И Ф В Э
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STATE RESEARCH CENTER OF RUSSIA

INSTITUTE FOR HIGH ENERGY PHYSICS

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V.E. Rochev

A MECHANISM FOR DYNAMICAL GENERATION OF $SU(2)$ GEORGI-GLASHOW MODEL

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Abstract

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A mechanism for the dynamical mass generation of a non-Abelian gauge field which is based on taking into account the contributions of the gauge field vacuum configurations into the formation of the physical vacuum is considered. For a model of the physical vacuum as a superposition of Abelian configurations the gauge field propagator is calculated in the leading order of $1/d$-expansion (d is a space-time dimension). One-particle spectrum of the model corresponds to the gauge sector of $SU(2)$ Georgi-Glashow model.

Аннотация

Рочев В.Е. Об одном механизме динамической генерации $SU(2)$-модели Джорджи-Глешоу: Препринт ИФВЭ 98-83. – Протвино, 1998. – 11 с., библиогр.: 9.

Рассмотрен механизм динамической генерации массы неабелева калибровочного поля, в основе которого лежит учет вкладов вакуумных конфигураций калибровочного поля в формирование физического вакуума теории. В модели физического вакуума как суперпозиции абелевых конфигураций вычислен пропагатор калибровочного поля в главном порядке $1/d$-разложения (d – размерность пространства-времени). Одночастичный спектр модели соответствует калибровочному сектору $SU(2)$-модели Джорджи-Глешоу.

1. Introduction

In the Standard Model the central role belongs to the Higgs mechanism which gives the masses to particles without violating the cardinal principles of theories, such as local gauge invariance and renormalizability. The Higgs mechanism is a well adjusted and efficient machine, and its description rightfully occupies its honorable place in any textbook on the modern high energy physics. However, in spite of all experimental efforts, no traces of the scalar sector of the Standard Model have been found hitherto, and the question on searching alternatives to the Higgs mechanism is not academic at all.

The generation of mass of a gauge field and the spontaneous symmetry breaking connected with it are defined by the structure of the ground state of the theory, that is a physical vacuum. Modeling of this structure is the main problem of the dynamical symmetry breaking description (see, for instance, [1]).

In this paper a mechanism for the dynamical mass generation of the non-Abelian gauge field is considered which does not require entering a scalar field and other additional fields. Here the gauge field itself, more exactly its vacuum constituent undertakes the role of an order parameter, which is played by the vacuum expectation of a scalar field in the usual Higgs mechanism. Appearance of such a vacuum constituent is a manifestation of a nontrivial structure of physical vacuum of the quantum field theory. This vacuum constituent arises quite naturally in constructing an iteration solution of the Schwinger-Dyson equations by the method, suggested in [2], [3], which we shall use in the present work as well.

To illustrate the method we examine the spontaneous symmetry breaking in the scalar theory. (As is well known, this phenomenon is a foundation for the Higgs mechanism in the Standard Model.) Consider the theory of a scalar field ϕ with the Lagrangian

$$\mathcal{L} = \partial_\mu \phi^* \partial_\mu \phi - m^2 \phi^* \phi - \frac{\lambda}{2} (\phi^* \phi)^2. \tag{1}$$

(We are working in the Minkowsky metric with $g_{00} = 1$, but will not distinguish between the upper and lower indices to simplify the notations.)

The Schwinger-Dyson equation (see, for instance, [4]) for the generating functional $G(j)$ of the Green functions of this model has the form

$$\frac{\lambda}{i}\frac{\delta^3 G}{\delta j^*\delta j\delta j^*} - (m^2+\partial^2)\frac{1}{i}\frac{\delta G}{\delta j^*} + jG = 0. \qquad (2)$$

Here $j(x)$ is a source of the field $\phi^*(x)$. At $\lambda = 0$ eq.(2) has the unique (up to a normalization factor) solution

$$G^{pert} = \exp\{i\int dxdyj^*(x)\Delta_c(x-y)j(y)\},$$

where $\Delta_c = (m^2+\partial^2)^{-1}$ is the free propagator.

This solution is a foundation for the iterative solution of eq.(2) in the form of the perturbation series. At $m^2 < 0$ such a solution is unstable — the physical vacuum in this case differs from the trivial vacuum of the perturbation theory. To describe the solution in this case, which corresponds the spontaneous symmetry breaking, let us consider another iterative scheme: an expansion near the point $j = 0$. As a leading approximation, consider the equation

$$\frac{\lambda}{i}\frac{\delta^3 G_0}{\delta j^*\delta j\delta j^*} - (m^2+\partial^2)\frac{1}{i}\frac{\delta G_0}{\delta j^*} = 0, \qquad (3)$$

and the iterative expansion for the generating functional

$$G = G_0 + G_1 + \cdots + G_n + \cdots,$$

is constructed by the subsequent solution of the iteration scheme equations

$$\frac{\lambda}{i}\frac{\delta^3 G_n}{\delta j^*\delta j\delta j^*} - (m^2+\partial^2)\frac{1}{i}\frac{\delta G_n}{\delta j^*} = -jG_{n-1}. \qquad (4)$$

Leading approximation equation (3) has the solution

$$G_0(j) = \exp i\int dx[v^*j + vj^*],$$

where v obeys the "characteristic equation"

$$(\lambda v^*v + m^2 + \partial^2)v = 0. \qquad (5)$$

Characteristic equation (5) has the form of a classical field equation of the theory with Lagrangian (1), and its solution v is the vacuum expectation of the field ϕ in the leading approximation of the given iteration scheme.

The trivial solution $v \equiv 0$ corresponds to the trivial perturbative vacuum, and the corresponding iterative expansion is the perturbation series, i.e. the usual perturbation theory is a partial case of the given iteration scheme. At $m^2 < 0$, $\lambda > 0$, the Green functions of the perturbation theory maintain tachyon poles, which indicates the instability of the trivial vacuum, but in this case another class of constant solutions of characteristic equation (5) exists

$$v^*v = -m^2/\lambda.$$

The iteration scheme based on this solution is a perturbation theory over spontaneously broken non-perturbative vacuum. For the construction of the iterations in this case, it is convenient to go over to the new sources

$$j_+ = \frac{1}{\sqrt{2v^*v}}(v^*j + vj^*),\ j_- = \frac{-i}{\sqrt{2v^*v}}(v^*j - vj^*).$$

In terms of these sources we obtain the solution of the first-step iteration scheme equations as $G_1 = P_1 G_0$, where

$$P_1 = \frac{i}{2}\int dxdy\{j_+\Delta_H j_+ + j_-\Delta_G j_-\}.$$

Here $\Delta_H = (2\lambda v^*v + \partial^2)^{-1}$ is the Higgs boson propagator, and $\Delta_G = \partial^{-2}$ is the Goldstone boson propagator.

Therefore, the adequate choice of the vacuum constituent — the solution v of characteristic equation (5) — defines the adequate structure of Green functions and one-particle spectrum of the theory.

We shall apply this scheme to the construction of the iterative solution for a non-Abelian gauge theory. Corresponding characteristic equations have a great number of various solutions, and a choice of a class of the solutions, i.e. the vacuum constituents, defines a choice of the candidate to the physical vacuum of the theory. Here we consider the simplest non-trivial class of the solutions ("Abelian solutions", see below) and demonstrate that in the leading order of $1/d$-expansion this class of solutions leads to the dynamical realization of $SU(2)$ Georgi-Glashow model [5].

2. Iteration scheme for non-Abelian gauge theory

Consider the theory of a gauge field $\mathbf{A}_\mu \equiv A^a_\mu$ with the Lagrangian

$$\mathcal{L} = -\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}_{\mu\nu}(\mathbf{A}) - \frac{1}{2\alpha}(\partial_\mu\mathbf{A}_\mu)^2 - \bar{\mathbf{c}}\partial_\mu\mathcal{D}_\mu(\mathbf{A})\mathbf{c}. \tag{6}$$

Here $\mathbf{F}_{\mu\nu}(\mathbf{A}) \equiv F^a_{\mu\nu}(\mathbf{A}) = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$ is the gauge field tensor; $\mathcal{D}_\mu(\mathbf{A}) \equiv \mathcal{D}^{ab}_\mu(\mathbf{A}) = \delta^{ab}\partial_\mu - gf^{abc}A^c_\mu$ is the covariant derivative; α is a gauge parameter; $\mathbf{c} \equiv c^a$ is the Faddeev-Popov ghost field; f^{abc} are structure constants of the gauge group.

Let us introduce the generating functional $G(J)$ of Green functions which depends on the gauge field source $\mathbf{J}_\mu \equiv J^a_\mu$ and the ghost field sources $\mathbf{j} \equiv j^a$ and $\bar{\mathbf{j}} \equiv \bar{j}^a$. The generating functional G is a solution of the system of equations in functional derivatives — the Schwinger-Dyson equations:

$$\mathcal{D}_\nu(\mathcal{A})\mathbf{F}_{\nu\mu}(\mathcal{A})G + \frac{1}{\alpha}\partial_\mu\partial_\nu\mathcal{A}_\nu G + g\mathbf{f}\mathbf{C}\partial_\mu\bar{\mathbf{C}}G + \mathbf{J}_\mu G = 0, \tag{7}$$

$$\partial_\mu \mathcal{D}_\mu(\mathcal{A})\mathbf{C}G = \mathbf{j}G. \tag{8}$$

Here the following notations are introduced

$$\mathcal{A}_\mu \equiv \mathcal{A}^a_\mu = \frac{1}{i}\frac{\delta}{\delta J^a_\mu}, \;\; \mathbf{C} \equiv C^a = \frac{1}{i}\frac{\delta}{\delta \bar{j}^a}, \;\; \bar{\mathbf{C}} \equiv \bar{C}^a = i\frac{\delta}{\delta j^a}.$$

For solving equations (7)-(8) we shall use the iteration scheme of [2], [3]. The leading approximation of this scheme is the system of equations

$$\mathcal{D}_\nu(\mathcal{A})\mathbf{F}_{\nu\mu}(\mathcal{A})G_0 + \frac{1}{\alpha}\partial_\mu\partial_\nu\mathcal{A}_\nu G_0 + g\mathbf{f}\mathbf{C}\partial_\mu\bar{\mathbf{C}}G_0 = 0, \tag{9}$$

$$\partial_\mu \mathcal{D}_\mu(\mathcal{A})\mathbf{C}G_0 = 0. \tag{10}$$

The equations of the n-th step of the iteration scheme have the form

$$\Big\{\mathcal{D}_\nu(\mathcal{A})\mathbf{F}_{\nu\mu}(\mathcal{A}) + \frac{1}{\alpha}\partial_\mu\partial_\nu\mathcal{A}_\nu + g\mathbf{f}\mathbf{C}\partial_\mu\bar{\mathbf{C}}\Big\}G_n = -\mathbf{J}_\mu G_{n-1}, \tag{11}$$

$$\partial_\mu \mathcal{D}_\mu(\mathcal{A})\mathbf{C}G_n = \mathbf{j}G_{n-1}. \tag{12}$$

The solution of leading approximation equations (9)-(10) is

$$G_0 = \exp i\Big\{\mathbf{J}_\mu \star \mathbf{V}_\mu + \bar{\mathbf{j}} \star \mathcal{G} + \bar{\mathcal{G}} \star \mathbf{j}\Big\}, \tag{13}$$

where $\mathbf{J}_\mu \star \mathbf{V}_\mu \equiv \int \; dx \; J^a_\mu(x) V^a_\mu(x)$, etc.

Coefficient functions $\mathbf{V}_\mu \equiv \; V^a_\mu$ и $\mathcal{G} \equiv \mathcal{G}^a$ (vacuum constituents) are solutions of the system of characteristic equations

$$\mathcal{D}_\nu(\mathbf{V})\mathbf{F}_{\nu\mu}(\mathbf{V}) + \frac{1}{\alpha}\partial_\mu\partial_\nu\mathbf{V}_\nu + g\mathbf{f}\mathcal{G}\partial_\mu\bar{\mathcal{G}} = 0, \tag{14}$$

$$\partial_\mu \mathcal{D}_\mu(\mathbf{V})\mathcal{G} = 0. \tag{15}$$

The solution of the n-th-step equations of the iteration scheme has the form

$$G_n = P_n G_0,$$

and taking into account characteristic equations (14)-(15), we obtain the system of equations for the functional P_n

$$\begin{aligned}\Big\{\Big(\mathcal{D}_\nu(\mathbf{V}) - g\mathbf{f}\mathcal{A}_\nu\Big)\Big(\mathcal{D}_\nu(\mathbf{V})\mathcal{A}_\mu - \mathcal{D}_\mu(\mathbf{V})\mathcal{A}_\nu + g\mathbf{f}\mathcal{A}_\mu\mathcal{A}_\nu\Big) - g\mathbf{f}\mathbf{F}_{\mu\nu}(\mathbf{V})\mathcal{A}_\nu \\ + \frac{1}{\alpha}\partial_\mu\partial_\nu\mathcal{A}_\nu + g\mathbf{f}\Big(\mathbf{C}\partial_\mu\bar{\mathbf{C}} + \mathcal{G}\partial_\mu\bar{\mathbf{C}} + \mathbf{C}\partial_\mu\bar{\mathcal{G}}\Big)\Big\}P_n = -\mathbf{J}_\mu P_{n-1}, \\ \partial_\mu\Big\{\Big(\mathcal{D}_\mu(\mathbf{V}) - g\mathbf{f}\mathcal{A}_\mu\Big)\mathbf{C} - g\mathbf{f}\mathcal{A}_\mu\mathcal{G}\Big\}P_n = \mathbf{j}P_{n-1}.\end{aligned}$$

Since $P_0 = 1$, the solution of this system for any n is a polynomial in sources $\mathbf{J}_\mu$ and $\mathbf{j}$. Coefficient functions of this polynomial define the Green functions. At each step the equations of the iteration scheme give a closed system of equations for these functions. The solution of the first-step equations is a quadratic polynomial, defining two-point functions (the propagators). At the second step the solution is a polynomial of the fourth degree, defining three-and four-point functions, as well as corrections to the propagators, etc. To eliminate ultraviolet divergences, it is necessary to modify the equations of the iteration scheme by introducing the corresponding counterterms (see [2]). Note that functions of the leading approximation and of the first step are ultraviolet-finite.

3. Physical vacuum and vacuum constituents

In the leading approximation of considered iteration scheme the ground state (the physical vacuum of the theory) is defined by a choice of solutions of the characteristic equation system (14)-(15). Each solution $(\mathbf{V}_\mu, \mathcal{G})$ of the characteristic equation system defines a partial solution $G(J \mid \mathbf{V}, \mathcal{G})$ of the iteration scheme. This solution will be referred to as corresponding to a partial mode $\mid \mathbf{V}, \mathcal{G} >$.

The trivial solution $\mathbf{V}_\mu = \mathcal{G} = 0$ corresponds to the leading approximation $G_0 = 1$. The iteration scheme based on this solution is a perturbation theory in the coupling constant over the trivial perturbative vacuum. Nontrivial solutions of the characteristic equations define non-perturbative modes. These solutions have a sense of the vacuum constituents of the quantum fields $\mathbf{A}_\mu$ and $\mathbf{c}$, like the vacuum constituent v of scalar field in the Higgs mechanism (see Introduction).

The choice of the approximation for the description of the physical vacuum $\mid\ 0 >$ must ensure general physical requirements, such as Poincaré-invariance, cluster decomposition, etc. In the Higgs mechanism it is sufficient to take the constant solution $v = \sqrt{-m^2/\lambda}$ for this purpose.

In the case under consideration the situation is more complicated. Obviously, the choice of a separate partial mode with $\mathbf{V}_\mu \neq 0$ as a leading approximation to the physical vacuum ("a candidate for the physical vacuum") does not ensure Poincaré-invariance of the theory. Notice, however, that Schwinger-Dyson equations (7)-(8) are the linear functional-differential equations for the generating functional, and any superposition of partial solutions $\sum G(J \mid \mathbf{V}, \mathcal{G})$ is also a solution of these equations. So we can choose a superposition of partial modes as a candidate for the physical vacuum, and choose the generating functional of the physical Green functions as the superposition

$$< 0 \mid 0 >_J = G(J) = \sum_{\{V, \mathcal{G}\}} G(J \mid \mathbf{V}, \mathcal{G}),$$

corresponding to a class $\{\mathbf{V}, \mathcal{G}\}$ of solutions of the characteristic equations. We shall suppose this superposition can be chosen in such a way that all the contributions, breaking the Poincaré-invariance, are mutually cancelled, and the resulting theory turns out to be Poincaré-invariant. For instance, the expectation value of the gauge field must disappear

$$< 0 \mid \mathbf{A}_\mu \mid 0 >= \frac{1}{i} \frac{\delta G}{\delta \mathbf{J}_\mu}\bigg|_{J=0} = 0,$$

in spite of the contributions of separate partial modes in this vacuum expectation can be different from zero. Further, the higher derivatives of the physical generating functional, defining multipoint functions, must be translational-invariant after switching off the sources, etc. The set of these conditions will ensure the Poincaré-invariance of the theory.

4. Ward-Slavnov-Taylor identities

Gauge invariance imposes the restrictions on Green functions which are known as Ward-Slavnov-Taylor identities. From the Jacobi identities for the structure constants of the gauge group the identity follows

$$\mathcal{D}_\mu(\mathcal{A})\mathcal{D}_\nu(\mathcal{A})\mathbf{F}_{\mu\nu}(\mathcal{A}) \equiv 0. \tag{16}$$

Acting by operator $\mathcal{D}_\mu(\mathcal{A})$ on Schwinger-Dyson equation (7) and taking into account (16) we obtain the generating relation for the Ward-Slavnov-Taylor identities

$$\frac{1}{\alpha}\mathcal{D}_\mu(\mathcal{A})\partial_\mu\partial_\nu\mathcal{A}_\nu G + g\mathcal{D}_\mu(\mathcal{A})\mathbf{f}\mathbf{C}\partial_\mu\bar{\mathbf{C}}G = -\mathcal{D}_\mu(\mathcal{A})\mathbf{J}_\mu G. \tag{17}$$

Differentiating this relation and switching off the sources, we get the desired restrictions on the Green functions. As relation (17) is an identity, and $G(J \mid \mathbf{V}, \mathcal{G})$ is a solution of Schwinger-Dyson equations, these restrictions must be fulfilled for each separate partial mode.

If $\mathbf{V}_\mu \neq 0$ or $\mathcal{G} \neq 0$ their form is certainly different from that of the usual Ward-Slavnov-Taylor identities. Relation (17) must be fulfilled in each order of the considered iteration scheme that is the chain of relations of type (17) must be satisfied, where in the left hand side G is changed for G_n, while in the right hand side – for G_{n-1}. In the leading approximation the consequence of (17) is the condition

$$\frac{1}{\alpha}\mathcal{D}_\mu(\mathbf{V})\partial_\mu\partial_\nu\mathbf{V}_\nu + g\mathcal{D}_\mu(\mathbf{V})\mathbf{f}\mathcal{G}\partial_\mu\bar{\mathcal{G}} = 0 \tag{18}$$

on the solutions $\mathbf{V}_\mu$ and $\mathcal{G}$ of characteristic equations (14)-(15).

5. Abelian configurations

Characteristic equations (14)- (15) possess a rich ensemble of various solutions, that is a reflection of the complex vacuum structure of non-Abelian gauge theory. In this paper we restrict ourselves to the analysis of the simplest solutions of these equations, bringing, however, to nontrivial physical effects. First of all note that if we impose the subsidiary condition

$$\partial_\mu\mathbf{V}_\mu = 0, \tag{19}$$

then, as is seen from (18), for the ghost vacuum field $\mathcal{G}$ we can restrict ourselves to the trivial solutions $\mathcal{G} = \bar{\mathcal{G}} = 0$ without contradicting the gauge invariance condition. Further, there exist two classes of the simplest solutions of characteristic equation (14):

I) Coordinate-independent solutions $\mathbf{V}_\mu$: $\partial_\nu \mathbf{V}_\mu = 0$. For such solutions eq.(14) reduces at $\mathcal{G} = \bar{\mathcal{G}} = 0$ to the condition

$$f^{abc} f^{cdh} V_\nu^b V_\nu^d V_\mu^h = 0. \tag{20}$$

(This class of solutions was considered in [3]).

II) "Abelian" solutions, for which the dependencies on the space-time coordinates and on isotopic variables are separated:

$$\mathbf{V}_\mu(x) = \mathbf{n} V_\mu(x), \tag{21}$$

where $\mathbf{n}$ is a unit vector in the isotopic space. For such vacuum configurations all nonlinear (non-Abelian) terms in eq. (14) disappear, and, with condition (19) taken into account, this equation becomes the d'Alambert equation $\partial^2 V_\mu = 0$.

For the gauge group $SU(2)$ the coordinate-independent solutions are a subset of Abelian solutions. Really, it is easy to prove that for the group $SU(2)$, when $f^{abc} = \epsilon^{abc}$, condition (20) is equivalent to the relation $\epsilon^{abc} V_\mu^b V_\nu^c = 0$. This relation means that all the vectors $\mathbf{V}_\mu$ are collinear in the isotopic space, i.e. there exists a selected isotopic direction, and choosing it as vector $\mathbf{n}$ we come to Abelian solutions (21).

6. First-step equations

Equations of the first step define gauge field and ghost field propagators. Polynomial P_1 is quadratic in sources and at $\mathcal{G} = \bar{\mathcal{G}} = 0$ has the form

$$P_1 = \frac{1}{2i} \mathbf{J}_\mu \star \mathbf{D}_{\mu\nu} \star \mathbf{J}_\nu + i \bar{\mathbf{j}} \star \mathbf{D} \star \mathbf{j}. \tag{22}$$

Here $\mathbf{D}_{\mu\nu} \equiv D_{\mu\nu}^{ab}(x, y \mid \mathbf{V})$ is the gauge field propagator; $\mathbf{D} \equiv D^{ab}(x, y \mid \mathbf{V})$ – the ghost propagator.

The iteration scheme gives the following equation for $\mathbf{D}$

$$\partial_\mu \mathcal{D}_\mu(\mathbf{V}) \mathbf{D} = 1. \tag{23}$$

From generating relation (17) one obtains the following relation for the longitudinal part of $\mathbf{D}_{\mu\nu}$

$$\frac{1}{\alpha} \mathcal{D}_\mu(\mathbf{V}) \partial_\mu \partial_\nu \mathbf{D}_{\nu\lambda} = \mathcal{D}_\lambda(\mathbf{V}). \tag{24}$$

Since from subsidiary condition (19) it follows that $[\partial_\mu, \mathcal{D}_\mu(\mathbf{V})] = 0$, then from (23) and (24) we get

$$\partial_\nu \mathbf{D}_{\nu\lambda} = \alpha \mathbf{D} \star \mathcal{D}_\lambda(\mathbf{V}). \tag{25}$$

At $\mathbf{V}_\mu = 0$: $\mathbf{D} = \partial^{-2}$, and identity (25) gains the familiar form

$$\partial_\nu \mathbf{D}_{\nu\lambda} = \alpha \frac{\partial_\lambda}{\partial^2}.$$

With taking into account (25), from the iteration scheme equations we obtain the equation for $\mathbf{D}_{\mu\nu}$:

$$\mathcal{P}_{\mu\nu}(\mathbf{V})\mathbf{D}_{\nu\lambda} = g_{\mu\lambda} - \partial_\mu \mathbf{D} \star \mathcal{D}_\lambda(\mathbf{V}), \tag{26}$$

where $\mathcal{P}_{\mu\nu}(\mathbf{V}) \equiv \Big\{\mathcal{D}^2(\mathbf{V})g_{\mu\nu} - \mathcal{D}_\mu(\mathbf{V})\mathcal{D}_\nu(\mathbf{V}) + 2\Big[\mathcal{D}_\mu(\mathbf{V}), \mathcal{D}_\nu(\mathbf{V})\Big]\Big\}$.

As has been pointed above, the four-point and three-point functions enter the second-step equations of the iteration scheme. So, for the four-point functions $\mathcal{F}^{abcd}_{\mu\nu\sigma\rho}(x,y,z,t)$ of the gauge field, we get the equation

$$\begin{aligned}\mathcal{P}_{\mu\nu}(\mathbf{V})\mathcal{F}_{\nu\lambda\sigma\rho}(x,y,z,t) = -(\{g_{\mu\lambda}\delta(x-y)\mathbf{1}\otimes \mathbf{D}_{\sigma\rho}(z,t)\} + \\ \{y \leftrightarrow z,\ \lambda \leftrightarrow \sigma\} + \{y \leftrightarrow t,\ \lambda \leftrightarrow \rho\}).\end{aligned} \tag{27}$$

Below we shall work in the transverse gauge $\partial_\nu \mathbf{D}_{\nu\lambda} = 0$. Besides, we restrict ourselves to the consideration of Abelian configurations (21) of the gauge group $SU(2)$. For the Abelian configurations it is convenient to introduce the orthogonal basis

$$u_0^{ab} = n^a n^b, \ \ u_\pm^{ab} = \frac{1}{2}(\delta^{ab} - n^a n^b \pm i\epsilon^{abc}n^c). \tag{28}$$

In basis (28) it is easy to separate the isotopic structure from the space-time one

$$\mathbf{D}_{\mu\nu} = \mathbf{u}_0 D^0_{\mu\nu} + \mathbf{u}_+ D_{\mu\nu} + \mathbf{u}_- \bar{D}_{\mu\nu},$$

$$\mathbf{D} = \mathbf{u}_0 D^0 + \mathbf{u}_+ D + \mathbf{u}_- \bar{D}.$$

For $D^0_{\mu\nu}$ и D^0 we get the free propagator equations, that is

$$D^0_{\mu\nu} = \frac{1}{\partial^2}(g_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}), \ \ D^0 = \frac{1}{\partial^2}.$$

The equation for D has the form

$$\partial_\mu \mathcal{D}_\mu(V) D = 1. \tag{29}$$

Here $\mathcal{D}_\mu(V) = \partial_\mu + igV_\mu$ is the "Abelian" covariant derivative. Equation for $\bar{D}$ is obtained from (29) by the substitution $\mathcal{D}_\mu \to \mathcal{D}^*_\mu$, $D \to \bar{D}$. For $D_{\mu\nu}$ we get the equation:

$$\Big\{\mathcal{D}^2(V)g_{\mu\nu} - \mathcal{D}_\mu(V)\mathcal{D}_\nu(V) + 2\Big[\mathcal{D}_\mu(V), \mathcal{D}_\nu(V)\Big]\Big\}D_{\nu\lambda} = g_{\mu\lambda} - \partial_\mu D \star \mathcal{D}_\lambda(V). \tag{30}$$

The equation for $\bar{D}_{\mu\nu}$ is obtained from (30) by the substitution $\mathcal{D}_\mu \to \mathcal{D}^*_\mu$, $D \to \bar{D}$, $D_{\mu\nu} \to \bar{D}_{\mu\nu}$.

7. Vacuum of Abelian configurations and 1/d expansion

As has been pointed in Section 3, when considering the nonperturbative modes with $\mathbf{V}_\mu \neq 0$ it is necessary to take a superposition of the non-perturbative modes as a candidate for the physical vacuum $\mid 0 >$ in order to preserve the Poincaré-invariance. As the simplest nontrivial variant for such a superposition, we will consider in the leading approximation a set of Abelian configurations $\{\mathbf{V}\}$ corresponding to Abelian solutions (21):

$$G_0(J) = \sum_{\{\mathbf{V}\}} G_0(J \mid \mathbf{V}) = \sum_{\{\mathbf{V}\}} \exp\ i\mathbf{J}_\mu \star \mathbf{V}_\mu .$$

The operation $\sum_{\{\mathbf{V}\}}$ must be chosen in such a manner that all the Poincaré-non-invariant contributions would disappear, in particular, the conditions below must be fulfilled

$$< 0 \mid \mathbf{V}_\mu \mid 0 >= \frac{1}{i}\frac{\delta G_0}{\delta \mathbf{J}_\mu}\bigg|_{J=0} = 0, \tag{31}$$

$$< 0 \mid \mathbf{V}_\mu(x)\mathbf{V}_\nu(y) \mid 0 >= -\frac{\delta^2 G_0}{\delta \mathbf{J}_\nu(y)\delta \mathbf{J}_\mu(x)}\bigg|_{J=0} = \mathbf{n}\otimes\mathbf{n}\cdot f_{\mu\nu}(x-y). \tag{32}$$

It is not difficult to make condition (31) true. For this we notice that for the Abelian configurations $-\mathbf{V}_\mu$ is a solution of the characteristic equation as well as $\mathbf{V}_\mu$ is, so for obeying (31) it is sufficient to take the superposition $G_0(J \mid \mathbf{V}) + G_0(J \mid -\mathbf{V}) \sim \cos\ \mathbf{J}_\mu \star \mathbf{V}_\mu$, or, in the general case, $\sum_{\{\mathbf{V}\}} \cos\ \mathbf{J}_\mu \star \mathbf{V}_\mu$. Note, that simultaneously the vacuum expectations of all the odd monomials in $\mathbf{V}_\mu$ also turn to zero: $< 0 \mid \mathbf{V}_{\mu_1} \cdots \mathbf{V}_{\mu_{2n+1}} \mid 0 >= 0$. Requirement (32) is less trivial. It is clear that required operation $\sum_{\{\mathbf{V}\}}$ should be continual, i.e., should correspond to some integration. But for the calculation of the vacuum expectation itself there is no necessity to specify this operation, if one is limited to configurations, for which

$$\mathbf{V}^2 \equiv V^a_\mu(x)V^a_\mu(x) = \mathcal{V}^2 = const,$$

that are the "equal-length" configurations [1]. Really, due to the characteristic equations and condition (19), the function $f_{\mu\nu}(x)$ must be a solution of the d'Alambert equation $\partial^2 f_{\mu\nu} = 0$ with the subsidiary condition $\partial_\mu f_{\mu\nu} = 0$ and the initial condition $f_{\mu\mu}(0) = \mathcal{V}^2$. The solution is unique

$$f_{\mu\nu} = \frac{\mathcal{V}^2}{d} g_{\mu\nu}. \tag{33}$$

Similarly, for the four-point monomial we get:

$$< 0 \mid V_\mu(x)V_\nu(y)V_\rho(z)V_\sigma(t) \mid 0 >= \frac{(\mathcal{V}^2)^2}{d(d+2)}(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}). \tag{34}$$

Let us turn to propagators. It is clear that the solutions of equations (29) and (30) can not be interpreted as particle propagators in the Poincaré-invariant theory. Physical

[1] The quantity $\mathcal{V}^2$ plays a role of the order parameter, and its sign must be defined from physical considerations.

propagators must be built by means of the same operations of partial mode superposition: $D(x-y \mid \mathcal{V}^2) = \sum_{\{\mathbf{V}\}} D(x,y \mid \mathbf{V})$.

Full solving of eqs. (29) and (30) with consequent transition to the physical vacuum presents a difficult problem. For its approximate solving notice that, as can be seen from formulae (32), (33) and (34), in the vacuum of Abelian configurations a small parameter arises, namely, $1/d$, where d is the dimension of space-time. It is easy to see that if one takes $D^{(0)} = \partial^{-2}$ as a leading approximation for equation (29), then after turning to the physical vacuum all the subsequent terms in the iterative solution $D = D^{(0)} + D^{(1)} + \cdots$ have a higher order in the parameter $1/d$. Slightly bulkier, but not complicated calculation shows that for $D_{\mu\nu}$ (see eq.(30)) the leading approximation of $1/d$-expansion is $D^{(0)}_{\mu\nu} = (\partial^2 - g^2\mathcal{V}^2)^{-1} \star \pi_{\mu\nu}$ (here $\pi_{\mu\nu}$ is a transverse projector).

Thus, in the leading order in $1/d$, we get in the momentum space

$$D(p) = \bar{D}(p) = -\frac{1}{p^2} + \mathcal{O}(1/d), \tag{35}$$

$$D_{\mu\nu}(p) = \bar{D}_{\mu\nu}(p) = -\frac{1}{p^2 + g^2\mathcal{V}^2}\Big(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\Big) + \mathcal{O}(1/d). \tag{36}$$

Stress once again that, unlike the partial solutions of equations (29)-(30), formulae (35) and (36) define the physical propagators of particles in the physical Poincaré-invariant vacuum.

As is well known, (see, for instance, [6]) in the lattice theories the $1/d$-expansion is, in essence, the mean-field expansion. Probably, as in this instance, the vacuum of Abelian configurations is a peculiar mean-field approximation to the true physical vacuum.

In the conclusion of this section let us touch on the cluster properties. For the scheme based on an approximation of the physical vacuum by the superposition of partial modes the cluster decomposition principle is a nontrivial property (see, for instance, [7]) and requires checking at each stage of calculations. We can state that this principle is satisfied for our model of physical vacuum at least in the leading order of $1/d$-expansion. So, for instance, from eq.(27) we get for the second-step four-point function $\mathcal{F}_{\mu\nu\sigma\rho}$ of the gauge field in the leading order in $1/d$: $\mathcal{F}_{\mu\nu\sigma\rho} = \mathbf{D}_{\mu\nu} \otimes \mathbf{D}_{\sigma\rho} + \{\nu \leftrightarrow \sigma\} + \{\nu \leftrightarrow \rho\}$, which is the usual disconnected part of the four-point function, in correspondence with the cluster decomposition principle.

8. Conclusion

In this paper it is found that for non-Abelian $SU(2)$-theories with the physical vacuum, representing a superposition of Abelian partial modes, the gauge field propagator in the leading approximation of $1/d$-expansion is

$$D^{ab}_{\mu\nu}(p) = \Big\{-\frac{1}{p^2}n^a n^b + \frac{1}{\mu^2 - p^2}(\delta^{ab} - n^a n^b)\Big\}\Big(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\Big), \tag{37}$$

where $\mu^2 = -g^2\mathcal{V}^2$.

If $\mathcal{V}^2 > 0$ the spectrum contains tachyons which is a sign of instability of this state [1].

At $\mathcal{V}^2 < 0$ this propagator corresponds to the mass spectrum of $SU(2)$ Georgi-Glashow model [5], and in this case the considered mechanism is a dynamical realization of the gauge sector for this model. As is well known, this model cannot be incorporated in the Standard Model phenomenology. From the viewpoint of our construction it means that the real physical vacuum of the Standard Model has a more complicated structure, and for its description it is necessary to take into account a wider (or other) class of solutions of the characteristic equations — vacuum constituents of fields. The ensemble of these solutions is highly extensive and various, and this variety allows one to hope for a possibility of the dynamical description of the mass generation in the Standard Model on the base of principles involved. Phenomenological consequences of the dynamical mass generation in Standard Model (see, for instance, [8], [9]) lead to the interesting physical results, and further studying of the dynamical mass generation mechanism seems quite actual.

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