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SPECTROSCOPY OF DOUBLY HEAVY BARYONS

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Abstract

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Spectra of masses for the families of doubly heavy baryons are calculated in the framework of nonrelativistic quark model with the QCD-induced potential by Buchmüller–Tye in accordance with both the quark-diquark structure for the wave functions and taking into account the splittings depending on the spins. We investigate the physical reasons pointing to the existence of quazi-stable excited states in the subsystem of heavy diquark for the heavy quarks with the identical flavors.

Аннотация

Герштейн С.С., Киселев В.В., Лиходед А.К., Онищенко А.И. Спетроскопия дваждытяжелых барионов: Препринт ИФВЭ 98-66. – Протвино, 1998. – 22 с., 4 рис., 9 табл., библиогр.: 16.

В рамках нерелятивистской кварковой модели с КХД-мотивированным потенциалом Бухмюллера–Тая проведены расчеты спектров масс семейств дваждытяжелых барионов в модели кварк-дикварковой структуры волновых функций с учетом расщепления, зависящего от спина. Исследуются физически обусловленные указания на существование квазистабильных возбужденных состояний в подсистеме тяжелого дикварка в случае тождественности ароматов тяжелых кварков.

Introduction

The first observation of B_c^+ meson in FNAL by the CDF collaboration [1] opens a new stage in the physics of hadrons, containing heavy quarks. On the one hand, this particle is the last heavy quarkonium ($Q\bar{Q}'$) and heavy flavored meson, which became accessible to the experimental investigations. On the other hand, it is the first one among the long-living hadrons with two heavy quarks. In this insight, the B_c^+ meson stands at the beginning of the families set, which would be continued by the doubly heavy baryons Ξ_{cc}, Ξ_{bc} and Ξ_{bb} (see the classification of hadrons in the framework of quark model by PDG [2]). The experimental discovery of B_c^+ was highlighted by the previous many-fold study of the meson spectroscopy as well as the mechanisms of its production and decay (see the review in [3]). In the same manner, to observe the doubly heavy baryons it is necessary to give detailed and reliable theoretical predictions of their properties. The first steps forward the performance of such program have been done:

1. in ref.[4], where the estimates for the lifetimes of Ξ_{cc}^+ and Ξ_{cc}^{++} baryons have been obtained in the framework of Operator Product Expansion over the inverse heavy quark mass,
2. in the set of papers [5], devoted to the investigation of differential and total cross-sections for the production of $\Xi_{QQ'}$ baryons in various interactions in the model of fragmentation, in the picture of intrinsic charm [6] (for the hadronic production of Ξ_{cc}) and in the framework of perturbative QCD calculations for the $O(\alpha_s^4)$ contributions, taking into account the hard nonfragmentational regime in addition to the fragmentation, which dominates at high transverse momenta $p_T \gg M$,
3. in refs.[7], where the masses of basic low-lying states of doubly heavy baryons have been estimated, and the excitations of Ξ_{cc} have been included into the consideration of ref.[8].

In the present paper we analyze the basic spectroscopic characteristics for the families of doubly heavy baryons $\Xi_{QQ'} = (QQ'q)$, where $q = u,\ d$ and $\Omega_{QQ'} = (QQ's)$.

The qualitative picture for the formation of bound states in the system of $(QQ'q)$ is determined by the presence of two scales of distances, which are given by the size of QQ'-diquark subsystem, $r_{QQ'}$, in the antitriplet color state as well as by the confinement scale for the light quark q, so that

$$r_{QQ'} \cdot \Lambda_{QCD} \ll 1, \quad \Lambda_{QCD} \ll m_Q.$$

Under such conditions, the compact diquark QQ' looks like a static source, approximated by the local colored QCD field, from the viewpoint of the light quark. Therefore, we can use a set of reliable results in models of mesons with a single heavy quark, i.e. with a local static source belonging to the antitriplet representation of $\mathrm{SU}(3)_c$ group: the potential models [9] and the Heavy Quark Effective Theory (HQET) [10] in the framework of expansion over the inverse heavy quark mass. We apply the nonrelativistic quark model with the potential by Buchmüller–Tye [11]. Then *theoretically* we can talk on the rough approximation for the light quark ($m_q^{QCD} \ll \Lambda_{QCD}$), being, hence, the relativistic one, in the system with a finite number of degrees of freedom and an instantaneous interaction $V(\mathbf{r})$. This disadvantage is caused by the fact that the confinement supposes: a) the generation of sea around the light quark, i.e. the presence of an infinite number of gluons and quark-antiquark pairs, and b) the nonperturbative effects with the correlation time $\tau_{QCD} \sim 1/\Lambda_{QCD}$, which is beyond the potential approach. However, *phenomenologically* the introduction of constituent mass $m_q^{NP} \sim \Lambda_{QCD}$ as a basic underlying parameter, determining the interaction with the QCD condensates, allows one to successfully justify the nonrelativistic potential model with a high accuracy ($\delta M \approx 30 - 40$ MeV) over the existing experimental data, which makes the approach quite a reliable tool for the prediction of masses for the hadrons, containing the heavy and light quarks.

As for the diquark QQ', it is completely analogous to the heavy quarkonium $Q\bar{Q}'$ except for the very essential peculiarities:

1. $(QQ')_{\bar{3}_c}$ is a system with the nonzero color charge,
2. for the quarks with the same flavor $Q = Q'$ it is necessary to take into account the Pauli principle for the identical fermions.

The second item simply turns out to lead to the exclusion of summed quark spin S=0 for the symmetric, P-even spacial wave functions in diquark, $\Psi_d(\mathbf{r})$ (the orbital momentum equals $L_d = 2n$, where $n = 0, 1, 2\dots$ is a non-negative integer number), as well as S=1 is forbidden for the antisymmetric, P-odd functions $\Psi_d(\mathbf{r})$ (i.e. $L_d = 2n + 1$). The nonzero color charge raises two problems.

First, we generally cannot apply the confinement hypothesis in the form of restricting potential (an infinite growth of energy with the increase of the system size) for the object under consideration. However, from the physical point of view, it is hard to imagine a situation, when a big colored object with the size $r > 1/\Lambda_{QCD}$ has a finitely restricted energy of self-action, and, for the same moment, it is confined inside a white hadron (the singlet over $\mathrm{SU}(3)_c$) with $r \sim 1/\Lambda_{QCD}$ due to the interaction with another colored source. Moreover, in the framework of well-justified picture of the hadronic string, the tension of such string for the diquark with the external leg inside the baryons is only two times less, than in the quark-antiquark pair inside the meson $q\bar{q}'$, and, in the same manner, the energy of diquark linearly grows with the increase of its size. So, the effect analogous to the confinement of quarks takes place in a similar way. Moreover, in the potential models one can suppose that the quark binding appears due to the effective single exchange by a colored object in the octet representation of $\mathrm{SU}(3)_c$ (one usually takes the sum of scalar exchange and vector one). Then again the potentials in the singlet $(q\bar{q}')$ and antitriplet (qq') states just differ by the factor 1/2, which means the presence of a confining potential with the linearly rising term in the QCD-motivated models for the heavy diquark $(QQ')_{\bar{3}_c}$. In the present work we use the nonrelativistic model with the Buchmüleer–Tye potential for the diquark, too.

Second, in the singlet color state $(Q\bar{Q}')$ there is the separate conservation of the summed spin S and the orbital moment L, since the QCD operators for the transitions between the levels, determined by these quantum numbers, are suppressed. Indeed, in the framework of multipole expansion in QCD [12], the amplitudes of chromo-magnetic and chromo-electric dipole transitions are suppressed by the inverse heavy quark mass, but in addition, the major sense is provided by: a) the necessity to emit a white object, i.e. at least two gluons, which results in the higher order in $1/m_Q$, and b) the projection to the real phase space in the physical spectrum of massive hadrons in contrast to the case of massless gluon. Furthermore, the probability of a hybrid state, say, the octet sybsystem $(Q\bar{Q}')$ and the additional gluon, i.e. the Fock state $|Q\bar{Q}'_{8_c}g\rangle$, is suppressed due to both the small size of system and the nonrelativistic motion of quarks (for a more strict and detailed consideration see ref.[13]). In the antitriplet color state, the emission of a soft nonperturbative gluon between the levels, determined by the spin S_d and the orbital momentum L_d in the diquark, is not excluded, if there are no other forbidding rules or small order-parameters. For the quarks with the identical flavors inside the diquark, the Pauli principle leads to the fact that the transitions are possible only between the levels, which either differ by the spin $(\Delta S_d = 1)$ and the orbital momentum $(\Delta L_d = 2n + 1)$, instantaniously, or stand in the same set of radial excitations or have $\Delta L_d = 2n$. In the latter two cases, the transition amplitudes are suppressed by a small recoil momentum of diquark in comparison with its mass. For the former case the transition operator, changing the diquark spin as well as its orbital momentum, has the higher order of smallness because of either the additional factor of $1/m_Q$ or the small size of diquark, which leads to the existence of quazi-stable states with the quantum numbers of S_d and L_d. In the diquark with the quarks of different flavors, (bc), the QCD operators of dipole transitions with the single emission of soft gluon are not forbidden, so that the lifetimes of levels can be comparable with the times for the forming of bound states or to the distances between the levels themselves. Then we cannot certainly state on the appearance of excitation system for the diquark with definite quantum numbers of the spin and orbital momentum[1].

[1] In other manner, the presence of gluon field inside the baryon Ξ_{bc} leads to the transitions between the states with the different excitations of diquark, like $|bc\rangle \to |bcg\rangle$ with $\Delta S_d = 1$ or $\Delta L_d = 1$, which are not suppressed.

Thus, in the present paper we use the presence of two physical scales in the form of factorization for the wave functions in the problem with the heavy diquark and light constituent quark in the framework of nonrelativistic quark model, so that the problem on the calculation of mass spectrum and characteristics of bound states in the system of doubly heavy baryon is reduced to two standard problems on the study of stationary levels of energy in the system of two bodies. After that, we take into account the relativistic corrections dependent on the quark spins in each of two subsystems under consideration. The natural boundary for the region of existence of stable states in the doubly heavy baryons can be assigned to the threshold energy for the decay into a heavy baryon and a heavy meson. As it was shown in [14], the appearance of such threshold in different systems can be provided by the existence of an universal characteristics in QCD, namely, a critical distance between the quarks, so that at the increase of distance the quark-gluon fields become unstable, i.e. the generation of valence quark-antiquark pairs from the sea takes place. In other manner, the hadronic string having a length greater, than the critical one, decay into the strings of smaller sizes with a high probability close to unit. In the framework of potential approach this effect can be incorporated due to that we will restrict the consideration of excited levels in the diquark by the region, wherein the size of diquark is less, than the critical distance, $r_{QQ'} < r_c \approx 1.4 - 1.5$ fm. Furthermore, the model of pair interactions with the isolated structure of diquark seems to be reliable, just if the size of diquark is less, than the distance to the light quark $r_{QQ'} < r_l$.

The peculiarity of quark-diquark picture for the doubly heavy baryon is the possibility of mixing between the states of higher excitations in the diquark, possessing different quantum numbers, because of the interaction with the light quark. Then, it is difficult to assign some definite quantum numbers to such excitations. We will discuss the mechanism and character of this effect in detail.

The paper is organized as follows. In Section 1 we describe the general procedure for the calculation of mass spectra for the doubly heavy baryons in the framework of assumptions drawn before with the account for the spin-dependent corrections to the potential motivated in QCD. The results of numerical estimates are presented in Section 2, and, finally, our conclusions are given in the last Section.

1. Nonrelativistic potential model

As noted in the Introduction, we reduce the problem for the calculation of mass spectra of baryons, containing two heavy quarks, to the subsequent computation of energy levels in the diquark and, then, for the point-like diquark, possessing the obtained parameters and interacting with the light constituent quark. At the each step of calculations we separate two stages in accordance with the effective models for the expansion of QCD interaction over the inverse heavy quark mass. So, in the first approximation, the nonrelativistic Schrödinger equation with the model potential, motivated by QCD, is solved numerically. After that, the spin-dependent corrections, suppressed by the quark masses, are introduced as perturbations.

1.1. Potential

As for the model potential, we use the Buchmüller–Tye one, which takes into account the coulomb-like corrections in the region of short distances, so that the effective coupling constant for the exchange by the octet color state between the quarks is approximated by the QCD running coupling constant in the two-loop accuracy. At large distances, there is the linear raise of interaction energy, which leads to the confinement, whereas these two regimes are the limits for the effective β-function by Gell-Mann–Low. It is proceeded in an explicit form. In the antitriplet state we incorporate the factor 1/2 due to the color structure of bound quark-quark state. For the interaction of diquark with the light constituent quark, the corresponding factor is equal to unity.

Table 1. The spectrum of (bb)-diquark levels without spin-dependent splittings: masses and mean-squared radii.

diquark level	mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)	diquark level	mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)
1S	9.74	0.33	2P	9.95	0.54
2S	10.02	0.69	3P	10.15	0.86
3S	10.22	1.06	4P	10.31	1.14
4S	10.37	1.26	5P	10.45	1.39
5S	10.50	1.50	6P	10.58	1.61
3D	10.08	0.72	4D	10.25	1.01
5D	10.39	1.28	6D	10.53	1.51
4F	10.19	0.87	5F	10.34	1.15
6F	10.47	1.40	5G	10.28	1.01
6G	10.42	1.28	6M	10.37	1.15

Note, that, as shown in [15], the nonperturbative constituent term, included into the mass of nonrelativistic quark, exactly coincides with the additive constant, subtracted from the coulomb potential. Therefore, we extract the masses of heavy quarks from the fitting of model to the actual spectra of charmonium and bottomonium:

$$m_c = 1.486 \text{ GeV}, \quad m_b = 4.88 \text{ GeV}, \tag{1}$$

so that the mass for the level in the heavy quarkonium has been calculated as $M(c\bar{c}) = 2m_c + E$, where E is the energy of static solution for the Schrödinger equation with the model potential V. Then, we have supposed, that the mass of meson with a single heavy quark is equal to $M(Q\bar{q}) = m_Q + m_q + E$, and $E = \langle T\rangle + \langle V - \delta V\rangle$, whereas the additive term to the potential is introduced because the constituent mass is determined as a part of interaction energy $\delta V = m_q$. In accordance to our fit on the masses of heavy mesons, we get $m_q = 0.385$ GeV.

The results of calculations for the eigen-energy levels in the Schrödinger equation with the Buchmüller–Tye potential are presented in Tabs. 1–3 for the various systems. The characteristics of corresponding wave functions are given in Tabs. 4–6.

Table 2. The spectrum of (bc)-diquark levels without spin-dependent splittings: masses and mean-squared radii.

diquark level	mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)	diquark level	mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)
1S	6.48	0.48	3P	6.93	1.16
2S	6.79	0.95	4P	7.13	1.51
3S	7.01	1.33	3D	6.85	0.96
2P	6.69	0.74	4D	7.05	1.35
4F	6.97	1.16	5F	7.16	1.52
5G	7.09	1.34	6H	7.19	1.50

We have checked that with a good accuracy *the binding energy and the wave function of light quark do not practically depend on the flavors of heavy quarks*, since the large value of diquark masses gives a small contribution to the reduced mass of the system and leads to weak corrections in the Schrödinger equation. Thus, the energies of levels for the light constituent quark in the states, lying below the threshold of doubly heavy baryon decay into the heavy baryon and heavy meson, are equal to

$$E(1s) = 0.38\ \text{GeV},\ \ E(2s) = 1.09\ \text{GeV},\ \ E(2p) = 0.83\ \text{GeV},$$

where the level energy has been determined by the sum of constituent mass and the eigen-value of energy for the stationary solution of Schrödinger equation. In HQET the value of $\bar{\Lambda} = E(1s)$ is generally introduced. Then we can conclude that our estimate is in a good agreement with calculations in other approaches, which, once again, confirms the reliability of such phenomenological predictions. For the corresponding radial wave functions at the origin, we find

$$R_{1S}(0) = 0.527\ \text{GeV}^{3/2},\ \ R_{2S}(0) = 0.278\ \text{GeV}^{3/2},\ \ R'_{2P}(0) = 0.127\ \text{GeV}^{5/2}.$$

The analogous characteristics of the bound states for the c-quark, interacting with the bb-diquark, are equal to

$$E(1s) = 1.42\ \text{GeV},\ \ E(2s) = 1.99\ \text{GeV},\ \ E(2p) = 1.84\ \text{GeV},$$

with the wave functions

$$R_{1S}(0) = 1.41\ \text{GeV}^{3/2},\ \ R_{2S}(0) = 1.07\ \text{GeV}^{3/2},\ \ R'_{2P}(0) = 0.511\ \text{GeV}^{5/2}.$$

For the binding of strange constituent quark, we add the current mass $m_s \approx 100 - 150$ MeV.

1.2. Spin-dependent corrections

According to [16], we introduce the spin-dependent corrections causing the splittings of nL-levels in both the diquark and the system of light constituent quark with the diquark

Table 3. The spectrum of (cc)-diquark levels without spin-dependent splittings: masses and mean-squared radii.

diquark level	mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)	diquark level	mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)
1S	3.16	0.58	3P	3.66	1.36
2S	3.50	1.12	4P	3.90	1.86
3S	3.76	1.58	3D	3.56	1.13
2P	3.39	0.88	4D	3.80	1.59

($n = n_r + L + 1$ is the principal number, n_r is the number of radial excitation, L is the orbital momentum). For the heavy diquark with the identical quarks we have

$$\begin{aligned} V_{SD}^{(d)}(\mathbf{r}) &= \frac{1}{2}\left(\frac{\mathbf{L_d}\cdot\mathbf{S_d}}{\mathbf{2m_Q^2}}\right)\left(-\frac{dV(r)}{rdr}+\frac{8}{3}\alpha_s\frac{1}{r^3}\right) \\ &+\frac{2}{3}\alpha_s\frac{1}{m_Q^2}\frac{\mathbf{L_d}\cdot\mathbf{S_d}}{\mathbf{r^3}}+\frac{4}{3}\alpha_s\frac{1}{3m_Q^2}\mathbf{S}_{Q1}\cdot\mathbf{S}_{Q2}[4\pi\delta(\mathbf{r})] \\ &-\frac{1}{3}\alpha_s\frac{1}{m_Q^2}\frac{1}{4\mathbf{L_d}^2-3}[6(\mathbf{L_d}\cdot\mathbf{S_d})^2+3(\mathbf{L_d}\cdot\mathbf{S_d})-2\mathbf{L_d}^2\mathbf{S_d}^2]\frac{1}{r^3}, \end{aligned} \tag{2}$$

where $\mathbf{L_d}$, $\mathbf{S_d}$ are the orbital momentum in the diquark system and the summed spin of quarks, composing the diquark, respectively. The account for the interaction with the light constituent quark gives ($\mathbf{S} = \mathbf{S_d} + \mathbf{S_l}$)

$$\begin{aligned} V_{SD}^{(l)}(\mathbf{r}) &= \frac{1}{4}\left(\frac{\mathbf{L}\cdot\mathbf{S_d}}{\mathbf{2m_Q^2}}+\frac{\mathbf{2L}\cdot\mathbf{S_l}}{\mathbf{2m_l^2}}\right)\left(-\frac{dV(r)}{rdr}+\frac{8}{3}\alpha_s\frac{1}{r^3}\right) \\ &+\frac{1}{3}\alpha_s\frac{1}{m_Q m_l}\frac{(\mathbf{L}\cdot\mathbf{S_d}+\mathbf{2L}\cdot\mathbf{S_l})}{\mathbf{r^3}}+\frac{4}{3}\alpha_s\frac{1}{3m_Q m_l}(\mathbf{S_d}+\mathbf{L_d})\cdot\mathbf{S_l}[4\pi\delta(\mathbf{r})] \\ &-\frac{1}{3}\alpha_s\frac{1}{m_Q m_l}\frac{1}{4\mathbf{L}^2-3}[6(\mathbf{L}\cdot\mathbf{S})^2+3(\mathbf{L}\cdot\mathbf{S})-2\mathbf{L}^2\mathbf{S}^2 \\ &-6(\mathbf{L}\cdot\mathbf{S_d})^2-3(\mathbf{L}\cdot\mathbf{S_d})+2\mathbf{L}^2\mathbf{S_d}^2]\frac{1}{r^3}, \end{aligned} \tag{3}$$

where the first term corresponds to the relativistic correction to the effective scalar exchange, and the latter ones appear because of corrections to the effective single-gluon exchange with the coupling constant α_s.

The value of effective parameter α_s can be determined in the following way. The splitting in the S-wave heavy quarkonium $(Q_1\bar{Q}_2)$ is given by the expression

$$\Delta M(ns) = \frac{8}{9}\alpha_s\frac{1}{m_1 m_2}|R_{nS}(0)|^2, \tag{4}$$

where $R_{nS}(r)$ is the radial wave function of quarkonium. From the experimental data on the system of $c\bar{c}$

$$\Delta M(1S, c\bar{c}) = 117 \pm 2 \text{ MeV}, \tag{5}$$

Table 4. The characteristics of radial wave function for the bb-diquark: $R_{d(ns)}(0)$ (GeV$^{3/2}$), $R'_{d(np)}(0)$ (GeV $V^{5/2}$).

nL	$R_{d(ns)}(0)$	nL	$R'_{d(np)}(0)$
1S	1.346	2P	0.479
2S	1.027	3P	0.539
3S	0.782	4P	0.585
4S	0.681	5P	0.343

and $R_{1S}(0)$ calculated in the model, we can determine $\alpha_s(\Psi)$.

Further, take into account the dependence of this parameter on the reduced mass of the system, (μ), in the framework of the one-loop approximation for the running coupling constant of QCD:

$$\alpha_s(p^2) = \frac{4\pi}{b \cdot \ln(p^2/\Lambda^2_{QCD})}, \tag{6}$$

where $b = 11 - 2n_f/3$ and $n_f = 3$ at $p^2 < m_c^2$. From the phenomenology of potential models it is well known, that the average kinetic energy of quarks in the bound state practically does not depend on the flavors of quarks, and it is given by the values:

$$\langle T_d \rangle \approx 0.2 \text{ GeV}, \tag{7}$$

$$\langle T_l \rangle \approx 0.4 \text{ GeV}, \tag{8}$$

for the antitriplet and singlet binding, correspondingly. Substituting the definition of the nonrelativistic kinetic energy

$$\langle T \rangle = \frac{\langle p^2 \rangle}{2\mu}, \tag{9}$$

we get

$$\alpha_s(p^2) = \frac{4\pi}{b \cdot \ln(2\langle T \rangle \mu/\Lambda^2_{QCD})}, \tag{10}$$

where numerically $\Lambda_{QCD} \approx 113$ MeV.

For the identical quarks inside the diquark, the scheme, well known for the corrections of LS-coupling in the heavy quarkonium, is applicable. Otherwise, for the interaction with the light quark we use the scheme of jj-coupling (here $\mathbf{LS_l}$ is of diagonal kind at the given $\mathbf{J_l}$, ($\mathbf{J_l} = \mathbf{L} + \mathbf{S_l}, \mathbf{J} = \mathbf{J_l} + \bar{\mathbf{J}}$), where $\mathbf{J}$ denotes the total spin of baryon, and $\bar{\mathbf{J}}$ is the total momentum of diquark, $\bar{\mathbf{J}} = \mathbf{S_d} + \mathbf{L_d}$).

Then, for the estimate of various contributions and mixing of states, we use the transformations of bases (in what follows $\mathbf{S} = \mathbf{S_l} + \bar{\mathbf{J}}$)

$$|J; J_l\rangle = \sum_S (-1)^{(\bar{J}+S_l+L+J)} \sqrt{(2S+1)(2J_l+1)} \left\{ \begin{array}{ccc} \bar{J} & S_l & S \\ L & J & J_l \end{array} \right\} |J; S\rangle \tag{11}$$

Table 5. The characteristics of radial wave function for the bc-diquark: $R_{d(ns)}(0)$ (GeV$^{3/2}$), $R'_{d(np)}(0)$ (GeV $V^{5/2}$).

nL	$R_{d(ns)}(0)$	nL	$R'_{d(np)}(0)$
1S	0.726	2P	0.202
2S	0.601	3P	0.240
3S	0.561	4P	

and

$$|J; J_l\rangle = \sum_{J_d} (-1)^{(\bar{J}+S_l+L+J)} \sqrt{(2J_d+1)(2J_l+1)} \left\{ \begin{array}{ccc} \bar{J} & L & J_d \\ S_l & J & J_l \end{array} \right\} |J; J_d\rangle. \tag{12}$$

Thus, we have defined the procedure of calculations for the mass spectra of doubly heavy baryons in detail. It leads to the results presented in the next section.

2. Numerical results

Here we present the results for the calculations of mass spectra with the account for the spin-dependent splittings of levels. In this way, as it has been clarified in the Introduction, the doubly heavy baryons with the identical heavy quarks allow quite a reliable interpretation in terms of quantum numbers for the excitations of diquark (the summed up spin and the orbital momentum). As for the excitations of bc-diquark, we show the results just for the spin-dependent splitting of the basic 1S-state, since the non-forbidden emission of a soft gluon breaks a simple picture in the classification of levels for the higher excitations of such diquark.

Obviously, the quark-diquark model of bound states in the doubly heavy baryons leads to the most reliable results for the system with the larger mass of quark, i.e. for Ξ_{bb}.

2.1. Ξ_{bb} baryons

For the classification of quantum numbers in the levels, we use the notations $n_d L_d n_l l_l$, i.e. we disignate the value of principal quantum number in the diquark, its orbital moment by a capital letter and the principal quantum number for the excitations of light quark and its orbital momentum by a lower-case letter. The splitting $\Delta^{(J)}$ of $1S2p$-level is determined by the following values:

$$\Delta^{(\frac{5}{2})} = 10.3 \text{ MeV}. \tag{13}$$

The states with the total spin $J = \frac{3}{2}$ (or $\frac{1}{2}$), can have the different J_l, and, hence, they acquire a nonzero mixing, when we perform the calculations in the perturbation theory, built over the states with the definite total moment of the light constituent quark. For

Table 6. The characteristics of radial wave function for the cc-diquark: $R_{d(ns)}(0)$ (GeV$^{3/2}$), $R^{'}_{d(np)}(0)$ (GeV $V^{5/2}$).

nL	$R_{d(ns)}(0)$	nL	$R'_{d(np)}(0)$
1S	0.530	2P	0.128
2S	0.452	3P	0.158

$J = \frac{3}{2}$, the mixing matrix equals

$$\left(\begin{array}{cc} -3.0 & -0.5 \\ -0.5 & 11.4 \end{array} \right) \text{ MeV}, \tag{14}$$

so that the mixing can be practically neglected, and the level shifts are determined by the values

$$\begin{aligned} \lambda'_1 &= -3.0 \text{ MeV}, \\ \lambda_1 &= 11.4 \text{ MeV}. \end{aligned} \tag{15}$$

For $J = \frac{1}{2}$, the mixing matrix has the form

$$\left(\begin{array}{cc} -5.7 & -17.8 \\ -17.8 & -14.9 \end{array} \right) \text{ MeV}, \tag{16}$$

with the eigenvectors, given by the form,

$$\begin{aligned} |1S2p(\frac{1}{2}^{'})\rangle &= 0.790|J_l = \frac{3}{2}\rangle - 0.613|J_l = \frac{1}{2}\rangle, \\ |1S2p(\frac{1}{2})\rangle &= 0.613|J_l = \frac{3}{2}\rangle + 0.790|J_l = \frac{1}{2}\rangle, \end{aligned} \tag{17}$$

and the eigenvalues equal

$$\begin{aligned} \lambda'_2 &= 8.1 \text{ MeV}, \\ \lambda_2 &= -28.7 \text{ MeV}. \end{aligned} \tag{18}$$

For the $2S2p$-level, the corresponding quantities are equal to

$$\Delta^{(\frac{5}{2})} = 10.3 \text{ MeV}, \tag{19}$$

and for $J = \frac{3}{2}$, the mixing matrix is equal to

$$\left(\begin{array}{cc} -3.6 & -0.5 \\ -0.5 & 12.4 \end{array} \right) \text{ MeV}, \tag{20}$$

so that

$$\begin{aligned}\lambda_1' &= -3.6 \text{ MeV}, \\ \lambda_1 &= 12.4 \text{ MeV}.\end{aligned} \tag{21}$$

For $J = \frac{1}{2}$, the matrix has the form

$$\begin{pmatrix} -6.1 & -17.6 \\ -17.6 & -13.5 \end{pmatrix} \text{ MeV}, \tag{22}$$

with the eigenvectors

$$\begin{aligned}|1S2p(\frac{1}{2}')\rangle &= 0.776|J_l = \frac{3}{2}\rangle - 0.631|J_l = \frac{1}{2}\rangle, \\ |1S2p(\frac{1}{2})\rangle &= 0.631|J_l = \frac{3}{2}\rangle + 0.776|J_l = \frac{1}{2}\rangle,\end{aligned} \tag{23}$$

possessing the eigenvalues

$$\begin{aligned}\lambda_2' &= 8.2 \text{ MeV}, \\ \lambda_2 &= -27.8 \text{ MeV}.\end{aligned} \tag{24}$$

We can straightforwardly see, that the difference between the wave functions, as it is caused by different masses of diquark subsystem, is, indeed, inessential.

The splitting of diquark, $\Delta^{(J_d)}$, has the form

$$\begin{aligned} & 3D1s: \\ \Delta^{(3)} &= -0.06 \text{ MeV}, \\ \Delta^{(2)} &= 0.2 \text{ MeV}, \\ \Delta^{(1)} &= -0.2 \text{ MeV}.\end{aligned} \tag{25}$$

$$\begin{aligned} & 4D1s: \\ \Delta^{(3)} &= -2.6 \text{ MeV}, \\ \Delta^{(2)} &= -0.8 \text{ MeV}, \\ \Delta^{(1)} &= -4.6 \text{ MeV}.\end{aligned} \tag{26}$$

$$\begin{aligned} & 5D1s: \\ \Delta^{(3)} &= 2.6 \text{ MeV}, \\ \Delta^{(2)} &= -0.9 \text{ MeV}, \\ \Delta^{(1)} &= -4.7 \text{ MeV}.\end{aligned} \tag{27}$$

$$
\begin{array}{rcl}
 & 5G1s: & \\
\Delta^{(5)} & = & -0.3\ \text{MeV}, \\
\Delta^{(4)} & = & 0.3\ \text{MeV}, \\
\Delta^{(3)} & = & 1.1\ \text{MeV}, \\
\Delta^{(2)} & = & 1.7\ \text{MeV}, \\
\Delta^{(1)} & = & 2.0\ \text{MeV}.
\end{array} \tag{28}
$$

$$
\begin{array}{rcl}
 & 6G1s: & \\
\Delta^{(5)} & = & 3.2\ \text{MeV}, \\
\Delta^{(4)} & = & -0.5\ \text{MeV}, \\
\Delta^{(3)} & = & -4.4\ \text{MeV}, \\
\Delta^{(2)} & = & -7.9\ \text{MeV}, \\
\Delta^{(1)} & = & -10.5\ \text{MeV}.
\end{array} \tag{29}
$$

Evidently, such corrections are inessential in the framework of method accuracy ($\delta M \approx 30 - 40$ MeV) for the excitations of diquark, whose sizes are less than the distance to the light quark, i.e. for the diquarks with a small value of the principal quantum number.

For the fine spin-spin splitting in the system of quark-diquark, we have

$$
\Delta^{(l)}_{h.f.} = \frac{2}{9}\Big[J(J+1) - \bar{J}(\bar{J}+1) - \frac{3}{4}\Big]\alpha_s(2\mu T)\frac{1}{m_c m_l}|R_l(0)|^2, \tag{30}
$$

where $R_l(0)$ is the radial wave function at the origin for the light constituent quark, and for the analogous shift of diquark level, we find

$$
\Delta^{(d)}_{h.f.} = \frac{1}{9}\alpha_s(2\mu T)\frac{1}{m_c^2}|R_d(0)|^2. \tag{31}
$$

The spectrum of Ξ^{+}_{bb} and Ξ^{0}_{bb} baryons is shown in Fig.1 and in Tab.7, wherein we restrict ourselves by the presentation of S-, P- and D-wave levels.

We can see in Fig.1, that the most reliable predictions are the masses of baryons $1S1s$ ($J^P = 3/2^+,\ 1/2^+$), $2P1s$ ($J^P = 3/2^-,\ 1/2^-$) and $3D1s$ ($J^P = 7/2^+, \dots 1/2^+$). Note, that the $2P1s$-level is quazistable, because the transition to the basic level requires the instantaneous change of both the orbital momentum and the summed spin of quarks inside the diquark. The analogous kind of transitions seems to be the transition between the states of ortho- and para-hydrogen in the molecule of H_2. The latter appears in the nonhomogeneous external field, produced by the magnetic moments of other molecules. For the transition of $2P1s \to 1S1s$, the role of such external field is played by the nonhomogeneous chromomagnetic field of the light quark. The corresponding perturbation has the form

$$
\begin{aligned}
\delta V &\sim \frac{1}{m_Q}[\mathbf{S}_1 \cdot \mathbf{H}_1 + \mathbf{S}_2 \cdot \mathbf{H}_2 - (\mathbf{S}_1 + \mathbf{S}_2) \cdot \langle \mathbf{H} \rangle] \\
&= \frac{1}{2m_Q}(\nabla \cdot \mathbf{r_d})\ (\mathbf{S}_1 - \mathbf{S}_2) \cdot \mathbf{H} \sim \frac{1}{m_Q}\frac{\mathbf{r_l} \cdot \mathbf{r_d}}{m_q r_l^5}\ (\mathbf{S}_1 - \mathbf{S}_2) \cdot \mathbf{J_l}\ f(r_l),
\end{aligned}
$$

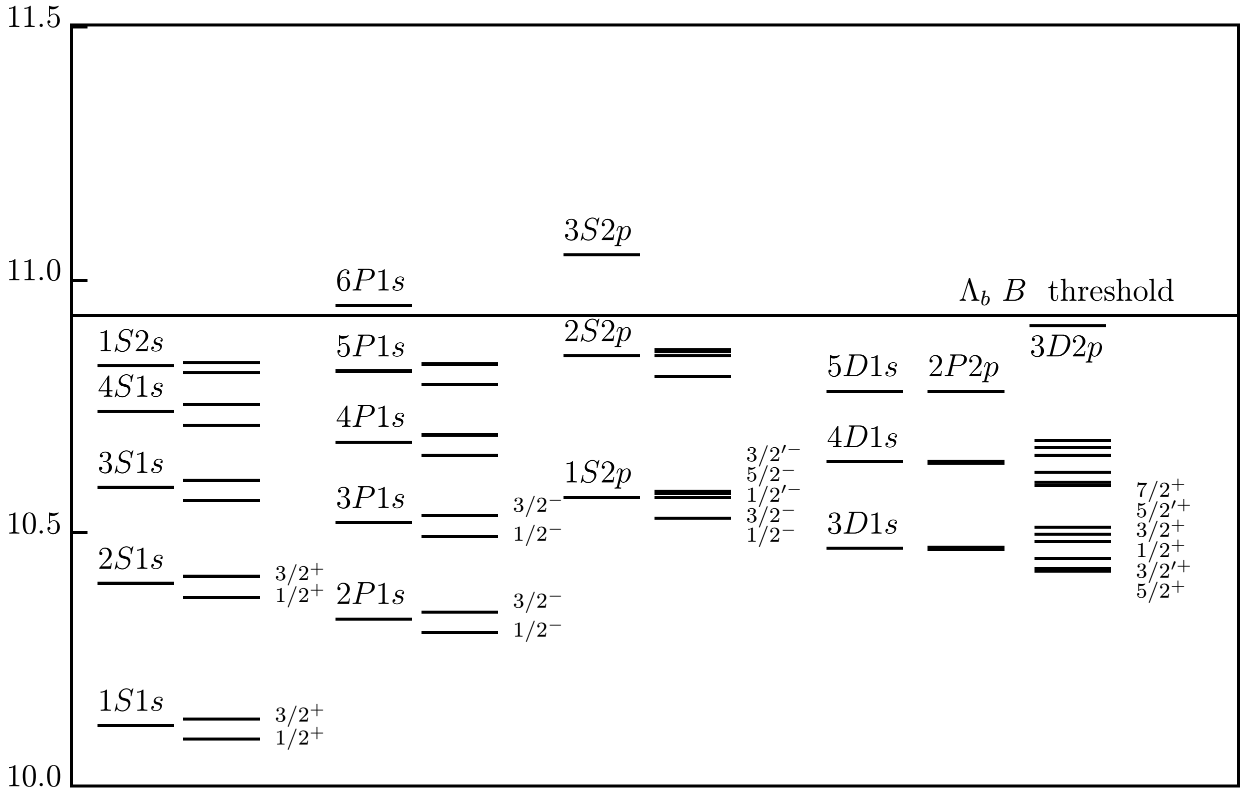

Fig. 1. The spectrum of baryons, containing two b-quarks: Ξ_{bb}^- and Ξ_{bb}^0, with the account for the spin-dependent splittings of low-lying excitations. The masses are given in GeV.

where $f(r_l)$ is a dimensionless nonperturbative function dependent on the coordinates of the light quark with respect to the diquark. Evidently, the δV operator changes the orbital momentum of light quark, too. It results in the mixing between the states with the same values of J^P. If the splitting is not small (for instance, $2P1s - 1S2p$, where $\Delta E \sim \Lambda_{QCD}$), then the mixing is suppressed: $\delta V/\Delta E \sim \frac{1}{m_Q m_q}\frac{r_d}{r_l^4}\frac{1}{\Delta E} \ll 1$. Since the admixture of $1S2p$ in the $2P1s$-state is low, the $2P1s$-levels are quazistable, i.e. their hadronic transitions with the emission of π-mesons to the basic level are suppressed, as it has been derived, though an additional suppression is given by a small value of phase space. Therefore, we have to expect the appearance of anomalously narrow resonances in spectra of pairs $\Xi_{bb}\pi$, as they are produced in the decays of quazistable states with $J^P = 3/2^-,\ 1/2^-$. The experimental observation of such levels could straightforwardly confirm the existence of diquark excitations and provide the information on the character of dependency in $f(r_l)$, i.e. on the genesis of the nonhomogeneous chromomagnetic field in the nonperturbative region.

Sure, the $3D1s$ $J^P = 7/2^+,\ 5/2^+$ states are also quazistable, since in the framework of multipole expansion of QCD they transform to the basic level due to the quadrupole emission of gluon (the E2-transition with the hadronization $gq \to q'\pi$).

As for the higher excitations, the $3P1s$-states are close to the $1S2p$-levels with $J^P = 3/2^-,\ 1/2^-$, so that even the operators, changing both the orbital moment of diquark and its spin and suppressed by the inverse heavy quark mass and the small size of diquark, can lead to an essential mixing with the amplitude $\delta V_{nn'}/\Delta E_{nn'} \sim 1$. However, we believe, that the mixing slightly shifts the masses of states. The most important thing is that a large admixture of $1S2p$ in $3P1s$ make the state unstable with respect to the transition into the basic $1S1s$-state with the emission of gluon (the E1-transition). In the physical spectra, the transition leads to decays, say, with the π-mesons[2]. The level $1S2p$ $J^P = 5/2^-$ has the definite quantum numbers of diquark and light quark, because in its vicinity, there are no levels with the same values of J^P. However, its width for the transition into the basic state with the emission of π-meson is not suppressed and seems to be large, $\Gamma \sim 100$ MeV.

Table 7. The mass spectrum of Ξ^-_{bb} and Ξ^0_{bb} baryons.

$(n_d L_d n_l L_l)$, J^P	mass (GeV)	$(n_d L_d n_l L_l)$, J^P	mass (GeV)
(1S 1s)$1/2^+$	10.093	(3P 1s)$1/2^-$	10.493
(1S 1s)$3/2^+$	10.133	(3D 1s)$5/2'^+$	10.497
(2P 1s)$1/2^-$	10.310	(3D 1s)$7/2^+$	10.510
(2P 1s)$3/2^-$	10.343	(3P 1s)$3/2^-$	10.533
(2S 1s)$1/2^+$	10.373	(1S 2p)$1/2^-$	10.541
(2S 1s)$3/2^+$	10.413	(1S 2p)$3/2^-$	10.567
(3D 1s)$5/2^+$	10.416	(1S 2p)$1/2'^-$	10.578
(3D 1s)$3/2'^+$	10.430	(1S 2p)$5/2^-$	10.580
(3D 1s)$1/2^+$	10.463	(1S 2p)$3/2'^-$	10.581
(3D 1s)$3/2^+$	10.483	(3S 1s)$1/2^+$	10.563

Note, that

$$\begin{aligned}
\frac{3}{2}^- &\to \frac{3}{2}^+ \pi \ \text{ in S} - \text{wave},\\
\frac{3}{2}^- &\to \frac{1}{2}^+ \pi \ \text{ in D} - \text{wave},\\
\frac{1}{2}^- &\to \frac{3}{2}^+ \pi \ \text{ in D} - \text{wave},\\
\frac{1}{2}^- &\to \frac{1}{2}^+ \pi \ \text{ in S} - \text{wave}.
\end{aligned}$$

The D-wave transitions are suppressed by low recoil momenta with respect to the masses of baryons.

The width of low-lying state $J^P = 3/2^+$ is completely determined by the radiative electromagnetic M1-transition into the basic $J^P = 1/2^+$ state.

[2]Remember, that the $\Xi_{QQ'}$-baryons are the iso-doublets.

2.2. Ξ_{cc} baryons

The calculation procedure described above leads to the following results for the doubly charmed baryons.

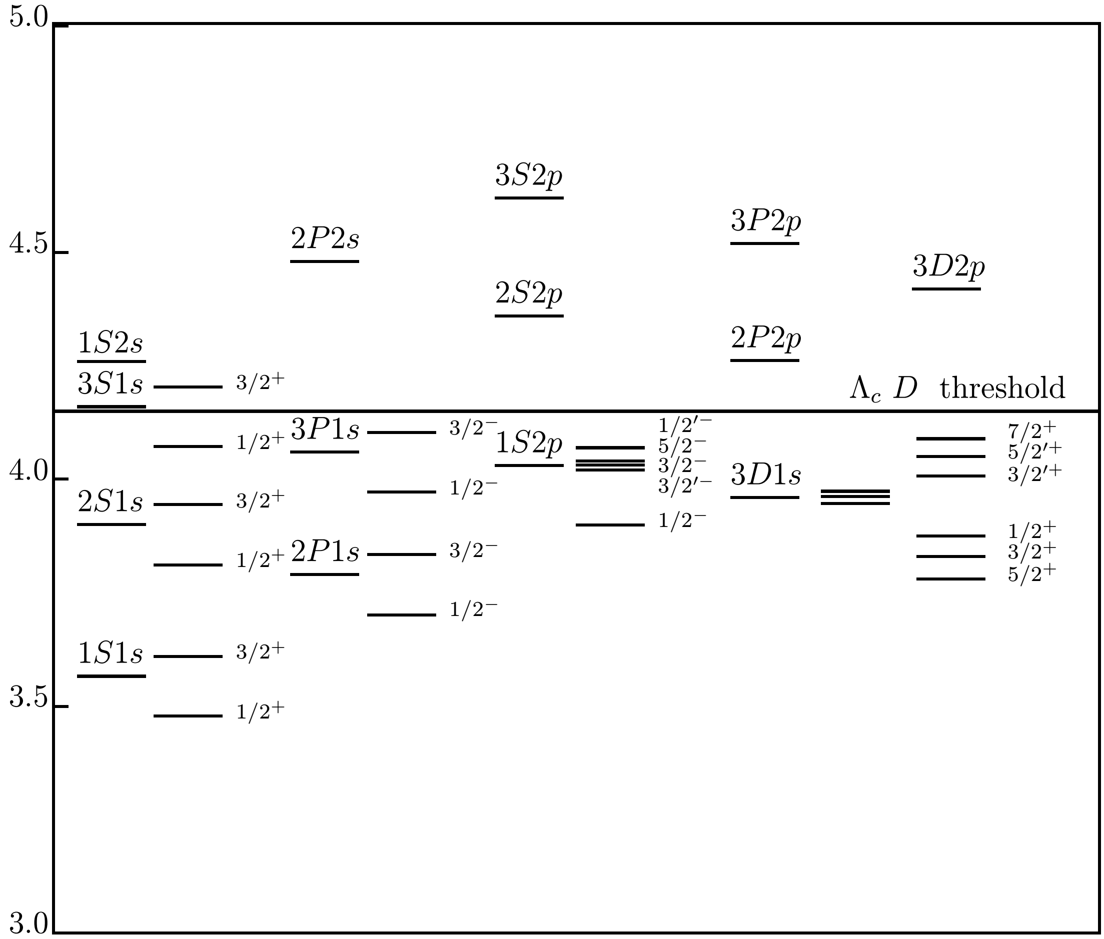

Fig. 2. The spectrum of Ξ_{cc}^{++} and Ξ_{cc}^{+} baryons. The masses are given in GeV.

For $1S2p$, the splitting is equal to

$$\Delta^{(\frac{5}{2})} = 17.4 \text{ MeV}. \tag{32}$$

For $J = \frac{3}{2}$, the mixing is determined by the matrix

$$\begin{pmatrix} 4.3 & -1.7 \\ -1.7 & 7.8 \end{pmatrix} \text{ MeV}, \tag{33}$$

so that the eigenvectors

$$\begin{aligned} |1S2p(\frac{3}{2}')\rangle &= 0.986|J_l = \frac{3}{2}\rangle + 0.164|J_l = \frac{1}{2}\rangle, \\ |1S2p(\frac{3}{2})\rangle &= -0.164|J_l = \frac{3}{2}\rangle + 0.986|J_l = \frac{1}{2}\rangle, \end{aligned} \tag{34}$$

have the eigenvalues

$$\begin{aligned} \lambda_1' &= 3.6 \text{ MeV}, \\ \lambda_1 &= 8.5 \text{ MeV}. \end{aligned} \tag{35}$$

For $J = \frac{1}{2}$, the mixing matrix equals

$$\begin{pmatrix} -3.6 & -55.0 \\ -55.0 & -73.0 \end{pmatrix} \text{MeV}, \tag{36}$$

where the vectors

$$\begin{aligned} |1S2p(\frac{1}{2}^{\prime})\rangle &= 0.957|J_l = \frac{3}{2}\rangle - 0.291|J_l = \frac{1}{2}\rangle, \\ |1S2p(\frac{1}{2})\rangle &= 0.291|J_l = \frac{3}{2}\rangle + 0.957|J_l = \frac{1}{2}\rangle, \end{aligned} \tag{37}$$

have the eigenvalues

$$\begin{aligned} \lambda_2' &= 26.8 \text{ MeV}, \\ \lambda_2 &= -103.3 \text{ MeV}. \end{aligned} \tag{38}$$

The splitting for the $3D$ diquark level gives

$$\begin{aligned} \Delta^{(3)} &= -3.02 \text{ MeV}, \\ \Delta^{(2)} &= 2.19 \text{ MeV}, \\ \Delta^{(1)} &= 3.39 \text{ MeV}. \end{aligned} \tag{39}$$

$$\tag{40}$$

Further, we have to take into account the fine spin-spin corrections in the quark-diquark system.

For the $1S$- and $2S$-wave levels of diquark, the shifts of vector states are equal to

$$\begin{aligned} \Delta(1S) &= 6.3 \text{ MeV}, \\ \Delta(2S) &= 4.6 \text{ MeV}. \end{aligned}$$

The mass spectra for the Ξ_{cc}^{++} and Ξ_{cc}^{+} baryons are presented in Fig.2 and Tab.8.

2.3. Ξ_{bc} baryons

As we have already noted in the Introduction, the heavy diquark, composed of the quarks of different flavors, turns out to be unstable with respect to the emission of soft gluons in the way, that in the Fock state of doubly heavy baryon there is a noticeable nonperturbative admixture of configurations, including the gluons and diquark with the various values of its spin S_d and orbital momentum L_d:

$$|B_{bcq}\rangle = O_B|bc_{\bar{3}_c}^{S_d,L_d}, q\rangle + H_1|bc_{\bar{3}_c}^{S_d\pm1,L_d}, g, q\rangle + H_2|bc_{\bar{3}_c}^{S_d,L_d\pm1}, g, q\rangle + \dots,$$

Table 8. The mass spectrum of Ξ_{cc}^{++} and Ξ_{cc}^{+} baryons.

$(n_d L_d n_l L_l)$, J^P	mass (GeV)	$(n_d L_d n_l L_l)$, J^P	mass (GeV)
(1S 1s)$1/2^+$	3.478	(3P 1s)$1/2^-$	3.972
(1S 1s)$3/2^+$	3.61	(3D 1s)$3/2'^+$	4.007
(2P 1s)$1/2^-$	3.702	(1S 2p)$3/2'^-$	4.034
(3D 1s)$5/2^+$	3.781	(1S 2p)$3/2^-$	4.039
(2S 1s)$1/2^+$	3.812	(1S 2p)$5/2^-$	4.047
(3D 1s)$3/2^+$	3.83	(3D 1s)$5/2'^+$	4.05
(2P 1s)$3/2^-$	3.834	(1S 2p)$1/2'^-$	4.052
(3D 1s)$1/2^+$	3.875	(3S 1s)$1/2^+$	4.072
(1S 2p)$1/2^-$	3.927	(3D 1s)$7/2^+$	4.089
(2S 1s)$3/2^+$	3.944	(3P 1s)$3/2^-$	4.104

where the amplitudes of H_1, H_2 are not small with respect to O_B. In the heavy quarkonium, the analogous operators for the octet-color states are suppressed by the probability for the emission by the nonrelativistic quarks inside the small volume, determined by the size of singlet-color system of quark and antiquark. In the baryonic system under consideration, the soft gluon is restricted only by the ordinary scale of confinement, and, hence, there is no suppression.

After that we suppose, that the mass calculations for the excited baryons of Ξ_{bc} are not justified in the given scheme. Therefore, we present only the result for the basic state with $J^P = 1/2^+$:

$$M_{\Xi'_{bc}} = 6.85 \text{ GeV}, \quad M_{\Xi_{bc}} = 6.82 \text{ GeV},$$

whereas for the vector diquark we have assumed, that the spin-dependent splitting due to the interaction with the light quark is determined by the standard contact coupling of magnetic moments for the point-like systems. The picture for the baryon levels with no account for the spin-dependent perturbation, suppressed by the heavy quark masses, in the higher excitations is shown in Fig.3.

2.4. The doubly heavy baryons with the strangeness: $\Omega_{QQ'}$

In the leading approximation we suppose that, with a good accuracy, the wave functions and the excitation energies of a strange quark in the field of diquark repeat the characteristics for the analogous baryons with the quarks u, d. Therefore, the level system of baryons $\Omega_{QQ'}$ reproduces that of $\Xi_{QQ'}$ with the accuracy of additive shift for the masses by the value of current mass of strange quark, $m_s \approx M(D_s) - M(D) \approx M(B_s) - M(B) \approx 0.1$ GeV.

Further, we suppose that the spin-spin splitting for the low-lying $\Omega_{QQ'}$ baryons in the levels $n_d S n_l s$ with $2P1s$ and $3D1s$ is 20-30% less than in $\Xi_{QQ'}$ (the factor of $m_{u,d}/m_s$). As for the $1S2p$-level, the procedure described above can be applied. So, in Ω_{bb}, the matrix

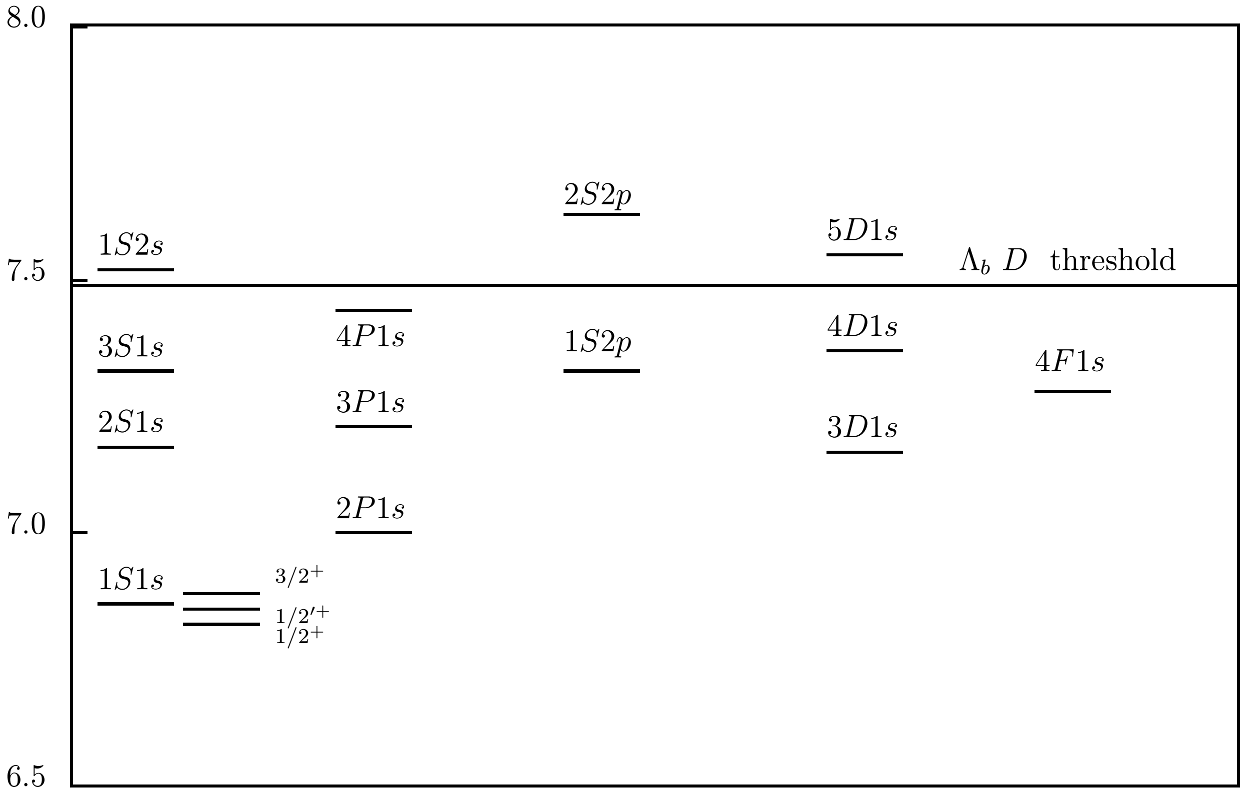

Fig. 3. The spectrum of Ξ_{bc}^{+} and Ξ_{bc}^{0} baryons without the splittings of higher excitations. The masses are given in GeV.

for the mixing of states with the different values of total moment for the light constituent quark can be practically assigned to be diagonal. The latter means, that the following term of perturbation is dominant:

$$\frac{1}{4}\left(\frac{\mathbf{2L}\cdot\mathbf{S_l}}{\mathbf{2m_l^2}}\right)\left(-\frac{dV(r)}{rdr}+\frac{8}{3}\alpha_s\frac{1}{r^3}\right).$$

Therefore, we can believe that the splitting of $1S2p$ is determined by the factor of $m_{u,d}^2/m_s^2$, i.e. it is 40 % less, than in Ξ_{bb}. Hence, the splitting is very low.

In the baryon Ω_{cc}, the factor of m_s/m_c is not small, and, hence, for $1S2p$, the mixing matrix for the states with the different values of total moment for the light constituent quark is not diagonal, so that the order in the spin states of $1S2p$ in Ω_{cc} can be slightly different from the arrangement in Ξ_{cc}.

It is of great interest to point to the following peculiarity of $\Omega_{QQ'}$: the low-lying S- and P-excitations of diquark will be stable with respect to the decays due to the strong interaction even after the account for the mixing of levels, possessing various spins and orbital moments. Indeed, the gluon emission causes its hadronization into the K-mesons (the transitions of $\Omega_{QQ'}\to\Xi_{QQ'}+K$), and the single emission of π-mesons is forbidden due to the conservation of iso-spin and strangeness, so that the corresponding hadronic transitions with kaons do not appear because of an insufficient splitting between the level masses in $\Omega_{QQ'}$ and $\Xi_{QQ'}$, and the decays with the emission of pion pairs, belonging to the iso-singlet state, are suppressed by a small phase space or even forbidden. Thus, the radiative electromagnetic transitions to the basic level are the dominant modes of decays for the low-lying excitations in $\Omega_{QQ'}$.

2.5. Ω_{bbc} baryons

In the framework of quark-diquark picture, we can build the model for the baryons with three heavy quarks: *bbc*. However, as we estimate, the size of diquark turns out to be comparable with the squared-mean distance to the charmed quark, so that the model assumption on the compact heavy diquark cannot be very accurate for the calculations of mass levels in this case. As for the force, depending on the spins, it is negligibly small for the interactions inside the diquark, as pointed out above. The spin-spin splittings for the vector diquark, interacting with the charmed quark, are equal to

$$\Delta(1s) = 33 \text{ MeV}, \ \Delta(2s) = 18 \text{ MeV},$$

and for $1S2p$, the level shifts are small, so that just for the state $J^P = 1/2$ we have to include the correction of -33 MeV. In the $3D1s$-state the splitting is determined by the spin-spin interaction. The characteristics of excitations for the charmed quark in the model with the potential by Buchmüller–Tye have been presented above. Finally, we obtain the picture for the levels of Ω_{bbc} baryons, as presented in Fig.4 and Tab.9.

Further, we would like to note that in some cases the excitations of basic Ω^0_{bbc} state can strongly mix at large amplitudes because of the small splittings between the levels, but they acquire small shifts of masses. It occurs for $3P1s - 1S2p$ with $J^P = 1/2^-,\ 3/2^-$, and $2S1s - 3D1s$ with $J^P = 1/2^+,\ 3/2^+$. We suppose the prediction for the states of $1S1s$ with $J^P = 1/2^+,\ 3/2^+$, $1S2p$ with $J^P = 5/2^-$ and $3D1s$ with $J^P = 5/2^+,\ 7/2^+$ to be quite reliable. Certainly, for these excitations, we might definitely predict the widths of their radiative electromagnetic transitions into the basic state in the framework of multipole expansion of QCD. The widths for the transitions, including the mixed states, will be essentially determined by the amplitudes of admixtures, which have a strong model dependence. In this respect, the experimental study of electromagnetic transitions in the family of Ω^0_{bbc} baryons could provide a valuable information on the mechanism of mixing for the different levels in the baryonic systems. Note, that the electromagnetic transitions combined with the emission of pion pairs, if the latter ones are not forbidden by the phase space, saturate the total widths for the excited levels of Ω^0_{bbc}. We believe that the characteristic value of the total width is at the level of magnitude like $\Gamma \sim 10 - 100$ keV.

Thus, the system of Ω^0_{bbc} can be characterized by the large number of narrow quazi-stable states.

Conclusions

In this paper we have performed the detailed calculations for the spectroscopic characteristics of baryons containing two heavy quarks in the model with the quark-diquark

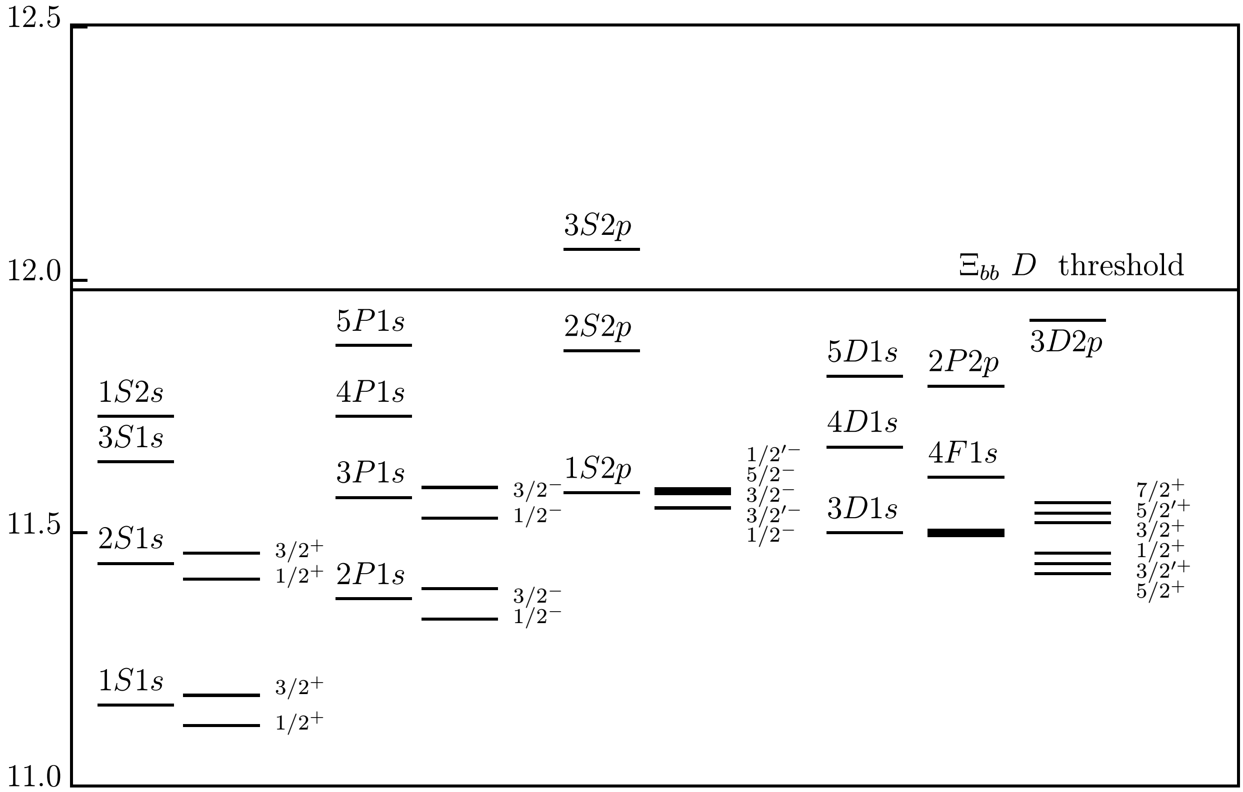

Fig. 4. The spectrum of Ω^0_{bbc} baryons with account for the spin-dependent splittings for the low-lying excitations. Masses are given in GeV.

factorization of wave functions, in the framework of nonrelativistic model of constituent quarks with the potential by Buchmüller–Tye. We have restricted the region for the application of such approximations.

We have taken into account the spin-dependent relativistic corrections to the potential in the subsystems of diquark and light quark-diquark, so that below the threshold of decay into the heavy baryon and heavy meson with the single heavy quark in both, there is the system of bound excited states, being quazi-stable with respect to hadronic transitions to the basic level. We have in detail considered the physical reasons for the quazi-stability, taking place for the baryons with two identical quarks. In accordance with the Pauli principle, the operators responsible for the hadronic decays and the mixing between the levels are suppressed by the inverse heavy quark mass and the small size of diquark. This suppression is caused by the necessity to instantaneously change both the spin and the orbital momentum of compact diquark. In the baryonic systems with two heavy quarks and the strange quark, the quazi-stability for the low-lying excitations of diquark is provided by the exclusion of transitions with the emission of both a single kaon because of small splitting between the levels and a single pion because of conservation for the iso-spin and strangeness.

Table 9. The mass spectrum of Ω^0_{bbc} baryons.

$(n_d L_d n_l L_l)$, J^P	mass (GeV)	$(n_d L_d n_l L_l)$, J^P	mass (GeV)
(1S 1s)$1/2^+$	11.12	(3D 1s)$3/2'^+$	11.52
(1S 1s)$3/2^+$	11.18	(3D 1s)$5/2'^+$	11.54
(2P 1s)$1/2^-$	11.33	(1S 2p)$1/2^-$	11.55
(2P 1s)$3/2^-$	11.39	(3D 1s)$7/2^+$	11.56
(2S 1s)$1/2^+$	11.40	(1S 2p)$3/2'^-$	11.58
(3D 1s)$5/2^+$	11.42	(1S 2p)$3/2^-$	11.58
(3D 1s)$3/2^+$	11.44	(1S 2p)$1/2'^-$	11.59
(3D 1s)$1/2^+$	11.46	(1S 2p)$5/2^-$	11.59
(2S 1s)$3/2^+$	11.46	(3P 1s)$3/2^-$	11.59
(3P 1s)$1/2^-$	11.52	(3S 1s)$1/2^+$	11.62

The characteristics of wave functions can be used in calculations of cross-sections for the doubly heavy baryons in the framework of quark-diquark approximation.

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