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γ–BEAM PROPAGATION IN THE ANISOTROPIC MEDIUM

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Abstract

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Propagation of γ-beam in the anisotropic medium is considered. The simplest example of this medium of a general type is a combination of the two linearly polarized monochromatic laser waves with different frequencies (dichromatic wave). The optical properties of this combination are described with the use of the permittivity tensor. The refractive indices and polarization characteristics of normal electromagnetic waves propagating in the anisotropic medium are found. The relations describing variations of γ-beam intensity and Stokes parameters as functions of the propagation length are obtained. The influence of laser wave intensity on the propagation process is calculated.

The γ-beam intensity losses in the dichromatic wave depend on the initial circular polarization of γ-quanta. This effect also applies to single crystals which are oriented in some regions of coherent pair production. In principle, the single crystal sensitivity to circular polarization can be used for determination of polarization of high energy (in tens GeV and more) γ-quanta and electrons.

Аннотация

Маишеев В.А. Распространение пучка γ-квантов в анизотропной среде: Препринт ИФВЭ 98-79. – Протвино, 1998. – 13 с., 4 рис., 1 табл., библиогр.: 18.

Рассмотрено распространение пучка γ-квантов в анизотропной среде. Простейшим примером такой среды общего вида является комбинация двух линейно-поляризованных монохроматических лазерных волн с разными частотами (дихроматическая волна). Оптические свойства такой комбинации описаны с помощью тензора диэлектрической проницаемости, что позволило определить показатели преломления и поляризационные характеристики γ-квантов, распространяющихся в анизотропной среде. Получены соотношения, описывающие изменение интенсивности и параметров Стокса пучка γ-квантов в зависимости от координаты. Рассмотрено влияние интенсивности лазерной волны на процесс распространения пучка.

Показано, что потери интенсивности пучка γ-квантов зависят от его начальной поляризации. Этот эффект имеет место и для монокристаллов ориентированных в некоторых областях, где имеет место когерентный механизм образования электрон-позитронных пар. Такая чувствительность кристаллов к циркулярной поляризации γ-квантов может быть использована для ее определения для пучков с энергиями в десятки ГэВ.

1. Introduction

Polarization phenomena [1,2] arising from the visible light propagation in the anisotropic or gyrotropic medium are well-known. Theory [3] makes prediction of the analogous phenomena for γ-quanta with energy > 1 GeV propagating in single crystals, which are anisotropic media by their very nature. The main absorption process of γ-quanta in the single crystals is the electron-positron pair production. The cross section of this process depends on the linear polarization of γ-beam with respect to crystallographic planes. As a result of interaction with the electric field of the single crystal, a monochromatic, linearly polarized γ-beam comprises two electromagnetic waves with different refractive indices, so that linear polarization is transformed into circular polarization or vice versa.

On the other hand, the process of e^+e^--pair production in single crystals is similar to the same process in a linearly polarized laser wave [4]. A possibility to use a bunch of linearly polarized laser photons as a "single crystal" is pointed out in [5], but no actual estimates of the effect are given.

In the recent paper [6] it has been shown that the polarization effects such as birefringence and rotation of polarization plane for γ- beams with energies of tens GeV or more take place for short (about some picoseconds) laser bunches with parameters whithin the realm of present-day engineering capabilities. In the cited paper the differential equations which determine the variation of Stokes parameters and intensity of γ-quanta traversing a bunch of arbitrary polarized laser photons are obtained. For these calculations the well-known scattering amplitudes for the process of elastic scattering light by light [7,8] were used. Then in [9] the process of γ-beam propagation in the field of a laser wave was investigated with the use of traditional optical methods. Bisides, the transformation of γ-beam linear polarization into circular in the anisotropic medium was discussed in [10].

The case of γ-quanta propagation in single crystals described in [3] is a special case of such a process. The general case of γ-propagation in single crystals oriented in the angle region corresponding to process of the coherent pair production was considered in [5]. In this paper it is shown that the propagating γ-beam is a superposition of the two elliptically polarized waves but the description of variation of the γ-beam polarization is unavailable.

In the present paper we study a general case of high energy γ-quanta propagation through the anisotropic medium. The anisotropic medium is determined as medium, whose optical properties can be described using a symmetric permittivity tensor [1,2]. As will be indicated the simplest example of the anisotropic medium of a general type is a superposition of the two linearly polarized laser waves with different frequencies moving in the same direction. We will study in detail the γ-quanta propagation in this combined laser wave with the goal of better understanding of this process in more complicated cases, such as with single crystals.

2. Permittivity tensor in anisotropic medium

We write the equations of the electromagnetic field in a medium (γ-beam propagating in a laser wave, single crystal, and the like) in the following form [1,2]:

$$
\begin{aligned}
rot\vec{B} &= \frac{1}{c}\frac{\partial \vec{D}}{\partial t}\,, \quad div\vec{D} = 0\,, \\
rot\vec{E} &= -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\,, \quad div\vec{B} = 0\,,
\end{aligned} \tag{1}
$$

$\vec{E}$ is the intensity of electric field and $\vec{D}$ and $\vec{B}$ are the electric and magnetic induction vectors, t is the time, c is the speed of light. All the properties of the medium are reflected in the relation between $\vec{B}, \vec{E}$ and $\vec{D}$. Eqs.(1) would suffice to describe the γ-beam propagation in a medium and such a property as the intensity of magnetic field is not needed [1,?]. We represent the relation between $\vec{D}$ and $\vec{E}$ in the form

$$
D_i(\omega) = \varepsilon_{ij} E_j(\omega)\,, \quad (i,j = 1,2,3)\,, \tag{2}
$$

where $\varepsilon_{ij} = \varepsilon'_{ij} + i\varepsilon''_{ij}$ is the complex permittivity tensor and ω is the frequency of γ-quanta.

By the example of the anisotropic medium let us consider the superposition of the two linearly polarized laser waves, moving in the same direction. The frequencies of these waves (photon energies) are different. In order to determine the permittivity tensor in the case of a monochromatic field (high energy γ-beam) $\vec{E}_0 e^{i(\vec{k}\vec{r}-\omega t)}$, propagating in the above-mentioned laser medium, where $\vec{k}$ is the wave vector of the γ-quanta, we find the average energy lost by the electromagnetic wave per unit volume V and per unit time [1,2]

$$
\tilde{q} = \frac{1}{4\pi V}\int_V \vec{E}\frac{\partial \vec{D}}{\partial t} dV = \frac{i\omega}{16\pi}(\varepsilon^*_{ij} - \varepsilon_{ji}) E^*_j E_i\,. \tag{3}
$$

The mechanism whereby the wave loses energy is e^+e^--pair production in the field of the laser wave [11]. The process is determined primarily by the transverse part of the permittivity tensor, while the longitudinal components of the tensor are higher-order infinitesimals in the interaction constant α [4,12]. Taking this into account, and in the coordinate system one axis of which is oriented parallel to the wave vector of γ-quanta, we have from (3)

$$\tilde{q} = \frac{i\omega J}{4}\{(\varepsilon_{11}{}^* - \varepsilon_{11})(1+\xi_3) + (\varepsilon_{12}{}^* - \varepsilon_{21})(\xi_1 - i\xi_2) + (\varepsilon_{21}{}^* - \varepsilon_{12})(\xi_1 + i\xi_2) + (\varepsilon_{22}{}^* - \varepsilon_{22})(1-\xi_3)\}\,, \qquad (4)$$

where $J = (E_1E_1{}^* + E_2E_2{}^*)/8\pi$, ξ_i are the Stokes parameters of γ-beam. On the other hand, knowing the cross section $\sigma_{\gamma\gamma}$ of the pair production process, we can write

$$\tilde{q} = 2cJ\{n_{l,1}\sigma_{\gamma\gamma,1} + n_{l,2}\sigma_{\gamma\gamma,2}\}\,, \qquad (5)$$

where $n_{l,1}$ is the number of photons per volume unit of the laser wave with the linear polarization equal to $P_{l,1}$, and $n_{l,2}$ is the similar number for the second wave with the linear polarization equal to $P_{l,2}$, $\sigma_{\gamma\gamma,1}$ and $\sigma_{\gamma\gamma,2}$ are the corresponding cross sections for e^+e^- -pair production in $\gamma\gamma$ -interactions, $P_{1,1}, P_{3,1}$ and $P_{1,2}, P_{3,2}$ are the Stokes parameters of the laser waves ($P_{1,1}^2 + P_{3,1}^2 = P_{l_1}^2$, $P_{1,2}^2 + P_{3,2}^2 = P_{l_2}^2$). Factor 2 in this formula is due to the counter-motion of the γ-beam and laser wave. Note, that Eq.(5) is true, when the intensity of laser wave are not high (see below).

We can write the cross section of e^+e^--pair production in the following form [4,6,7,11]

$$\sigma_{\gamma\gamma}(z) = \sigma_0(z) + \sigma_l(z)(\xi_1P_1 + \xi_3P_3)\,,\ \ 0 < z \leq 1\,, \qquad (6)$$

$$\sigma_0(z) = \pi r_e^2 z\{(1 + z - z^2/2)L_- - \sqrt{1-z}(1+z)\}\,, \qquad (7)$$

$$\sigma_l(z) = \frac{\pi r_e^2 z^3}{2}(L_- + 2\sqrt{1-z}/z)\,, \qquad (8)$$

$$L_- = \ln\frac{1+\sqrt{1-z}}{1-\sqrt{1-z}}\,,$$

where $z = \frac{m^2c^4}{E_\gamma E_l}$ is the invariant variable, E_γ is the γ-quantum energy, m and r_e are the mass and classical radius of electron, E_l and P_1, P_3 are the energy and Stokes parameters of the laser photon. It is well known that the pair production is a threshold process and, because of this, the laser wave is a transparent medium for γ-beam , when $E_\gamma E_l < m^2c^4$ or $z > 1$. There are two different photon energies $E_{l,1}$ and $E_{l,2}$ in the case of dichromatic laser wave. Because of this, it is convenient to use the two corresponding invariant variables $z_1 = \frac{m^2c^4}{E_\gamma E_{l,1}}$ and $z_2 = \frac{m^2c^4}{E_\gamma E_{l,2}}$. It is evident that $z_1/z_2 = E_{l,2}/E_{l,1}$. Comparing Eqs.(4) and (5), we can find the components of permittivity tensor caused by γ-quanta absorption.

Then we can determine the other components of the tensor with the help of the following dispersion relations [2]:

$$\varepsilon'_{ij} - \delta_{ij} = \frac{2}{\pi}\mathcal{P}\int_0^\infty \frac{x\varepsilon''_{ij}(x)\,dx}{x^2-\omega^2}\,, \qquad (9)$$

$$\varepsilon''_{ij} = -\frac{2\omega}{\pi}\mathcal{P}\int_0^\infty \frac{(\varepsilon'_{ij} - \delta_{ij})\,dx}{x^2-\omega^2}\,, \qquad (10)$$

where δ_{ij} is the Kronecker δ-function. Comparing Eqs.(4) and (5), we get

$$\varepsilon''_{11} + \varepsilon''_{22} = 4c(n_{l,1}\sigma_0(z_1) + n_{l,2}\sigma_0(z_2))/\omega\,, \qquad (11)$$

$$\varepsilon''_{11} - \varepsilon''_{22} = 4c(n_{l,1}\sigma_l(z_1)P_{3,1} + n_{l,2}\sigma_l(z_2)P_{3,2})/\omega\,, \tag{12}$$
$$\varepsilon''_{12} + \varepsilon''_{21} = 4c(n_{l,1}\sigma_l(z_1)P_{1,1} + n_{l,2}\sigma_l(z_2)P_{1,2})/\omega\,, \tag{13}$$
$$\varepsilon'_{12} = \varepsilon'_{21} \tag{14}$$

It easy to verify that $\varepsilon_{12} = \varepsilon_{21}$. The same result is evident from the theory of generalized susceptibilities [13].

The other components of the permittivity tensor can be calculated with the help of relations (9)-(10). The calculational results of the components ε_{ij} for the arbitrary coordinate system, one axis of which is oriented parallel to the wave vector of the γ-quanta, are presented below

$$\varepsilon'_{11} - \varepsilon'_{22} = \frac{\alpha}{2\pi E_o^2}(< E_1^2 > P_{3,1}z_1^2 F_1{}'(z_1) + < E_2^2 > P_{3,2}z_2^2 F_1{}'(z_2)) \tag{15}$$

$$\varepsilon'_{11} + \varepsilon'_{22} = 2 + \frac{2\alpha}{\pi E_o^2}(< E_1^2 > z_1^2 F_2{}'(z_1,1) + < E_2^2 > z_2^2 F_2{}'(z_2,1))\,, \tag{16}$$

$$\varepsilon'_{12} = \varepsilon'_{21} = \frac{\alpha}{4\pi E_o^2}(< E_1^2 > P_{1,1}z_1^2 F_1{}'(z_1) + < E_2^2 > P_{1,2}z_2^2 F_1{}'(z_2)) \tag{17}$$

$$\varepsilon''_{11} - \varepsilon''_{22} = -\frac{\alpha}{4E_o^2}(< E_1^2 > P_{3,1}F_1{}''(z_1) + < E_2^2 > P_{3,2}F_1{}''(z_2))\,, \tag{18}$$

$$\varepsilon''_{11} + \varepsilon''_{22} = \frac{\alpha}{E_o^2}(< E_1^2 > F_2{}''(z_1,1) + < E_2^2 > F_2{}''(z_2,1)\,, \tag{19}$$

$$\varepsilon''_{12} = \varepsilon''_{21} = -\frac{\alpha}{8E_o^2}(< E_1^2 > P_{1,1}F_1{}''(z_1) + < E_2^2 > P_{1,2}F_1{}''(z_2))\,, \tag{20}$$

where $< E_i^2 >= 4\pi n_{l,i}$, $E_{l,i}$ $(i = 1,2)$ is the mean square of intensity electric field for each laser wave and the functions F_1', F_2', F_1'', F_2'', are equal to

$$F_1{}'(z) = \begin{cases} [\sqrt{1-z} + \frac{z}{2}L_-]^2 + [\sqrt{1+z} - \frac{z}{2}L_+]^2 - \frac{\pi^2 z^2}{4},\ 0 < z \leq 1, \\ -[\sqrt{z-1} - z\,\mathrm{arccot}\,\sqrt{z-1}]^2 + [\sqrt{1+z} - \frac{z}{2}L_+]^2,\ z > 1. \end{cases} \tag{21}$$

$$F_2{}'(z,\mu) = \begin{cases} -2 - \mu - (1 + \mu(z - \frac{z^2}{2}))\frac{1}{4}L_-^2 - (1 - \mu(z + \frac{z^2}{2}))\frac{1}{4}L_+^2 + \\ + \frac{(1+\mu z)\sqrt{1-z}}{2}L_- - \frac{(\mu z - 1)\sqrt{z+1}}{2}L_+ + \frac{\pi^2}{4}(1 + \mu(z - \frac{z^2}{2})),\ 0 < z \leq 1, \\ -2 - \mu + (1 + \mu(z - \frac{z^2}{2}))\,\mathrm{arccot}^2(\sqrt{z-1}) - (1 - \mu(z + \frac{z^2}{2}))\frac{1}{4}L_+^2 + \\ +(1+\mu z)\sqrt{z-1}\,\mathrm{arccot}\,\sqrt{z-1} - \frac{(\mu z - 1)\sqrt{1+z}}{2}L_+,\ z > 1. \end{cases} \tag{22}$$

$$F_1{}''(z) = \begin{cases} z^4(L_- + \frac{2\sqrt{1-z}}{z}),\ 0 < z \leq 1, \\ 0,\ z > 1. \end{cases} \tag{23}$$

$$F_2{}''(z,\mu) = \begin{cases} z^2((1 + \mu(z - \frac{z^2}{2}))L_- - \sqrt{1-z}(1 + \mu z)),\ 0 < z \leq 1, \\ 0,\ z > 1 \end{cases} \tag{24}$$

The function L_+ is equal to

$$L_+ = ln\frac{\sqrt{1+z}+1}{\sqrt{1+z}-1}\,. \tag{25}$$

The constant $E_o = \frac{m^2c^3}{e\hbar}$ is the critical field of quantum electrodynamics. The presented data determine completely the permittivity tensor for high energy γ-quanta traversing a dichromatic linearly polarized bunch of laser photons.

In a number of problems in optics it is more convenient to employ the tensor η_{ij}, which is the inverse of the tensor ε_{ij}. When $|\varepsilon_{ij} - \delta_{ij}| \ll 1$, these tensors are related by

$$\eta_{ij} + \varepsilon_{ij} = 2\delta_{ij}. \tag{26}$$

The following peculiarities of the permittivity tensor should be noted:

1) Our description can be extended to the case when the laser bunch is the superposition of more a two linearly polarized waves. It is obvious that the analogous terms should be added in the tensor components in these cases.

2) Let $E_{l,1} > E_{l,2}$. Then the laser bunch is a transparent medium at $z_1 > 1$. In this case all the components ε''_{ij} are equal to zero.

3) In a general case the symmetric complex tensor ε''_{ij} is not reduced to principal axes [1,2,5] (i.e., there does not exist a coordinate system in which the tensors ε'_{ij} and ε''_{ij} are both diagonal).

4) In the case, when the two waves have the same direction of linear polarization or their polarizations are orthogonal, the tensor ε_{ij} can be reduced to a diagonal form. The permittivity tensor for monochromatic linearly polarized wave can be always reduced to the diagonal form.

The above is also true for tensor η_{ij}.

3. Refractive indices of γ-quanta

The main problem in the of anisotropic medium optics is to investigate the propagation of monochromatic plane waves, characterized by definite values of the frequency ω and wave vector $\vec{k}$. Such waves, satisfying a homogeneous wave equation, are called normal electromagnetic waves [2], and they have the form

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\vec{r} - \omega t)}, \vec{k} = \omega \tilde{n} \vec{s}/c\,,$$

where $\vec{E}_0$ is the complex vector, independent of coordinates $\vec{r}$ and the time, $\tilde{n}$ is the complex index of refraction and $\vec{s} = \vec{k}/|k|$ is the real unit vector. The vectors $\vec{D}$ and $\vec{B}$ have the same form.

From Maxwell's equations (1) we obtain the wave equation

$$rotrot\vec{E} + \frac{1}{c}\frac{\partial^2 \vec{D}}{\partial t^2} = 0\,.$$

Taking into account the relation between $\vec{D}$ and $\vec{E}$ in a system of coordinates in which the axis x is oriented parallel to the wave vector, we obtain

$$\begin{aligned} \eta_{11}\frac{\partial^2 D_1}{\partial x^2} + \eta_{12}\frac{\partial^2 D_2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 D_1}{\partial t^2} &= 0, \\ \eta_{21}\frac{\partial^2 D_1}{\partial x^2} + \eta_{22}\frac{\partial^2 D_2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 D_2}{\partial t^2} &= 0. \end{aligned} \tag{27}$$

For a monochromatic plane wave it follows from these equations that

$$(\tilde{n}^{-2}\delta_{ij} - \eta_{ij})D_j = 0, \; i, j = 1, 2. \tag{28}$$

From the condition that the two homogeneous equations are compatible, we find the index of refraction of the γ-quanta

$$\tilde{n}^{-2} = \frac{S}{2} \pm \sqrt{\frac{S^2}{4} - D_\eta} = (\eta_{11} + \eta_{22})/2 \pm \sqrt{(\eta_{11} - \eta_{22})^2/4 + \eta_{12}\eta_{21}}\,, \tag{29}$$

where S and D_η are, respectively, the trace and determinant of the matrix η_{ij}. Thus, in the general case the γ-beam propagates through the laser wave as the superposition of two electromagnetic waves with different refractive indices. Note, that the two roots of (30) which have the form $-1 +$ *small quantity*, are superfluous. They correspond to the γ-quanta motion in the opposite direction.

The refractive indices are complex quantities in the general case. However, they are real values, when the laser bunch is a transparent medium for γ-quantum (all components of the permittivity tensor are the real numbers in this case). Fig.1 illustrates the refractive indices as functions of the invariant variable z_1 (the laser wave parameters are in the caption).

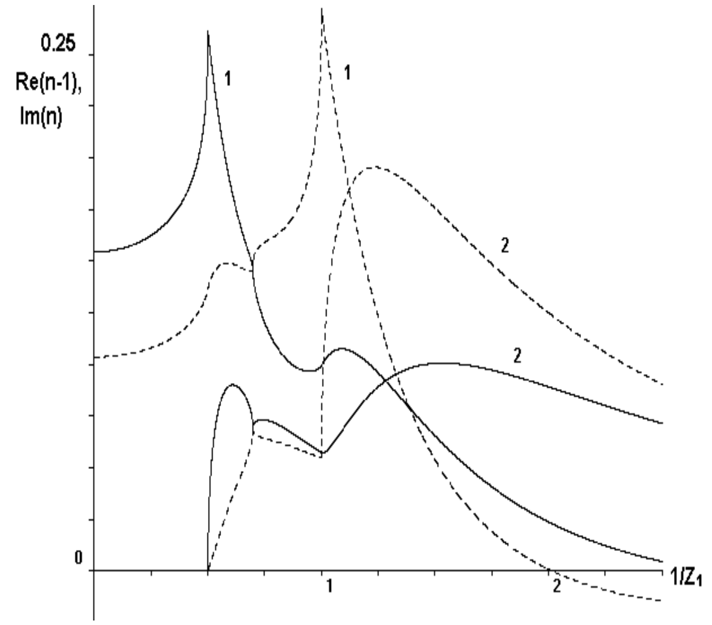

Fig. 1. The real (1) and imaginary (2) parts of refractive indices for dichromatic laser wave ($P_{1,1} = 1$, $P_{3,2} = 1$, $r = 0.658$, $z_2/z_1 = 2$) as functions of the invariant parameter z_1. For obtaining absolute quantity the ordinate value is multiplied by the factor $E_o^2/\alpha < E_1^2 >$.

4. Polarization properties of γ-beam propagation in laser wave

Here we consider the polarization properties of one of two normal electromagnetic waves. From dispersion equations (29) we find the ratios of the components of the vector $\vec{D}$

$$\frac{D_1}{D_2} = \kappa = \frac{\tilde{n}^{-2} - \eta_{22}}{\eta_{21}} = \frac{|D_1|}{|D_2|}e^{i\delta}, \tag{30}$$

where δ is the phase shift between D_1 and D_2. This ratio κ can be reduced to zero or to the form $\kappa = i\rho$ (since $|D_1||D_2|\sin\delta = b_1 b_2$, where b_1 and b_2 are the semiaxes of the ellipse and $|\rho| = b_1/b_2$ [14]) by the rotation of the coordinate system around the wave vector of γ-quanta (the x-axis is constantly aligned with the wave vector). The first case corresponds to the propagation of a linearly polarized wave and the second case corresponds to an elliptically polarized wave; in addition, $\rho > 0 (\rho < 0)$ corresponds to left (right) - hand polarization of γ-quanta.

The different cases of γ-beam propagation in the anisotropic medium, whose optical properties described by the symmetric tensor, were considered in [5]. In the case when a permittivity tensor may be reduce to principal axes (i.e., there is a coordinate system in which $\varepsilon_{12} = 0$) the normal electromagnetic waves are linearly polarized. In general case the permittivity tensor is not reduced to principal axes and the normal waves are elliptically polarized. The propagation of γ-beam, which is the superposition of the two linearly polarized waves, was considered in [6,9,15]. Because of this, in the following we will consider the case when γ-beam is the superposition of the two elliptically polarized normal waves.

In the case under consideration the refractive indices are the complex values. Because of this, the value κ is also complex and we get the following relation between two normal waves

$$\kappa^{(1)}\kappa^{(2)} = -1\,, \tag{31}$$

where the indices in parentheses refer to the waves with refractive indices $\tilde{n}_1$ and $\tilde{n}_2$. In what follows we will use only one of two values, namely, the $\kappa = \kappa^{(1)}$ (without pointing any indices). In our case one can obtain

$$D_1^{(1)}D_1^{(2)} + D_2^{(1)}D_2^{(2)} = 0\,, \tag{32}$$

$$D_1^{(1)}D_1^{*(2)} + D_2^{(1)}D_2^{*(2)} = D_2^{(1)}D_2^{*(2)}(\kappa^* - \kappa)/\kappa^*\,, \tag{33}$$

where the indices in parentheses refer to the waves with refractive indices $\tilde{n}_1$ and $\tilde{n}_2$. Herefrom we can see that $\vec{D}^{(1)}$ and $\vec{D}^{(2)}$ vectors are orthogonal but $\vec{D}^{(1)}$ and $\vec{D}^{*(2)}$ vectors are not orthogonal if the value $\kappa^* - \kappa$ is not equal to zero. Let us name the Stokes parameters of the normal wave with the refractive indices $\tilde{n}_1$ and $\tilde{n}_2$ respectively as X_1, X_2, X_3 and Y_1, Y_2, Y_3. Then we get

$$X_1 = \frac{\kappa + \kappa^*}{1 + \kappa\kappa^*}\,, \tag{34}$$

$$X_2 = \frac{i(\kappa - \kappa^*)}{1 + \kappa\kappa^*}\,, \tag{35}$$

$$X_3 = \frac{\kappa\kappa^* - 1}{1 + \kappa\kappa^*}\,. \tag{36}$$

We have also the following relations $Y_1 = -X_1, Y_2 = X_2, Y_3 = -X_3$. The angle of ellipse turn φ is found from relation $tg 2\varphi = X_1/X_3 = Y_1/Y_3$.

Fig.2 illustrates the results of calculations of $|P_{circ}| = |X_2|$ as functions of the invariant variable z_1 under various conditions.

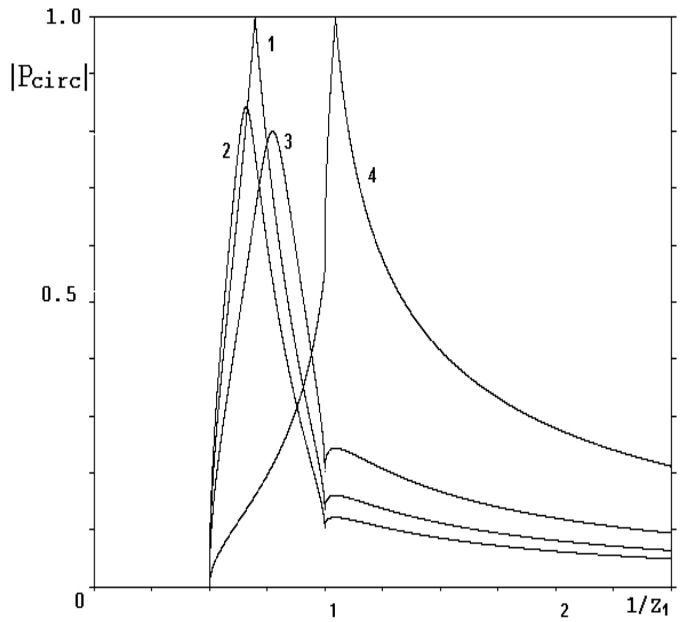

Fig. 2. The absolute value of normal waves circular polarizition $|P_{circ}| = |X_2|$ as function of the invariant parameter z_1. The polarization state of laser wave is as in Fig.1. The ratios r are equal to 0.658,0.5,1.0 and 4.12 for curves 1-4. The ratio z_1/z_2 is equal to 2.

5. γ-quanta propagation in the laser wave

Now we can find the relations describing the variations of intensity and Stokes parameters of γ-quanta propagating in the uniform ($n_l = const$) laser wave. Then, representing the γ-beam as the superposition of two normal waves corresponding to the polarization state of a laser wave, we get the following relations:

$$J_\gamma(x) = J_1(x) + J_2(x) + 2J_3(x)\,, \tag{37}$$

$$\xi_1(x) = (X_1 J_1(x) + Y_1 J_2(x) + p_1 J_4(x))/J_\gamma(x)\,, \tag{38}$$

$$\xi_2(x) = (X_2 J_1(x) + Y_2 J_2(x) + p_2 J_3(x))/J_\gamma(x)\,, \tag{39}$$

$$\xi_3(x) = (X_3 J_1(x) + Y_3 J_2(x) + p_3 J_4(x))/J_\gamma(x)\,, \tag{40}$$

where $J_\gamma, \xi_1, \xi_2, \xi_3$ are the intensity and Stokes parameters of γ-quanta on the laser bunch thickness equal to x. The partial intensities $J_i(x), (i = 1-4)$ have the following form (the physical sense of these values is easy to understand, if the relation $(\vec{D}^{(1)} + \vec{D}^{(2)})(\vec{D}^{*(1)} + \vec{D}^{*(2)})$ is written in the component-wise form):

$$J_1(x) = J_1(0)\exp(-2\,\mathrm{Im}(\tilde{n}_1)\omega x/c)\,, \tag{41}$$

$$J_2(x) = J_2(0)\exp(-2\,\mathrm{Im}(\tilde{n}_2)\omega x/c)\,, \tag{42}$$

$$\begin{aligned} J_3(x) = \exp(-\,\mathrm{Im}(\tilde{n}_1 + \tilde{n}_2)\omega x/c)\{&J_3(0)\cos(\mathrm{Re}(\tilde{n}_1 - \tilde{n}_2)\omega x/c) + \\ &+J_4(0)\sin(\mathrm{Re}(\tilde{n}_1 - \tilde{n}_2)\omega x/c)\}\,, \end{aligned} \tag{43}$$

$$\begin{aligned} J_4(x) = -\exp(-\,\mathrm{Im}(\tilde{n}_1 + \tilde{n}_2)\omega x/c)\{&J_3(0)\sin(\mathrm{Re}(\tilde{n}_1 - \tilde{n}_2)\omega x/c) - \\ &-J_4(0)\cos(\mathrm{Re}(\tilde{n}_1 - \tilde{n}_2)\omega x/c)\}\,. \end{aligned} \tag{44}$$

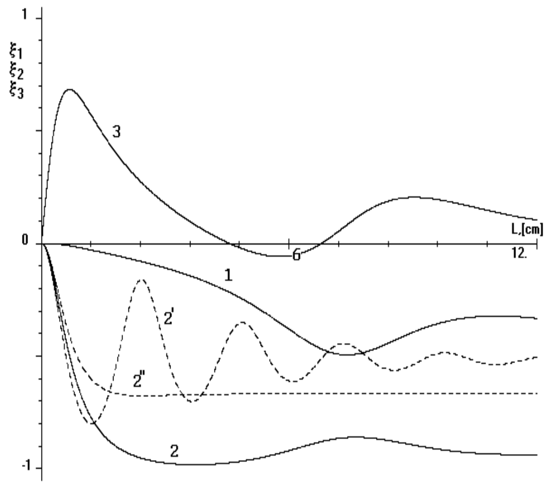

Fig. 3. The Stokes parameters of initialy unpolarized γ-beam as functions of a laser bunch thickness. The polarization state of laser wave is as in Fig.1. The number near each curve corresponds to index i of Stokes parameter ξ_i. The solid curves correspond to $z_1 = 1.4$. The curves $2'$ and 2" correspond to $z_1 = 1.2, 1.6$. $r = 0.658$, $z_1/z_2 = 2$, $n_{l,1} = 5\,10^{25} cm^{-3}$, $E_{l,1} = 1eV$.

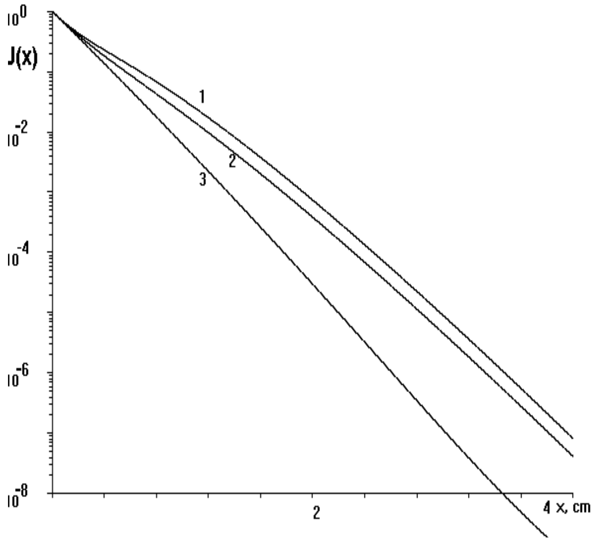

Fig. 4. The intensity of γ-beam as function of the laser bunch thickness. The polarization state of laser wave is as in Fig.1. Curves 1-3 correspond to initial circular polarization $\xi_2(0)$ of γ-beam equal to 1, 0 and -1. $r = 0.658$, $z_1/z_2 = 2$, $n_{l,1} = 5\,10^{25} cm^{-3}$, $E_{l,1} = 1eV$.

The initial partial intensities are defined from the following relations:

$$J_1(0) = \frac{\xi_2(0) - q}{2(X_2 - q)} + \frac{\xi_3(0) - f\xi_1(0)}{2(X_3 - fX_1)}\,, \tag{45}$$

$$J_2(0) = \frac{\xi_2(0) - q}{2(X_2 - q)} - \frac{\xi_3(0) - f\xi_1(0)}{2(X_3 - fX_1)}\,, \tag{46}$$

$$J_3(0) = 1/2 - \frac{(\xi_2(0) - q)}{2(X_2 - q)}\,, \tag{47}$$

$$J_4(0) = \frac{\xi_1(0)X_3 - \xi_3(0)X_1}{p_1X_3 - p_3X_1}\,. \tag{48}$$

The relations between X_i and Y_i values were used, because of this the Y_i-values are absent in Eqs.(45)-(48). Besides, we assume that $J_\gamma(0) = 1$. The parameters f, q, p_1, p_2, p_3 have the following form:

$$p_1 = \frac{2i(1 - \kappa\kappa^*)}{\kappa^* - \kappa}\,, \quad p_2 = \frac{2i(1 + \kappa\kappa^*)}{\kappa^* - \kappa}\,, \quad p_3 = \frac{2i(\kappa + \kappa^*)}{\kappa^* - \kappa}\,.$$
$$f = p_3/p_1\,, \qquad q = p_2/2\,.$$

Eqs.(37)-(40) describe the general case of γ-beam propagation, when the variations intensity and Stokes parameters are determined by the imaginary values of refractive indices and the difference of their real quantities. However, these equations do not described these cases, when the relation $\kappa - \kappa^* = 0$ takes place. In the case $\kappa + \kappa^* = 0$, one can use the known relations from papers [6,9,15] or one can find the limits of Eqs.(37)-(40) obtained here. For example, one can offer $\kappa = \delta + i\rho$ and δ direct to zero.

Figs.3,4 illustrate the variations of Stokes parameters and the intensity of an initially unpolarized γ-beam moving in the dichromatic laser wave.

6. Influence of the laser wave intensity on γ-quanta propagation

The influence of the laser wave intensity on the e^+e^--pair production was studied in a number of papers (see Ref.[4] and literature therein). The degree of intensity of a dichromatic laser wave can be characterized by the dimensionless parameter [4] $\xi^2 = \xi_1^2 + \xi_2^2$ where $\xi_1^2 = \frac{<E_1^2>}{E_o{}^2}\frac{m^2c^4}{E_{l,1}{}^2}$ and $\xi_2^2 = \frac{<E_2^2>}{E_o{}^2}\frac{m^2c^4}{E_{l,2}{}^2}$. Here we have considered the case of a relatively low-intensity of a laser wave, when $\xi^2 \ll 1$. Previously obtained results [4,11] can be used to write the components of the permittivity tensor with the allowance for series expansion in ξ. A key issue in this treatment of the intensity is the replacement of the variables z_i, $(i = 1, 2)$ by variables, which we denote by $\tilde{z}_i$, $(i = 1, 2)$, such that $\tilde{z}_i = z_i(1 + \xi^2)$. On the whole, the corresponding components of the permittivity tensor retain their form, but the variables z_i are replaced by the variables $\tilde{z}_i$, and the critical field E_o is replaced by $\tilde{E}_o = \frac{m^2c^3(1+\xi^2)}{e\hbar}$. The functions $F_2'(z_i, 1), F_2''(z_i, 1), F_1'(z_i)$ and $F_(''z_i)$ are replaced by the functions $F_2'(\tilde{z}_i, \mu), F_2''(\tilde{z}_i, \mu), F_1'(\tilde{z}_i)$ and $F_1''(\tilde{z}_i)$, where $\mu = 1/(1+\xi^2)$. A new condition for the pair production threshold for components of the dichromatic wave is $\tilde{z}_i < 1$. It means that the threshold energy of γ-beam enhances (at the fixed frequency of laser photons). In a strict sense the field of application of these more refined relations satisfies the condition $\xi^2 \ll 1$. Nevertheless, we can receive the important information in this case [9].

In the case, when the parameter $\xi^2 \gg 1$, the pair production process is similar to an analogous process in the constant electromagnetic field. The permittivity tensor for these fields was found in Ref.[16] and some particular calculations of γ-quanta propagation are in [17].

7. Discussion

The optical properties of an anisotropic medium can be described by the use of the symmetric permittivity tensor. In the general case the tensor components are complex values. It means, generally speaking, that the permittivity tensor is not reduced to principal axes with the result that the normal electromagnetic waves (the eigenfunctions of the problem) present two elliptically polarized waves. Because of this some peculiarities in γ-quanta propagation in anisotropic medium appeared. The simplest example of such a medium of the general type is the dichromatic laser wave involving two linearly polarized waves with different frequencies. The other example is a single crystal oriented in a region of the coherent pair production process [4]. The permittivity tensor and polarization characteristics of normal waves in single crystals were obtained in [5], and it was shown that orientation regions were in single crystals, where the circular polarization of normal waves was high ($\approx$ 90% in maximum). However, in single crystals the components of permittivity tensor are a sum of sufficiently large number of terms that makes the investigation more difficult. Note that Eqs.(37-40) for variations of γ-beam intensity and Stokes parameters are true for an arbitrary anisotropic medium, when $\kappa - \kappa^* \neq 0$

We made some calculations of γ-beam propagation in the field of dichromatic wave in the case, when the frequency ratio $E_{l,2}/E_{l,1}$ is equal to 2 (see Figs.(1-4)). We take the values $P_{1,1} = 1, P_{3,2} = 1$ for the polarization state of the dichromatic wave. It means that the angle between directions of these two polarizations is equal to 45^o. One can see from Fig.2 that the value $|P_{circ}| = |X_2|$ depends on the ratio of the electric field intensities $r = \sqrt{< E_2^2 > / < E_1^2 >}$. The value $|X_2| = 1$, when r=0.658 or 4.12 (curves 1 and 4 on Fig.2). The refractive indices for r=0.658 are shown in Fig.1. One can see that the real and imaginary parts of refractive indices of two normal electromagnetic waves are equal in magnitude at $z_1 = 1.43$. This is the so-called in classical crystal optics case of singular axis [1,2].

Now we can make a conclusion that in single crystals the high degree of circular polarization of normal waves [5] is due to the common action of the two "strong" crystallographic planes with the 45^o angle between them. The (110) and (010) planes are responsible for the effect under conditions of the cited paper.

Fig.3 illustrates the variations of Stokes parameters as functions of the laser bunch thickness. The behavior of these curves is easily to understand. As has already been noted, the γ-beam in a medium can be presented as a superposition of two normal electromagnetic waves with different refractive indices. Because of this, one normal wave is adsorbed to greater extend than other one and after propagation of some thickness x only this wave would then be left behind and $\xi_1(x) \approx X_1, \xi_2(x) \approx X_2, \xi_3(x) \approx X_3$ or $\xi_1(x) \approx Y_1, \xi_2(x) \approx Y_2, \xi_3(x) \approx Y_3$. Referring to Fig.3 notice that such a thickness is enough large when $1 < z_1 < 1.45$. The reason is that the difference of imaginary parts of refractive indices is a small value in the pointed region of the variable z_1(see Fig.1).

Table 1. The 50 GeV γ-beam intensities in Si single crystal as a function of thickness. Here the value ξ_2 is the initial circular polarization of the beam, and $\xi_1 = 0, \xi_3 = 0$. The direction of motion of the beam is near $< 001 >$ axis of the single crystal. $A_S = |I(\xi_2 = 1) - I(\xi_2 = 0)|/I(\xi_2 = 0) = |I(\xi_2 = -1) - I(\xi_2 = 0)|/I(\xi_2 = 0)$.

x, cm	$I(\xi_2 = 0)$	$I(\xi_2 = -1)$	$I(\xi_2 = 1)$	A_S
0	1.000000	1.000000	1.000000	0.000
10	0.195701	0.197923	0.193479	0.0114
20	0.039152	0.040871	0.037433	0.0439
30	0.007990	0.008739	0.007242	0.0936
40	0.001658	0.001916	0.001401	0.1552
50	0.000349	0.000427	0.000271	0.2232

The dichromatic wave or single crystals [5] are sensitive to the circular polarization of γ-beam (see Figs.3,4). So, initially an unpolarized γ-beam moving in an anisotropic medium become a circularly polarized one. This case differs from the known one [3] of the γ-beam propagation in the anisotropic medium. As is shown in Ref.[3] the only linearly polarized beam is transformed in a circularly polarized one. In principle, the single crystal sensitivity to a circular polarization can be used for determination of polarization of high energy (in tens GeV and more) γ-quanta and electrons. Table 1 illustrates the dependence of $\gamma - beam$ intensity on the initial circular polarization in silicon single crystals. The direction of motion of γ-beam is defined with the use of the angles $\phi_H = \theta \cos\alpha = 1.50\, mrad$ and $\phi_V = \theta \sin\alpha = 1.78\, mrad$, where the θ is the angle with respect to $< 001 >$ axis of the single crystal and α is the azimuth angle around this axis ($\alpha = 0$, when the γ-quanta momentum lies in (110) - plane). More detailed consederation of this process in single crystals one can find in paper [18].

It is believed that a γ-beam propagation in the linearly polarized monochromatic laser wave moving in the magnetic field (normally to it direction) is similar to such a propagation in a dichromatic wave. Another analogous example is two monochromatic laser waves with the equal frequencies moving at an nonzero angle between them.

The propagation of γ-quanta through a laser wave (when $\xi^2 \ll 1$) is similar to the same process in single crystals for the region of coherent pair production. For example, the permittivity tensor in single crystals [5] is determined by the functions F_1', F_2', F_1'', F_2'' as in a laser wave. However, the existence of some frequencies of pseudophotons and incoherent pair production in single crystals is the main difference between these two cases. Note that the permittivity tensor components (see also Ref.[9]) can be presented as a linear combinations of the invariant helicity amplitudes for the forward light by light scattering [6,7,8].

Note that there are no experiments yet in support of the transformation of γ-beam polarization in single crystals and laser waves. Nevertheless, a number of proposals on the investigation and utilization of this phenomenon [12] is available.

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