoff removing.

The triviality of ϕ_4^4-interaction leads to the fact, that the mass of Higgs particle is not a fully independent parameter but it is connected with other parameters of the model such as intermediate boson masses, t-quark mass, etc. In the frameworks of different approaches (see, for example, [2], [3] and refs. therein) this fact leads to different estimates of the Higgs boson mass, and the absence of experimental data does not favor over any approach.

The common feature for all investigations of the Higgs mechanism is the fundamental proposition about the existence of non-zero Higgs field vacuum expectation value

$$< 0 \mid \phi \mid 0 > \equiv v \neq 0, \tag{1}$$

which defines the Higgs boson and gauge field masses. The existence of such a non-zero expectation value means that the physical vacuum of the electroweak interaction is a non-trivial medium — the relativistic superconductor. The structure of this medium is not investigated in full measure, and there are no prior foundations to suppose that such a relativistic medium is completely similar to non-relativistic Ginzburg-Landau supercon-ductor. For example, the triviality of the ϕ_4^4-interaction is a purely relativistic quantum field phenomenon, which has no analogue in non-relativistic case.

Usually the role of the gauge field in the formation of the physical vacuum is fully ignored when one considers the Higgs mechanism. This is made on the base of a quite evident observation that the non-zero vacuum expectation values of the vector gauge

field destroy the Poincaré-invariance of the theory. On the other hand, however, it is necessary to take into account that the quantum field theory equations possess the variety of solutions, and the choice of a unique physical solution is, in fact, the definition of the above-mentioned relativistic medium — the physical vacuum. The example is the Higgs mechanism itself in its traditional formulation: besides the solution over non-symmetrical physical vacuum (1), the usual symmetrical solution over the trivial ("perturbative") vacuum always exists, and the choice in favor of either solution is defined by the sign of the quadratic term coefficient of Higgs field Lagrangian. But the complete set of solutions of quantum field theory equations is not exhausted by these two classes of solutions. A variety of other solutions exists, and among them there are those with non-zero expectation values of vector field. Due to linearity of basic equations the general solution is a superposition of the partial solutions including the solutions with non-zero expectation values of vector field. Each of the partial solutions defines its ("partial") vacuum. The physical vacuum is a superposition of partial vacua, which satisfies the certain physical conditions including the Poincaré-invariance. The Poincaré-invariance of the theory does not mean, generally speaking, the absence of the partial vacua with non-zero vacuum expectations of the fields in the superposition but only means the mutual compensation of their contributions in the physical vacuum expectation value of the vector field.

In the present work the generalization of the Higgs mechanism is considered, which takes into account the contribution of the partial solutions of quantum field theory equations (Schwinger-Dyson equations) with non-zero expectations of both scalar and vector fields. A method of iteration solution of Schwinger-Dyson equations for the generating functional of Green functions, which permits taking into account the contribution of such solutions, was developed in [4]-[7]. This method does not require the effective potential be introduced, whose use for a theory with vector fields run across the known difficulties. The method has been applied to the description of spontaneous symmetry breaking for scalar theory [4], [5] and Gross-Neveu model at N finite [5], [6], and also for non-Abelian gauge theory [7] (for constant vacuum configurations).

In Section 2 the method is described by an example of the complex scalar field. In section 3 the method is applied to the Abelian Higgs model. Taking into account the vacuum configurations of gauge fields allows one to model the triviality of the ϕ_4^4-interaction. In this case a very strict triviality bound ($m_H \leq 1.15 m_A$) arises. In the concluding Section a possibility of generalization for non-Abelian theory is briefly discussed.

2. Complex scalar field with self-action

As the first step to the Higgs model and for the illustration of the investigation method consider the theory of the complex scalar field ϕ with Lagrangian

$$\mathcal{L} = \partial_\mu \phi^* \partial_\mu \phi - m^2 \phi^* \phi - \frac{\lambda}{2}(\phi^* \phi)^2. \tag{2}$$

We work in Minkowski space with $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, but for the notation simplicity do not distinguish the upper and lower Lorentz vector indices. The generating functional of Green functions $G(j)$ is a solution of Schwinger-Dyson equation (SDE)

$$\frac{\lambda}{i} \frac{\delta^3 G}{\delta j^* \delta j \delta j^*} - (m^2 + \partial^2) \frac{1}{i} \frac{\delta G}{\delta j^*} + jG = 0. \tag{3}$$

Here $j(x)$ is a source of field $\phi^*(x)$. Green functions (vacuum expectation values of T-product) are the functional derivatives of G at the source switched off:

$$G(x_1 \ldots x_n \mid y_1 \ldots y_n) = < 0 \mid T\phi(x_1) \cdots \phi(x_n)\phi^*(y_1) \cdots \phi^*(y_n) \mid 0 > =$$
$$= \frac{1}{i^{2n}} \frac{\delta^{2n} G}{\delta j(y_n) \cdots \delta j(y_1) \delta j^*(x_n) \cdots \delta j^*(x_1)} \Big|_{j=0}.$$

At $\lambda = 0$ equation (3) has a solution

$$G^{pert} = \exp\{i \int dx dy j^*(x) \Delta_c(x-y) j(y)\},$$

where $\Delta_c = (m^2 + \partial^2)^{-1}$ is the free propagator. This solution is the foundation for the iteration scheme of the coupling constant perturbation theory. We shall use an iteration scheme of solving SDE which is based on an alternative principle and allows one to investigate the non-perturbative effects (see also [4]–[7]). This iteration scheme is based on the following considerations: since the purpose of the calculations is the Green functions, it is sufficient for us to know the generating functional $G(j)$ near the point $j = 0$. Therefore it is reasonable to take as a leading approximation of SDE an equation with "constant coefficients", i.e. to put $j = 0$ in the coefficient functions of eq.(3). The leading approximation equation has the form

$$\frac{\lambda}{i} \frac{\delta^3 G_0}{\delta j^* \delta j \delta j^*} - (m^2 + \partial^2) \frac{1}{i} \frac{\delta G_0}{\delta j^*} = 0. \tag{4}$$

This equation has a solution

$$G_0(j) = \exp i \int dx [v^*(x)j(x) + v(x)j^*(x)], \tag{5}$$

where $v(x)$ obeys the "characteristic equation"

$$(\lambda v^* v + m^2 + \partial^2)v = 0. \tag{6}$$

Functional (5), considered as a leading ("vacuum") approximation, is a foundation for the linear iteration scheme

$$G = G_0 + G_1 + \cdots + G_n + \cdots,$$

where G_n is defined by the iteration scheme equation

$$\frac{\lambda}{i} \frac{\delta^3 G_n}{\delta j^* \delta j \delta j^*} - (m^2 + \partial^2) \frac{1}{i} \frac{\delta G_n}{\delta j^*} = -jG_{n-1}. \tag{7}$$

3

A solution of eq.(7) is the functional

$$G_n(j) = P_n(j)G_0(j),$$

where $P_n(j)$, with taking into account the leading approximation equations (5)-(6), is a solution of the equation

$$\frac{1}{i}(2\lambda v^* v + m^2 + \partial^2)\frac{\delta P_n}{\delta j^*} + \frac{\lambda v^2}{i}\frac{\delta P_n}{\delta j} - \quad (8)$$

$$-\lambda(2v\frac{\delta}{\delta j} + v^*\frac{\delta}{\delta j^*})\frac{\delta P_n}{\delta j^*} - \frac{\lambda}{i}\frac{\delta^3 P_n}{\delta j^* \delta j \delta j^*} = jP_{n-1}.$$

Since $P_0 = 1$, the solution of eq.(8) is a polynomial in the source j at any n. Coefficient functions of this polynomial define the Green functions of the corresponding step of the iteration scheme. Thus, the solution of the first-step equation is a quadratic polynomial in j:

$$P_1(j) = i\int dxdy\{j^*(x)\Delta(x,y)j(y) + \frac{1}{2}(j(x)d(x,y)j(y)+$$

$$+j^*(x)\bar{d}(x,y)j^*(y))\} + i\int dx(\bar{\Phi}(x)j(x) + j^*(x)\Phi(x)).$$

The solution of the second-step equation is a polynomial of the fourth order and so on. At any step equation (8) defines a closed system of equations for coefficient functions of the polynomial $P_n(j)$.

For the ultraviolet divergences removing, it is necessary to modify SDE by introducing counterterms, that is by making the following substitution in eq.(3):

$$\lambda \to \lambda + \delta\lambda, \ m^2 \to m^2 + \delta m^2, \ \partial^2 \to (1 + \delta z)\partial^2.$$

Here $\delta\lambda, \delta m^2$ and δz are counterterms of the coupling constant, mass and wave function renormalization. Eq.(7) and eq.(8) for P_n are modified by introducing counterterms of corresponding orders of the iteration scheme. There is no need to introduce the counterterms in the leading order, and leading approximation equation (4) and l.h.s. of eq.(7) are not changed. The first-step equation with the counterterms has the form

$$\frac{\lambda}{i}\frac{\delta^3 G_1}{\delta j^* \delta j \delta j^*} - (m^2 + \partial^2)\frac{1}{i}\frac{\delta G_1}{\delta j^*} =$$

$$= -jG_0 - \frac{\delta\lambda_1}{i}\frac{\delta^3 G_0}{\delta j^* \delta j \delta j^*} + (\delta m_1^2 + \delta z_1\partial^2)\frac{1}{i}\frac{\delta G_0}{\delta j^*}.$$

and, correspondingly, the equation for P_1 is modified as

$$\frac{1}{i}(2\lambda v^* v + m^2 + \partial^2)\frac{\delta P_1}{\delta j^*} + \frac{\lambda v^2}{i}\frac{\delta P_1}{\delta j} - \lambda(2v\frac{\delta}{\delta j} + v^*\frac{\delta}{\delta j^*})\frac{\delta P_1}{\delta j^*} =$$

$$= j - \delta\lambda_1 v^* v^2 - (\delta m_1^2 + \delta z_1\partial^2)v.$$

4

For the first-step coefficient functions Δ, d и Φ we obtain the system of equations

$$(2\lambda v^* v + m^2 + \partial^2)\Delta(x,y) + \lambda v^2 d(x,y) = \delta(x-y), \tag{9}$$

$$(2\lambda v^* v + m^2 + \partial^2)d(x,y) + \lambda(v^*)^2\Delta(x,y) = 0, \tag{10}$$

$$(2\lambda v^* v + m^2 + \partial^2)\Phi + \lambda v^2\bar{\Phi} - i\lambda(2v\Delta(x,x) + v^*\bar{d}(x,x)) = \tag{11}$$
$$= -\delta\lambda_1 v^* v^2 - (\delta m_1^2 + \delta z_1 \partial^2)v.$$

The particle propagators are defined by subsystem (9)-(10), while eq.(11) is a connection for first-step counterterms. Note, that solutions of eqs.(9) and (10) are evidently ultraviolet-finite, and therefore the first-step counterterms seem to be superfluous ones. But they are necessary for removing the ultraviolet divergences at the *following* steps of the iteration scheme (see [4]–[6] for more detailed discussion of the renormalization in the given iteration scheme). Since in the present work we limit ourselves to studying the first-step equations, we shall not write the counterterms and terms of P_1 which are linear in sources, as well as the connections among them, because they are necessary only at the following steps of the calculations.

Let us discuss leading approximation (5) in more detail. All the functions of the leading approximation are fully disconnected and they are simply the products of functions v, consequently, the role of the leading approximation is reduced to the definition of structure of the ground state — the physical vacuum of the theory. Firstly, notice that eq.(6) possesses a variety of solutions $\{v\}$, i.e., there exists a variety of "v-vacua" and each of them defines a solution of eq.(4), and, correspondingly, iterative solution of SDE (3). The trivial solution of the characteristic equation $v \equiv 0$ (i.e., $G_0 = 1$) leads, nevertheless, to the non-trivial solution of SDE. In this case the first-step Green functions are $\Delta = (m^2 + \partial^2)^{-1} = \Delta_c$, $d \equiv \Phi \equiv 0$, and the considered iteration scheme defines the reconstructed series of the perturbation theory in coupling constant over the trivial perturbative vacuum.

Though *a priori* one cannot exclude coordinate-dependent solution of eq.(6), for our purposes it is sufficient to limit ourselves to the class of solutions $v = \text{const}$. Each coordinate-indepedned v defines the Poincaré-invariant theory. At $m^2 < 0$ this class of solutions describes the phase with spontaneously broken $U(1)$-symmetry. Really, eq.(6) takes the familiar form

$$\lambda v^* v + m^2 = 0.$$

In this case eqs. (9) and (10) have the solutions (in the momentum space)

$$\Delta(p) = \frac{1}{2}\left(\frac{1}{2\lambda v^* v - p^2} - \frac{1}{p^2}\right), \ d(p) = \frac{1}{2}\frac{v^*}{v}\left(\frac{1}{2\lambda v^* v - p^2} + \frac{1}{p^2}\right).$$

Linear combinations

$$\Delta_H = \Delta + \frac{v}{v^*}d = \frac{1}{2\lambda v^* v - p^2}, \ \Delta_G = \Delta - \frac{v}{v^*}d = -\frac{1}{p^2}$$

correspond to Higgs boson and Goldstone boson propagators.

All the picture completely coincides with the traditional approach based on the study of the effective potential for model (2) in the leading quasiclassical approximation.

5

3. Higgs model

The principle of local gauge invariance requires a gauge field A_μ to be introduced, and Lagrangian (2) for the gauge theory is changed to the Lagrangian of Abelian Higgs model

$$\mathcal{L} = (\partial_\mu + ieA_\mu)\phi^*(\partial_\mu - ieA_\mu)\phi - m^2\phi^*\phi - \frac{\lambda}{2}(\phi^*\phi)^2 - \tag{12}$$

$$-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2\alpha}(\partial_\mu A_\mu)^2.$$

In the theory with Lagrangian (12) the system of SDEs for the generating functional $G(j, J_\mu)$ is of the form

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{\alpha}\partial_\mu\partial_\nu)\frac{1}{i}\frac{\delta G}{\delta J_\nu} + \tag{13}$$

$$+ie\frac{\delta}{\delta j^*}(\partial_\mu + e\frac{\delta}{\delta J_\mu})\frac{\delta G}{\delta j} - ie\frac{\delta}{\delta j}(\partial_\mu - e\frac{\delta}{\delta J_\mu})\frac{\delta G}{\delta j^*} + J_\mu G = 0,$$

$$\frac{\lambda}{i}\frac{\delta^3 G}{\delta j^*\delta j\delta j^*} - (m^2 + (\partial_\mu - e\frac{\delta}{\delta J_\mu})^2)\frac{1}{i}\frac{\delta G}{\delta j^*} + jG = 0. \tag{14}$$

Here $J_\mu(x)$ is the source of field $A_\mu(x)$.

Let us apply the method of the preceding Section to the system of eqs.(13)-(14) and choose as a leading approximation a system of equations with "constant coefficients"

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{\alpha}\partial_\mu\partial_\nu)\frac{1}{i}\frac{\delta G_0}{\delta J_\nu} +$$

$$+ie\frac{\delta}{\delta j^*}(\partial_\mu + e\frac{\delta}{\delta J_\mu})\frac{\delta G_0}{\delta j} - ie\frac{\delta}{\delta j}(\partial_\mu - e\frac{\delta}{\delta J_\mu})\frac{\delta G_0}{\delta j^*} = 0,$$

$$\frac{\lambda}{i}\frac{\delta^3 G_0}{\delta j^*\delta j\delta j^*} - (m^2 + (\partial_\mu - e\frac{\delta}{\delta J_\mu})^2)\frac{1}{i}\frac{\delta G_0}{\delta j^*} = 0.$$

A solution of this system is the functional

$$G_0(j, J_\mu) = \exp i \int dx[V_\mu(x)J_\mu(x) + v^*(x)j(x) + j^*(x)v(x)],$$

where V_μ and v are the solutions of the system of characteristic equations

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{\alpha}\partial_\mu\partial_\nu)V_\nu - iev(\partial_\mu + ieV_\mu)v^* + iev^*(\partial_\mu - ieV_\mu)v = 0,$$

$$\lambda v^*v^2 + (m^2 + \partial^2 - 2ieV_\mu\partial_\mu - ie\partial_\mu V_\mu - e^2V^2)v = 0.$$

Below we shall consider only the class of solutions $v = \text{const} \neq 0$. If, in addition, one imposes the subsidiary condition $\partial_\mu V_\mu = 0$, the system of characteristic equations has the form

$$(\mu^2 + \partial^2)V_\mu = 0, \tag{15}$$

$$\lambda v^* v + m^2 - e^2 V^2 = 0, \tag{16}$$

where $\mu^2 = 2e^2 v^* v$.

A system of iteration scheme equations is

$$(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu + \frac{1}{\alpha}\partial_\mu\partial_\nu)\frac{1}{i}\frac{\delta G_n}{\delta J_\nu} + ie\frac{\delta}{\delta j^*}(\partial_\mu + e\frac{\delta}{\delta J_\mu})\frac{\delta G_n}{\delta j} -$$

$$-ie\frac{\delta}{\delta j}(\partial_\mu - e\frac{\delta}{\delta J_\mu})\frac{\delta G_n}{\delta j^*} = -J_\mu G_{n-1} + \text{counterterms},$$

$$\frac{\lambda}{i}\frac{\delta^3 G_n}{\delta j^*\delta j\delta j^*} - (m^2 + (\partial_\mu - e\frac{\delta}{\delta J_\mu})^2)\frac{1}{i}\frac{\delta G_n}{\delta j^*} = -jG_{n-1} + \text{counterterms}.$$

A solution of the first-step equations ($n = 1$) will be looked for the form

$$G_1 = P_1 G_0,$$

where P_1 is a second-order polynomial in sources j and J_μ:

$$P_1(j, J_\mu) = \int dxdy\{ij^*(x)\Delta(x,y)j(y) + \frac{i}{2}(j(x)d(x,y)j(y) +$$

$$+ j^*(x)\bar{d}(x,y)j^*(y)) + \frac{1}{2i}J_\mu(x)D_{\mu\nu}(x,y)J_\nu(y) + \tag{17}$$

$$+i(J_\mu(x)B_\mu(x\mid y)j(y) + J_\mu(x)\bar{B}_\mu(x\mid y)j^*(y))\} + \text{linear terms}.$$

With characteristic eqs. (15)-(16) taken into account, the first-step equations lead to the system of equations for the coefficient functions of polynomial (17):

$$L_{\mu\nu}D_{\nu\lambda} + ie(v\partial_\mu B_\lambda^T - v^*\partial_\mu \bar{B}_\lambda^T) - 2e^2 V_\mu(vB_\lambda^T + v^*\bar{B}_\lambda^T) = g_{\mu\lambda}, \tag{18}$$

$$L\bar{B}_\nu^T + \lambda v^2 B_\nu^T + v(ie\partial_\mu + 2e^2 V_\mu)D_{\mu\nu} = 0, \tag{19}$$

$$L_{\mu\nu}B_\nu + ie\partial_\mu(v^*\Delta - vd) + 2e^2 V_\mu(v^*\Delta + vd) = 0, \tag{20}$$

$$L_{\mu\nu}\bar{B}_\nu + ie\partial_\mu(v^*\bar{d} - v\Delta^T) + 2e^2 V_\mu(v^*\bar{d} + v\Delta^T) = 0, \tag{21}$$

$$L\Delta + \lambda v^2 d - v(ie\partial_\mu + 2e^2 V_\mu)B_\mu = 1, \tag{22}$$

$$L\bar{d} + \lambda v^2 \Delta^T - v(ie\partial_\mu + 2e^2 V_\mu)\bar{B}_\mu = 0. \tag{23}$$

Here $L_{\mu\nu} \equiv (\mu^2 + \partial^2)g_{\mu\nu} - \partial_\mu\partial_\nu + \frac{1}{\alpha}\partial_\mu\partial_\nu$ and $L \equiv \partial^2 + \lambda\, v^* v - 2ieV_\mu\partial_\mu$. The upper index T means transposition: $\Delta^T(x,y) \equiv \Delta(y,x)$ and so on. Besides eqs. (18)-(23), three conjugated equations exist which follow from the Schwinger-Dyson equation

$$\frac{\lambda}{i}\frac{\delta^3 G}{\delta j\delta j^*\delta j} - (m^2 + (\partial_\mu + e\frac{\delta}{\delta J_\mu})^2)\frac{1}{i}\frac{\delta G}{\delta j} + j^* G = 0,$$

which is conjugated to eq. (14). These equations differ from eqs. (19), (22) and (23) in the substitution

$$L \to L^*, \ B_\lambda \leftrightarrow \bar{B}_\lambda, \ \Delta \leftrightarrow \Delta^T, \ d \leftrightarrow \bar{d}, \ v \leftrightarrow v^*, \ i \to -i.$$

For the investigation of the system of equations for coefficient functions it is useful to introduce the linear combinations

$$C_\lambda^\pm = v B_\lambda^T \pm v^* \bar{B}_\lambda^T, \ \Delta_H = \Delta + \frac{v}{v^*} d, \ \Delta_G = \Delta - \frac{v}{v^*} d.$$

First of all, notice, that with the formula

$$\partial_\mu \ L_{\mu\nu} = (\mu^2 + \frac{1}{\alpha}\partial^2)\partial_\nu$$

one can easy calculate the longitudinal part of gauge field propagator $\partial_\mu \ D_{\mu\nu}$ from eqs. (18), (19) and from an equation which is conjugated to eq.(19). The result is

$$\partial_\mu D_{\mu\nu} = \alpha \frac{\partial_\nu}{\partial^2}.$$

Therefore, the longitudinal part of gauge field propagator is not renormalized. Below we shall work in the transverse gauge $\alpha = 0$. At $\alpha \to 0$

$$L_{\mu\nu} D_{\nu\lambda} = (\mu^2 + \partial^2) D_{\mu\lambda} + \partial_\mu \partial_\lambda / \partial^2,$$

$$\partial_\mu C_\mu^\pm = 0,$$

and the system of equations is simplified. Excluding the combination C_λ^+ from the system of eqs. (18), (19) and from one conjugated to (19), we obtain the system of equations

$$(\mu^2 + \partial^2) D_{\mu\nu} + 4e^2\mu^2 V_\mu \Delta_1 \star (V_\rho D_{\rho\nu}) + \tag{24}$$

$$+ie[\partial_\mu C_\nu^- + 4e^2 V_\mu \Delta_1 \star (V_\rho \partial_\rho C_\nu^-)] = \pi_{\mu\nu},$$

$$\partial^2 C_\nu^- + 4e^2 V_\mu \partial_\mu \Delta_1 \star (V_\rho \partial_\rho C_\nu^-) = 4ie\mu^2 V_\mu \partial_\mu \Delta_1 \star (V_\rho D_{\rho\nu}). \tag{25}$$

Here $\Delta_1 \equiv (\partial^2 + 2\lambda \ v^* v)^{-1}$ and $\pi_{\mu\nu} \equiv g_{\mu\nu} - \partial_\mu \partial_\nu / \partial^2$. The symbol $\star$ denotes the operator multiplication: $(A \star B)(x, y) \equiv \int \ dz \ A(x, z) B(z, y)$.

Excluding B_μ and Δ_G from eqs.(20), (22) and from that conjugated to (23), one obtains (after simple transformations) the equation for Higgs propagator

$$(\partial^2 + 2\lambda v^* v)\Delta_H + 4e^2 V_\mu \partial_\mu \Delta_0 \star (V_\nu \partial_\nu \Delta_H) + \tag{26}$$

$$+4e^2\mu^2 V_\mu D_{\mu\nu}^c \star (V_\nu \Delta_H) = 1 + 2ie V_\mu \partial_\mu \Delta_0.$$

Here $D_{\mu\nu}^c \equiv (\mu^2 + \partial^2)^{-1} \star \pi_{\mu\nu}$ and $\Delta_0 \equiv 1/\partial^2$.

At $V_\mu \equiv 0$ eqs.(24)-(26) have the solution

$$D_{\mu\nu} = (\mu^2 + \partial^2)^{-1} \star \pi_{\mu\nu}, \ \Delta = (\partial^2 + 2\lambda \ v^* v)^{-1}, \ C_\nu^- = 0,$$

which exactly corresponds to the result of Higgs model in transverse gauge for usual approach when the effective potential is used.

Let us consider now a generalization of the Higgs mechanism taking into account the solutions with $V_\mu \not\equiv 0$. Firstly, notice, that such solutions exist. Indeed, besides the solution $V_\mu = 0$, $\lambda(v^*v) = -m^2$, the characteristic equations (15)-(16) have the class of solutions

$$V_\mu(x) = V_\mu^c \cos ax + V_\mu^s \sin ax, \tag{27}$$

where constant vectors V_μ^c, V_μ^s and a_μ satisfy the conditions

$$(V^c)^2 = (V^s)^2 = \frac{\lambda v^*v + m^2}{e^2}, \ a^2 = \mu^2, \ (V^c V^s) = (aV^c) = (aV^s) = 0. \tag{28}$$

The solutions G_V of SDEs with $V_\mu \not\equiv 0$ violate the Poincaré-invariance. But since these equations are linear, the arbitrary superposition

$$G = \sum G_V$$

is also a solution of these equations and it can be chosen in such a way that the corresponding Poincaré-noninvariant contributions will cancel, and the vacuum expectation values, which violate the Poincaré-invariance, will disappear. (For example, in the leading order of the given iteration scheme it is sufficient to take the linear combinations $\cos \int dx\, V_\mu(x) J_\mu(x)$ for terminating the vacuum expectation of the vector field.) A choice of such a superposition is equivalent, in essence, to some averaging and it defines the physical vacuum of the theory. Physical vacuum $\mid 0 >$ should be Poincaré-invariant, therefore

$$< 0 \mid A_\mu \mid 0 > = \frac{\delta G}{\delta J_\mu} = \sum \frac{\delta G_V}{\delta J_\mu} = 0$$

at the sources switched off, though some terms in the sum can be non-zero. Therefore the characteristic equation solutions with $V_\mu \not\equiv 0$ and the corresponding solutions of iteration scheme equations can take part in the formation of physical vacuum without destroying the Poincaré-invariance of the theory.

At $V_\mu \not\equiv 0$ the system of eqs.(24)-(26) is the complicated system of integro-differential equations, and so we resort to the mean-field approximation. This approximation describes properly many statistical systems and with extraordinary success it works in microscopical theory of superconductivity [8], [9]. The physical vacuum of the Higgs model is a relativistic superconductor, and it is reasonable to suppose this approximation provides an adequate description in the given case as well. In the mean-field approximation we change the product $V_\mu V_\nu$ by its average value on the physical vacuum:

$$V_\mu(x)V_\nu(y) = f_{\mu\nu}(x - y) + \delta_{\mu\nu}(x, y),$$

where

$$f_{\mu\nu}(x - y) = < 0 \mid V_\mu(x)V_\nu(y) \mid 0 > .$$

The equation for the Higgs propagator Δ_H in the mean-field approximation has the form

$$(\partial^2 + 2\lambda v^* v)\Delta_H^{mf} + 4e^2(f_{\mu\nu}\partial_\mu\partial_\nu\Delta_0) \star \Delta_H^{mf} + 4e^2\mu^2(f_{\mu\nu}D_{\mu\nu}^c) \star \Delta_H^{mf} = 1, \qquad (29)$$

and

$$\Delta_H = \Delta_H^{mf} + \mathcal{O}(\delta_{\mu\nu}^2).$$

due to $< 0 \mid \delta_{\mu\nu} \mid 0 >= 0$. Here and in what follows Δ_H does not denote a partial solution of eq.(26) at some partial value V_μ, but it stands for the physical Higgs propagator in the physical Poincaré-invariant vacuum. The same refers to the gauge field propagator $D_{\mu\nu}$. Since $\partial_\mu f_{\mu\nu} = 0$, it follows from eq.(25) that in the mean-field approximation $C_\nu^- = 0$, and the equation for the gauge field propagator takes the form

$$(\mu^2 + \partial^2)D_{\mu\nu}^{mf} + 4e^2\mu^2(f_{\mu\rho}\Delta_1) \star D_{\rho\nu}^{mf} = \pi_{\mu\nu}. \qquad (30)$$

Equations (29) and (30) are translational-invariant and are easily solved in the momentum space. For subsequent calculations it is necessary to define the function $f_{\mu\nu}$. A simplest way to do this is the following: due to subsidiary condition $\partial_\mu V_\mu = 0$ and characteristic eqs. (15)-(16) this function has the form

$$f_{\mu\nu} = \frac{\lambda v^* v + m^2}{3e^2}\pi_{\mu\nu} \star f, \qquad (31)$$

where the scalar function $f(x - y)$ is a solution of the equation

$$(\mu^2 + \partial^2)f = 0$$

with boundary condition $f(0) = 1$, i.e.

$$f(x) = \frac{2}{\sqrt{\mu^2 x^2}}J_1(\sqrt{\mu^2 x^2}), \qquad (32)$$

where $J_1(z)$ is the Bessel function.

Note that above we suppose the translational invariance of the vacuum expectation $f_{\mu\nu} =< 0 \mid V_\mu(x)V_\nu(y) \mid 0 >$. One can avoid this supposition (i.e., actually to prove it) by direct averaging solution (27). Averaging products of constant vectors V_μ^c и V_μ^s with conditions (28) taken into account, we obtain

$$< V_\mu(x)V_\nu(y) >_a = \frac{\lambda v^* v + m^2}{3e^2}(g_{\mu\nu} - \frac{a_\mu a_\nu}{\mu^2})\cos a(x - y). \qquad (33)$$

Subsequent averaging is reduced to the application of the formula

$$< (ax)^{2n} >= (\mu^2 x^2)^n \frac{\Gamma(n + 1/2)}{(n + 1)!\Gamma(1/2)} \qquad (34)$$

(and similar ones, see [7]). It leads to the same result for $f_{\mu\nu}$, i.e. to formulae (31)-(32).

Further calculation of the propagators in the momentum space is reduced to the calculation of "single-loop integrals" like $f_{\mu\nu}(x)D_{\mu\nu}^c(x)$ and is quite elementary. (For the simplification of the calculations it is convenient to use the intermediate result (33) and then to make the averaging with the help of formulae similar to (34).) A location of propagator poles defines masses of particles. A distinctive feature of the generalized Higgs mechanism in comparison with the usual one is the possibility to model the triviality of ϕ_4^4-theory, i.e., we can tend λ to zero but the masses of Higgs and gauge bosons will retain non-zero values. At $\lambda \to 0$ the admissible values of the parameter m^2 lie in the region

$$-\infty < m^2 \leq -6\mu^2.$$

(At other values of the parameter m^2 the particle masses became complex-valued.) When m^2 lies in the above region, the Higgs boson mass m_H and the gauge boson m_A vary in the limits

$$4.7\mu^2 > m_H^2 \geq 4\mu^2,$$
$$\infty > m_A^2 \geq 3.04\mu^2,$$

and we obtain the triviality bound

$$m_H \leq 1.15 m_A. \tag{35}$$

4. Conclusion

In the present work we have considered a generalization of the Higgs mechanism which takes into account the vacuum configurations with non-zero expectation values of vector field. A contribution of such configurations does not violate the Poincaré-invariance of the physical vacuum and gives a possibility to keep a principal physical result of the Higgs model — a generation of gauge field mass — in spite of the triviality of ϕ_4^4-interaction. This generalized Higgs mechanism imposes on the Higgs mass the very strict triviality bound (35). A distinctive feature of the proposed approach is the fact that the triviality bound exists even for the simple Abelian Higgs model and does not require, contrary to the usual approach [2], [3], the scalar sector to be treated as an effective approximation for some Grand unified theory.

The proposed generalization of the Higgs mechanism is also possible, of course, for a non-Abelian theory. A main complication for a non-Abelian theory is a drastic growing of the "non-linearity degree" of the characteristic equations. Nevertheless, the strict triviality bound similar to (35) may remain for a non-Abelian theory too. At the same time, taking into account the vector vacuum configuration for a non-Abelian theory we acquire a principally new possibility of avoiding the consideration of the scalar sector at all: a generation of gauge field mass can be made dynamically. A consideration of such models became quite actual in connection with unsuccessful searches of the Higgs boson and leads to non-trivial phenomenological consequences (see, for example, [10]).

11

References

[1] *Acciarri M. et al.* (L3 Collaboration) Preprint CERN-PPE/97-81, 1997.

[2] *Callaway D.J.E.//* Phys.Reports. C167(1988)241.

[3] *Choudhury S.R. and Mamta//* Int.J.Mod.Phys. A12(1997)1847.
Montvay I. Preprint DESY 97-034, 1997.
Krasnikov N.V., Kreyerhoff G. and Rodenberg R. hep-ph/9711286, 1997.

[4] *Rochev V.E.//* J.Phys.A: Math.Gen. 30(1997)3671 (hep-th/9606155).

[5] *Rochev V.E.//* Quantum Chromodynamics: Collisions, Confinement and Chaos//
Eds. H.M. Fried and B. Müller. Singapore: World Scientific, 1997, p.354 (Preprint
IHEP 96-61, 1996).

[6] *Rochev V.E. and Saponov P.A.* hep-th/9710006, 1997 (Int.J.Mod.Phys.A., in print).

[7] *Rochev V.E.//* J.Phys.A: Math.Gen. 31(1998)409 (hep-th/9705168).

[8] *Bogoliubov N.N., Zubarev D.N. and Tserkovnikov Yu.A.//* JETP, 39(1960)120.

[9] *Tinkham M.//* Introduction to superconductivity. N.Y.: McGraw Hill, 1975.

[10] *Arbuzov B.A.//* Phys.Lett. B288(1992)179; Phys.Lett. B353(1995)532.