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THE NLO DGLAP EXTRACTION OF $\alpha_{\mathbf{s}}$ AND HIGHER TWIST TERMS FROM CCFR $\mathbf{xF_3}$ AND $\mathbf{F_2}$ STRUCTURE FUNCTIONS DATA FOR νN DIS

Abstract

Alekhin S.I., Kataev A.L. The NLO DGLAP extraction of α_s and higher twist terms from CCFR xF_3 and F_2 structure functions data for νN DIS: IHEP Preprint 98-87. – Protvino, 1998. – p. 9, figs. 1, tables 2, refs.: 37.

We perform a detailed NLO analysis of the combined CCFR xF_3 and F_2 structure functions data and extract the value of α_s, parameters of distributions and higher-twist (HT) terms using a direct solution of the DGLAP equation. The value of $\alpha_s(M_Z) = 0.1222 \pm 0.0048(exp) \pm 0.0040(theor)$ is obtained. Our result has a larger central value and errors than the original one of the CCFR collaboration due to model independent parametrization of the HT contributions. The x-shapes of the HT contributions to xF_3 and F_2 are in agreement with the results of other model-independent extractions and are in qualitative agreement with the predictions of the infrared renormalon model. We also argue that the low x CCFR data might have inaccuracies, since their inclusion into the fits leads to the following low x-behaviour of the gluon distribution $xG(x, 9\ \text{GeV}^2) \sim x^{0.092\pm0.0073}$, in contradiction with the results of its extraction from low x HERA data.

Аннотация

Алехин С.И., Катаев А.Л. Извлечение α_s и вклада высших твистов из данных CCFR по структурным функциям глубоко неупругого νN рассеяния F_2 и xF_3 в нелидирующем порядке DGLAP: Препринт ИФВЭ 98-87. – Протвино, 1998. – 9 с., 1 рис., 2 табл., библиогр.: 37.

Проведен детальный анализ данных CCFR по структурным функциям F_2 и xF_3 с использованием решений уравнений DGLAP и извлечены величина α_s, параметры распределений и вклад высших твистов (ВТ). Получена величина $\alpha_s(M_Z) = 0.1222\pm0.0048(эксп)\pm 0.0040(теор.)$. Среднее значение и ошибки этой величины больше, чем результат самой коллаборации CCFR из-за того, что в нашем анализе вклад ВТ учтен в модельно независимой форме. Зависимость вклада ВТ от x согласуется с результатами других модельно-независимых анализов и не противоречит предсказаниям модели инфракрасного ренормалона. Отмечено, что данные CCFR в малых x могут иметь неточности, т.к. поведение глюонного распределения в малых x, определяемое этими данными $xG(x, 9\ \text{GeV}^2) \sim x^{0.092\pm0.0073}$, противоречит результатам анализа данных HERA.

1. Study of the possibility to separate power suppressed terms (namely, higher-twist (HT) effects) from the perturbation theory logarithmic corrections in the analysis of scaling violation of the deep-inelastic scattering (DIS) processes has a rather long history (see e.g. Refs. [1,2,3] and Ref. [4] for the review). In the recent years interest to this problem was renewed, mainly due to the possibility to model the HT terms in different processes using the infrared-renormalon (IRR) technique (see e.g. Refs. [5]-[10],[11] and, especially, Ref. [12] for the review).

On the other hand, the experimentalists improve their data precision and achieve, sometimes, a percent level of accuracy. For example, very precise data on xF_3 and F_2 from the νN DIS experiment, performed at Tevatron by the CCFR collaboration, recently appeared [13,14]. The CCFR data on xF_3 were analyzed in Ref. [15] in the leading order (LO), with inclusion of the next-to-leading-order (NLO), and with an approximate next-to-next-to-leading order (NNLO) corrections. For the latter the NNLO QCD corrections to the coefficient function [16] were taken into account. The NNLO corrections to the anomalous dimensions of a limited set of even non-singlet moments [17] were also taken into account. The NNLO corrections to the anomalous dimensions of odd moments, which are not still explicitly calculated, were obtained using smooth interpolation procedure proposed in Ref. [18] and improved in Ref. [19]. The aim of Ref. [15] was to attempt the first NNLO determination of $\alpha_s(M_Z)$ from DIS and to extract the HT terms from the data on xF_3 within the framework of the IRR-model [7]. Alongside, the model-independent extraction of the HT terms was made, similarly to the analysis of the combined SLAC-BCDMS data [20], which was performed in the NLO approximation. Theoretical uncertainties of the analysis of Ref. [15] were further estimated in Refs. [21,22] in the N^{3}LO approximation using the method of Padé approximants. It has been found in Refs. [15,21,22] that the inclusion of the NNLO corrections leads to the decrease of the HT terms, so that at the NNLO its x-shape come closer to zero.

In these analyses only statistical errors of data were taken into account. However, the systematic errors of the CCFR data are not small [14] and can dominate some parameters errors. In this paper we filled in this gap and performed the NLO analysis of the CCFR data with the help of QCD DGLAP evolution code, developed in Ref. [23]. (Remind that the analyses of Refs. [15,21,22] were performed with the help of the Jacobi polynomial variant [24,25,26] of the DGLAP equation [27]). In addition, we included in our analysis the CCFR data on the singlet structure function F_2. It should be stressed that the code [23]

was tested using the procedure proposed in Ref. [28] and demonstrated the accuracy at the level of $O(0.1\%)$ in the kinematic region covered by the analyzed data. It was already applied to the nonsinglet DGLAP analysis of the combined SLAC-BCDMS data on F_2 [29].

2. Our fits were made in the NLO approximation within the modified-minimal-subtraction $(\overline{MS})$ factorization and renormalization schemes. The Q^2 dependence of the strong coupling constant α_s was defined from the following equation:

$$\frac{1}{\alpha_s(Q)} - \frac{1}{\alpha_s(M_Z)} = \frac{\beta_0}{2\pi}\ln\left(\frac{Q}{M_Z}\right) + \beta\ln\left[\frac{\beta + 1/\alpha_s(Q)}{\beta + 1/\alpha_s(M_Z)}\right], \tag{1}$$

where $\beta = \frac{4\pi\beta_1}{\beta_0}$ and β_0 and β_1 are the coefficients of the QCD β-function, defined as

$$\beta(\alpha_s) = \frac{1}{4\pi}\mu\frac{\partial\alpha_s}{\partial\mu} = -2\sum_{i\geq 0}\beta_i(\frac{\alpha_s}{4\pi})^{i+2}, \tag{2}$$

where $\beta_0 = 11 - (2/3)n_f$ and $\beta_1 = 102 - (38/3)n_f$. Note that the explicit solution of Eq.(1) can be expressed through the Lambert function [30]. However, we did not use this explicit representation and solved Eq.(1) numerically. The effective number of flavours n_f was chosen to be $n_f = 4$ for Q^2 less than the definite scale M_5^2 and increased to $n_f = 5$ at larger values of Q^2 keeping the continuity of α_s [31]. The value of the effective matching scale M_5 was varied from $M_5 = m_b$ to $M_5 = 6.5m_b$. The latter choice was advocated in Ref. [32] on the basis of the DIS sum rules consideration. The dependence of the fit results on the choice of the matching point gives one of the sources of theoretical uncertainties inherent to our analysis.

The leading twist term $xF_3^{LT}(x,Q)$ was obtained by direct integration of the DGLAP equation [27]

$$\frac{dxq^{NS}}{d\ln Q} = \frac{\alpha_s(Q)}{\pi}\int_x^1 dz P_{qq}^{NS}(z)\frac{x}{z}q^{NS}(x/z,Q), \tag{3}$$

where $P_{qq}^{NS}(x)$ denotes the NLO splitting function, taken from Ref. [34]. The function xF_3 is determined by the subsequent convolution with the NLO coefficient function $C_{3,q}(x)$:

$$xF_3^{LT}(x,Q) = \int_x^1 dz C_{3,q}(z)\frac{x}{z}q^{NS}(x/z,Q). \tag{4}$$

The boundary condition at the reference scale $Q_0^2 = 5$ GeV^2 was chosen in the form analogous to the ones, used in Refs. [14,15]:

$$xq^{NS}(x,Q_0) = \eta_{NS}x^{b_{NS}}(1-x)^{c_{NS}}(1+\gamma x)\frac{3}{A_{NS}}, \tag{5}$$

where

$$A_{NS} = \int_0^1 x^{b_{NS}-1}(1-x)^{c_{NS}}(1+\gamma x)dx, \tag{6}$$

and η_{NS} is the measure of the deviation of the Gross-Llewellyn Smith integral [35] from its quark-parton value equal to 3. The expression for the xF_3 that includes the HT contribution looks as follows:

$$xF_3^{HT}(x,Q) = xF_3^{LT,TMC}(x,Q) + \frac{H_3(x)}{Q^2}, \tag{7}$$

where $F_3^{LT,TMC}(x,Q)$ is $F_3^{LT}(x,Q)$ with the target mass correction [36] applied.

2

At the first stage of this work, in order to perform the cross-checks with the results of Refs. [14,15], we fitted the data on xF_3 in the kinematical region $Q^2 > 5$ GeV^2, $W^2 > 10$ GeV^2, $x < 0.7$ (the number of data points (NDP) is 86). We made three fits with various ways of taking into account the HT effects. The first fit with no HT, i.e. $H_3(x) = 0$, was performed to compare our results with the Table 1 of Ref. [15], which was obtained using different method [24,25,26] and different computer code. In the second fit HT contribution was chosen as one-half of the IRR model predictions [7], i.e.

$$H_3(x) = A'_2 \int_x^1 dz C_2^{IRR}(z) \frac{x}{z} F_3^{LT}(x/z, Q), \tag{8}$$

where

$$C_2^{IRR}(z) = -\frac{4}{(1-z)_+} + 2(2+z+2z^2) - 5\delta(1-z) - \delta'(1-z) \tag{9}$$

and $A'_2 = -0.1$ GeV^2, as advocated for the first time in [8]. The aim of this fit was to compare our results with Table 7.9 of Ref. [14], where the computer code developed by Duke and Owens [33] was used. In the third fit we used the model independent HT-expression, i.e. $H_3(x)$ parametrized at $x = 0., 0.2, 0.4, 0.6, 0.8$ with linear interpolation between these points. It was performed to compare our results with Table 3 of Ref. [15].

All results of these our fits are given in Table 1. We observed a good agreement of our results on α_s with those from referenced papers. However, we found a certain discrepancy of the x-shape parameters values with the results of Ref. [15]. In particular, the value of γ, as given in column I of Table 1, is $\gamma = 0.26 \pm 0.30$, meanwhile the analogous parameter in Ref. [15] is $\gamma = 1.96 \pm 0.36$. At the same time our x-shape parameters are in agreement with those, given in Ref. [14], within errors. In addition, we made the fit releasing parameter A'_2 and obtained the value of $A'_2 = -0.12 \pm 0.05$ that agrees with the results of Ref. [15].

Table 1. The results of the fits to data on xF_3 with statistical errors only; I) without HT-terms; II) with HT accounted as one-half of the IRR model predictions; III) with model independent HT-contributions, $H_3^{(0),(2),(4),(6),(8)}$ are the values of $H_3(x)$ at $x = 0., 0.2, 0.4, 0.6, 0.8$.

	I	II	III
χ^2/NDP	88.5/86	81.9/86	70.3/86
b	0.789 ± 0.024	0.786 ± 0.024	0.805 ± 0.067
c	4.02 ± 0.11	4.00 ± 0.11	4.24 ± 0.21
γ	0.29 ± 0.30	0.26 ± 0.30	0.61 ± 0.71
η_{NS}	0.927 ± 0.014	0.949 ± 0.014	0.927 ± 0.030
$\alpha_s(M_Z)$	0.1193 ± 0.0025	0.1219 ± 0.0024	0.1216 ± 0.0066
$H_3^{(0)}$	–	–	0.18 ± 0.19
$H_3^{(2)}$	–	–	-0.26 ± 0.12
$H_3^{(4)}$	–	–	-0.21 ± 0.31
$H_3^{(6)}$	–	–	0.11 ± 0.26
$H_3^{(8)}$	–	–	0.90 ± 0.47

3

3. The next step of our analysis was to take into account the point-to-point correlations of the data due to systematic errors, which, as we have mentioned above, can be crucial for the estimation of total experimental errors of the parameters (see, in particular, Ref. [29], where the value $\alpha_s(M_Z) = 0.1180 \pm 0.0017$ (stat+syst) was obtained as a result of the combined fit to the SLAC-BCDMS data with HT included). The systematic errors were taken into account analogously to the earlier works [23,29]. The total number of the independent systematic errors sources for the analyzed data is 18 and all of them were convoluted into a general correlation matrix, which was used for the construction of the minimized χ^2. The results of the fits to xF_3 data with the model independent HT and with the systematic errors taken into account are given in the first column of Table 2. One can see that the account of systematic errors leads to a significant increase of the experimental uncertainties of the HT parameters and the shift of their central values (compare the first column of Table 2 with the third one of Table 1). However, even in this case, there is a definite agreement with the results on HT-behaviour of Ref. [15], obtained in NLO. Moreover, these results do not contradict the IRR-model prediction of Ref. [7], since, releasing A_2', we obtained $A_2' = -0.10 \pm 0.09$.

Table 2. The results of the fits with account of systematic errors and model independent HT-effects. $H_{2,3}^{(0),(2),(4),(6),(8)}$ are the values of $H_2(x)$ and $H_3(x)$ $x = 0., 0.2, 0.4, 0.6, 0.8$; I) xF_3 with the cut $Q^2 > 5$ GeV^2, $Q_0^2 = 5$ GeV^2; II) $xF_3 \& F_2$ with the cut $Q^2 > 5$ GeV^2, $Q_0^2 = 9$ GeV^2; III) $xF_3 \& F_2$ with the cut $Q^2 > 1$ GeV^2, $Q_0^2 = 9$ GeV^2.

	I	II	III
χ^2/NDP	55.7/86	154.9/172	204.2/220
c_{NS}	4.24 ± 0.21	4.060 ± 0.068	4.131 ± 0.056
γ	0.75 ± 0.79	0.	0.
η_{NS}	0.945 ± 0.043	0.922 ± 0.027	0.920 ± 0.025
$\alpha_s(M_Z)$	0.1269 ± 0.0065	0.1248 ± 0.0048	0.1131 ± 0.0045
η_S	$-$	0.1785 ± 0.0077	0.1796 ± 0.0065
b_S	$-$	0.	-0.034 ± 0.023
c_S	$-$	8.37 ± 0.21	8.00 ± 0.29
b_G	$-$	0.	0.092 ± 0.073
c_G	$-$	7.5 ± 2.6	11.50 ± 0.90
η_G	$-$	0.69 ± 0.35	1.08 ± 0.19
$H_2^{(0)}$	$-$	-0.23 ± 0.56	0.09 ± 0.11
$H_2^{(2)}$	$-$	-0.28 ± 0.18	-0.239 ± 0.094
$H_2^{(4)}$	$-$	-0.14 ± 0.18	0.17 ± 0.13
$H_2^{(6)}$	$-$	-0.03 ± 0.13	0.204 ± 0.097
$H_2^{(8)}$	$-$	0.21 ± 0.18	0.14 ± 0.18
$H_3^{(0)}$	0.28 ± 0.21	0.34 ± 0.11	0.115 ± 0.031
$H_3^{(2)}$	-0.22 ± 0.19	-0.24 ± 0.16	-0.16 ± 0.16
$H_3^{(4)}$	-0.42 ± 0.35	-0.22 ± 0.22	0.28 ± 0.19
$H_3^{(6)}$	-0.09 ± 0.28	-0.05 ± 0.17	0.19 ± 0.15
$H_3^{(8)}$	1.21 ± 0.50	0.89 ± 0.44	0.88 ± 0.44

4

Trying to minimize the errors of the parameters, we added the CCFR data on the structure function F_2 to the analysis. To perform the QCD evolution of F_2, one is to involve into the analysis the singlet and gluon distributions:

$$F_2^{LT}(x,Q) = \int_x^1 dz \Big[C_{2,q}(z)\frac{x}{z}(q^{NS}(x/z,Q) + q^{PS}(x/z,Q)) + C_{2,G}(z)\frac{x}{z}G(x/z,Q) \Big]. \quad (10)$$

The distributions $q^{PS}(x,Q)$ and $G(x,Q)$ were obtained by integrating the system

$$\frac{dxq^{PS}}{d\ln Q} = \frac{\alpha_s(Q)}{\pi} \int_x^1 dz \Big[P_{qq}^{PS}(z)\frac{x}{z}q^{PS}(x/z,Q) + P_{qG}(z)\frac{x}{z}G(x/z,Q) \Big] \quad (11)$$

$$\frac{dxG}{d\ln Q} = \frac{\alpha_s(Q)}{\pi} \int_x^1 dz \Big[P_{Gq}(z)\frac{x}{z}q^{PS}(x/z,Q) + P_{GG}(z)\frac{x}{z}G(x/z,Q) \Big] \quad (12)$$

with the boundary conditions

$$xq^{PS}(x,Q_0) = \eta_S x^{b_S}(1-x)^{c_S}/A_S, \quad (13)$$

$$xG(x,Q_0) = \eta_G x^{b_G}(1-x)^{c_G}/A_G, \quad (14)$$

where

$$A_S = \int_0^1 x^{b_S}(1-x)^{c_S}dx, \quad (15)$$

$$A_G = \frac{1- <xQ(x)>}{\int_0^1 x^{b_G}(1-x)^{c_G}dx} \quad (16)$$

and $<xQ(x)>$ is the total momentum carried by quarks.

In order to provide the straightforward way for comparison of our results with Ref. [23], the initial reference scale $Q_0^2 = 9$ GeV^2 was chosen. In addition to the point-to-point correlation of the data due to systematic errors, the statistical correlations between F_2 and xF_3 were also taken into account. Performing the trial fits we got convinced that the introduction of the factor $(1 + \gamma x)$ into the reference expressions for the the gluon and singlet distributions do not improve the quality of the fit. Also, we fixed parameters γ_{NS}, b_S and b_G at zero because this increase the value of χ^2 by few units only while χ^2/NDP remained less than unity. The HT contribution to F_2 was accounted analogously to xF_3:

$$F_2^{HT}(x,Q) = F_2^{LT,TMC}(x,Q) + \frac{H_2(x)}{Q^2},$$

where $H_2(x)$ was parametrized in the model independent form. The results of the fit on $H_2(x)$ and $H_3(x)$ parameters are given in the second column of Table 2 and in Fig. 1. One can note that, comparing with the fit to xF_3 data only, the HT parameters errors decrease. Within the errors, the parameters that describe the boundary distributions are compatible with ones of Ref. [14]. The $H_3(x)$ coefficients are in agreement with the NLO results of Ref. [15] and the behaviour of $H_2(x)$ qualitatively reproduce the HT contribution to F_2 that was obtained from the combined fits to the SLAC-BCDMS data on F_2 [20,29].

5
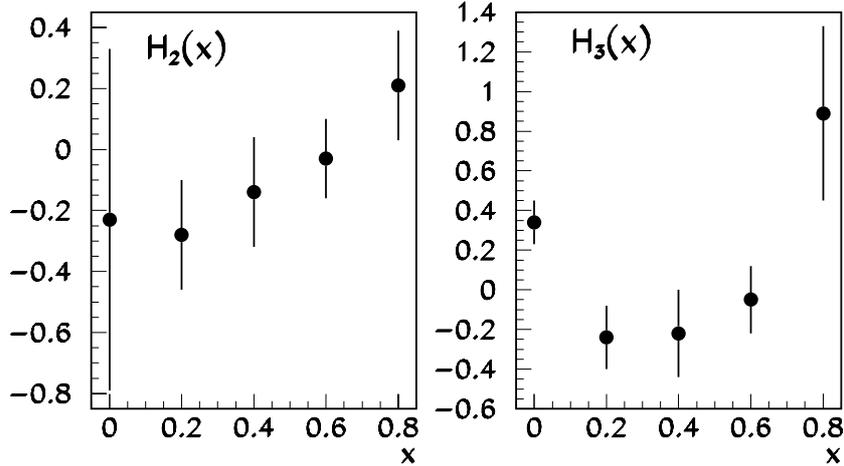
Fig. 1.   The high-twist contribution to the structure functions F_2 F_3.

When the matching scale M_5 was changed from m_b to $6.5m_b$, the value of $\alpha_s(M_Z)$ shifted down by 0.0052 and, hence, the theoretical error in $\alpha_s(M_Z)$ due to uncertainty of b-quark threshold can be estimated as 0.0026. This uncertainty is in agreement with the results of the NLO Jacobi-polynomial fits to the CCFR data obtained within the so-called spline $\overline{MS}$ prescription [37]. One more source of the theoretical uncertainty due to the truncation of higher QCD orders was evaluated following the way, which was proposed in Ref. [20]. In accordance with their procedure, one can introduce renormalization scale k_R into QCD evolution equations in the way, given below for non-singlet evolution:

$$\frac{dxq^{NS}}{d\ln Q} = \frac{\alpha_s(k_R Q)}{\pi} \int_x^1 dz \Big\{ P_{qq}^{NS,(0)}(z) +$$

$$+ \frac{\alpha_s(k_R Q)}{2\pi} \Big[P_{qq}^{NS,(1)}(z) + \beta_0 P_{qq}^{NS,(0)}(z) \ln(k_R) \Big] \Big\} \frac{x}{z} q^{NS}(x/z, Q), \qquad (17)$$

where $P^{NS,(0)}$ and $P^{NS,(1)}$ denote the LO and the NLO parts of the splitting function P^{NS}. The dependence of the results on k_R would signal an incomplete account of the perturbation series. The shift of $\alpha_s(M_Z)$ resulting from the reasonable variation of k_R leads to an additional error of over 0.003 due to the renormalization scale uncertainty. Having taken $Q_0^2 = 20$ GeV^2 as an initial scale, we checked that our results obtained were quite stable to the variation of the factorization point.

The NLO value of α_s is finally given as

$$\alpha_s(M_Z) = 0.1222 \pm 0.0048 \text{ (stat + syst)} \pm 0.0040 \text{ (thresh + ren.scale)} \qquad (18)$$

It differs a bit from the NLO value $\alpha_s(M_Z) = 0.119 \pm 0.002$ (stat+syst)± 0.004 (theory) obtained in the CCFR analysis [13]. The increase of the experimental error is due to that CCFR group used model-dependent form of the HT contributions, while we considered them as the additional free parameters and extracted them from the fit.

6

In order to decrease the errors further, we repeated the fits to the combined xF_3 and F_2 data, applying the less stringent cut $Q^2 > 1$ GeV^2. The obtained results are given in the third column of Table 2. In this fit the parameters b_s and b_G were released since their values turned out to be statistically different from zero. We found that the values of α_s and b_G were correlated (the correlation coefficient was equal to –0.65). When we had fixed b_G at zero, the value $\alpha_s(M_Z) = 0.1172 \pm 0.0029$ was obtained, and when we had kept b_G as the free parameter, we obtained a low value of α_s, namely $\alpha_s(M_Z) = 0.1131 \pm 0.0045$. The analogous effect of correlations was observed for the fits with the cut $Q^2 > 5$ GeV^2, although with less statistical significance. It should be underlined that when we had released b_G in the fit with the cut $Q^2 > 1$ GeV^2, another problem was faced: b_G value turned out to be $b_G = 0.092 \pm 0.073$, which is in the evident contradiction with the results obtained in the analysis of HERA data (for example, the combined analysis of DIS data from HERA and CERN-SPS gives the value $b_G = -0.267 \pm 0.043$ [23]).This problem might be related to the well-known discrepancy between CCFR and NMC-BCDMS data at small x.

In conclusion, we would like to stress that in order to perform a similar analysis at the NNLO level, it is necessary to calculate yet unknown Altarelli-Parisi kernels to the corresponding DGLAP equations. Therefore, we are unable to obtain the results similar to the NNLO ones of Refs. [15,22]. We hope that future progress of theoretical calculations will allow us to generalize our results up to the NNLO approximation.

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