



STATE RESEARCH CENTER OF RUSSIA
INSTITUTE FOR HIGH ENERGY PHYSICS

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V.V.Kiselev, A.I.Onishchenko

**TWO-LOOP ANOMALOUS DIMENSIONS FOR CURRENTS
OF BARYONS WITH TWO HEAVY QUARKS IN NRQCD**

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Abstract

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We present analytical results on the two-loop anomalous dimensions of currents for baryons, containing two heavy quarks $J = [Q^{iT} C \Gamma \tau Q^j] \Gamma' q^k \varepsilon_{ijk}$ with arbitrary Dirac matrices Γ and Γ' in the framework of NRQCD in the leading order over both the relative velocity of heavy quarks and the inverse heavy quark mass. It is shown, that in this approximation the anomalous dimensions do not depend on the Dirac structure of the current under consideration.

Аннотация

Киселев В.В., Онищенко А.И. Двухпетлевые аномальные размерности для токов барионов с двумя тяжелыми кварками в NRQCD: Препринт ИФВЭ 98-64. – Протвино, 1998. – 13 с., 1 рис., библиогр.: 22.

Представлены аналитические результаты для двухпетлевых аномальных размерностей токов барионов, содержащих два тяжелых кварка $J = [Q^{iT} C \Gamma \tau Q^j] \Gamma' q^k \varepsilon_{ijk}$ с произвольными дираковскими матрицами Γ и Γ' в рамках NRQCD в ведущем порядке как по относительной скорости тяжелых кварков, так и по их обратной массе. Показано, что в этом приближении аномальные размерности не зависят от дираковской структуры рассматриваемых токов.

Introduction

The necessary feature of QCD applications to various fields of particle physics is the study of a scale dependence for operators as it is governed by the renormalization-group (RG). In the present paper we investigate the RG properties of currents for baryons with two heavy quarks in the framework of Non-Relativistic Quantum Chromodynamics (NRQCD) [1],[2] and its dimensionally regularized version [3]. In the two-loop approximation we analytically calculate the anomalous dimensions of currents associated with the ground-state baryons, containing two heavy quarks.¹ The dependence of QCD operators and matrix elements on the relative velocity v of heavy quarks inside the hadron as well as on the inverse heavy quark mass $1/M_Q$ can be systematically treated in the framework of justified effective expansions in QCD. So, we apply the expansion in $1/M_Q$, as it was developed in Heavy Quark Effective Theory (HQET) [7,8,9] for operators, corresponding to the interaction of heavy quarks with the light quark. For the heavy-heavy subsystem, a powerful tool is the NRQCD-expansion in both the relative velocity and the inverse mass. Here we consider the leading order in both v and $1/M_Q$, which can serve as a good approximation for the anomalous dimensions of currents under consideration.

The anomalous dimensions of composite operators can be desirably used in QCD sum rules [10], which will allow us to evaluate the masses of these baryons together with their residues in terms of basic non-perturbative QCD parameters. For example, calculating the two-point correlators of baryonic currents in the Operator Product Expansion (OPE) in NRQCD, we have to insert the anomalous dimensions, obtained here in the static approximation, to relate the result to QCD.

This procedure is caused by a different ultraviolet behavior of loop corrections in the full QCD and the effective theory. The latter contains the divergences absent in QCD, since it was constructed in the way to provide correct infrared properties of local QCD fields. The regularized quantities of effective theory depend on the normalization point under the RG equations with the corresponding anomalous dimensions. The ambiguity in the initial conditions of such differential equations is eliminated by the matching to the full QCD at a scale, which is generally chosen as the heavy quark mass. The latter

¹We do not consider the problems concerning the spectroscopy, decays and production mechanisms of baryons with two heavy quarks. This can be found in [4],[5] and [6], correspondingly.

procedure means, that using the effective theory, we can systematically take into account the virtualities greater than the heavy quark mass. So, the knowledge of the two-loop anomalous dimensions is also important, when one discusses the matching of baryonic currents, obtained in this approximation with the corresponding currents in the full QCD.

Our analysis in this paper is close to what was presented in [11], devoted to the baryons with a single heavy quark² While being very similar, these analyses also have some differences, which we would like to stress. The main technical obstacle of calculations is related to the fact that the kinetic term is thought to be a necessary ingredient in a quark propagator for the evaluation of RG quantities in NRQCD, unlike HQET,

$$\frac{1}{k_0 + i\varepsilon} \longrightarrow \frac{1}{k_0 - \frac{\mathbf{k}^2}{2m} + i\varepsilon}. \quad (1)$$

If a hard cut-off is used ($\mu \ll m$), we can easily see that such NRQCD-calculations can be performed just like in HQET, since $k^0 \gg \mathbf{k}^2/m$ in the ultraviolet regime. However, if the dimensional regularization is used, the high energy modes ($k > m$) are not explicitly suppressed and they give non-vanishing contributions. This can be seen because the behavior of the NRQCD propagator changes at energies greater than the mass. In spite of this, one would like to use dimensional regularization because it keeps all of the QCD symmetries and, moreover, the calculations are technically simpler.

The difference between NRQCD and HQET can be explicitly highlighted in the consideration of an effective Lagrangian, derived at the tree level in the $1/m$ -expansion:

$$\begin{aligned} \mathcal{L}_{\text{NRQCD}} = & \psi^\dagger \left(iD^0 + \frac{\vec{D}^2}{2m} \right) \psi + \frac{1}{8m^3} \psi^\dagger \vec{D}^4 \psi - \frac{g_s}{2m} \psi^\dagger \vec{\sigma} \cdot \vec{B} \psi \\ & - \frac{g_s}{8m^2} \psi^\dagger \left(\vec{D} \cdot \vec{E} - \vec{E} \cdot \vec{D} \right) \psi - \frac{ig_s}{8m^2} \psi^\dagger \vec{\sigma} \cdot \left(\vec{D} \times \vec{E} - \vec{E} \times \vec{D} \right) \psi \\ & + O(1/m^3) + \text{antiquark terms} + \mathcal{L}_{\text{light}}. \end{aligned} \quad (2)$$

For a single heavy quark, interacting at low virtualities $D \sim \Lambda_{QCD}$, the kinetic term is suppressed and can be treated perturbatively. This results in the HQET prescription to the heavy quark propagator. However, in the heavy-heavy system there is the Coulomb-like interaction, wherein $D^0 \sim \vec{D}^2/m \sim \alpha_s^2 m$. Therefore, we must include the kinetic term into the initial "free" Lagrangian of NRQCD. So, the loop corrections in α_s look different in HQET and NRQCD. Nevertheless, the physical reason to distinguish these effective theories is still the Coulomb-like corrections near the production threshold, which should make no influence on the ultraviolet properties. We would note that the question is, in a sense, analogous to that in the theory of massive gauge fields in the spontaneously broken theories, where the explicit introduction of mass seems to destroy good RG properties of massless vector fields (the question was removed by the appropriate redefinitions of fields due to the surviving of the gauge invariance).

²We generally accept a set of basic notations used in [11].

Several authors have approached a similar problem of NRQCD in connection with matching calculations [12], and recently an appealing solution has been proposed [13]: It is claimed that the matching in NRQCD, which use the dimensional regularization, should be performed just as in HQET, namely, the kinetic term must be treated as a perturbation vertex

$$\frac{1}{k_0 - \frac{\mathbf{k}^2}{2m} + i\varepsilon} = \frac{1}{k_0} + \frac{\mathbf{k}^2}{2m(k_0)^2} + \dots \quad (3)$$

The derivation is based on the appropriate redefinition of the heavy quark field [13]

$$Q \longrightarrow \left[1 - \frac{D_\perp^2}{8m^2} - \frac{g\sigma_{\alpha\beta}G^{\alpha\beta}}{16m^2} + \frac{D_\perp^\alpha(iv \cdot D)D_{\alpha\perp}}{16m^3} + \frac{gv_\lambda D_{\perp\alpha}G^{\alpha\lambda}}{16m^3} - i\frac{\sigma_{\alpha\beta}D_\perp^\alpha(iv \cdot D)D_\perp^\beta}{16m^3} - i\frac{gv_\lambda\sigma_{\alpha\beta}D_\perp^\alpha G^{\beta\lambda}}{16m^3} \right] Q, \quad (4)$$

where the σ matrices are projected by $P_v\sigma P_v$, $P_v = \frac{1+\hat{v}}{2}$ and $D_\perp^\mu = D^\mu - v^\mu v \cdot D$. The substitution converts the HQET Lagrangian into the NRQCD one, so that the loop renormalization of perturbative terms is the same.

Here, we propose to use the same prescription for the heavy quark propagator as it stands in (3), not only in the matching procedure, but also for the calculations of anomalous dimensions for the NRQCD currents in $\overline{\text{MS}}$ -renormalization scheme. To support this point, let us consider the matching procedure in more detail. The matching condition can be written down as

$$Z_{J,QCD}^{-1} Z_{2,QCD}^{on-shell} Z_{V,QCD}^{h.m.} \Gamma'_{QCD} = C_0 Z_{2,NRQCD}^{on-shell} Z_{J,NRQCD}^{-1} \Gamma_{NRQCD}^{(0)}, \quad (5)$$

$$Z_{V,QCD}^{h.m.} \Gamma'_{QCD} = \Gamma_{QCD}^{(0)}, \quad (6)$$

where $Z_V^{h.m.}$ denotes the poles, associated with the hard momenta region for the bare single-particle irreducible vertex $\Gamma_{QCD}^{(0)}$ in full QCD, $Z_{J,QCD}$ and $Z_{J,NRQCD}$ are the renormalization constants of currents in QCD and NRQCD, correspondingly, $Z_{2,QCD}$ and $Z_{2,NRQCD}$ include the renormalization of wave functions, and, finally, $\Gamma_{NRQCD}^{(0)}$ denotes the bare vertex in NRQCD. At this stage we use prescription (3) for treating the heavy quark propagators. On the other hand, one can write the following identity:

$$\Gamma_{QCD} = Z_{J,QCD}^{-1} Z_{2,QCD}^{\overline{\text{MS}}} Z_{V,QCD}^{h.m.} Z_{V,QCD}^{s.m.} \Gamma'_{QCD}, \quad (7)$$

where we have collected all the divergences in Z -factors and used the convention of (3) for the expansion of heavy quark propagators in powers of the kinetic term. The $Z_{V,QCD}^{s.m.}$ denotes the contribution from a small momenta region. Calculating the contribution from the small momenta, we have to set the external legs to be off-shell in order to exclude the contribution from the infrared region as it was done in the case of matching. To proceed further, let us introduce the following definitions:

$$Z_{2,QCD}^{on-shell} = Z_{2,QCD}^{\overline{\text{MS}}} Z_{inf.r.}, \quad (8)$$

$$Z_{2,NRQCD}^{on-shell} = Z_{2,NRQCD}^{\overline{\text{MS}}} Z_{inf.r.}, \quad (9)$$

where $Z_{inf.r.}$ is the contribution to the wave-function renormalization from the infrared region, which is the same in both theories. Using these notations and the fact that $Z_{2,NRQCD}^{on-shell} = 1$, we can rewrite Eq. (7) as

$$\Gamma_{QCD} = Z_{J,QCD}^{-1} Z_{2,QCD}^{on-shell} Z_{V,QCD}^{h.m.} Z_{V,QCD}^{s.m.} Z_{2,NRQCD}^{\overline{MS}} \Gamma_{QCD}'' \quad (10)$$

Now we can easily see from Eqs. (5) and (10), that the NRQCD anomalous dimensions in the \overline{MS} -renormalization scheme can be computed in two ways: Either from the matching condition (5) or using the HQET Feynman rules and setting the external legs off-shell in order to avoid the infrared divergencies. We have explicitly checked this conjecture to one-loop for the heavy-heavy vector current, however, for a full confidence we feel the need for such a check in the two-loop approximation.

So, in our approach we exploit the same reasoning for the calculation of RG quantities and work in the leading order of this expansion. Moreover, it is theoretically sound, because in the \overline{MS} -renormalization scheme used, the anomalous dimensions of currents do not depend on the masses of particles. The following fact also supports our claim: The values of Wilson coefficients, calculated in the matching procedure, are directly connected to the anomalous dimensions of operators multiplying these coefficients in the Lagrangian. And, finally, the high energy behavior in the effective theory with several scales does not depend on a relative weight of the lower scales. Thus, we only need

$$m \gg |\mathbf{p}|, E, \Lambda_{QCD}, \quad (11)$$

where there is no matter what the relations between $|\mathbf{p}|$, E and Λ_{QCD} are.

So, in our calculations we use the HQET propagators for the heavy quarks, setting the quark momenta in a way to avoid infrared divergencies. As will be explained in detail below, the two-loop contribution to the anomalous dimensions of currents under consideration consists of three parts. The first corresponds to the set of graphs, wherein the two-loop contributions are associated with one of the heavy-light subsystems. For this contribution we use the result of [11]. Then, there is the subset of two-loop graphs that are associated with the heavy-heavy system. The expression for this contribution is a generalization of what was obtained in [14]. And, finally, there are the irreducible contributions, where the two-loops connect the three quark lines. This contribution is calculated in this paper. We evaluate the two loop diagrams with the use of package, written by us on MATHEMATICA, and the recurrence-relations in HQET [15].

This paper is organized as follows. In section 1 we discuss the choice of currents for the baryons with two heavy quarks and give some comments on the renormalization properties of composite operators under consideration. In section 2 we furnish some remarks on the anomalous dimensions and present the results on the one-loop anomalous dimensions. In section 3 we discuss general features of two-loop renormalization procedure and present our two-loop results. We work in the \overline{MS} -renormalization scheme throughout the paper. As concerning the treatment of γ_5 we will show that the final expression does not depend on the scheme used. In Conclusion the results are summarized.

1. Baryonic Currents

The currents of baryons with two heavy quarks Ξ_{cc}^\diamond , Ξ_{bb}^\diamond and Ξ_{bc}^\diamond , where \diamond means different charges depending on the light quark charge, are associated with the spin-parity quantum numbers $j_d^P = 1^+$ and $j_d^P = 0^+$ for the heavy diquark system with the symmetric and antisymmetric flavor structure, respectively. Adding the light quark to the heavy quark system, one obtains $j^P = \frac{1}{2}^+$ for the Ξ_{bc}^\diamond baryons and the pair of degenerate states $j^P = \frac{1}{2}^+$ and $j^P = \frac{3}{2}^+$ for the baryons Ξ_{cc}^\diamond , Ξ_{bc}^\diamond , Ξ_{bb}^\diamond and $\Xi_{cc}^{*\diamond}$, $\Xi_{bc}^{*\diamond}$, $\Xi_{bb}^{*\diamond}$. The structure of baryon currents with two heavy quarks is generally chosen as

$$J = [Q^{iT} C \Gamma \tau Q^j] \Gamma' q^k \varepsilon_{ijk}. \quad (12)$$

Here T means transposition, C is the charge conjugation matrix with the properties $C\gamma_\mu^T C^{-1} = -\gamma_\mu$ and $C\gamma_5^T C^{-1} = \gamma_5$, i, j, k are colour indices and τ is a matrix in the flavor space. The effective static field of the heavy quark is denoted by Q . To obtain the corresponding NRQCD currents one has to perform the above-mentioned redefinition of the local field. But as we are working in the leading order over both the relative velocity of heavy quarks and their inverse masses, this local redefinition does not change the structure of the currents.

Here, unlike the case of baryons with a single heavy quark, there is the only independent current component J for each of the ground state baryon currents. They equal

$$\begin{aligned} J_{\Xi_{QQ'}^\diamond} &= [Q^{iT} C \tau \gamma_5 Q^{j'}] q^k \varepsilon_{ijk}, \\ J_{\Xi_{QQ}^\diamond} &= [Q^{iT} C \tau \vec{\gamma} Q^j] \cdot \vec{\gamma} \gamma_5 q^k \varepsilon_{ijk}, \\ J_{\Xi_{QQ}^{*\diamond}} &= [Q^{iT} C \tau \vec{\gamma} Q^j] q^k \varepsilon_{ijk} + \frac{1}{3} \vec{\gamma} [Q^{iT} C \vec{\gamma} Q^j] \cdot \vec{\gamma} q^k \varepsilon_{ijk}, \end{aligned} \quad (13)$$

where $J_{\Xi_{QQ}^{*\diamond}}$ satisfies the spin-3/2 condition $\vec{\gamma} J_{\Xi_{QQ}^{*\diamond}} = 0$. The flavor matrix τ is antisymmetric for Ξ_{bc}^\diamond and symmetric for Ξ_{cc}^\diamond and Ξ_{bb}^\diamond . The currents written down in Eq. (6) are taken in the rest frame of hadrons. The corresponding expressions in a general frame moving with a velocity four-vector v^μ can be obtained by the substitution of $\vec{\gamma} \rightarrow \gamma_\perp^\mu = \gamma^\mu - \hat{v} v^\mu$.

Now we would like to give some comments on the renormalization properties of these currents. As we have the only light leg in this problem, all of γ matrices, which will appear in calculations, will stay on a single side of our composite operators, without touching their Dirac structure. This will lead to the fact that the anomalous dimensions of all our currents in this approximation are the same, i.e. they do not depend on Γ -matrices in (4). From this reasoning, we also can conclude that the result does not depend on the γ_5 scheme used.

2. Common notations in renormalization

The local operators O_0 composed of bare physical fields contain the ultra-violet divergences, which can be absorbed by the renormalization factors Z_O , being a series in

powers of coupling constant, so that $O = Z_O O_0$ is a finite quantity, while the regularization parameters do not tend to peculiar values. In the dimensional regularization using the $\overline{\text{MS}}$ -scheme of subtractions in $D = 4 - 2\epsilon$ dimensions [16], Z_O is expanded in inverse powers of ϵ , so that

$$Z = 1 + \sum_{m=1}^{\infty} \sum_{k=1}^m \left(\frac{\alpha_s}{4\pi} \right)^m \frac{1}{\epsilon^k} Z_{m,k} = 1 + \sum_{k=1}^{\infty} \frac{1}{\epsilon^k} Z_k. \quad (14)$$

The dependence on the dimensionful subtraction point μ defines the anomalous dimension of renormalized operator O

$$\gamma = \frac{d \ln Z(\alpha(\mu), a; \epsilon)}{d \ln(\mu)}, \quad (15)$$

where a is the renormalized gauge parameter in the general covariant gauge (with a gluon propagator proportional to $-g_{\mu\nu} + (1-a)k_\mu k_\nu/k^2$) and $\alpha(\mu)$ is the renormalized coupling constant in four-dimensional space, so that

$$\alpha_0 = \alpha(\mu) \mu^{2\epsilon} Z_\alpha(\alpha(\mu), a; \epsilon), \quad a_0 = a Z_3(\alpha(\mu), a; \epsilon), \quad (16)$$

and the corresponding $Z_{\{\alpha,3\}}$ -factors determine the anomalous dimensions, which are generally denoted by $\{-\beta, -\delta\}$, respectively.

The γ -quantities are finite at $D \rightarrow 4$, so we define the coefficients of series

$$\gamma = \sum_{m=1}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^m \gamma^{(m)}. \quad (17)$$

One can check that [17]

$$\gamma = -2 \frac{\partial Z_1}{\partial \ln \alpha_s}, \quad (18)$$

and for $k > 0$

$$-2 \frac{\partial Z_{k+1}}{\partial \ln \alpha_s} = \left(\gamma - \beta \frac{\partial}{\partial \ln \alpha_s} - \delta \frac{\partial}{\partial \ln a} \right) Z_k. \quad (19)$$

The latter provides the consistency condition, when the former produces a simple extraction of the anomalous dimensions to the two-loop accuracy

$$\gamma^{(1)} = -2Z_{1,1} \quad \text{and} \quad \gamma^{(2)} = -4Z_{2,1}. \quad (20)$$

2.1. One-loop result

Consider the one-loop renormalization of currents of baryons with two heavy quarks. In the $\overline{\text{MS}}$ -scheme with $D = 4 - 2\epsilon$ space-dimensions, we have the following squares of renormalization factors for the bare quark fields:

$$Z_q = 1 - a_0 \frac{g_0^2 C_F}{(4\pi)^2 \epsilon}, \quad Z_Q = 1 + (3 - a_0) \frac{g_0^2 C_F}{(4\pi)^2 \epsilon}, \quad (21)$$

where we use the usual definitions for $SU(N)$, i.e. $C_F = (N_c^2 - 1)/2N_c$, $C_A = N_c$, $C_B = (N_c + 1)/2N_c$, and $T_F = 1/2$ for $N_c = 3$, N_F being the number of light quarks. One-loop $\overline{\text{MS}}$ -results for the factors Z_α and Z_3 have been given e.g. in [17]

$$Z_\alpha = 1 - \frac{\alpha_s}{4\pi\epsilon} \left[\frac{11}{3}C_A - \frac{4}{3}T_F N_F \right], \quad (22)$$

$$Z_3 = 1 + \frac{\alpha_s}{4\pi\epsilon} \left[\frac{13 - 3a}{6}C_A - \frac{4}{3}T_F N_F \right]. \quad (23)$$

The bare current is renormalized by the factor Z_J

$$J_0 = (Q_0^T C \Gamma \tau Q_0) \Gamma' q_0 = Z_Q Z_q^{1/2} Z_V J = Z_J J, \quad (24)$$

which straightforwardly means that

$$\gamma_J = 2\gamma_Q + \gamma_q + \gamma_V, \quad (25)$$

i.e. the anomalous dimension of the full current J is a sum of three terms given by the renormalization of the light and heavy quark fields, and the renormalization of the vertex.

For the vertex $(Q_0^T C \Gamma \tau Q_0) \Gamma' q_0$, we find

$$Z_V = 1 + \frac{\alpha_s C_B}{4\pi\epsilon} (3a - 3), \quad (26)$$

which results in

$$\gamma_V^{(1)} = -2C_B(3a - 3). \quad (27)$$

The one-loop anomalous dimensions $\gamma_q^{(1)}$ and $\gamma_Q^{(1)}$ are equal to

$$\gamma_q^{(1)} = C_F a, \quad \gamma_Q^{(1)} = C_F(a - 3). \quad (28)$$

Thus, the one-loop anomalous dimension of the baryonic current is given by

$$\gamma_J = \frac{\alpha_s}{4\pi} \left(-2C_B(3a - 3) + 3C_F(a - 2) \right) + O(\alpha_s^2). \quad (29)$$

3. Two-loop calculations

In this section we apply the two-loop renormalization of the baryon current with two heavy quarks in the $\overline{\text{MS}}$ -scheme and restrict ourselves by the Feynman gauge. The two-loop anomalous dimensions of the quark fields are given by [15,18,19,20,21]

$$\gamma_q^{(2)} = C_F \left(\frac{17}{2}C_A - 2T_F N_F - \frac{3}{2}C_F \right), \quad \gamma_Q^{(2)} = C_F \left(-\frac{38}{3}C_A + \frac{16}{3}T_F N_F \right). \quad (30)$$

Since the baryonic currents are renormalized multiplicatively in the effective theory³, the Dirac structure of vertex repeats the Born-term. Technically we perform the calculations in terms of bare coupling and gauge parameter, so that to isolate the two-loop

³See discussion in ref.[11].

contribution to the anomalous dimension, we need also the one-loop result, wherein we have to include the one-loop expressions written down through the renormalized quantities α_s and a , which will add the contribution to the corresponding α_s^2/ϵ -term. The procedure described leads to the relations

$$Z_{1,1} = V_{1,1}, \quad Z_{2,2} = V_{2,2}, \quad Z_{2,1} = V_{2,1} - V_{1,1}V_{1,0}. \quad (31)$$

As expected, $Z_{2,1}$ has to include the one-loop contributions.

In the Introduction we have described three subgroups of two-loop diagrams for the vertex, which can be expressed as

$$V_0 = 2V_0^{(hl)} + V_0^{(hh)} + V_0^{(ir)}, \quad (32)$$

whose evaluation is presented in the rest of this section.

3.1. The heavy-light subsystem

As for the problem of evaluation the bare proper vertex $V^{(hl)}$ of composite operator (qQ) with a massless quark field q and the effective static heavy quark field Q , we can easily see that the result does not depend on the Dirac structure of the vertex. For this reason, in our calculations we have used the result of [11], where this vertex was calculated to two-loop order in the Feynman gauge ($a = 1$) with the use of algorithm developed in [15]:

$$\begin{aligned} V_{1,1}^{(hl)} &= C_B a, & V_{1,0}^{(hl)} &= 0, \\ V_{2,2}^{(hl)} &= C_B \left(\frac{1}{2} C_B - C_A \right), & V_{2,1}^{(hl)} &= -C_B (C_B (1 - 4\zeta(2)) - C_A (1 - \zeta(2))). \end{aligned} \quad (33)$$

Then from relations (31) one can calculate the coefficients $Z_{n,k}$, which determine the two-loop anomalous dimension for the subset of the heavy-light graphs

$$\gamma_{(hl)}^{(2)} = C_B^2 (4 - 16\zeta(2)) - C_B C_A (4 - 4\zeta(2)). \quad (34)$$

It is worth noting that this expression was calculated for antisymmetric baryonic color configuration $q^i Q^j Q^k \epsilon_{ijk}$ unlike the case of colour-singlet $\bar{q}^i Q^j \delta_j^i$ mesonic configuration. The expression for the latter case can be obtained by the substitution of $C_B \rightarrow C_F$, which reconstructs the required results, as it was checked by the authors of [11].

3.2. The heavy-heavy subsystem

To evaluate this contribution we have used the results of [14], where the expression for the anomalous dimension of NRQCD mesonic vector current was presented

$$\begin{aligned} \gamma_J^M &= 2\gamma_Q + \gamma_{(hh)} = \frac{d \ln Z_J}{d \ln \mu} \\ &= -C_F (2C_F + 3C_A) \frac{\pi^2}{6} \left(\frac{\alpha_s}{\pi} \right)^2 + O(\alpha_s^3). \end{aligned} \quad (35)$$

In our case we have a similar problem, but a different color structure. Thus, following [14] we can consider the matching of QCD vector current with the antisymmetric color structure on the NRQCD one. Unlike the meson case with the singlet-color structure, the QCD vector current with the antisymmetric color structure need not be conserved, so we allow for its renormalization. In terms of the on-shell matrix elements, the matching equation can be written down as⁴

$$Z_{2,QCD}Z_{J,QCD}^{-1}\Gamma_{QCD} = C_0Z_{2,NRQCD}Z_{J,NRQCD}^{-1}\Gamma_{NRQCD} + O(v^2), \quad (36)$$

where $Z_{J,QCD}$ has the following expression [11]

$$\begin{aligned} Z_{J,QCD} = & 1 - \frac{C_B C_F}{\epsilon^2} \left(\frac{\alpha_s}{4\pi} \right)^2 + \frac{1}{\epsilon} \left((C_B - C_F) \left(\frac{\alpha_s}{4\pi} \right) + \right. \\ & \left. \left(-\frac{1}{4} C_B (-17C_A + 3C_B + 4(1 + N_F) T_F) \right) \right. \\ & \left. + \frac{1}{4} C_F (-17C_A + 3C_F + 4(1 + N_F) T_F) \right) \left(\frac{\alpha_s}{4\pi} \right)^2. \end{aligned} \quad (37)$$

The anomalous dimension of NRQCD current, obtained in this way, may be used in the calculations of anomalous dimensions for the baryonic currents with two heavy quarks, as it does not depend on the Dirac structure of the vertex. The contributions of different two-loop diagrams with the antisymmetric color structure of the vertex $q^i Q^j Q^k \epsilon_{ijk}$ in the notations of [14] are shown in Appendix. To obtain the anomalous dimension $\gamma_{(hh)}^{(2)}$ of composite operator under consideration, one has to perform the following steps.

1) To sum up all of these contributions, including the one-loop term, multiplied by the two-loop QCD on-shell wave function renormalization constant [22], $Z_{J,QCD}^{-1}$ and one-loop NRQCD-current renormalization constant,

2) To carry out the one-loop renormalization of coupling and mass.

After these manipulations the coefficient at $\frac{1}{\epsilon}$ multiplied by -4 will give us the sum $\gamma_{(hh)}^{(2)} + 2\gamma_Q^{(2)}$. For the two-loop anomalous dimension $\gamma_{(hh)}^{(2)}$ in the heavy-heavy subsystem, we find the following result:

$$\gamma_{(hh)}^{(2)} = -\frac{4}{3} C_B ((-19 + 6\pi^2) C_A + 4(\pi^2 C_B + 2N_F T_F)). \quad (38)$$

3.3. The light-heavy-heavy irreducible vertex

In this case one needs to calculate the three-quark irreducible vertex $V_0^{(ir)}$. There are 8 diagrams in the two-loop order. We have shown four of them in Fig. 1, the other four can be obtained by exchanging two heavy quark legs. We set the heavy quarks off shell in order to avoid any infrared singularities. Using the partial fractioning of the integrand in

⁴Since the matching coefficient contains only short-distance effects, the matching can be done by comparing the matrix elements of these currents over a free quark-antiquark pair of the on-shell quarks at a small relative velocity.

momentum integrals and recurrence-relations of [15], we arrive at the following expressions for the diagrams depicted on Fig.1:

$$V_0^{(ir)[1]} = 2C_B^2 \left(\frac{\alpha_s}{4\pi} \right)^2 \left(\frac{1}{2\epsilon^2} - \left(1 + \frac{\pi^2}{3}\right) \frac{1}{\epsilon} + \frac{216 + 35\pi^2 - 48\psi^{(2)}(1) - 96\psi^{(2)}(2)}{36} \right), \quad (39)$$

$$V_0^{(ir)[2]} = 0, \quad (40)$$

$$V_0^{(ir)[3]} = 2C_B^2 \left(\frac{\alpha_s}{4\pi} \right)^2 \left(-\frac{1}{\epsilon^2} - \frac{2}{\epsilon} - 4 - \frac{\pi^2}{6} \right), \quad (41)$$

$$V_0^{(ir)[4]} = 2C_B^2 \left(\frac{\alpha_s}{4\pi} \right)^2 \left(-\frac{1}{\epsilon^2} + \frac{2}{\epsilon} - 4 - \frac{3\pi^2}{2} \right), \quad (42)$$

where $\psi^{(n)}(z) = d^n \psi(z)/dz^n$, $\psi(z) = \Gamma'(z)/\Gamma(z)$ and the factor of 2 accounts for the contributions of the remaining four reflected diagrams not included in Fig.1. For the Z -factors and anomalous dimension, we obtain

$$Z_{22}^{(ir)} = -3 \cdot C_B^2, \quad (43)$$

$$\gamma_{(ir)}^{(2)} = -4Z_{2,1}^{(ir)} = 8 \cdot C_B^2 \left(1 + \frac{\pi^2}{3}\right). \quad (44)$$

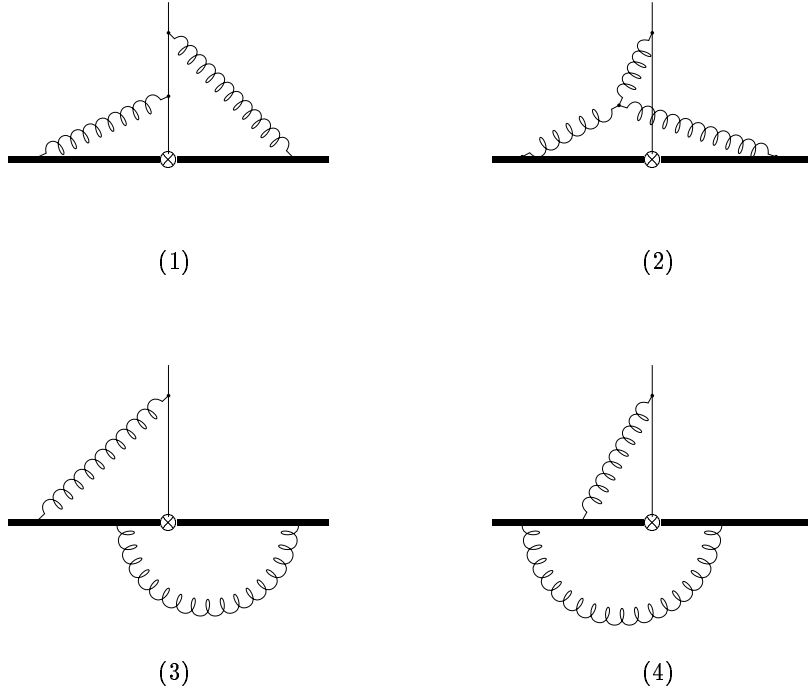


Fig. 1. The two-loop contribution to the light-heavy-heavy irreducible vertex with the spinor lines directed outside.

3.4. Anomalous dimension combined

Now we are ready to calculate the anomalous dimension of baryonic currents with two heavy quarks. As we have already said above it does not depend on the Dirac structure of the current under consideration. Collecting the results for the heavy-light, heavy-heavy and irreducible light-heavy-heavy vertices, we find

$$\gamma_V^{(2)} = -\frac{4}{3}C_B((-13 + 30\zeta(2))C_A + 6(-2 + 6\zeta(2))C_B + 8N_F T_F). \quad (45)$$

And, finally, to obtain the full two-loop result for the anomalous dimension, one has to add the anomalous dimensions of heavy and light quarks. The result is

$$\begin{aligned} \gamma_J^{(2)} = & \frac{1}{6}(-48(-2 + 6\zeta(2))C_B^2 + C_A((104 - 240\zeta(2))C_B - 101C_F) \\ & - 64C_B N_F T_F + C_F(-9C_F + 52N_F T_F)). \end{aligned} \quad (46)$$

With this formula we are finishing our analytical calculations.

Turning to the SU(3) group of QCD, we get

$$\gamma^{(1)} = -4, \quad (47)$$

$$\gamma^{(2)} = -\frac{254}{9} - \frac{152\pi^2}{9} + \frac{20}{9}N_F \approx -194.909 + 2.222N_F, \quad (48)$$

which indicates a rather strong sensitivity of those currents to the choice of reference scale μ .

Conclusions

We have calculated the two-loop anomalous dimensions of NRQCD baryonic currents with two heavy quarks in the leading order in both the relative velocity of heavy quarks and the inverse heavy quark mass. It is shown, that the results do not depend on the Dirac structure of the currents and on the γ_5 prescription used in the calculations. These results will be useful for the derivation of QCD sum rules for baryons with two heavy quarks in the same static approximation in both the leading and next-to-leading orders. We suppose to approach this problem in the nearby future.

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4. Appendix

In this appendix we present the generalization of expressions for the hard contributions to the diagrams of Fig. 1 of [14] with the antisymmetric color structure of vertex evaluated at the threshold $q^2 = 4m^2$. Below you can find the coefficients of $(\alpha_s/\pi)^2(e^{\gamma_E} m_Q^2/(4\pi\mu^2))^{-2\epsilon}$

$$D_1 = C_B^2 \left[\frac{9}{32\epsilon^2} - \left(\frac{27}{64} + \frac{5\pi^2}{24} \right) \frac{1}{\epsilon} - \frac{81}{128} - \frac{133\pi^2}{96} - \frac{5\pi^2 \ln 2}{12} - \frac{35\zeta(3)}{8} \right], \quad (49)$$

$$D_2 = C_B C_F \left[-\frac{3}{16\epsilon^2} - \frac{43}{32} \frac{1}{\epsilon} + \frac{733}{192} + \frac{971\pi^2}{576} \right], \quad (50)$$

$$D_3 = C_B C_A \left[\frac{15}{32\epsilon^2} - \left(\frac{5}{64} + \frac{\pi^2}{16} \right) \frac{1}{\epsilon} + \frac{715}{384} - \frac{319\pi^2}{576} - \frac{\pi^2 \ln 2}{8} - \frac{21\zeta(3)}{16} \right], \quad (51)$$

$$D_4 = C_B (C_A - 2C_B) \left[\left(\frac{3}{16} - \frac{\pi^2}{16} \right) \frac{1}{\epsilon} - \frac{39}{32} - \frac{251\pi^2}{1152} - \frac{3\pi^2 \ln 2}{8} - \frac{31\zeta(3)}{16} \right], \quad (52)$$

$$D_5 = C_B (C_A - 2C_F) \left[-\frac{9}{32\epsilon^2} - \frac{19}{64} \frac{1}{\epsilon} + \frac{761}{384} + \frac{1157\pi^2}{1152} + \frac{\pi^2 \ln 2}{6} - \frac{3\zeta(3)}{4} \right], \quad (53)$$

$$D_6 = C_B T_F N_F \left[-\frac{1}{8\epsilon^2} + \frac{5}{48} \frac{1}{\epsilon} - \frac{355}{288} - \frac{5\pi^2}{48} \right], \quad (54)$$

$$D_7 = C_B C_A \left[\frac{19}{128\epsilon^2} - \frac{53}{768} \frac{1}{\epsilon} + \frac{6787}{4608} + \frac{95\pi^2}{768} \right], \quad (55)$$

$$D_8 = C_B C_A \left[\frac{1}{128\epsilon^2} + \frac{1}{768} \frac{1}{\epsilon} + \frac{361}{4608} + \frac{5\pi^2}{768} \right], \quad (56)$$

$$D_9 = C_B T_F \left[-\frac{1}{4\epsilon^2} + \frac{13}{48} \frac{1}{\epsilon} - \frac{145}{96} + \frac{5}{72} \right]. \quad (57)$$

В.В.Киселев, А.И.Онищенко

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