IHEP 98-70

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MASSIVE FIELDS
OF ARBITRARY HALF-INTEGER SPIN
IN CONSTANT ELECTROMAGNETIC FIELD

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Protvino 1998

Abstract

Klishevich S.M. Massive Fields of Arbitrary Half-Integer Spin in Constant Electromagnetic Field: IHEP Preprint 98-70. – Protvino, 1998. – p. 15, refs.: 21.

We study the interaction of gauge fields of arbitrary half-integer spins with the homogeneous electromagnetic field. We reduce the problem of obtaining the gauge-invariant Lagrangian and transformations of the half-integer spin fields in the external field to a purely algebraic problem of finding a set of operators with certain features using the representation of the higher-spin fields as vectors in a pseudo-Hilbert space. We consider such construction at linear order in the external electromagnetic field and also present an explicit form of interaction Lagrangians and gauge transformations for the massive particles of spins $\frac{3}{2}$ and $\frac{5}{2}$ in terms of symmetric spin-tensor fields. The obtained result is valid for space-time of arbitrary even dimension.

Аннотация

Клишевич С.М. Массивные поля произвольного полуцелого спина в постоянном электромагнитном поле: Препринт ИФВЭ 98-70. – Протвино, 1998. – 15 с., библиогр.: 21.

Мы изучаем взаимодействие массивных калибровочных полей произвольных полуцелых спинов с однородным электромагнитным полем. Основываясь на представлении полей высоких спинов в виде векторов некоторого псевдогильбертового пространства, мы сводим проблему получения калибровочно-инвариантного лагранжиана полей полуцелых спинов во внешнем поле к чисто алгебраической задаче отыскания некоторого набора операторов с определенными свойствами. Мы рассматриваем такое построение в линейном порядке по напряженности электромагнитного поля и приводим явный вид лагранжианов взаимодействия для массивных частиц со спинами $\frac{3}{2}$ и $\frac{5}{2}$ в терминах симметричных спин-тензорных полей. Полученный результат имеет место для поизвольного пространства-времени четной размерности.

1. Introduction

At present there are a lot of different approaches to the description of free higher-spin fields (see for, example, [1]-[7]). But the investigation of interactions usually faces significant difficulties. Very often the study of the higher-spin fields ends at the free level. In this work we develop the algebraic approach that allows us to construct an interaction for all the massive half-integer spin fields at once.

As is well-known, the massless fields of spins $s \geq \frac{3}{2}$ do not have the "minimal" interaction with electromagnetic field in asymptotically flat space-time. This is related to the impossibility to construct linear approximation in such a case [8]. But the massive higher-spin fields can have the interaction [9]-[12]. Therefore, we will study the massive case only.

In the literature the electromagnetic interaction of arbitrary spin fields was considered at the lowest order [13]. Investigating the interactions, the authors start from the free theory of massive fields in the conventional form [1]. The "minimal" introduction of the interaction leads to contradictions. Therefore, it is necessary to include non-minimal terms into an interaction Lagrangian. Since the massive Lagrangian for the higher-spin fields [1] is not gauge invariant, there are no restrictions on the form of the non-minimal interaction in such approach and additional restrictions have to be proposed in order to build a consistent theory. So, for instance, studying the electromagnetic interaction,[13] the authors have used the requirement that tree-level scatering amplitudes must possess a smooth $M \to 0$ fixed-charge limit for any theory describing the interaction of arbitrary-spin massive particles with photons. Under such requirement, the amplitudes do not violate unitarity up to center-of-mass energies $E \gg M/e$. This restriction leads to the gyromagnetic ratio $g = 2$ for massive particles of any spin. In Ref. [14] the authors were investigating the electromagnetic interaction of massive spin-2 field using the compactification of the 5 dimension gravity. But that approach does not work (in any case in the asymptotically flat space) for the fields of the higher spins since it implies the existence of an consistent theory of the interaction for the massless higher-spin fields. In Ref. [9] the way to obtain the e.m. interaction of the massive fields of arbitrary spins has been

proposed. This method is founded on the formulation of the open bosonic string in the external constant field. But in Ref. [11] it has been shown that such approach allows one to get the e.m. interaction of the fields for the whole string mass level rather than for the single massive field, because the presence of the interaction mixes the states of given mass level.

Here we go along the line of Ref. [12] where the massive integer-spin fields were investigated. We consider the interaction of massive fields of arbitrary half-integer spin with the the homogeneous electromagnetic field at linear order.

We represent free state with the half-integer spin $s+\frac{1}{2}$ as state $|\Psi^s\rangle$ in a pseudo-Hilbert space[1]. This space contains the bosonic one [12] as subspace. Coefficient functions of the state $|\Psi^s\rangle$ are spin-tensor fields corresponding to a particle of spin $s+\frac{1}{2}$. In the considered Fock space we introduce a set of even and odd operators. By means of these operators we define the gauge transformations and the necessary constraints for the state $|\Psi^s\rangle$. Like the bosonic case the gauge-invariant Lagrangian has the form of the expectation value of a Hermitian operator in state $|\Psi^s\rangle$ but the operator is odd in this case.

In the considered approach the gauge invariance is a consequence of commutation relations of the introduced operators. The introduction of the interaction by means of the "minimal" coupling prescription induces a change of algebraic features of the operators and, as a consequence, leads to the loss of the gauge invariance. The problem of restoring the invariance is reduced to the algebraic problem of finding such modified operators that restore the initial commutation relations. We should note that in the massless case one cannot realize such a construction. This relates to the fact that when the interaction is present, the massless limit does not exist.

In section 4 we construct the set of the operators having the necessary features at linear order in the external e.m. field. Besides in the next section we give an explicit form of the interaction Lagrangian and the transformations for the massive spin-$\frac{3}{2}$ and spin-$\frac{5}{2}$ fields in this approximation.

2. Gauge Massive Fields with Integer Spins

Here we will briefly discuss a description of the massive fields with an integer spin using an auxiliary Fock space.

Let us consider the Fock space generated by creation and annihilation operators $\bar{a}_\mu$, a_μ with a Lorentz vector index and by the scalar ones $\bar{b}$ and b. These operators have the commutation relations

$$
\begin{aligned}
[a_\mu, \bar{a}_\nu] &= g_{\mu\nu}, \quad a_\mu^\dagger = \bar{a}_\mu, \\
\left[b, \bar{b}\right] &= 1, \quad b^\dagger = \bar{b}.
\end{aligned}
\tag{1}
$$

where $g_{\mu\nu}$ is the metric tensor on space-time $\mathcal{M}_D$ of arbitrary dimension D with the signature$\|g_{\mu\nu}\| = \mathrm{diag}(-1, 1, 1, ..., 1)$. Since the metric is indefinite, the Fock space, which

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[1] In a similar way the representation of massless free fields was considered in Refs [7,15] for an arbitrary integer spin and in Ref. [16] for any half-integer spin.

realizes the representation of the Heisenberg algebra (1), is pseudo-Hilbert. Therefore, to exclude states with negative norms, we have to impose additional conditions on the physical states.

We will consider the states of type

$$|\Phi^s\rangle = \sum_{n=0}^{s} \Phi_{\mu_1...\mu_n}(x)\bar{b}^{s-n} \prod_{i=1}^{n} \bar{a}_{\mu_i}|0\rangle, \tag{2}$$

where $|0\rangle$ is the usual Fock vacuum. The coefficient functions $\Phi_{\mu_1...\mu_n}(x)$ are symmetric tensor fields of rank n on $\mathcal{M}_\mathcal{D}$. At $s \to \infty$ such states span the whole Fock space.

In order to properly describe the physical state of spin s by vector (2), we should impose a restriction on this state. For that we introduce in our pseudo-Hilbert space the following operators:

$$L_1 = p_\mu a^\mu + mb, \quad L_{-1} = L_1^\dagger,$$
$$L_2 = \tfrac{1}{2}\left(a_\mu a^\mu + b^2\right), \quad L_{-2} = L_2^\dagger, \tag{3}$$
$$L_0 = p^2 + m^2.$$

Here $p_\mu = i\partial_\mu$ is the momentum operator, which acts on the coefficient functions.

Operators (3) satisfy the following commutation relations:

$$\begin{array}{llll} [L_1, L_{-2}] &=& L_{-1}, & [L_1, L_2] &=& 0, \\ [L_2, L_{-2}] &=& N + \frac{D+1}{2}, & [L_0, L_n] &=& 0, \\ [L_1, L_{-1}] &=& L_0, & [N, L_n] &=& -nL_n, \quad n = 0, \pm 1, \pm 2, \end{array} \tag{4}$$

where $N = \bar{a}_\mu a^\mu + \bar{b}b$. Vectors of type (2) are eigenvector of this operator, i.e. N defines the spin of the state

$$N|\Phi^s\rangle = s|\Phi^s\rangle.$$

Let us impose the following condition on state (2)

$$(L_2)^2|\Phi^s\rangle = 0. \tag{5}$$

This corresponds to the usual condition that the tensor fields describing massive (massless) higher-spin particles [10,12] is twice-traceless.

In order to avoid the presence of the redundant states, we must also have the gauge transformations for state (2) in the form

$$\delta|\Phi^s\rangle = L_{-1}|\Lambda^{s-1}\rangle. \tag{6}$$

Here the gauge Fock vector

$$|\Lambda^{s-1}\rangle = \sum_{n=0}^{s-1} \Lambda_{\mu_1...\mu_n}\bar{b}^{s-n-1} \prod_{i=1}^{n} \bar{a}_{\mu_i}|0\rangle, \tag{7}$$

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is eigenvector of operator N with eigenvalue $s - 1$. This vector satisfies the condition

$$L_2|\Lambda\rangle = 0. \tag{8}$$

It is easy to verify that these relations define the usual gauge transformations for the coefficient functions.

The gauge Lagrangian for the massive fields with the integer spin can be written as the expectation value of a Hermitian operator in state (2)

$$\mathcal{L}_s = \langle\Phi^s|\mathcal{L}(L)|\Phi^s\rangle, \quad \langle\Phi^s| = |\Phi^s\rangle^\dagger, \tag{9}$$

where

$$\begin{aligned}
\mathcal{L}(L) &= \frac{1}{2}L_0 - \frac{1}{2}L_{-1}L_1 - L_{-2}L_0L_2 - \frac{1}{2}L_{-2}L_{-1}L_1L_2 \\
&\quad + \frac{1}{2}\left\{L_{-2}L_1L_1 + \text{h.c.}\right\}.
\end{aligned} \tag{10}$$

Lagrangian (9) is invariant with respect to transformations (6) as a consequence of (8) and of the relation

$$\mathcal{L}(L)L_{-1} \sim (...)L_2.$$

In the free case one can regard this construction as the dimensional reduction $\mathcal{M}_{D+1} \to \mathcal{M}_D \otimes S^1$ of the massless theory with the radius of the sphere $R \sim 1/m$ (refer also to [7,15]). We should note that this statement is not valid when an interaction is present because in this case the terms corresponding to the interaction are proportional to an inverse degree of the mass parameter. For example, operator L_1 deformed in the presence of the constant electromagnetic field [12] has the following form at linear order

$$\begin{aligned}
L_1^{(1)} &= \frac{1}{m}\left(1 - d_2\right)\left(\bar{\alpha}F\alpha\right)\beta + \frac{1}{m^2}\left\{\left(\mathcal{P}F\alpha\right)\left(d_1\left(\frac{1}{2} - \bar{\beta}\beta\right)e^{-2\bar{\beta}\beta}\right.\right. \\
&\quad \left.\left. + d_2\left(\frac{1}{2} + \bar{\beta}\beta\right)\right) + \left(\bar{\alpha}F\mathcal{P}\right)\left(d_1e^{-2\bar{\beta}\beta} + d_2\right)b^2\right\},
\end{aligned} \tag{11}$$

where α_μ and β are normal symbols of operators a_μ and b, correspondingly. Obviously, in linear approximation action (9) contains the terms proportional to the inverse degree of mass parameter m as well. Thereby one cannot perform the smooth transition to the massless case in the presence of the interaction.

3. Massive Half-integer spin fields

In this section we develop a similar construction for the gauge fields with half-integer spins.

The massless fermionic gauge field with spin $s + 1/2$ are usually described by means of symmetric spin-tensor $\Psi_{\mu_1...\mu_s}$.

Henceforth we will use the following definition:

$$\Psi' = \gamma^\mu \Psi_{\mu\mu_2...\mu_s}.$$

The massless gauge field satisfies the condition

$$\Psi''' = 0. \tag{12}$$

The gauge transformation for this field is

$$\delta\Psi_{\mu_1...\mu_s} = \partial_{(\mu_1}\xi_{\mu_2...\mu_s)}, \tag{13}$$

where ξ is a fermionic gauge parameter, which obeys the condition

$$\xi' = 0. \tag{14}$$

Transformation (13) and conditions (12), (14) unambiguously determine the Lagrangian for the free fields up to surface terms

$$\begin{aligned}
\mathcal{L}^F_{s+1/2} &= i\left\{ \bar{\psi} \cdot \hat{\partial}\psi + s\bar{\psi}' \cdot \hat{\partial}\psi' - \frac{1}{4}s(s-1)\bar{\psi}'' \cdot \hat{\partial}\psi'' \right. \\
&\quad \left. - 2s\bar{\psi}' \cdot (\partial \cdot \psi) + s(s-1)\bar{\psi}'' \cdot (\partial \cdot \psi') \right\}.
\end{aligned} \tag{15}$$

One can easily see that the Lagrangian does not depend on space-time dimensionality explicitly.

In Ref. [17] the authors derived a gauge description of the massive free half-integer spin fields from the dimensional reduction of action (15). Our way to obtain the action for the fermionic fields resembles this construction in non-interacting case but we use an auxiliary Fock space for that (for the massless fields see also Ref. [16,18]).

In order to describe the massive fermionic fields in a Fock space, we enlarge the pseudo-Hilbert space generated by operators (1) by means of the anticommuting operators which obey the following relations

$$\{\Gamma_\mu, \Gamma_\nu\} = 2g_{\mu\nu}, \qquad \left\{\Gamma_\mu, \bar{\Gamma}\right\} = 0, \qquad \bar{\Gamma}^2 = 1. \tag{16}$$

We will consider these operators as the Hermitian ones. We will also consider space-time of even dimensionality only. This implies that the operator $\bar{\Gamma}$ is not independent

$$\Gamma \sim \Gamma_0\Gamma_1...\Gamma_{D-1}.$$

In order to realize a representation of algebra (16), we introduce a spinor vacuum vector. This vector transforms as spinor under the Lorentz transformations and, therefore, it carries spinor index

$$|0\rangle_F = |0\rangle_\alpha \tag{17}$$

Let us define the action of operators (16) on the spinor vacuum by

$$
\begin{aligned}
\Gamma_\mu |0\rangle_\alpha &= (\bar{\gamma}\gamma_\mu)_{\beta\alpha} |0\rangle_\beta, \\
\bar{\Gamma} |0\rangle_\alpha &= \bar{\gamma}_{\beta\alpha} |0\rangle_\beta.
\end{aligned}
\tag{18}
$$

Here, matrixes γ_μ and $\bar{\gamma}$ have the usual properties

$$
\{\gamma_\mu, \gamma_\nu\} = -2g_{\mu\nu}, \qquad \{\gamma_\mu, \bar{\gamma}\} = 0, \qquad \gamma_0 \gamma_\mu^\dagger \gamma_0 = \gamma_\mu,
$$
$$
\gamma = (-1)^{\frac{1}{4}(D-2)} \gamma_0 \gamma_1 ... \gamma_{D-1}, \qquad \bar{\gamma}^\dagger = \bar{\gamma}.
\tag{19}
$$

We also define the dual vacuum vector and the scalar product in the following way:

$$
{}_\alpha\langle 0|0\rangle_\beta = (\gamma_0 \bar{\gamma})_{\alpha\beta}, \qquad {}_\alpha\langle 0| = |0\rangle_\alpha^\dagger.
\tag{20}
$$

From definitions (18-20) it is not difficult to check the operators Γ_μ and $\bar{\Gamma}$ are Hermitian indeed.

The whole vacuum vector in our Fock space is the tensor product of the bosonic and fermionic vacua

$$
|0\rangle = |0\rangle_B \otimes |0\rangle_F.
$$

By means of operators (16) we define the following odd operators:

$$
\begin{aligned}
F_1 &= \frac{1}{2}\left(\hat{a} + \bar{\Gamma}b\right), \\
F_0 &= \hat{p} + m\bar{\Gamma},
\end{aligned}
\tag{21}
$$

where the notation $\hat{A} = A_\mu \Gamma^\mu$ has been used.

Using relations (1), (3), (16) and (21), one can easily verify that operators L_i and F_i have the following commutation relations:

$$
\left\{F_1, \bar{F}_0\right\} = L_1, \quad F_0^2 = L_0, \quad F_1^2 = \tfrac{1}{2}L_2
$$
$$
\left[L_1, \bar{F}_1\right] = \tfrac{1}{2}F_0, \quad \left[L_2, \bar{F}_1\right] = F_1, \quad [F_1, L_0] = 0.
\tag{22}
$$

One can regard relations (4) and (22) as a finite-dimensional truncation of the infinite superconformal algebra [19].

We will consider the fermionic state with spin $s + \frac{1}{2}$ in the form of a vector of type

$$
|\Psi^s\rangle = \sum_{n=0}^{s} \Psi_{\mu_1 ... \mu_n}(x) \bar{b}^{s-n} \prod_{i=1}^{n} \bar{a}_{\mu_i} |0\rangle.
\tag{23}
$$

Here we imply that coefficient functions $\Psi_{\mu_1 ... \mu_n}(x)$ are spin-tensor fields on $\mathcal{M}_D$, i.e. they have a spinor index which contracts with the one of the fermionic vacuum. Henceforth we will suppress the spinor index and will assume $m = 1$ as well.

Vector (23) contains an redundant states like the bosonic case. To eliminate them we impose the condition

$$
(F_1)^3 |\Psi\rangle = 0
\tag{24}
$$

on state $|\Psi^s\rangle$ and require the presence of the gauge invariance

$$\delta|\Psi^s\rangle = \bar{L}_1|\xi^{s-1}\rangle, \tag{25}$$

where gauge vector

$$|\xi^{s-1}\rangle = \sum_{n=0}^{s-1} \xi_{\mu_1...\mu_n} \bar{b}^{s-n-1} \prod_{i=1}^{n} \bar{a}_{\mu_i}|0\rangle \tag{26}$$

obeys condition

$$F_1|\xi\rangle = 0. \tag{27}$$

In terms of the coefficient functions relations (24)-(27) look like (12)-(14), correspondingly, after the dimensional reduction up to some redefinitions [17]. In limit $m \to 0$ they restore its usual form. We would like to note that this transition is smooth, i.e. the number of physical degree of freedom does not change in contrast to the classical non-gauge formulation [1].

Like the bosonic case we will search for the Lagrangians for the massive half-integer spin fields in the form of the expectation value of a Hermitian operator[2] in state (23)

$$\mathcal{L} = \langle\Psi^s|\mathcal{L}(F,L)|\Psi^s\rangle. \tag{28}$$

Operator $\mathcal{L}(F,L)$ is defined by gauge transformation (25) and conditions (24), (27) unambiguously and have the form

$$\mathcal{L}(F,L) = F_0 + 4\bar{F}_1 F_0 F_1 - \bar{L}_2 F_0 L_2 - 2\left(\bar{L}_1 F_1 - \bar{L}_2 F_1 L_1 + h.c.\right) \tag{29}$$

Having calculated expectation (28) we shall obtain the Lagrangians for the fermions in terms of the spin-tensor fields. This corresponds to the dimensional reduction of action (15) in space-time with odd dimensionality $D + 1$. Similar to the bosonic case we have this correspondence only in non-interacting case.

4. Electromagnetic Interaction of Massive Half-Integer Spin Fields

In this section we will construct the interaction of the fermionic fields of any spins with homogeneous e.m. field in the gauge-invariant manner in the linear approximation.

We introduce the interaction by means of the "minimal" coupling prescription, i.e. we replace the usual momentum operators by the $U(1)$-covariant ones $p_\mu \to \mathcal{P}_\mu$. The commutator of the covariant momenta defines the electromagnetic field strength

$$[\mathcal{P}_\mu, \mathcal{P}_\nu] = F_{\mu\nu}. \tag{30}$$

For convenience we have included the imaginary unit and the coupling constant into the definition of the strength tensor.

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[2]Such an operator must be odd in the fermionic case.

In the definition of operators (3) and (21) we replace the usual momenta by the covariant ones as well. As a result the operators cease to obey algebra (4), (22) and, correspondingly, Lagrangian (28) loses the invariance under transformations (25).

In order to restore the gauge invariance, we do not need to restore whole algebra (4) and (22) it is sufficient to ensure the existence of the following relations:

$$[L_1, L_{-1}] = L_0, \quad [L_2, L_{-1}] = L_1, \quad F_0^2 = L_0,$$
$$[L_1, F_{-1}] = \frac{1}{2}F_0, \quad [L_0, F_1] = 0. \tag{31}$$

In order to restore the relations, we represent operators (3) and (21) as normal ordered functions of the bosonic creation and annihilation operators, the odd ones $\Gamma_\mu, \bar{\Gamma}$ as well as of the electromagnetic field, i.e.

$$L_i = L_i\left(\bar{a}_\mu, \bar{b}, a_\mu, b, \Gamma_\mu, \bar{\Gamma}, F_{\mu\nu}\right),$$
$$F_i = F_i\left(\bar{a}_\mu, \bar{b}, a_\mu, b, \Gamma_\mu, \bar{\Gamma}, F_{\mu\nu}\right). \tag{32}$$

The exact form of these operators will be defined from the condition of restoring commutation relations (31). We should note that it is sufficient to define the form of operators L_1, L_2, and F_1, since the others can be determined from (31).

Since we consider the deformation[3] of the operators we should take into account an arbitrariness in the definition of operators a, b, and Γ which have also appeared. Besides, in the right-hand side of (1) and (16) we should admit the presence of arbitrary operator functions depending on a, b, Γ, and $F_{\mu\nu}$. In this, one must require the condition that the deformed operators would not break the Jacobi identities and they restore the initial algebra in the limit $F_{\mu\nu} \to 0$. The subsequent analysis shows that the Jacobi identities and the arbitrariness in the definition of operators a, b, and Γ allow one to restore initial algebra (1) and (16) at linear order. This means that we can consider the deformation of operators L_i and F_i only.

We shall search for operators L_1, L_2, and F_1 as series in the strength tensor of e.m. field that is equivalent to the expansion in coupling constant.

Let us consider linear approximation.

Operator L_1 should be no higher than linear in operator $\mathcal{P}_\mu$, since the presence of a greater number of these operators changes the type of gauge transformations (25) and, as a consequense, changes the number of the physical degrees of freedom. Therefore, at this order we shall search for them in the form

$$L_1^{(1)} = (\bar{a}Fa)h_0(\bar{b}, b)b + (\mathcal{P}Fa)h_1(\bar{b}, b) + (\bar{a}F\mathcal{P})h_2(\bar{b}, b)b^2 + \hat{F}h_3(\bar{b}, b)b$$
$$+(aF\Gamma)\bar{\Gamma}h_4(\bar{b}, b) + (\bar{a}F\Gamma)\bar{\Gamma}h_5(\bar{b}, b)b^2 + (\mathcal{P}F\Gamma)\bar{\Gamma}h_6(\bar{b}, b)b, \tag{33}$$

where $\hat{F} = (\Gamma F \Gamma)$. At the same time the operators L_2 and F_1 can not depend on the momentum operators at all, since conditions (5) and (24) must define purely algebraic

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[3]The deformation means that we have passed to the extended universal enveloping algebra of the Heisengerg ones.

constraints on the coefficient functions. Therefore, at this order we choose the operators in the following form:

$$
\begin{aligned}
L_2^{(1)} &= (\bar{a}Fa)\,h_7(\bar{b},b)b^2 + \hat{F}h_8(\bar{b},b)b^2 + (aF\Gamma)\bar{\Gamma}h_9(\bar{b},b)b \\
&\quad + (\bar{a}F\Gamma)\bar{\Gamma}h_{10}(\bar{b},b)b^3 \\
F_1^{(1)} &= (aF\Gamma)h_{11}(\bar{b},b) + (\bar{a}F\Gamma)h_{12}(\bar{b},b)b^2 + (\bar{a}Fa)\bar{\Gamma}h_{13}(\bar{b},b)b \\
&\quad + \hat{F}\bar{\Gamma}h_{14}(\bar{b},b)b
\end{aligned}
\tag{34}
$$

Here $h_i(\bar{b},b)$ are normal ordered operator functions of type

$$
h_i(\bar{b},b) = \sum_{n=0}^{\infty} h_n^i \bar{b}^n b^n,
$$

where h_n^i are the arbitrary real coefficients. We consider only the real coefficients since the operators with purely imaginary coefficients do not give any contribution to the "minimal" interaction.

Let us define a particular form of functions h_i from the condition restoring commutation relations (31) by operators (33) and (34).

Having calculated (31) and passing to normal symbols of the creation and annihilation operators, we get a system of linear differential equations of the second order for functions $h_i(x)$, where $x = \bar{\beta}\beta$ and β is the normal symbol of operator b. The number of the equations is 36 for 20 unknowns. We will not write down them here.

Of course, the system of linear differential equations is overdetermined but nevertheless it is solvable. From these equations we obtain as a result

$$
\begin{aligned}
L_2^{(1)} &= -2c_2\,(\Gamma F\alpha)\,\bar{\Gamma}e^{-2\bar{\beta}\beta}\beta \\
L_1^{(1)} &= (1+2c_1)\,(\bar{\alpha}F\alpha)\,\beta - c_1\,(\mathcal{P}F\alpha)\left(1 + 2\bar{\beta}\beta\right) + 2c_1\,(\bar{\alpha}F\mathcal{P})\,\beta^2 \\
&\quad + (\Gamma F\alpha)\,\bar{\Gamma}\left(\frac{1}{4} + c_2\left(1 - 2\bar{\beta}\beta\right)e^{-2\bar{\beta}\beta}\right) + (\mathcal{P}F\Gamma)\,\bar{\Gamma}\left(c_1 + c_2 e^{-2\bar{\beta}\beta}\right)\beta \\
&\quad - 2c_2\,(\bar{\alpha}F\Gamma)\,\bar{\Gamma}e^{-2\bar{\beta}\beta}\beta^2 + \frac{1}{2}\left(\frac{1}{4} + c_1\right)\hat{F}\beta \\[6pt]
F_1^{(1)} &= c_2\,(\bar{\alpha}F\alpha)\,\bar{\Gamma}e^{-2\bar{\beta}\beta}\beta + \frac{1}{2}c_2\hat{F}\bar{\Gamma}e^{-2\bar{\beta}\beta}\beta - \frac{1}{2}c_2\,(\Gamma F\alpha)\,e^{-2\bar{\beta}\beta} \\
F_0^{(1)} &= \left(\frac{1}{2} + 2c_1\right)(\bar{\alpha}F\alpha)\,\bar{\Gamma} - \frac{1}{2}\left(\frac{1}{4} - c_1\right)\hat{F}\bar{\Gamma} + \left\{(\Gamma F\alpha)\,\bar{\beta}\left(\frac{1}{2} - 2c_2 e^{-2\bar{\beta}\beta}\right)\right. \\
&\quad \left. - 2\,(\mathcal{P}F\alpha)\,\bar{\Gamma}\left(c_1 - c_2 e^{-2\bar{\beta}\beta}\right) + h.c.\right\}.
\end{aligned}
\tag{35}
$$

Thus, we have obtained the general form of operators L_i and F_i which satisfy relations (31) in linear approximation. This means that Lagrangian (29) is invariant under transformations (25) at this order. In this, our solution has the two-parametric arbitrariness.

In conclusion of this section we note that it is possible to restore whole algebra (4), (22) if one sets $c_2 = 0$.

5. Examples

Here we apply the general result of the previous section to the description of the electromagnetic interaction of massive fields with spins $\frac{3}{2}$ and $\frac{5}{2}$.

Spin $\frac{3}{2}$. The massive spin-$\frac{3}{2}$ field is the simplest one among the other gauge fermionic fields. Besides, interactions of this field are the best-studied in the literature Therefore, it is useful to represent the interaction Lagrangian that one can derive in our approach.

The state that corresponds to the massive spin-$\frac{3}{2}$ field is

$$|3/2\rangle = \left((\chi \cdot \bar{a}) + \eta \bar{b} \right) |0\rangle, \tag{36}$$

while the gauge vector is

$$|\Lambda, 3/2\rangle = \xi |0\rangle.$$

Having calculated expectation value (28) in this state, we easily derive the free Lagrangian for the massive field with spin $\frac{3}{2}$

$$
\begin{aligned}
\mathcal{L}_{3/2}^{(0)} &= \bar{\chi} \cdot (\hat{p} + 1) \chi + \bar{\eta} (\hat{p} + 1) - (\bar{\chi}' + \bar{\eta}) (\hat{p} - 1) (\chi' + \eta) \\
&\quad - \left\{ ((\bar{\chi} \cdot p) + \bar{\eta}) (\chi' + \eta) + h.c. \right\}.
\end{aligned}
\tag{37}
$$

Here the "hat" denotes the contraction of a vector index with the usual γ-matrixes. The kinetic terms of (37) are non-diagonal[4] but one can diagonalize them by the field redefinition of type $\chi \to \chi + \gamma \eta$.

Then, from (25) we obtain the following gauge transformations for the coefficient functions of state (36)

$$
\begin{aligned}
\delta_0 \chi_\mu &= p_\mu \xi, \\
\delta_0 \eta &= \xi.
\end{aligned}
$$

One can easily see from the transformations that field η can be completely gauged away. Having performed this, we obtain the usual Rarita-Schwinger action [20] for the massive field with spin $\frac{3}{2}$.

As the next step we calculate the Lagrangian of the interaction and the transformations in the linear approximation. For that we have to modify operators F_i and L_i according to solution (35) and keep only linear in the strength of the e.m. field terms in operator (29). Then, having computed the expectation value of this deformed operator in state (36) one derives the interaction Lagrangian

$$
\begin{aligned}
\mathcal{L}_{3/2}^{(1)} &= \left(\frac{1}{2} + 2c_1 \right) (\bar{\chi} F \chi) + \frac{1}{2} \left(\frac{1}{4} - c_2 \right) \left(\bar{\chi} \hat{F} \chi + \bar{\chi}' \hat{F} \chi' \right) + \frac{1}{4} \bar{\eta} \hat{F} \eta \\
&\quad + \left\{ c_2 \left((\bar{\chi} \cdot \mathcal{P}) - \bar{\chi}' - \bar{\eta} \right) \left((\gamma F \chi) + \hat{F} \eta \right) - (c_1 - 2c_2) \bar{\eta} (\mathcal{P} F \chi) \right.
\end{aligned}
$$

---

[4]This is an usual situation for the dimensional reduction.

$$+\bar{\chi}'\left(c_1\left(\mathcal{P}F\chi\right)+\left(c_1+c_2\right)\left(\mathcal{P}F\gamma\right)\eta\right)+\frac{3}{4}\bar{\eta}\left(\gamma F\chi\right)$$
$$+\left(\frac{1}{4}+c_2\right)\bar{\chi}'\hat{F}\eta+\frac{1}{4}\bar{\chi}'\left(\gamma F\chi\right)+h.c.\bigg\}.$$

Applying (25) to state (36) we obtain the following expression for the gauge transformation

$$\delta_1\chi_\mu = c_1\left(\mathcal{P}F\right)_\mu\xi+\left(\frac{1}{4}+c_2\right)\left(F\gamma\right)_\mu\xi,$$
$$\delta_1\eta = \left(c_1+c_2\right)\left(\gamma F\mathcal{P}\right)\xi-\frac{1}{2}\left(\frac{1}{4}+c_1\right)\hat{F}\xi. \tag{38}$$

Obviously, constraints (24) and (27) are trivial for the case of the massive spin-$\frac{3}{2}$ field.

One can notice that the Lagrangian and the transformation formally contain one derivative more then in the supergravity theories with spontaneous breaking.

It would be interesting to investigate the causality of the obtained construction. Such an exploration will be made elsewhere.

Spin $\frac{5}{2}$. Now we pass to the description of the massive field with spin $\frac{5}{2}$. This case is less studied and therefore, it is even more interesting.

A state describing the massive field with spin $\frac{5}{2}$ has the form:

$$|s=5/2\rangle = \left(\left(\bar{a}\cdot\Psi\cdot\bar{a}\right)+\left(\chi\cdot\bar{a}\right)\bar{b}+\eta\bar{b}^2\right)|0\rangle, \tag{39}$$

where $\Psi_{\mu\nu}$ is a symmetric spin-tensor. At the same time the corresponding gauge vector is

$$|\Lambda,5/2\rangle = \left(\left(\xi\cdot\bar{a}\right)+\varepsilon\bar{b}\right)|0\rangle. \tag{40}$$

Constraint (24) is still trivial for state (39), while (27) imposes on the coefficient functions of (40) the following restriction:

$$\xi'+\varepsilon = 0. \tag{41}$$

We discard ε by means of this relation.

Having calculated expectation value (28) in state (39), we derive the Lagrangian describing the propagation of the free massive spin-$\frac{5}{2}$ field

$$\begin{aligned}
\mathcal{L}_{5/2}^{(0)} &= 2\bar{\Psi}\left(\hat{p}+1\right)\Psi+\bar{\chi}\left(\hat{p}+1\right)\chi+2\bar{\eta}\left(\hat{p}+1\right)\eta\\
&\quad-\left(2\bar{\Psi}'+\bar{\chi}\right)\left(\hat{p}-1\right)\left(2\Psi'+\chi\right)-\left(\bar{\chi}'+2\bar{\eta}\right)\left(\hat{p}-1\right)\left(\chi'+2\eta\right)\\
&\quad-\left(\bar{\Psi}''-\bar{\eta}\right)\left(\hat{p}+1\right)\left(\Psi''-\eta\right)\\
&\quad+\Big\{\left(2\left(\bar{\Psi}\cdot p\right)+\bar{\chi}\right)\left(2\Psi'+\chi\right)+\left(\left(\bar{\chi}\cdot p\right)+2\bar{\eta}\right)\left(\chi'+2\eta\right)\\
&\quad+\left(\bar{\Psi}''-\bar{\eta}\right)\left(2\left(p\cdot\Psi'\right)+\left(p\cdot\chi\right)+\chi'+2\eta\right)+h.c.\Big\}. \tag{42}
\end{aligned}$$

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And having computed (25) in a proper way, we obtain the free gauge transformations

$$
\begin{aligned}
\delta\Psi_{\mu\nu} &= p_{(\mu}\xi_{\nu)}, \\
\delta\chi_\mu &= -p_\mu\xi' + \xi_\mu, \\
\delta\eta &= -\xi'.
\end{aligned}
$$

The same result can be obtained up to insignificant redefinitions by the dimensional reduction [21].

Now let us pass to the case when the interaction is present.

According to the general scheme we have to use the deformed operators. Keeping in (28) the linear in the strength terms we arrive at the result

$$
\begin{aligned}
\mathcal{L}_{5/2}^{(1)} =\ & \left(\frac{1}{4}-c_1\right)\left(\bar\Psi\hat F\Psi + \frac{1}{2}\bar\chi\hat F\chi + \bar\eta\hat F\eta\right) + 2\left(1+4c_1\right)\bar\Psi'F\Psi' \\
& + \left(\frac{1}{2}-2c_1\right)\bar\Psi'\hat F\Psi' - \frac{1}{8}\left(1-4c_1\right)\left(\bar\Psi''-\bar\eta\right)\hat F\left(\Psi''-\eta\right) \\
& + \left(\frac{1}{2}+c_1\right)\bar\chi F\chi + \frac{1}{2}\left(\frac{1}{4}-c_1\right)\left(\bar\chi'+2\bar\eta\right)\hat F\left(\chi'+2\eta\right) \\
& + \Big\{(1-4c_2)\bar\chi\left(\gamma F\Psi\right) - 4(c_1-c_2)\bar\chi\left(\mathcal{P}F\Psi\right) \\
& + \left(\bar\chi'+2\bar\eta\right)\left((1-4c_2)\left(\gamma F\Psi'\right) - 2(c_1-c_2)\left(\mathcal{P}F\Psi'\right)\right) \\
& + 2c_1\left(\bar\Psi''-\eta\right)\left(\hat{\mathcal{P}}+1\right)\left(\gamma F\chi\right) + \left(\frac{1}{2}+2c_2\right)\bar\Psi'\left(\gamma F\Psi\right) \\
& + 2c_2\bar\Psi'(\hat{\mathcal{P}}-1)\left(\left(\gamma F\Psi\right) - 2\left(F\chi\right) - \hat F\chi\right) \\
& + 2c_2\left(2\left(\bar\Psi\cdot\mathcal{P}\right)+\bar\chi\right)\left(\left(\gamma F\Psi\right) - \left(F\chi\right) - \frac{1}{2}\hat F\chi\right) \\
& + 2(c_1+c_2)\bar\Psi'\left(\mathcal{P}F\gamma\right)\chi - 2(1+2c_1)\left(\bar\Psi F\chi\right) \\
& + \left(\frac{1}{4}+c_1\right)\bar\Psi'\hat F\chi - 8c_1\left(\bar\Psi'F\mathcal{P}\right)\eta - 2c_2\left(\bar\Psi'F\gamma\right)\eta \\
& + 2c_2\left(2\left(\bar\Psi\cdot\mathcal{P}\right)+\bar\chi\right)\left(\left(\gamma F\Psi\right) - \left(F\chi\right) + \frac{1}{2}\hat F\chi\right) \\
& + \left(\bar\Psi''-\bar\eta\right)\Big(2c_2(\gamma F(\mathcal{P}\cdot\Psi)) + 3c_1(\mathcal{P}F\chi) + c_2\hat F(\mathcal{P}\cdot\chi) \\
& - (c_1+c_2)(\mathcal{P}F\gamma)\chi' - 2(3c_1-c_2)(\mathcal{P}F\gamma)\eta \\
& - \left(\frac{1}{4}+c_2\right)\left(\gamma F\Psi'\right) - \left(\frac{3}{4}+2c_1-4c_2\right)\left(\gamma F\chi\right) \\
& + \frac{1}{2}\left(\frac{1}{4}+c_1\right)\hat F\chi' + \left(\frac{1}{4}+c_1-2c_2\right)\hat F\eta\Big) \\
& + 2c_2\left(\bar\chi F\gamma\right)\left(2\left(\mathcal{P}\cdot\Psi\right) + \left(\mathcal{P}\cdot\chi\right) + \chi' + 2\eta\right)
\end{aligned}
$$

$$- 4(c_1 + c_2)\bar{\eta} \left(\mathcal{P}F\chi\right) + (1 + 4c_2)\bar{\eta} \left(\gamma F\chi\right)$$

$$- c_2 \left(\bar{\chi}' + 2\bar{\eta}\right) \left(\hat{\mathcal{P}} - 1\right) \left((\gamma F\chi) - 2\hat{F}\eta\right)$$

$$+ \left((\bar{\chi} \cdot \mathcal{P}) + 2\bar{\eta}\right) \left((\gamma F\chi) - 2\hat{F}\eta\right)$$

$$+ \left(\bar{\chi}' + 2\bar{\eta}\right) \left(3c_1 \left(\mathcal{P}F\chi\right) + \left(\frac{1}{4} + 3c_2\right) (\gamma F\chi)\right.$$

$$+ 2(c_1 - c_2) \left(\mathcal{P}F\gamma\right)\eta + \left(\frac{1}{4} + c_1\right) \hat{F}\eta\bigg)$$

$$- \left((\bar{\chi} \cdot \mathcal{P}) + 2\bar{\eta}\right) \left((\gamma F\chi) - 2\hat{F}\eta\right) + h.c.\bigg\}. \tag{43}$$

And from (25) we get the deformed gauge transformation of the component fields

$$\delta_1 \Psi_{\mu\nu} = c_2 (\mathcal{P}F)_{(\mu}\xi_{\nu)} + \left(\frac{1}{2} + c_2\right) (F\gamma)_{(\mu}\xi_{\nu)},$$

$$\delta_1 \chi_\mu = \frac{1}{2}c_2 \mathcal{P}_\mu (\gamma F\xi) - \frac{1}{2}c_2 \hat{F}\mathcal{P}_\mu \xi' + (c_1 + c_2)(\gamma F\mathcal{P})\xi_\mu + 3c_2 (F\mathcal{P})_\mu \xi'$$

$$+ (1 + 2c_1)(F\xi)_\mu - \frac{1}{2}\left(\frac{1}{4} + c_1\right) \hat{F}\xi_\mu - \left(\frac{1}{4} - 3c_2\right) (\gamma F)_\mu \xi'$$

$$\delta_1 \eta = 2c_1 (\mathcal{P}F\xi) + (c_1 - c_2)(\mathcal{P}F\gamma)\xi' - \frac{3}{2}c_2 (\gamma F\xi)$$

$$+ \frac{1}{2}\left(\frac{1}{4} + c_1 - c_2\right) \hat{F}\xi'. \tag{44}$$

In this, we have resolved the following relation with respect to ε

$$\xi' - \frac{1}{2}c_2 (\gamma F\xi) + \left(1 - \frac{1}{2}c_2 \hat{F}\right) \varepsilon + \mathcal{O}\left(F^2\right) = 0, \tag{45}$$

which corresponds to (41) in the linear approximation.

Having compared the considered cases, one can see that the case of the massive spin-$\frac{3}{2}$ field is rather marginal, because transformations (38) can be reduced to the trivial ones by the redefinition of field χ_μ, η of type

$$\chi_\mu \rightarrow s_1 \chi_\mu + s_2 \gamma_\mu \eta,$$
$$\eta \rightarrow s_3 \eta + s_4 \chi',$$

i.e. in the presence of the constant electromagnetic field all the information about the non-minimal interaction can be transferred into the Lagrangian. In this sense the situation for the spin-$\frac{5}{2}$ and higher-spin fields is different, because from (44) it is clear that this transformation cannot be reduced to the trivial ones by any redefinitions of the fields.

6. Conclusion

Here we have extended the algebraic framework of the description of the massive bosonic fields of any spins to the case of the massive fermionic ones. In such a framework

we have got the explicit form of the operators by means of which we have obtained the Lagrangian and the gauge transformations describing the interaction of arbitrary massive half-integer fields with constant electromagnetic field in linear approximation. We have also applied our approach to the particular cases of the spin-$\frac{3}{2}$ and spin-$\frac{5}{2}$ fields and have derived the explicit gauge-inariant Lagrangian and the transformations for each case at linear order in the external e.m. field $F_{\mu\nu}$.

Of course, there are open important questions: the existence of next approximation and the causality. We hope the subsequent investigations will shed light upon these questions.

It is worth noting that the case of the constant Abelian field can be easily extended to a non-Abelian case. If so, we have to consider the external field as covariantly constant one. In this, one should take the whole vacuum as $|0\rangle \otimes e^i$, where e^i are basis vectors in space of linear representation of a non-Abelian group. The covariant derivative has the form $\partial_\mu + A_\mu^a T^a$, where T^a are the operators realizing the representation. Such modification does not change the algebraic features of our scheme in linear approximation. Therefore, all the results derived are valid as well.

In conclusion we would like to note that our approach allows one to construct not only the electromagnetic interaction for the fields of an arbitrary spin, but it also allows one to describe the propagation of the fields in a special Riemann space. For the integer spin fields such a construction will be considered elsewere.

Acknowledgements

References

[1] L. P. C. Singh and C.R. Hagen. *Phys. Rev*, **D9**:898, 910, 1974.

[2] J. Fang and C. Fronsdal. Massless fields with half integral spin. *Phys. Rev.*, **D18**:3630, 1978.

[3] C. Fronsdal. Massless fields with integer spin. *Phys. Rev.*, **D18**:3624, 1978.

[4] M. A. Vasilev. 'Gauge' form of description of massless fields with arbitrary spin. (in Russian). *Yad. Fiz.*, **32**:855, 1980.

[5] Yu. M. Zinoviev. Gauge invariant description of massive high spin particles. IHEP preprint 83-91 (Protvino, 1983).

[6] S. D. Rindani, D. Sahdev, and M. Sivakumar. Dimensional reduction of symmetric higher spin actions i: Bosons. *Mod. Phys. Lett.*, **A4**:265, 1989.

[7] A.I. Pashnev. Composite system and free field theory of the regge trajectory. *Teor. Mat. Fiz.*, **78**, 1989 (in Russian).

[8] Yu. M. Zinoviev, *The Role of Poincare Group in Elementary Particle Physics*, XVII Seminar on High Energy Physics and Field Theory, June 1994, Protvino.

[9] P. C. Argyres and C. R. Nappi. Massive spin 2 bosonic string states in an electromagnetic background. *Phys. Lett.*, **B224**:89, 1989.

[10] S. M. Klishevich and Yu. M. Zinoviev. On electromagnetic interaction of massive spin-2 particle. 1997, hep-th/9708150 (to be published in Phys. At. Nucl.).

[11] S. M. Klishevich. Electromagnetic interaction of massive spin-3 state from string theory. 1998, IHEP 98-20, hep-th/9805174.

[12] S. M. Klishevich. Massive fields of any integer spin in homogeneous e.m. field. 1998, IHEP 98-24.

[13] Sergio Ferrara, Massimo Porrati, and Valentine L. Telegdi. g = 2 as the natural value of the tree level gyromagnetic ratio of elementary particles. *Phys. Rev.*, **D46**:3529–3537, 1992.

[14] C. R. Nappi and L. Witten. Interacting lagrangian for massive spin two field. *Phys. Rev.*, **D40**:1095, 1989.

[15] A. Pashnev and M. Tsulaya. Dimensional reduction and BRST approach to the description of a regge trajectory. *Mod. Phys. Lett.*, **A12**:861–870, 1997, hep-th/9703010.

[16] Thomas Domeij. A BRST approach to massless gauge fields of half-integer spin. *Int. J. Mod. Phys.*, **A5**, 1990.

[17] S. D. Rindani, D. Sahdev, and M. Sivakumar. Dimensional reduction of symmetric higher spin actions ii: Fermions. *Mod. Phys. Lett.*, **A4**:275, 1989.

[18] Thomas Domeij. The free massless higher spin gauge field equations in anti-de sitter space from BRST quantization. GOTEBORG-90-25.

[19] M. B. Green, J. H. Schwarz, and E. Witten. *Superstring Theory*. Cambridge Universite Press, 1987.

[20] W. Rarita and J. Schwinger. On a theory of particles with half integral spin. *Phys. Rev.*, **60**:61, 1941.

[21] S. D. Rindani and M. Sivakumar. Gauge-invariant description of massive higher-spin particles by dimensional reduction. *Phys. Rev.*, **D32**:3238, 1985.