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ON OBSERVABILITY OF SIGNAL OVER BACKGROUND

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Abstract

Bityukov S.I., Krasnikov N.V. On observability of signal over background: IHEP Preprint 98-48. – Protvino, 1998. – p. 13, figs. 8, tables 4, refs.: 8.

Several statistics used by physicists to declare the signal observability over the background are compared. It is shown that the frequentist method of testing a precise hypothesis allows one to estimate the power value of criteria with specified level of significance for the considered statistics by Monte Carlo calculations. The application of this approach for the analysis of discovery potential of experiments is discussed.

Аннотация

Битюков С.И., Красников Н.В. К вопросу о наблюдаемости сигнала над фоном: Препринт ИФВЭ 98-48. – Протвино, 1998. – 13 с., 8 рис., 4 табл., библиогр.: 8.

В данной работе сравниваются несколько статистик, используемых физиками для характеристики возможности наблюдения новых явлений. Показано, что при проверке гипотез возможно использование Монте-Карло вычислений для построения функций плотности вероятности распределения сложных статистик. Обсуждаются приложения данного подхода при определении потенциала открытия эксперимента.

Introduction

One of the common tasks for searching experiments is the detection of a predicted new Phenomenon. As a rule the estimations of an expected mean N_s for the signal events of new Phenomenon and N_b for the background events are known. Then we want to know is the given experiment able to detect new Phenomenon or not. To check the statement about the observation of Phenomenon a researcher uses some function of the observed number of events – a statistic. The value of this statistic for detected x events allows one to find the degree of confidence of the discovery statement. After having drawn a conclusion on the observation of Phenomenon, two possibilities for mistake are available: to state that Phenomenon is absent but in real life it exists (Type I error), or to state that Phenomenon exists but it is absent (Type II error).

In this paper we compare the "signal significances" used by the researchers for the hypothesis testing about the observation of Phenomenon:

(a) "significance" $S_1 = \dfrac{N_s}{\sqrt{N_b}}$ [1],
(b) "significance" $S_2 = \dfrac{N_s}{\sqrt{N_s + N_b}}$ [2,3],
(c) "significance" $S_{12} = \sqrt{N_s + N_b} - \sqrt{N_b}$ [4],
(d) likelihood ratio as is defined in references [5,6].

For this purpose we formulate the null and alternative hypotheses, construct the statistical test, determine the rejection region by Monte Carlo calculations, make the decision and find the power of test for the criteria with a specified level of significance. We also use an equal-tailed test to study the behaviour of Type I and Type II errors versus N_s and N_b for specified values of S_1 and S_2. The hypotheses testing results obtained by Monte Carlo calculations are compared with result obtained by the direct calculations of probability density functions.

1. Notations

Let us study a physical process during a fixed time. The estimations of the average number of signal events which indicate new Phenomenon (N_s) and of the average number of background events (N_b) in the experiment are given. We suppose that the events have the Poisson distributions with the parameters N_s and N_b, i.e. the random variable $\xi \sim Pois(N_s)$ describes the signal events and the random variable $\eta \sim Pois(N_b)$ describes the background events. Say we observed x events – the realization of the studying process $X = \xi + \eta$ (x is the sum of signal and background events in the experiment). Here N_s, N_b are non-negative real numbers and x is an integer. The classical frequentist methods of testing a precise hypothesis allow one to construct a rejection region and determine associated error probabilities for the following "simple" hypotheses:

H_0 : $X \sim Pois(N_s + N_b)$ versus H_1 : $X \sim Pois(N_b)$, where $Pois(N_s + N_b)$ and $Pois(N_b)$ have the probability density functions (p.d.f.'s)
$f_0(x) = \dfrac{(N_s + N_b)^x}{x!} e^{-(N_s+N_b)}$ for the case of presence and $f_1(x) = \dfrac{(N_b)^x}{x!} e^{-(N_b)}$ for the case of absence of signal events in the universe population.

In Fig.1 the p.d.f.'s $f_0(x)$ (a) and $f_1(x)$ (b) for the case $N_s+N_b = 104$ and $N_b = 53$ ([3], Table.13, cut 6) are shown. As is seen the intersection of these p.d.f.'s takes place. Let us denote the threshold (critical value) that divides the abscissa in Fig.1 into the rejection region and the area of accepted hypothesis H_0 via N_{ev}. The incorrect rejection of the null hypothesis H_0, the Type I error (the statement that Phenomenon is absent, but it is present), has the probability $\alpha = \sum\limits_{x=0}^{N_{ev}} f_0(x)$, and the incorrect acceptance of H_0, the Type II error (the statement that Phenomenon exists, but it is absent), has the probability $\beta = \sum\limits_{x=N_{ev}+1}^{\infty} f_1(x)$. The dependence of α and β on the value of N_{ev} for above example is presented in Fig.2.

2. Hypothesis testing

In this Section we show the procedure of the rejection region construction for the likelihood ratio [5].

We denote by $B(x) = \dfrac{f_0(x)}{f_1(x)}$ the likelihood ratio of H_0 to H_1 in the area of existing $B(X)$. The decision to either reject or accept H_0 will depend on the observed value of $B(x)$, where small values of $B(x)$ correspond to the rejection of H_0. For the traditional frequentist the classical most powerful test of the simple hypothesis is determined by some critical value c such that

if $B(x) \le c$, reject H_0,

if $B(x) > c$, accept H_0.

In compliance with this test, the frequentist reports Type I and Type II error probabilities as $\alpha = P_0(B(X) \le c) \equiv F_0(c)$ and $\beta = P_1(B(X) > c) \equiv 1 - F_1(c)$, where F_0 and F_1 are cumulative distribution functions of $B(X)$ under H_0 and H_1, respectively. For a conventional equal-tailed test with $\alpha = \beta$, the critical value c satisfies $F_0(c) \equiv 1 - F_1(c)$.

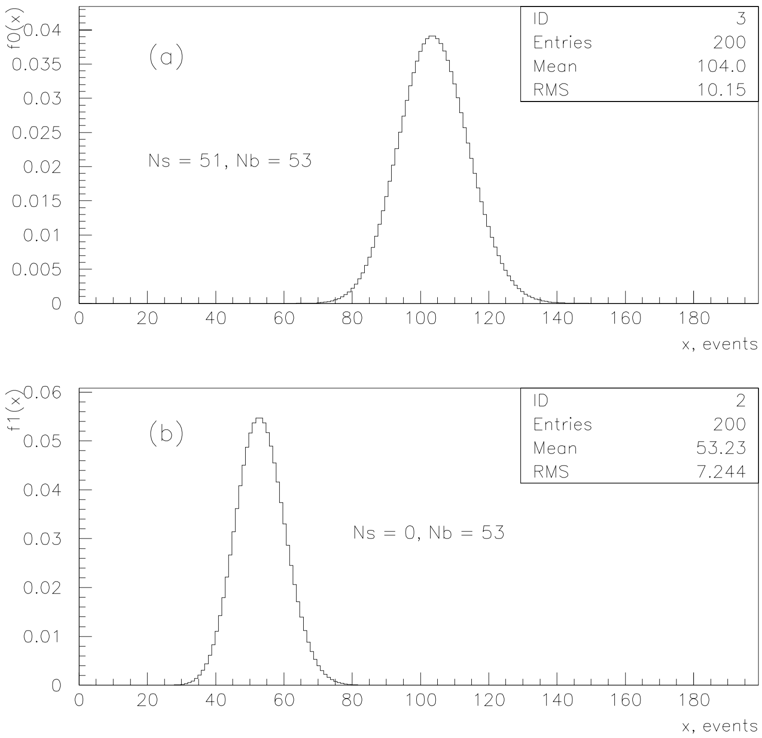

Fig. 1. The probability density functions $f_0(x)$ (a) and $f_1(x)$ (b) for the case of 51 signal events and 53 background events obtained by direct calculations of the probabilities.

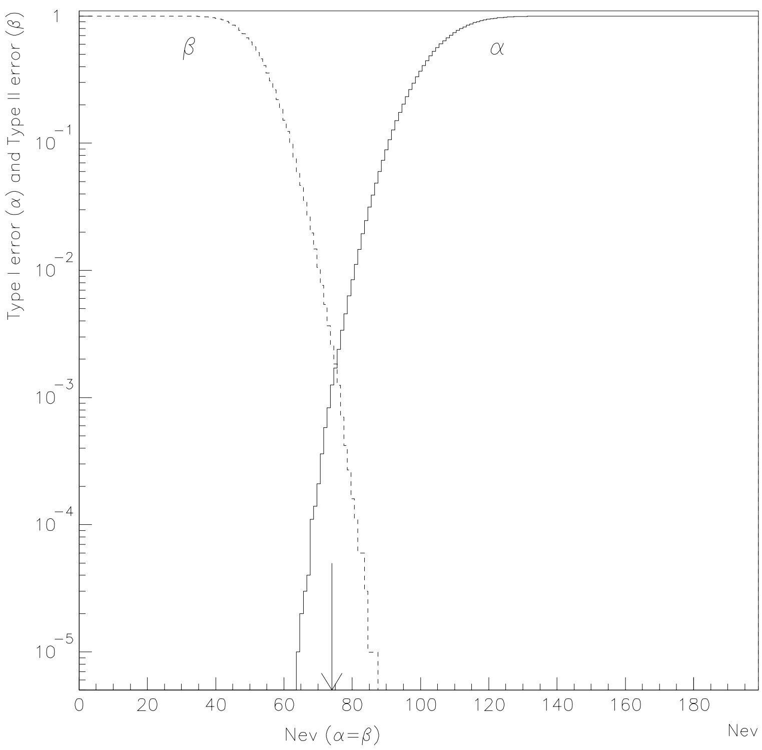

Fig. 2. The dependence of Type I α and Type II β errors on N_{ev} for the case of 51 signal events and 53 background events.

In the same way we can construct the rejection region, find the critical values c_1, c_2 and c_{12}, the probabilities α and β for the statistics $s_1 = \dfrac{x - N_b}{\sqrt{N_b}}$ (for "significance" S_1), $s_2 = \dfrac{x - N_b}{\sqrt{x}}$ (for "significance" S_2) and $s_{12} = \sqrt{x} - \sqrt{N_b}$ (for "significance" S_{12}). Here, the value of $x - N_b$ is the estimation of the number of signal events. Note that "significance" S_{12} depends on S_1 and S_2, namely, $S_{12} = \dfrac{S_1 \cdot S_2}{S_1 + S_2}$ [4].

3. Determination of probability density functions for statistics

The probability density functions of statistics under consideration can be obtained in an analytical form. Another way to obtain the p.d.f. is the calculations by a Monte Carlo simulation of the results of a large number of experiments (see as an example [7,6,8]) for the given values N_s and N_b. In this study we use the latter approach. The p.d.f.'s for $N_s + N_b = 104$ and $N_b = 53$ obtained by this way are shown in Fig.3 (these distributions are the result of 10^5 simulation experiments for random variables ξ and η). The difference between these p.d.f.'s and p.d.f.'s resulting from direct calculations of the probabilities (Fig.1) is extremely small.

In Fig.4 the p.d.f.'s of statistic s_2 for the case of $N_s = 51$, $N_b = 53$ (a) and the case of $N_s = 0$, $N_b = 53$ (b) are shown. The behaviour of probabilities α and β versus the critical value c_2 for the statistic s_2 is also presented in Fig.4 (c).

It is worth to stress that this approach allows one to construct the p.d.f.'s and, correspondingly, the acceptance and the rejection regions for complicated statistics with account for the systematic errors and the uncertainties in N_b and N_s estimations.

4. Comparison of different statistics

We compare the statistic s_1, the statistic s_2, the statistic s_{12} and the likelihood ratio ($B(x - N_b)$ in our case). The reason for the comparison is the existence of a opinion that the value of such type statistic ($s_1,\ s_2,\ s_{12}$) characterizes the difference between the samples with and without signal events in terms of "standard deviations" ($1\ \sigma,\ 2\ \sigma, \ldots,\ 5\ \sigma$) [1]. To anticipate a little, the values of α and β corresponding to these "standard deviations" depend on the value of the sample and for S_1, for example, α and β have a perceptible value even if N_s and N_b satisfy the condition $S_1 = 5$.

The Type I error α is also called a significance level of the test. The value for β is meaningful only when it is related to an alternative hypothesis H_1. The dependence $1 - \beta$ is referred to as a power function that allows one to choose a preferable statistic for the hypothesis testing. It means that for the specified significance level we can determine the critical value c (correspondingly, $c_1,\ c_2,\ c_{12}$) and find the power $1 - \beta$ of this criterion. The greater the value $1 - \beta$, the better statistic separates hypotheses for the specified value of α.

[1] If $f_1(x)$ is the standard normal distribution, then the $1\ \sigma$ deviation from 0 corresponds the area of tail that is equal to 0.1587, $2\ \sigma$ – 0.0228, $3\ \sigma$ – 0.00135, $4\ \sigma$ – 0.000032 and $5\ \sigma$ – 0.000003.

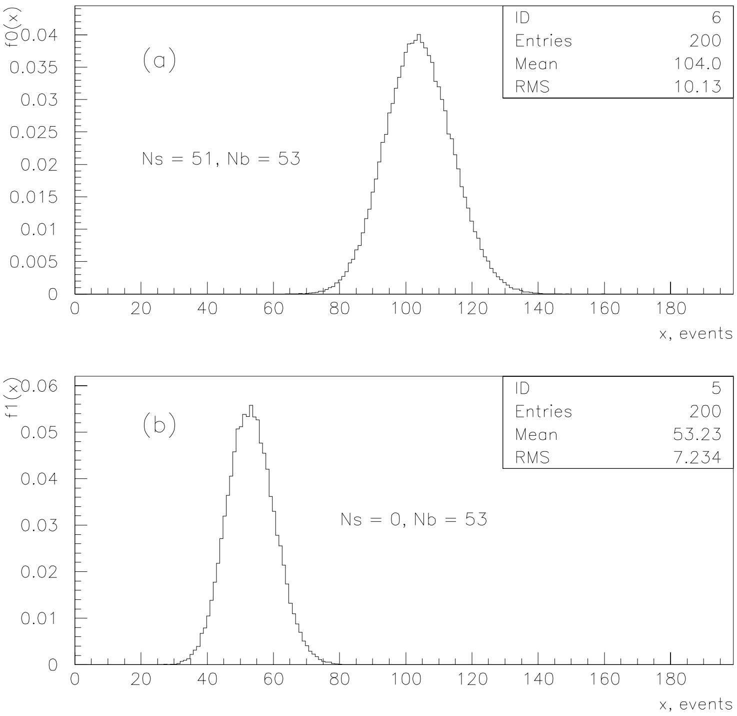

Fig. 3. The probability density functions $f_0(x)$ (a) and $f_1(x)$ (b) for the case of 51 signal events and 53 background events obtained by Monte Carlo simulation.

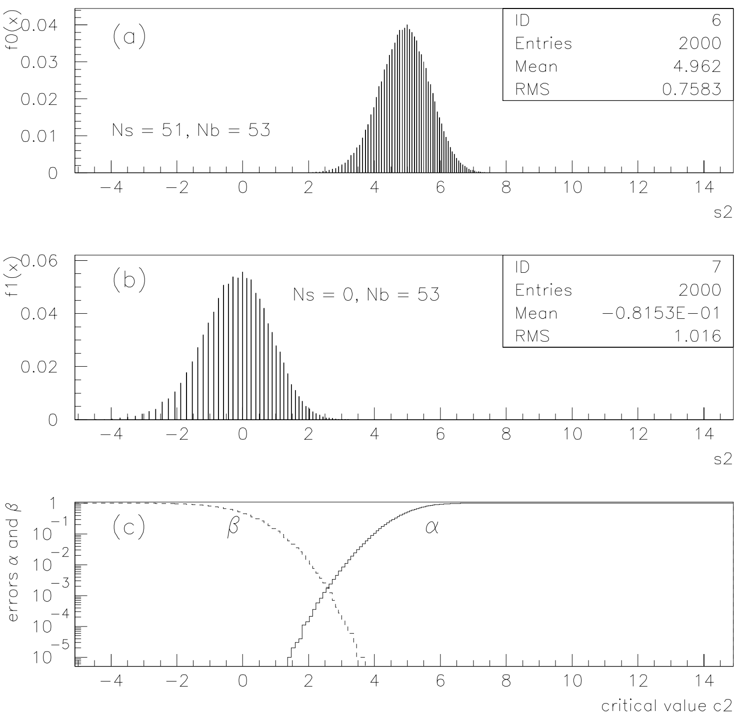

Fig. 4. The probability density functions $f_0(x)$ (a) and $f_1(x)$ (b) of statistic s_2. The dependence of Type I and Type II errors on critical value c_2 (c) for the case of 51 signal events and 53 background events.

Table 1. The comparison of power of criteria for different statistics. The values c_1, c_2, c_{12} and c are the critical values of statistics s_1, s_2, s_{12} and likelihood ratio for $\alpha = 0.01$. The values $1-\beta$ are the power for corresponding critical values.

	statistic:	s_1		s_2		s_{12}		likelihood	ratio
N_s	N_b	c_1	$1-\beta$	c_2	$1-\beta$	c_{12}	$1-\beta$	c	$1-\beta$
10	5	0.89	0.762	0.75	0.762	0.3	0.762	0.035	0.760
15		2.23	0.968	1.58	0.968	0.8	0.968	0.078	0.968
20		4.02	0.999	2.40	0.999	1.4	0.999	2.563	0.999
25		5.81	1.000	3.06	1.000	1.9	1.000	110.0	1.000
15	10	1.26	0.864	1.06	0.866	0.4	0.865	0.045	0.864
20		2.52	0.986	1.88	0.986	0.9	0.985	0.269	0.986
25		3.79	0.999	2.55	0.999	1.4	0.999	3.939	0.999
30		5.05	1.000	3.13	1.000	1.8	1.000	307.0	1.000
15	15	0.77	0.750	0.70	0.747	0.2	0.750	0.040	0.749
20		1.80	0.947	1.49	0.947	0.7	0.948	0.117	0.947
25		2.84	0.994	2.15	0.994	1.1	0.994	0.667	0.994
30		3.87	0.999	2.73	1.000	1.5	1.000	7.795	1.000
20	55	0.13	0.535	0.00	0.479	-0.1	0.483	0.052	0.536
25		0.67	0.733	0.64	0.733	0.2	0.735	0.049	0.731
30		1.21	0.873	1.12	0.874	0.4	0.843	0.074	0.873
35		1.88	0.963	1.68	0.962	0.7	0.950	0.231	0.962
40		2.42	0.989	2.10	0.988	1.0	0.988	0.512	0.989
45		2.96	0.997	2.60	0.998	1.3	0.998	2.894	0.998
50		3.64	1.000	2.98	1.000	1.5	1.000	9.957	1.000

Table 2. The dependence of α and β determined by using equal-tailed test on N_s and N_b for $S_1 = 5$. The κ is the area of intersection of probability density functions $f_0(x)$ and $f_1(x)$.

N_s	N_b	α	β	κ
5	1	0.0620	0.0803	0.1423
10	4	0.0316	0.0511	0.0828
15	9	0.0198	0.0415	0.0564
20	16	0.0141	0.0367	0.0448
25	25	0.0162	0.0225	0.0383
30	36	0.0125	0.0225	0.0333
35	49	0.0139	0.0164	0.0303
40	64	0.0114	0.0171	0.0278
45	81	0.0124	0.0136	0.0260
50	100	0.0106	0.0143	0.0245
55	121	0.0114	0.0120	0.0234
60	144	0.0100	0.0126	0.0224
65	169	0.0106	0.0109	0.0216
70	196	0.0095	0.0115	0.0209
75	225	0.0101	0.0102	0.0203
80	256	0.0091	0.0107	0.0198
85	289	0.0096	0.0097	0.0193
90	324	0.0088	0.0101	0.0189
95	361	0.0081	0.0106	0.0185
100	400	0.0086	0.0097	0.0182
150	900	0.0078	0.0084	0.0162
500	10^4	0.0068	0.0068	0.0136
5000	10^6	0.0062	0.0065	0.0125

In Table 1 the comparison result is shown. For several values of N_s and N_b (significance level $\alpha = 0.01$) [2] the critical values c_1, c_2, c_{12}, c and the corresponding values of power $1 - \beta$ of these criteria for the statistics s_1, s_2, s_{12} and the likelihood ratio are presented. As is seen from Table I there is no visible difference in the power values for the considered statistics, i.e. we can use in an equivalent manner either of these statistics for the hypotheses testing.

5. Equal-tailed test

Of concern to us is the question: What is meant by the statement that
$S_1 = \dfrac{N_s}{\sqrt{N_b}} = 5$ or $S_2 = \dfrac{N_s}{\sqrt{N_s + N_b}} = 5$?

Tables 2 and 3 give the answer to this question. In Tables 2 and 3 the values N_s and N_b corresponding to the above condition, the values α and β determined by applying equal-tailed test (in this study we use the conditions $min(\beta - \alpha)$ and $\alpha \leq \beta$) are presented. One can see the dependence of α (or β) on the value of sample. The case of $N_s = 5$ and $N_b = 1$ for S_1 (Fig.5) is perhaps the most dramatic example. We have 5σ deviation, however, if we reject the hypothesis H_0, we are mistaken in 6.2% of cases and if we accept the hypothesis H_0 we are mistaken in 8.0% of cases.

Table 3. The dependence of α and β determined by using equal-tailed test on N_s and N_b for $S_2 \approx 5$. The κ is the area of intersection of probability density functions $f_0(x)$ and $f_1(x)$.

N_s	N_b	α	β	κ
26	1	$0.519 \cdot 10^{-5}$	$0.102 \cdot 10^{-4}$	$0.154 \cdot 10^{-4}$
29	4	$0.661 \cdot 10^{-4}$	$0.764 \cdot 10^{-4}$	$0.142 \cdot 10^{-3}$
33	9	$0.127 \cdot 10^{-3}$	$0.439 \cdot 10^{-3}$	$0.440 \cdot 10^{-3}$
37	16	$0.426 \cdot 10^{-3}$	$0.567 \cdot 10^{-3}$	$0.993 \cdot 10^{-3}$
41	25	$0.648 \cdot 10^{-3}$	$0.118 \cdot 10^{-2}$	$0.172 \cdot 10^{-2}$
45	36	$0.929 \cdot 10^{-2}$	$0.193 \cdot 10^{-2}$	$0.262 \cdot 10^{-2}$
50	49	$0.133 \cdot 10^{-2}$	$0.185 \cdot 10^{-2}$	$0.314 \cdot 10^{-2}$
55	64	$0.178 \cdot 10^{-2}$	$0.179 \cdot 10^{-2}$	$0.357 \cdot 10^{-2}$
100	300	$0.317 \cdot 10^{-2}$	$0.428 \cdot 10^{-2}$	$0.735 \cdot 10^{-2}$
150	750	$0.445 \cdot 10^{-2}$	$0.450 \cdot 10^{-2}$	$0.894 \cdot 10^{-2}$

One can point out that for a good deal of events the values of α for S_1 and S_2 approach each other. A simple argument explains such dependence. The $x - N_b$ has the variation equal to $\sqrt{N_s + N_b}$ for nonzero signal events, and to $\sqrt{N_b}$ if signal events are absent. Correspondingly, if $N_b \gg N_s$, the contribution of N_s to the variation is very small. Therefore, the standard deviation tends to unity both for the distribution of s_1 (Fig.6) and for the distribution of s_2. It means that for the sufficiently large N_b, the values of α and β obtained by equal-tailed test have a constant value close to 0.0062. These distributions also can be approximated by a standard Gaussian $\mathcal{N}(0,1)$ [3] for the pure background and Gaussian $\mathcal{N}(5,1)$ for the signal mixed with the background.

[2] The conditions $min(0.01 - \alpha)$ and $\alpha \leq 0.01$ are performed.

[3] It is a conventional notation for normal distribution $\mathcal{N}$(mean,variance).

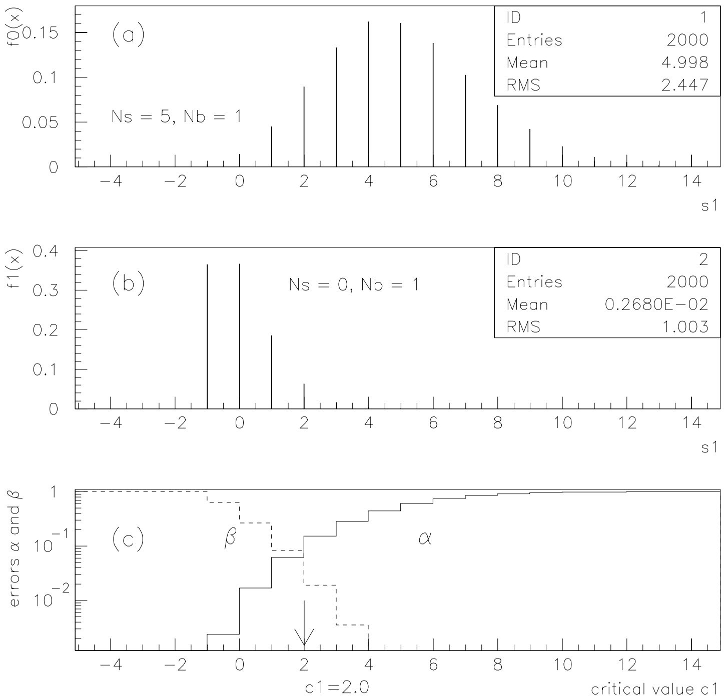

Fig. 5. The probability density functions $f_0(x)$ (a) and $f_1(x)$ (b) of statistic s_1. The dependence of Type I and Type II errors on critical value c_1 (c) for the case of 5 signal events and 1 background events.

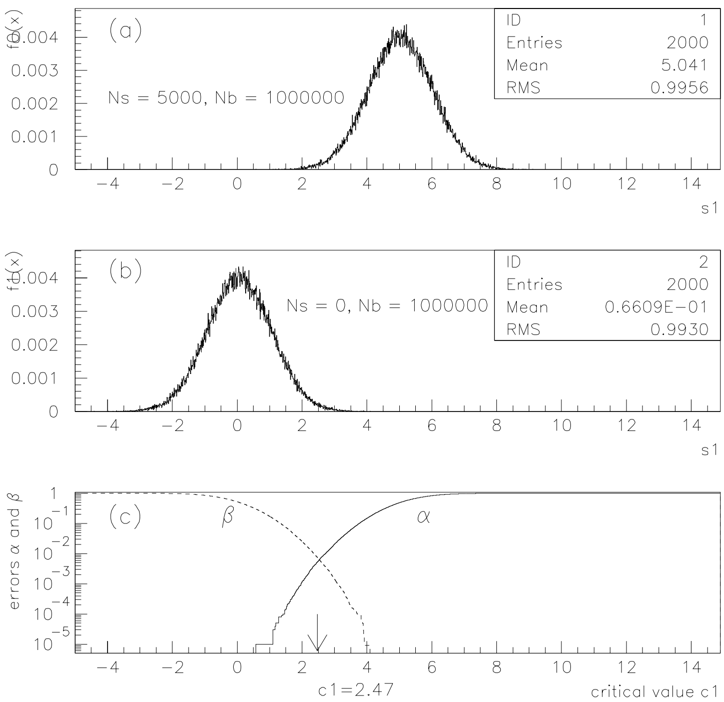

Fig. 6. The probability density functions $f_0(x)$ (a) and $f_1(x)$ (b) of statistic s_1. The dependence of Type I and Type II errors on critical value c_1 (c) for the case of 5000 signal events and 10^6 background events.

Therefore, the equal-tailed test for the normal distributions gives $c_1 = 2.5$ and $\alpha = \beta = 0.0062$. These are the limiting values of α and β for the requirement $S_1 = 5$ or $S_2 = 5$ (by the way S_{12} equals 2.5 in this case).

In a similar way we can determine the behaviour of the Type I and Type II errors depending on N_s and N_b for a small number of events and we can predict the limiting values of α and β for a large number of events in case of other statements about statistic s_1 (Table 4) or any other estimator.

Table 4. The dependence of α and β determined by using equal-tailed test on N_s and N_b for $S_1 = 2$, $S_1 = 3$, $S_1 = 4$, $S_1 = 6$ and $S_1 = 8$. The κ is the area of intersection of probability density functions $f_0(x)$ and $f_1(x)$.

S_1	N_s	N_b	α	β	κ
2	2	1	0.199	0.265	0.4634
	4	4	0.192	0.216	0.4061
	6	9	0.184	0.199	0.3817
	8	16	0.179	0.188	0.3680
	∞	∞	0.1587	0.1587	0.3174
3	3	1	0.0906	0.263	0.3184
	6	4	0.0687	0.216	0.2408
	9	9	0.0917	0.123	0.2159
	12	16	0.0722	0.131	0.1952
	∞	∞	0.0668	0.0668	0.1336
4	4	1	0.0400	0.263	0.2050
	8	4	0.0459	0.110	0.1406
	12	9	0.0424	0.0735	0.1130
	16	16	0.0407	0.0572	0.0977
	∞	∞	0.0228	0.0228	0.0456
6	6	1	0.0301	0.0806	0.1008
	12	4	0.0217	0.0217	0.0434
	18	9	0.0089	0.0224	0.0271
	24	16	0.00751	0.0132	0.0198
	∞	∞	0.00135	0.00135	0.0027
8	8	1	0.0061	0.0822	0.0402
	16	4	0.0049	0.0081	0.0131
	24	9	0.0016	0.0052	0.00567
	32	16	0.00128	0.00237	0.00331
	∞	∞	0.000032	0.000032	0.000064

Right column in Tables 2, 3 and 4 contains the value of probability κ [4]. The κ is a characteristic of the observability of Phenomenon for the given N_s and N_b. In particular, it is the fraction of p.d.f. $f_0(x)$ for statistic x that can be described by the fluctuation of background in case of the absence of Phenomenon. The value of κ equals the area of intersection of probability density functions $f_0(x)$ and $f_1(x)$ (Fig.1). Clearly, if we superimpose the p.d.f.'s $f_0(x)$ and $f_1(x)$ and choose the intersection point of curves (point $N_{ev} = [\dfrac{N_s}{ln(1+\frac{N_s}{N_b})}]$) as a critical value for the hypotheses testing [4] we have $\kappa \equiv \alpha + \beta$.

[4]Notice that in this point $f_0(N_{ev}) = f_1(N_{ev})$ (in our case conditions $min(f_0(N_{ev}) - f_1(N_{ev}))$ and

As is seen from Tables 2, 3 and 4 the value of κ is also close to the sum $\alpha+\beta$ determined by using the equal-tailed test.

The accuracy of determination of the critical value by Monte Carlo calculations depends on the number of Monte Carlo trials and on the level of significance defined by the critical value. To illustrate, Fig.7 shows the distribution of the estimations of the value $\frac{\alpha+\beta}{2}$ for the case $N_s = 100$, $N_b = 500$ and for the 10^5 Monte Carlo trials in each estimation (equal-tailed test is used). The result obtained via the direct calculations of p.d.f.'s is also shown in this Figure. Thus, this method is accurate enough to give reliable results for estimation of the discovery potential of the experiment.

The approach to the determination of the critical region in the hypotheses testing by Monte Carlo calculation of p.d.f.'s can be used to estimate the integrated luminosity which is necessary for detection the predicted effects with sufficient accuracy. In Fig.8 (a) the dependence of N_{ev} on integrated luminosity ([3], Table.12, cut.5, $m_{\chi_1} = 85\ GeV$, $N_s = 45$, $N_b = 45$) is shown. The corresponding values of α and β are presented in Fig.8 (b). As evident from Figure the integrated luminosity $L = 8\cdot 10^4 pb^{-1}$ is sufficient to detect sleptons under the requirement that the probability $\kappa \approx \alpha+\beta$ less than 1%.

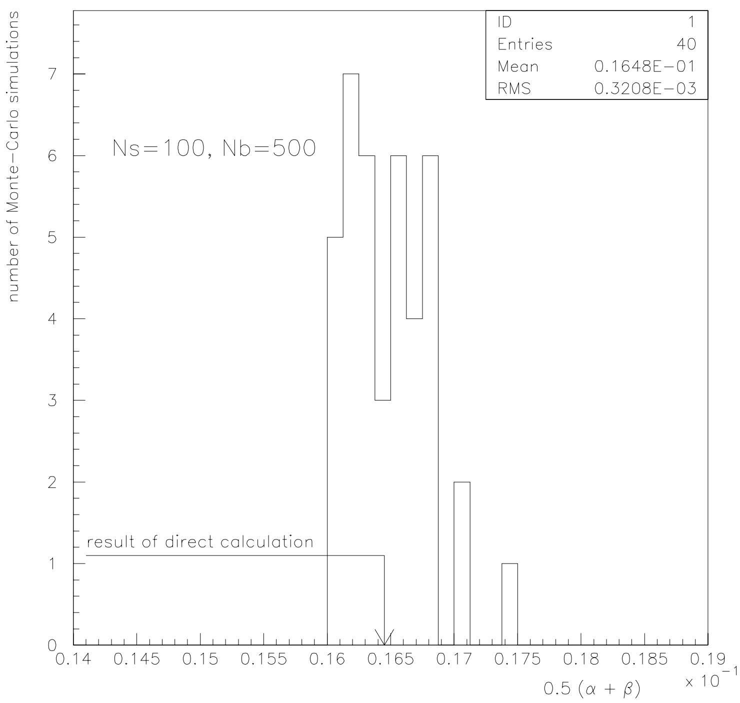

Fig. 7. The variation of $\frac{\alpha+\beta}{2}$ in the equal-tailed hypotheses testing ($N_s = 100$, $N_b = 500$ and $N_s = 0$, $N_b = 500$ in 40 Monte Carlo simulations of probability density functions).

$f_1(N_{ev}) \le f_0(N_{ev})$ are performed). By this is meant that this checking can be named as the equal probability test. Of course, if we use the hypotheses testing we can also determine N_{ev} having found the minimum of the sum of α and β or having found the minimum of the sum of weighted α and β or having exploited any other condition in accordance with the requirements of experiment. The κ may be thought of as independing of these requirements.

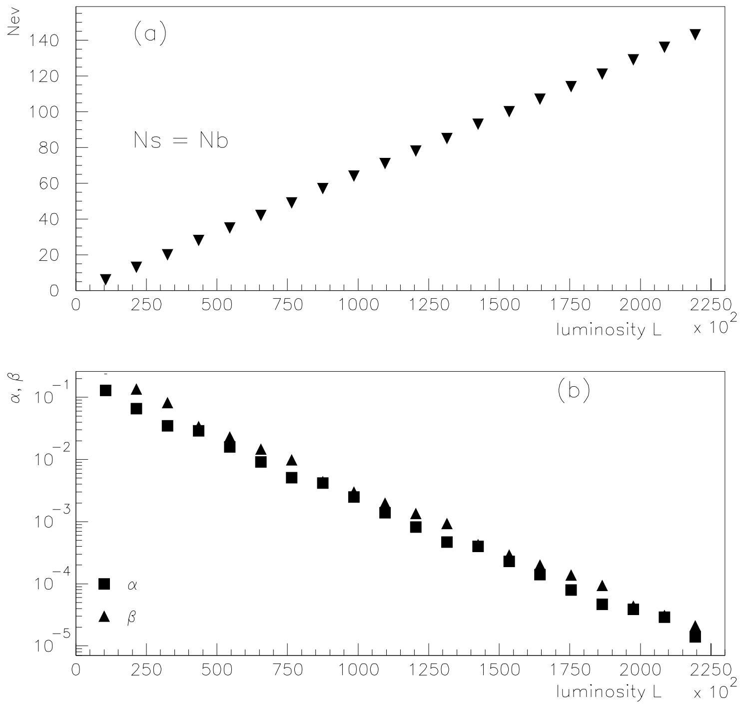

Fig. 8. The dependence of the critical value N_{ev} (a), Type I and Type II errors (b) on integrated luminosity L for the case $N_s = N_b$ and $N_s = 45$ for $L = 10^5 pb^{-1}$ (equal-tailed test).

Conclusion

In this paper the discussion on the observation of new Phenomenon is restricted to the testing of simple hypotheses in case of the predicted values N_s and N_b and the observed value x. As is stressed in [5], the precise hypothesis testing should not be done by forming a traditional confidence interval and simply checking whether or not the precise hypothesis is compatible with the confidence interval. A confidence interval [8] is usually of considerable importance in determining where the unknown parameter is likely to be, given that the alternative hypothesis is true, but it is not useful in determining whether or not a precise null hypothesis is true.

To compare several statistics used for the hypotheses testing, we employ the method that allows one to construct the rejection regions via the determination the probability density functions of these statistics by Monte Carlo calculations. As is shown, the considered statistics have close values of power for the specified significance level and can be used for the hypotheses testing in an equivalent manner. Also, it has been shown that the estimations of Type I and Type II errors obtained by this method have a reasonable accuracy. The method was used to make the inferences on the observability of some predicted phenomena.

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