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MAGNETIC CATALYSIS AND OSCILLATING EFFECTS IN THE NAMBU – JONA-LASINIO MODEL AT NONZERO CHEMICAL POTENTIAL

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Abstract

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Phase structure of the four dimensional Nambu – Jona-Lasinio model has been investigated in two cases: 1) in nonsimply connected space-time of the form $R^3 \times S^1$ (space coordinate is compactified and the length of the circle S^1 is L) with nonzero chemical potential μ and 2) in the Minkowski space-time at nonzero values of μ, H, where H is the external magnetic field. In both cases on phase portraits of the model there are infinitely many massless chirally symmetric phases as well as massive ones with spontaneously broken chiral invariance. Such phase structure leads unavoidably to oscillations of some physical parameters at $L \to \infty$ or $H \to 0$, including magnetization, pressure and particle density of the system as well as quark condensate and a critical curve of chiral phase transitions. Phase transitions of the 1st and 2nd orders and several tricritical points have been shown to exist on phase diagrams of the model.

Аннотация

Клименко К.Г. Магнитный Катализ и Осцилляционные Явления в Модели Намбу – Йона-Лазинио при ненулевом Химическом Потенциале: Препринт ИФВЭ 98-56. – Протвино, 1998. – 18 с., 3 рис., библиогр.: 30.

Исследована фазовая структура четырехмерной модели Намбу – Йона-Лазинио в двух случаях: 1) в неодносвязном пространстве-времени вида $R^3 \times S^1$ (компактифицирована пространственная координата, и окружность S^1 имеет длину L) с ненулевым химическим потенциалом μ и 2) в пространстве Минковского при ненулевых значениях μ, H, где H - внешнее магнитное поле. В обоих случаях на фазовых портретах модели существуют бесконечно много безмассовых кирально симметричных, а также массивных со спонтанно нарушенной киральной инвариантностью фаз. Такая фазовая структура приводит к осцилляциям некоторых физических параметров при $L \to \infty$ или $H \to 0$ таких, как намагниченность, давление и плотность частиц в системе, а также фермионного конденсата и критической кривой киральных фазовых переходов. Фазовые переходы первого и второго родов, а также несколько трикритических точек существуют на фазовых диаграммах модели.

1. Introduction[1]

The concept of dynamical chiral symmetry breaking (DCSB) plays an essential role in elementary particle physics and quantum field theory (QFT). In QFT this phenomenon is well observed in Nambu – Jona-Lasinio (NJL) type models – four-dimensional models with four-fermionic interactions [3,4]. The simplest one is presented by the Lagrangian

$$L_{\psi}=\sum_{k=1}^{N}\bar{\psi}_k i\hat{\partial}\psi_k+\frac{G}{2N}[(\sum_{k=1}^{N}\bar{\psi}_k\psi_k)^2+(\sum_{k=1}^{N}\bar{\psi}_k i\gamma_5\psi_k)^2], \tag{1}$$

which is invariant under continuous chiral transformations

$$\psi_k\to e^{i\theta\gamma_5}\psi_k \quad ; \quad (k=1,...,N). \tag{2}$$

(In order to apply a large N-expansion technique we use here N-fermionic version of the model.)

Since there are no closed physical systems in nature, the influence of different external factors on the DCSB mechanism is of great interest. In these realms, special attention have been given to the analysis of the vacuum structure of the NJL type models at nonzero temperature and chemical potential [5,1], in the presence of external (chromo-)magnetic fields [6,7,8], with allowance for the curvature and nontrivial space-time topology [9,2]. Combined action of external electromagnetic and gravitational fields on the DCSB effect in four-fermion field theories were investigated in [10,11].

In the present paper we consider the phase structure and related oscillating effects of the four dimensional Nambu – Jona-Lasinio model in two cases: 1) in nonsimply connected space-time of the form $R^3\times S^1$ (space coordinate is compactified) with nonzero chemical potential μ and 2) in the Minkowski space-time at nonzero values of μ, H, where H is the external magnetic field.

[1]This report is based on works done in collaboration with A.K.Klimenko, M.A.Vdovichenko, A.S.Vshivtsev and V.Ch.Zhukovskii [1,2].

1.1 NJL model at $\mu \neq 0$

First of all let us prepare the basis for investigations in the following sections and consider a phase structure of the model (1) at $\mu \neq 0$ in the Minkowski space-time.

Recall some well - known vacuum properties of the theory (1) at $\mu = 0$. The introduction of an auxiliary Lagrangian

$$\tilde{L} = \bar{\psi} i\hat{\partial}\psi - \bar{\psi}(\sigma_1 + i\sigma_2\gamma_5)\psi - \frac{N}{2G}(\sigma_1^2 + \sigma_2^2) \tag{3}$$

greatly facilitates the problem under consideration. (In (3) and other formulae below we have omitted the fermionic index k for simplicity.) Theory (3) is equivalent to the (1) for auxiliary bosonic fields $\sigma_{1,2}$ which are solutions of the equations of motion.

From (3) it follows in the leading order of $1/N$-expansion:

$$\exp(iNS_{eff}(\sigma_{1,2})) = \int D\bar{\psi}D\psi \exp(i\int \tilde{L}d^4x),$$

where

$$S_{eff}(\sigma_{1,2}) = -\int d^4x \frac{\sigma_1 + \sigma_2^2}{2G} - i\ln\det(i\hat{\partial} - \sigma_1 - i\gamma_5\sigma_2).$$

Supposing that in this formula $\sigma_{1,2}$ are independent of the space-time points, we have by definition:

$$S_{eff}(\sigma_{1,2}) = -V_0(\sigma_{1,2})\int d^4x,$$

where ($\Sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$):

$$V_0(\sigma_{1,2}) = \frac{\Sigma^2}{2G} + 2i\int \frac{d^4p}{(2\pi)^4}\ln(\Sigma^2 - p^2) \equiv V_0(\Sigma). \tag{4}$$

Introducing in (4) the Euclidean metrics ($p_0 \to ip_0$) and cutting off the range of integration ($p^2 \leq \Lambda^2$), we obtain:

$$V_0(\Sigma) = \frac{\Sigma^2}{2G} - \frac{1}{16\pi^2}\left\{\Lambda^4\ln\left(1 + \frac{\Sigma^2}{\Lambda^2}\right) + \Lambda^2\Sigma^2 - - \Sigma^4\ln\left(1 + \frac{\Lambda^2}{\Sigma^2}\right)\right\}. \tag{5}$$

The stationary equation for the effective potential (5) has the form:

$$\frac{\partial V_0(\Sigma)}{\partial\Sigma} = 0 = \frac{\Sigma}{4\pi^2}\left\{\frac{4\pi^2}{G} - \Lambda^2 + \Sigma^2\ln\left(1 + \frac{\Lambda^2}{\Sigma^2}\right)\right\} \equiv \frac{\Sigma}{4\pi^2}F(\Sigma). \tag{6}$$

Now one can easily see that at $G < G_c = 4\pi^2/\Lambda^2$ eq. (6) has no solutions apart from $\Sigma = 0$. Hence, in this case the fermions are massless, and chiral invariance (2) is not broken.

If $G > G_c$, then Eq. (6) has one nontrivial solution $\Sigma_0(G,\Lambda) \neq 0$ such that $F(\Sigma_0) = 0$. In this case Σ_0 is a point of global minimum for the potential $V_0(\Sigma)$. This means that the spontaneous breaking of the symmetry (2) takes place. Moreover, the fermions acquire mass $M \equiv \Sigma_0(G,\Lambda)$.

Let us now imagine that $\mu > 0$ and temperature $T \neq 0$. In this case one can find the an effective potential $V_{\mu T}(\Sigma)$ if the measure of integration in (4) is transformed according to the rule

$$\int \frac{dp_0}{2\pi} \to iT \sum_{n=-\infty}^{\infty}, \; p_0 \to i\pi T(2n+1) + \mu.$$

Summing there over n [12] and directing the temperature to zero in the obtained expression, we have:

$$V_\mu(\Sigma) = V_0(\Sigma) - 2\int \frac{d^3p}{(2\pi)^3}\theta(\mu - \sqrt{\Sigma^2+p^2})(\mu - \sqrt{\Sigma^2+p^2}), \tag{7}$$

where $\theta(x)$ is the step function. Integrating in (7), we find

$$V_\mu(\Sigma) = V_0(\Sigma) \; - \; \frac{\theta(\mu-\Sigma)}{16\pi^2}\left\{\frac{10}{3}\mu(\mu^2-\Sigma^2)^{3/2} - \right.$$
$$\left. -2\mu^3\sqrt{\mu^2-\Sigma^2} \; + \; \Sigma^4 \ln\left[\,(\,\mu + \sqrt{\mu^2-\Sigma^2}\,)^2/\Sigma^2\,\right]\right\}. \tag{8}$$

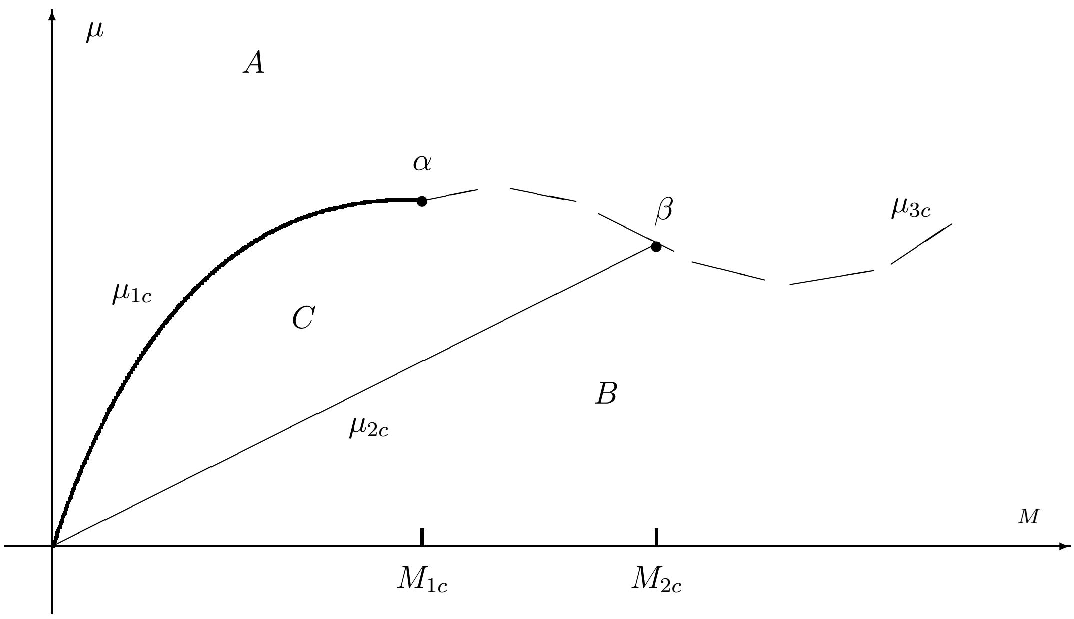

Fig.1. Phase portrait of the NJL model at nonzero μ and for arbitrary values of fermionic mass M. Phases B and C are massive nonsymmetric phases, A is chirally symmetric phase. Here $\mu_{2c} = M$, $\mu_{1c} = \sqrt{\frac{1}{2}M^2\ln(1+\Lambda^2/M^2)}$, $M_{2c} = \Lambda/(2.21...)$, M_{1c} is the solution of equation $\mu_{1c}^2(M_{1c}) = \Lambda^2/(4e)$. In phase B the particle density in the ground state is equal to zero. However, in phase C the particle density is not zero.

It follows from (8) that in the case $G < G_c$ and at arbitrary values of chemical potential chiral symmetry (2) is not broken. However, at $G > G_c$ the model has a rich phase structure, which is presented in Fig.1 in terms of μ and M. (At $G > G_c$ one can use the fermionic mass M as an independent parameter of the theory. Three quantities G, M and Λ are connected by Eq. (6).) In this Figure the solid and dashed lines represent the critical curves of the second- and first-order phase transitions, respectively. Furthermore, there are two tricritical points α and β, two massive phases B and C with spontaneously broken chiral invariance as well as the symmetric massless phase A on the phase portrait of the NJL model (for detailed calculations of the vacuum structure of the NJL model see [1]).

2. Phase structure of the NJL model at $\mu \neq 0$ and in the $R^3 \times S^1$ space-time

It is well-known that the unified theory of all forces (including gravitation) of nature has yet to be constructed. Since in the early Universe the gravity was sufficiently strong and one should take it into account, a lot of physicists study quantum field theories in space-times with nontrivial metric and topology. In this, the NJL model is the object of special attention (see review [10]), because the idea of dynamical chiral symmetry breaking is the underlying concept of elementary particle physics. There is a copious literature on this subject [9,10,11,13]. In particular, the investigation of four-fermion theories in the space-time of the form $R^d \times S^1 \times \cdots \times S^1$ is of great interest [13]. The matter is that such space-time topology occurs in superstring theories, in the description of Casimir type effects and so on.

In the present section the NJL model in the $R^3 \times S^1$ space-time and at $\mu \neq 0$ is considered since a great amount of physical phemomena take place at nonzero particle density, i.e. at nonzero chemical potential. Here the space coordinate is compactified and the circumference S^1 has the length L. For simplicity we study only the case with periodic boundary conditions: $\psi(t, x + L, y, z) = \psi(t, x, y, z)$.

2.1 Phase structure

In order to find the effective potential $V_{\mu L}(\Sigma)$ at $\mu \neq 0$ and $L \neq \infty$, we need to transform the integration over p_1 in (7) into a summation over discrete values p_{1n} according to the rule

$$\int \frac{dp_1}{2\pi} f(p_1) \to \frac{1}{L} \sum_{n=-\infty}^{\infty} f(p_{1n}); \quad p_{1n} = 2\pi n/L \ , \ n = 0, \pm 1, \pm 2, ...$$

The resulting expression is

$$V_{\mu L}(\Sigma) = V_L(\Sigma) - \frac{\lambda}{6\pi} \sum_{n=0}^{\infty} \alpha_n \theta(\mu - \sqrt{\Sigma^2 + (2\pi\lambda n)^2}\ ) \cdot \\ \cdot (\mu - \sqrt{\Sigma^2 + (2\pi\lambda n)^2}\ )^2 (\mu + 2\sqrt{\Sigma^2 + (2\pi\lambda n)^2}\ ), \tag{9}$$

where

$$V_L(\Sigma) = V_0(\Sigma) - \frac{2}{\pi^2 L}\int\limits_0^\infty dx x^2 \ln[\,1 - \exp(-L\sqrt{x^2+\Sigma^2}\,)\,], \tag{10}$$

$\alpha_n = 2 - \delta_{n0}$ and $V_0(\Sigma)$ is given in (5). The stationary equation for function (9) has the form

$$\begin{aligned} \frac{\partial V_{\mu L}(\Sigma)}{\partial \Sigma} &= \frac{\partial V_L(\Sigma)}{\partial \Sigma} + \frac{\lambda\Sigma}{\pi}\sum_{n=0}^{\infty}\alpha_n \theta(\mu - \sqrt{\Sigma^2 + (2\pi\lambda n)^2}\,)\cdot \\ & \cdot(\mu - \sqrt{\Sigma^2 + (2\pi\lambda n)^2}\,) \equiv \frac{2\Sigma}{\pi^2}\phi(\Sigma) = 0. \end{aligned} \tag{11}$$

The case $\mu = 0, \lambda \equiv 1/L > 0$. Putting μ equals zero in (9), we obtain the effective potential $V_L(\Sigma)$ (11) in the case under consideration. This function at $G > G_c$ has a global minimum point $\Sigma_0(\lambda) > 0$, which means that for all the values of $\lambda \geq 0$ chiral invariance of the model is spontaneously broken. Obviously, $\Sigma_0(\lambda) \to M$ at $\lambda \to 0$.

When $G < G_c$, $\Sigma_0(\lambda) \equiv 0$ at $\lambda < \lambda_0$ and $\Sigma_0(\lambda) > 0$ at $\lambda > \lambda_0$ (here $\frac{\pi^2}{2G} - \frac{\Lambda^2}{8} \equiv \frac{\pi^2}{6}\lambda_0^2$). In the point $\lambda = \lambda_0$ there is a second order phase transition from a symmetric to a nonsymmetric phase of the model, because at $\lambda \to \lambda_{0+}$

$$\Sigma_0(\lambda) = \frac{2}{3}\pi(\lambda - \lambda_0) + o(\lambda - \lambda_0),$$

i.e. the order parameter $\Sigma_0(\lambda)$ is a continuous function in the point $\lambda = \lambda_0$. At $\lambda \to \infty$ for all the values of coupling constant G we have

$$\Sigma_0(\lambda) \sim 2\pi\lambda(2.719...).$$

Details of above calculations and of the following ones are presented in [2].

The general case $\mu, \lambda \neq 0$. We shall find a one-to-one correspondence between the points of the plane (λ, μ) and the phase structure of initial model. It is very convinient to divide this plane into regions ω_k:

$$(\mu, \lambda) = \bigcup_{k=0}^{\infty} \omega_k; \;\; \omega_k = \{(\mu, \lambda) : 2\pi\lambda k \leq \mu < 2\pi\lambda(k+1)\}. \tag{12}$$

In ω_0 only the first term from a series in (9) is nonzero, in ω_1 only the first and the second terms are nonzero and so on. In order to obtain a phase structure, one should study step by step the global minimum point of the function $V_{\mu L}(\Sigma)$ in regions ω_0,ω_1,.... Omitting calculational details we show at once the resulting phase portraits at $G_1 \equiv (0.917...)G_c < G < G_c$ (see Fig.2) and at $G_c < G < (1.225...)G_c \equiv G_2$ (see Fig.3) as well [2].

One can see in Fig.2 only two massive nonsymmetric phases B and C. In contrast, there are infinitely many massive phases $C_k (k = 0, 1, ...)$ in Fig.3. In phase B the particle density is identically zero, but in C and in all C_k phases this quantity is not zero. In both figures there are also infinitely many symmetric massless phases $A_k (k = 0, 1, ..)$ of the NJL model.

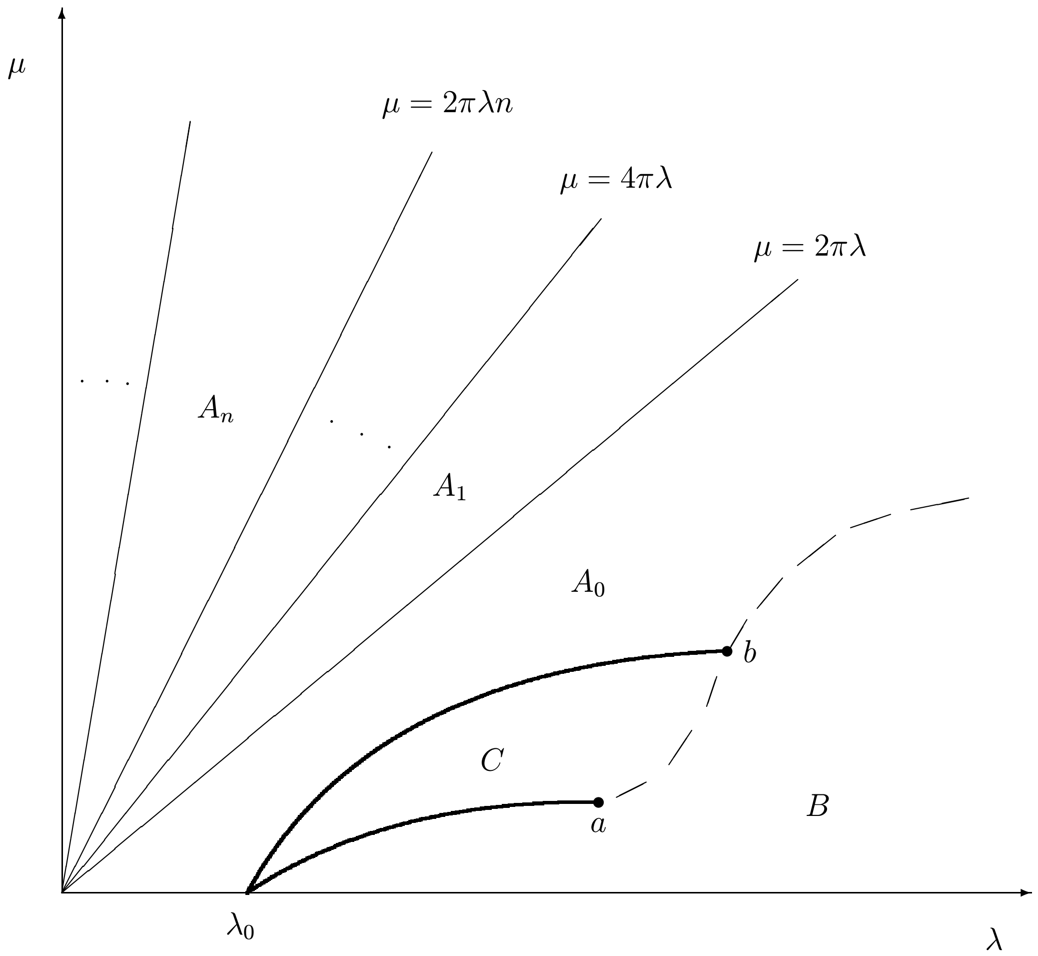

Fig.2. Phase portrait of the $R^3 \times S^1$ NJL model at $\mu \neq 0$ and $G_1 < G < G_c$ ($\lambda = 1/L$). The dashed lines are the critical curves of the first-order phase transition, the solid lines correspond to the second-order critical curves. Points a and b are the tricritical ones. There is a cascade of massless symmetric phases A_k ($k = 0, 1, 2...$).

The line $\mu_0(0)c_n c_2 c_1 b$ in Fig.3 is the critical line $\mu_c(\lambda)$ of the second-order phase transitions, where chiral symmetry is restored. The $\mu_c(\lambda)$ is defined by the equation

$$\phi(0) = 0, \qquad (13)$$

where $\phi(\Sigma)$ is given in (11). Critical lines $l_1, l_2, ...$ are the solutions of equations

$$\phi(\mu_k) \equiv \phi(\sqrt{\mu^2 - (2\pi k\lambda)^2}) = 0$$

for $k = 1, 2, ...$, respectively. Boundaries between massless phases in both figures are the boundaries between regions ω_k from (12).

Phase structure of the NJL model at other values of the coupling constant G is presented in [2], where one can also find more detailed description of above phase portraits at $G_1 < G < G_2$.

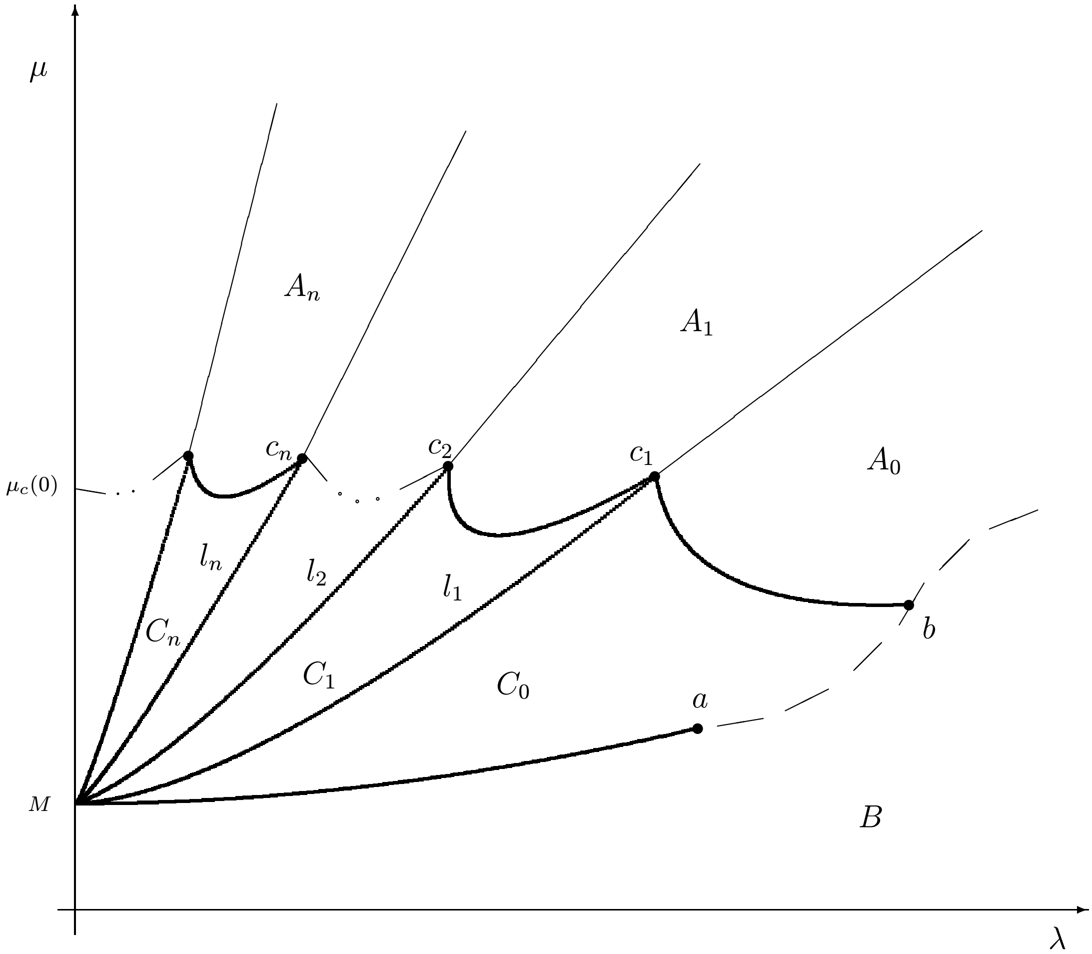

Fig.3. Phase portrait of the $R^3 \times S^1$ NJL model at $\mu \neq 0$ and $G_c < G < G_2$ ($\lambda = 1/L$). The dashed lines are the critical curves of the first-order phase transition, the solid lines correspond to the second-order critical curves. Points a and b are the tricritical points. There are cascades of massless symmetric phases A_k as well as massive phases C_k ($k = 0, 1, 2...$). The line Ma is $\mu = \Sigma_0(\lambda)$ and $\mu_c(0) = 2\pi\bar{\lambda}_0/\sqrt{6}$, where $\frac{\pi^2}{2G} - \frac{\Lambda^2}{8} \equiv -\frac{\pi^2}{6}\bar{\lambda}_0^2$.

2.1 Effects of oscillations

Now let us show that, due to the presence in a phase structure of the NJL model of cascades of massless A_k as well as massive C_k phases, one can observe oscillations of some physical parameters. We shall consider only the case $G_c < G < G_2$.

The continuous physical quantity $f(x)$ is called an oscillating one at $x \to a$, if there exist a monotonically increasing (decreasing) sequence $\{x_n\}$ such that: i) $x_n \to a$ at $n \to \infty$, ii) $f(x)$ is a continuous function at each point x_n and iii) $f'(x)$ is a discontinuous function at points x_n.

At zero temperature the oscillating quantity satisfies, as a rule, this definition (see, for example, magnetic oscillations in quantum electrodynamics [14,15]). Of course, at nonzero temperature one can observe a smoother behaviour of oscillating parameters. Since we shall deal with the zero temperature case only, the above cited definition of oscillations is well suited in the framework of the present paper.

Oscillations of the critical curve $\mu_c(\lambda)$. Recall that $\mu_c(\lambda)$ is the solution of the equation (13). Evidently, inside an arbitrary region ω_k (see (12)) this function has the form

$$\mu_c(\lambda)\Big|_{\omega_k} \equiv \mu_{(k)}(\lambda) = \frac{2\pi\{[6k(k+1)+1]\lambda^2 + \bar{\lambda}_0^2\}}{6(2k+1)\lambda}, \tag{14}$$

where $\bar{\lambda}_0$ is given in the caption to Fig.3. Hence,

$$\mu_c(\lambda) = \mu_{(k)}(\lambda) \qquad \text{at} \qquad t_{k+1} \le \lambda \le t_k \quad , \quad k = 1, 2, 3, ..., \tag{15}$$

where t_k is such a value of parameter λ, that the curve $\mu_c(\lambda)$ crosses the line $\mu = 2\pi k\lambda$, i.e. the right boundary of ω_k:

$$t_k = \frac{\bar{\lambda}_0}{\sqrt{6k^2-1}}. \tag{16}$$

Note, $\mu_{(k)}(t_k) = \mu_{(k-1)}(t_k)$, so the function $\mu_c(\lambda)$ (15) is a continuous one at $\lambda > 0$. It follows also from (15) that

$$\frac{d\mu_{(k-1)}(\lambda)}{d\lambda}\Bigg|_{\lambda\to t_{k+}} = \frac{\pi(2-6k)}{3(2k-1)} < 0,$$

$$\frac{d\mu_k(\lambda)}{d\lambda}\Bigg|_{\lambda\to t_{k-}} = \frac{\pi(2+6k)}{3(2k+1)} > 0.$$

the last inequalities mean that at an infinite set of points t_k $(k = 1, 2, ...)$ the function $\mu_c(\lambda)$ is not differentiated. According to the above given definition, the critical curve $\mu_c(\lambda)$ oscillates at $\lambda \to 0$ or, equivalently, at $L \to \infty$ (see Fig.3).

Finally, let us present this oscillations of the $\mu_c(\lambda)$ in a manifest form. We need the following Poisson summation formula [16]:

$$\sum_{n=0}^{\infty} \alpha_n \Phi(n) = 2\sum_{k=0}^{\infty} \alpha_k \int_0^{\infty} \Phi(x)\cos(2\pi kx)dx, \tag{17}$$

where $\alpha_n = 2 - \delta_{n0}$. Using it in equation (13), one can easily find at $\lambda \to 0$:

$$\mu_c(\lambda) \approx \frac{2\pi\bar{\lambda}_0}{\sqrt{6}}\left\{1 + \frac{3\lambda^2}{\pi^2\bar{\lambda}_0^2}\sum_{n=1}^{\infty}\frac{\cos(n\pi\bar{\lambda}_0 L/\sqrt{6})}{n^2}\right\}, \tag{18}$$

From (18) it follows that $\mu_c(\lambda)$ has an oscillating part, which oscillates at $L \to \infty$ with frequency $\bar{\lambda}_0/(2\sqrt{6})$.

Oscillations of the fermionic condensate. Fermionic condensate is defined as $<\bar{\psi}\psi>$, and in the NJL model it is proportional to $<\Sigma>$. Since the last quantity is the global minimum point $\Sigma(\mu, \lambda)$ of an effective potential, we should study nontrivial solution of the stationary equation (11). A detailed analysis of $\Sigma(\mu,\lambda)$ was carried out in [2], and this quantity at $M<\mu<\mu_c(0)$ and at $\lambda\to 0$ ($L\to\infty$) behaves as

$$\Sigma(\mu,\lambda)=m(\mu)+\frac{\lambda^2\sqrt{\mu^2-m^2(\mu)}}{\mu f'(m(\mu))}\sum_{n=1}^{\infty}\frac{\cos(n\sqrt{\mu^2-m^2(\mu)}\ L)}{n^2}+o(\lambda^2), \qquad (19)$$

where $m(\mu)$ equals $\Sigma(\mu,0)$, $f'(m)$ is the derivative of the function

$$f(m)\equiv F(m)+\frac{\mu}{4}\sqrt{\mu^2-m^2}-\frac{m^2}{4}\ln\left(\frac{\mu+\sqrt{\mu^2-m^2}}{m}\right), \qquad (20)$$

and $F(\Sigma)$ is given in (6). It is clear from (19) that $\Sigma(\mu,\lambda)$ has an oscillating part, which oscillates at $L\to\infty$ with frequency $\sqrt{\mu^2-m^2(\mu)}/(2\pi)$.

Oscillations of the particle density. Suppose, $M<\mu<\mu_c(0)$. Then the thermodynamic potential (TDP) $\Omega(\mu,\lambda)$ of the NJL system is equal to the value of its effective potential at the global minimum point $\Sigma(\mu,\lambda)$, i.e. $\Omega(\mu,\lambda)=V_{\mu L}(\Sigma(\mu,\lambda)$. It is well-known that the thermodynamic potential defines the particle density $n(\mu,\lambda)$ through the relation: $n(\mu,\lambda)=-\partial\Omega(\mu,\lambda)/\partial\mu$. Hence,

$$n(\mu,\lambda)=-\left\{\frac{\partial V_{\mu L}(\Sigma)}{\partial\mu}+\frac{\partial V_{\mu L}(\Sigma)}{\partial\Sigma}\frac{\partial\Sigma}{\partial\mu}\right\}\Bigg|_{\Sigma=\Sigma(\mu,\lambda)}$$

$$=\frac{\lambda}{2\pi}\sum_{n=0}^{\infty}\alpha_n\Theta(\mu-\sqrt{\Sigma^2(\mu,\lambda)+(2\pi\lambda n)^2}\ )(\mu^2-\Sigma^2(\mu,\lambda)-(2\pi\lambda n)^2\ ). \qquad (21)$$

Using in (21) the Poisson summation formula (17) [2], we see that at $\mu=const$ and $L\to\infty$

$$n(\mu,\lambda)=\frac{(\mu^2-m^2(\mu))^{3/2}}{3\pi^2}+\lambda^2\left[\frac{m(\mu)(\mu^2-m^2(\mu))}{\mu f'(m(\mu))}-2\sqrt{\mu^2-m^2(\mu)}\right]\cdot$$

$$\cdot\sum_{n=1}^{\infty}\frac{\cos(n\sqrt{\mu^2-m^2(\mu)}L)}{\pi^2n^2}+o(\lambda^2), \qquad (22)$$

where $m(\mu)$ is the fermion mass at $\lambda=1/L=0$ (see (19)), $f(m)$ is defined in (20). From (22) one can easily see that the particle density in the ground state of the NJL model oscillates with frequency $\sqrt{\mu^2-m^2(\mu)}/(2\pi)$.

Oscillations of the pressure. Let us suppose that $\mu>\mu_c(0)$. Then for the sufficiently large values of L the global minimum point of the effective potential equals zero. In this case the TDP of the model is equal to $V_{\mu L}(0)$. So, at $L\to\infty$ the TDP $\Omega(\mu,\lambda)$ oscillates with frequency $\mu/(2\pi)$, because it looks like [2]:

$$\Omega(\mu,\lambda)=V_L(0)-\frac{\mu^4}{12\pi^2}-\sum_{k=0}^{\infty}\left[\frac{4\lambda^4}{\pi^2k^4}-\frac{2\mu\lambda^3}{\pi^2k^3}\sin(\mu kL)-\frac{4\lambda^4}{\pi^2k^4}\cos(\mu kL)\right].$$

In our case the pressure in the system is defined as $p = -\partial(L\Omega)/\partial L$. Using the above expression for $\Omega(\mu, \lambda)$, we see that the pressure in the vacuum of the NJL model also oscillates with frequency $\mu/(2\pi)$.

One can interpret the case under consideration as the ground state of the NJL system, located between two parallel plates with periodic boundary conditions. The force which acts on each of plates is known as a generalized Casimir force. Evidently, this force is proportional to the pressure in the ground state of the system. Hence, at a nonzero chemical potential the Casimir force of the constrained fermionic vacuum oscillates at $L \to \infty$.

3. Phase structure of the NJL model at $\mu \neq 0$ and in the presence of external magnetic field

In the present section we shall study the magnetic properties of the NJL vacuum. At $\mu = 0$ this problem was considered in [6,8]. It was shown in [6] that at $G > G_c$ the chiral symmetry is spontaneously broken for arbitrary values of external magnetic field H, and even for $H = 0$. At $G < G_c$ the NJL system has a symmetric vacuum at $H = 0$. However, if the external (arbitrary small) magnetic field is switched on, then for all $G \in (0, G_c)$ one has a spontaneous breaking of initial symmetry [8]. This is the so called effect of dynamical chiral symmetry breaking catalysis by external magnetic field.

The brief history of this effect is the following: First of all, such property of external magnetic field was discovered in (2+1) - dimensional Gross-Neveu (3DGN) model [17,18]. At $H = 0$ there exist two phases in the 3DGN model: one of which is a massless chirally invariant phase ($G < G_c$), and the other is a massive phase with spontaneously broken chiral symmetry ($G > G_c$). However, for each value of $H \neq 0$ as well as for all the values of the bare coupling constant $G > 0$, the symmetric phase is absent in the 3DGN theory, and the chiral symmetry is broken down [17,18] [2]. Of course, in [17,18] special consideration was taken for the case $G < G_c$, where the magnetic field induces the DCSB even for the weakest attractive interaction between fermions (the magnetic catalysis of DCSB). It turns out, that the external chromomagnetic field is also a magnetic catalyst of DCSB [19]. The influence of temperature and chemical potential on this effect has been studied in [18]-[21], where the restoration of chiral symmetry at sufficiently large values of T and μ was predicted. Later, in [22] the explanation of this phenomenon on the basis of dimensional reduction mechanism was found in the framework of 3DGN model. The magnetic catalysis takes place in the four-dimensional NJL model [8,23,24] as well as in other theories, and now it is under intensive consideration (see, e.g. [10,25,26] and references therein).

(Authors of [24,26] declare that in our paper [18] only a "fact that external magnetic field enhances a fermion dynamical mass" was established. Hence, they assert that in

[2] The consideration in [18] is performed in terms of parameter g, such that $\frac{1}{g} = \frac{1}{g(m)} - \frac{2m}{\pi}$, where m is a normalization point, $g(m)$ is a renormalized coupling constant. The connection between g and G was established in [19]: $\frac{1}{g} = \frac{1}{G} - \frac{1}{G_c}$. So, at $g < 0$ and $g > 0$, one has $G > G_c$ and $G < G_c$, respectively.

[18] only the case $G > G_c$ was considered. In fact, in [17,18,19] the action of external (chromo-)magnetic field on the 3DGN model was studied for the arbitrary values of bare coupling constant. The spontaneous breakdown of chiral symmetry was found there for all $G \in (0, \infty)$, including the case $G < G_c$, and even the case of arbitrary small values of $G > 0$ (the magnetic catalysis of DCSB).)

In the present section we continue the investigation of magnetic catalysis effect and this time turn to the consideration of the four-dimensional NJL model at $H, \mu \neq 0$.

3.1 Magnetic catalysis at $\mu \neq 0$

Let us recall some aspects of the problem at $\mu = 0$. Using a well-known proper-time method [27] or momentum-space calculations [28], one can find the effective potential $V_H(\Sigma)$ of the NJL model at $H \neq 0$:

$$V_H(\Sigma) = \frac{\Sigma^2}{2G} + \frac{eH}{8\pi^2}\int_0^\infty \frac{ds}{s^2}\exp(-s\Sigma^2)\ \coth(eHs).$$

After identical transformations we have

$$V_H(\Sigma) = V_0(\Sigma) + \tilde{V}_H(\Sigma) + Z(\Sigma), \tag{23}$$

where

$$\begin{aligned} V_0(\Sigma) &= \frac{\Sigma^2}{2G} + \frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3}\exp(-s\Sigma^2), \\ Z(\Sigma) &= \frac{(eH)^2}{24\pi^2}\int_0^\infty \frac{ds}{s}\exp(-s\Sigma^2), \\ \tilde{V}_H(\Sigma) &= \frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3}\exp(-s\Sigma^2)\left[\ (eHs)\coth(eHs) - 1 - \frac{(eHs)^2}{3}\ \right]. \end{aligned} \tag{24}$$

The potential $V_0(\Sigma)$ in (24) up to an infinite additive constant is equal to function (4). Hence, the UV-regularized expression for it looks like (5).

The function $Z(\Sigma)$ is also an UV-divergent one, so we need to regularize it:

$$\begin{aligned} Z(\Sigma) &= \frac{(eH)^2}{24\pi^2}\int_0^\infty \frac{ds}{s}(\exp(-s\Sigma^2) - \exp(-s\Lambda^2)) + \frac{(eH)^2}{24\pi^2}\int_0^\infty \frac{ds}{s}\exp(-s\Lambda^2) \\ &= -\frac{(eH)^2}{24\pi^2}\ln\frac{\Sigma^2}{\Lambda^2} + \frac{(eH)^2}{24\pi^2}\int_0^\infty \frac{ds}{s}\exp(-s\Lambda^2). \end{aligned} \tag{25}$$

The last infinite term in (25) contributes to the renormalization of an electric charge and magnetic field as well, similar as it occures in quantum electrodynamics [27].

The potential $\tilde{V}_H(\Sigma)$ in (25) has no UV divergences, so it is easily calculated with the help of the table of integrals [29]. The final expression for $V_H(\Sigma)$ is:

$$V_H(\Sigma) = V_0(\Sigma) - \frac{(eH)^2}{2\pi^2}\Big\{\zeta'(-1, x) - \frac{1}{2}[x^2 - x]\ln x + \frac{x^2}{4}\Big\}, \tag{26}$$

where $x = \Sigma^2/(2eH)$, $\zeta(\nu, x)$ is the generalized Riemann zeta-function and $\zeta'(-1, x) = d\zeta(\nu, x)/d\nu|_{\nu=-1}$. The global minimum point of this function is the solution of the stationary equation:

$$\frac{\partial}{\partial\Sigma}V_H(\Sigma) = \frac{\Sigma}{4\pi^2}\{F(\Sigma) - I(\Sigma)\} = 0, \tag{27}$$

where $F(\Sigma)$ is given in (6), and

$$\begin{aligned} I(\Sigma) &= 2eH\{\ln\Gamma(x) - \frac{1}{2}\ln(2\pi) + x - \frac{1}{2}(2x-1)\ln x\} \\ &= \int_0^\infty \frac{ds}{s^2}\exp(-s\Sigma^2)[\, eHs\coth(eHs) - 1]. \end{aligned} \tag{28}$$

For the arbitrary fixed values of H, G there is only one nontrivial solution $\Sigma_0(H)$ of equation (27), which is the global minimum point of $V_H(\Sigma)$.

Hence, at $G < G_c$ and $H = 0$ the NJL vacuum is chirally symmetric one, but an arbitrary small value of external magnetic field H induces the DCSB, and fermions acquire nonzero mass $\Sigma_0(H)$ (the effect of magnetic catalysis of DCSB).

In the present paper we shall consider only the case $G < G_c$. Therein, $\Sigma_0(H)$ is a monotonically increasing function versus H. Besides, at $H \to \infty$

$$\Sigma_0(H) \approx \frac{eH}{\pi}\sqrt{\frac{G}{12}} \tag{29}$$

and at $H \to 0$

$$\Sigma_0^2(H) \approx \frac{eH}{\pi}\exp\{-\frac{1}{eH}(\frac{4\pi^2}{G} - \Lambda^2)\}. \tag{30}$$

Now let us consider a more general case, when $H \neq 0, \mu \neq 0$. In one of our prevoius papers [21] the effective potential of a 3DGN model at nonzero H, μ and T was obtained. Similarly, one can find an effective potential in the NJL model at $H, T, \mu \neq 0$:

$$V_{H\mu T}(\Sigma) = V_H(\Sigma) - \frac{TeH}{4\pi^2}\sum_{k=0}^{\infty}\alpha_k\int_{-\infty}^{\infty} dp\ln\left\{\left[1 + \exp^{-\beta(\varepsilon_k+\mu)}\right]\left[1 + \exp^{-\beta(\varepsilon_k-\mu)}\right]\right\}, \tag{31}$$

where $\beta = 1/T$, $\alpha_k = 2 - \delta_{0k}$, $\varepsilon_k = \sqrt{\Sigma^2 + p^2 + 2eHk}$, and a function $V_H(\Sigma)$ is given in (26). With the temperature in (31) tending to the zero, we have the effective potential of NJL model at $H, \mu \neq 0$:

$$V_{H\mu}(\Sigma) = V_H(\Sigma) - \frac{eH}{4\pi^2}\sum_{k=0}^{\infty}\alpha_k\int_{-\infty}^{\infty} dp(\mu - \varepsilon_k)\theta(\mu - \varepsilon_k), \tag{32}$$

which can be easily transformed to the form

$$V_{H\mu}(\Sigma) = V_H(\Sigma) - \frac{eH}{4\pi^2}\sum_{k=0}^{\infty}\alpha_k\theta(\mu - s_k)\left\{\mu\sqrt{\mu^2 - s_k^2} - s_k^2\ln\left[\frac{\mu + \sqrt{\mu^2 - s_k^2}}{s_k}\right]\right\}, \tag{33}$$

where $s_k = \sqrt{\Sigma^2 + 2eHk}$. Finally, let us present the stationary equation for the potential (33):

$$\frac{\partial}{\partial \Sigma} V_{H\mu}(\Sigma) = \frac{\Sigma}{4\pi^2}\left\{F(\Sigma) - I(\Sigma) + 2eH\sum_{k=0}^{\infty}\alpha_k\theta(\mu - s_k)\ln\left[\frac{\mu + \sqrt{\mu^2 - s_k^2}}{s_k}\right]\right\} = 0. \quad (34)$$

In order to get a phase portrait of the model, one should find a one-to-one correspondence between points of the (μ, H)-plane and global minimum points of function (33), i.e. we need to solve equation (34), find a global minimum $\Sigma(\mu, H)$ for potential (33), to study properties of $\Sigma(\mu, H)$ versus (μ, H).

In order to greatly simplify this problem, let us divide the plane (μ, H) into a set of regions ω_k:

$$(\mu, H) = \bigcup_{k=0}^{\infty}\omega_k; \;\; \omega_k = \{(\mu, H) : 2eHk \leq \mu^2 \leq 2eH(k+1)\}. \quad (35)$$

In the region ω_0 only a first term is nonzero from a series in (34-35). So, one can find that for the points $(\mu, H) \in \omega_0$ which are above the line L=$\{(\mu, H) : \mu = \Sigma_0(H)\}$, the global minimum is at the point $\Sigma = 0$. Just under the curve L the point $\Sigma = \Sigma_0(H)$ is a local minimum of the potential (33), and $\Sigma = \Sigma_0(H)$ transforms to the global minimum when (μ, H) lies under the critical curve of the first order phase transitions $\mu = \mu_c(H)$, which is defined by the following equation:

$$V_{H\mu}(0) = V_{H\mu}(\Sigma_0(H)). \quad (36)$$

In the region ω_0 one can easily solve this equation:

$$\mu_c(H) = \frac{2\pi}{\sqrt{eH}}[V_H(0) - V_H(\Sigma_0(H))]^{1/2}. \quad (37)$$

Hence, we have shown that at $\mu > \mu_c(H)$ ($G < G_c$) there is a massless symmetric phase of the NJL model (numerical investigations of (33-34) give us the zero global minimum point for the potential $V_{H\mu}(\Sigma)$ in other regions $\omega_1, \omega_2, ...$ as well). The external magnetic field ceases to induce the DCSB at $\mu > \mu_c(H)$ (or at sufficiently small values of magnetic field $H < H_c(\mu)$, where $H_c(\mu)$ is the inverse function to $\mu_c(H)$). But, under the critical curve (37) (or at $H > H_c(\mu)$) due to the presence of external magnetic field the chiral symmetry is spontaneously broken. Here magnetic field induces dynamical fermion mass $\Sigma_0(H)$, which is not μ-dependent value.

At last, we should remark that in the NJL model the magnetic catalysis effect takes place only in the phase with zero particle density, i.e. at $\mu < \mu_c(H)$. If $\mu > \mu_c(H)$, we have a symmetric phase with nonzero particle density, but here the magnetic field cannot induce DCSB.

3.2 Magnetic oscillations

In a previous case we have shown that points (μ, H), which are above the critical curve $\mu = \mu_c(H)$, correspond to the chirally symmetric ground state of the NJL model. One-fermionic excitations of such a vacuum have zero masses. At first sight, properties of this symmetric vacuum slightly vary, when parameters μ and H change. However, this is not the case and in the region $\mu > \mu_c(H)$ we have infinitely many massless symmetric phases of the theory as well as a variety of critical curves of the second order phase transitions. In the experiment this cascade of phases is identified with oscillations of such physical quantities as magnetization and particle density. Let us prove it.

It is well-known that the state of the thermodynamic equilibrium ($\equiv$ the ground state) of arbitrary quantum system is described by the thermodynamic potential (TDP) Ω, which is a value of the effective potential in its global minimum point. In the case under consideration the TDP $\Omega(\mu, H)$ at $\mu > \mu_c(H)$ has the form

$$\Omega(\mu, H) \equiv V_{H\mu}(0) = V_H(0) -$$
$$-\frac{eH}{4\pi^2}\sum_{k=0}^{\infty}\alpha_k\theta(\mu-\epsilon_k)\{\,\mu\sqrt{\mu^2-\epsilon_k^2}-\epsilon_k^2\ln[(\sqrt{\mu^2-\epsilon_k^2}+\mu)/\epsilon_k]\,\}, \tag{38}$$

where $\epsilon_k = \sqrt{2eHk}$. We shall use the following criterion of the phase transitions: If, at least, one first (second) partial derivative of $\Omega(\mu, H)$ is a discontinuous function at a point, then it is a point of the first (second) order phase transition.

Using this criterion let us show that the boundaries of ω_k regions (35), i.e. lines $l_k = \{(\mu, H) : \mu = \sqrt{2eHk}\}$ $(k = 1, 2, ...)$, are the critical lines of second order phase transitions. In the arbitrary region ω_k the TDP (38) has the form:

$$\Omega(\mu, H)\Big|_{\omega_k} \equiv \Omega_k = V_H(0) -$$
$$-\frac{eH}{4\pi^2}\sum_{i=0}^{k}\alpha_i\theta(\mu-\epsilon_i)\left\{\,\mu\sqrt{\mu^2-\epsilon_i^2}-\epsilon_i^2\ln\left[\frac{(\sqrt{\mu^2-\epsilon_i^2}+\mu)}{\epsilon_i}\right]\right\}. \tag{39}$$

From (39) one can easily find

$$\frac{\partial\Omega_k}{\partial\mu}\Bigg|_{(\mu,H)\to l_{k+}} - \frac{\partial\Omega_{k-1}}{\partial\mu}\Bigg|_{(\mu,H)\to l_{k-}} = 0, \tag{40}$$

as well as:

$$\frac{\partial^2\Omega_k}{(\partial\mu)^2}\Bigg|_{(\mu,H)\to l_{k+}} - \frac{\partial^2\Omega_{k-1}}{(\partial\mu)^2}\Bigg|_{(\mu,H)\to l_{k-}} = -\frac{eH\mu}{2\pi^2\sqrt{\mu^2-\epsilon_k^2}}\Bigg|_{\mu\to\epsilon_{k+}} \to -\infty. \tag{41}$$

Equality (40) means that the first derivative $\partial\Omega/\partial\mu$ is a continuous function on all lines l_k. However, the second derivative $\partial^2\Omega/(\partial\mu)^2$ has an infinite jump on each line l_k (see (41)), so these lines are the critical curves of the second order phase transitions. (Similarly, one can prove the discontinuity of $\partial^2\Omega/(\partial H)^2$ and $\partial^2\Omega/\partial\mu\partial H$ on all lines l_n.)

Let the chemical potential be fixed, i.e. $\mu = const$. Then on the plane (μ, H) we have a line, that crosses critical lines $l_1, l_2, ...$ at points $H_1, H_2, ...$ correspondingly. The particle density n and the magnetization m of any thermodynamic system are defined by the TDP in the following way: $n = -\partial\Omega/\partial\mu$, $m = -\partial\Omega/\partial H$. At $\mu = const$ these quantities are continuous functions over external magnetic field only, i.e. $n \equiv n(H)$, $m \equiv m(H)$. We know that all the second derivatives of $\Omega(\mu, H)$ are discontinuous on every critical line l_n. So, functions $n(H)$ and $m(H)$, continuous on the interval $H \in (0, \infty)$, have derivatives broken on infinite set of points $H_1, ..., H_k, ...$. According to the definition given in section 2.1, the particle density and magnetization oscillate at $H \to 0$.

In order to present oscillating parts of $n(H)$ and $m(H)$ in a manifest analytical form, we shall use the technique elaborated in [15], where a manifest analytical expression was found for oscillating part of $\Omega(\mu, H)$ for perfect relativistic electron-positron gas. This technique can be used without any difficulties in our case as well.

Hence, one can rewrite the TDP (38) in the following form:

$$\Omega(\mu, H) = \Omega_{mon}(\mu, H) + \Omega_{osc}(\mu, H), \tag{42}$$

where ($\nu = \mu^2/(eH)$):

$$\Omega_{mon} = V_H(0) - \frac{\mu^4}{12\pi^2} - \frac{(eH)^2}{4\pi^3}\int_0^\nu dy \sum_{k=1}^{\infty} \frac{1}{k} P(\pi k y), \tag{43}$$

$$\Omega_{osc} = \frac{\mu}{4\pi^{3/2}} \sum_{k=1}^{\infty} \left(\frac{eH}{\pi k}\right)^{3/2} [Q(\pi k\nu)\cos(\pi k\nu + \pi/4) + P(\pi k\nu)\cos(\pi k\nu - \pi/4)]. \tag{44}$$

(To find (43-44) it is sufficient to bring the electronic mass to zero in formula (19) from [15].) Functions $P(x)$ and $Q(x)$ in (43-44) are connected with Fresnel's integrals $C(x)$ and $S(x)$ [30]:

$$\begin{aligned} C(x) &= \frac{1}{2} + \sqrt{\frac{x}{2\pi}}[P(x)\sin x + Q(x)\cos x] \\ S(x) &= \frac{1}{2} - \sqrt{\frac{x}{2\pi}}[P(x)\cos x - Q(x)\sin x]. \end{aligned}$$

They have at $x \to \infty$ the following asymptotics [30]:

$$P(x) = x^{-1} - \frac{3}{4}x^{-3} + ..., \quad Q(x) = -\frac{1}{2}x^{-2} + \frac{15}{8}x^{-4} + ...$$

Formula (44) presents the exact oscillating part of the TDP (38) for the NJL model at $G < G_c$. Since in the present case the TDP is propotional to the pressure of the system, one can conclude that the pressure in the NJL model oscillates, when $H \to 0$. It follows from (44) that frequency of oscillations at large values of a parameter $(eH)^{-1}$ equals $\mu^2/2$. Then, starting from (44) one can easily find a manifest expression for oscillating parts of $n(H)$ and $m(H)$. These quantities oscillate at $H \to 0$ with the same frequency $\mu^2/2$ and have rather involved form, so we do not present it here.

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