STATE RESEARCH CENTER OF RUSSIA

INSTITUTE FOR HIGH ENERGY PHYSICS

IHEP 99–15

Yu.F. Pirogov[1], O.V. Zenin[2]

QUARK MASSES AND MIXINGS IN THE STANDARD MODEL WITH HEAVY VECTOR-LIKE FAMILIES

Submitted to Nucl. Phys. B

[1] E-mail: pirogov@mx.ihep.su
[2] E-mail: mpti9302@mx.ihep.su

Protvino 1999

UDK 539.1.01 M–24

Abstract

Pirogov Yu.F., Zenin O.V. Quark masses and mixings in the standard model with the heavy vector-like families: IHEP Preprint 99–15. – Protvino, 1999. – p. 16, tables 3, refs.: 11.

The extension of the standard model with pairs of the vector-like families is studied. The quark mixing matrices for the left- and right-handed charged currents, as well as those for the flavour changing neutral currents, the Z and Higgs mediated, are found. Both the model independent parametrization for an arbitrary case and an explicit realization for the case with one pair of the heavy vector-like families are presented. The extension opens new prospects for studying deviations from the standard model in the future experiments at high energies.

Аннотация

Пирогов Ю.Ф., Зенин О.В. Массы и смешивание кварков в стандартной модели с тяжелыми векторноподобными семействами: Препринт ИФВЭ 99–15. – Протвино, 1999. – 16 с., 3 табл., библиогр.: 11.

Рассмотрено расширение стандартной модели парами векторноподобных семейств. Найдены матрицы смешивания кварков для лево- и правоспиральных заряженных токов, а также для нейтральных токов с нарушением аромата, переносимых Z и хиггсовским бозоном. Представлены модельно-независимая параметризация для произвольного случая и явная реализация для случая с единственной парой тяжелых векторноподобных семейств. Данное расширение открывает новые возможности для изучения отклонений от стандартной модели в будущих экспериментах при высоких энергиях.

1. Introduction

At present we know of three quark-lepton chiral families in the standard model (SM). Their mixing within the present experimental accuracy is well known to be described by the 3×3 unitary matrix [1]. But beyond it, whether there are extra families and, if so, what their masses and mixings are — this is yet an unsolved problem.

A recent two-loop renormalization group analysis [2] of the SM shows that subject to the precision experiment restriction on the Higgs mass, $M_H \leq 215$ GeV at 95% C.L. [3], the forth chiral family, if alone, is excluded[1]. In fact, it does not depend on whether this extra family has the normal chiral structure or the mirror one. But as it is noted in Ref. [2], a pair of the opposite chirality families with the relatively low Yukawa couplings evades the SM self-consistency restrictions and could still exist. In order to conform to observations, these extra families, which otherwise can be considered as the vectorial ones, should get large direct masses and drop out of the light particle spectrum of the SM in the decoupling limit. Nevertheless, at not too high masses, say, in the TeV region, such families could result in observable corrections to the SM interactions through mixing with the light fermions.

Various vector-like fermions are generic in many extensions of the SM like the superstring and grand unified theories, composite models, etc. Many issues concerning those fermions, both the electroweak doublets and singlets, the latter ones of the up and down types, were considered in the literature [5], [6]. On the other hand, there are numerous studies of the $n > 3$ chiral family extensions of the SM [7], [8]. Some topics concerning the SM extensions with the vector-like families are studied in Ref. [9].

In a previous letter [10] we presented the results for the SM light quark masses and mixings in the presence of the extra vector-like families. In the current paper we give the complete results including those for the heavy quarks. In Section 2 we carry out the model independent analysis for the general case. In Section 3 an explicit realization for the case with a pair of the heavy vector-like families is presented. In Appendix we give the technical details of the diagonalization procedure and the explicit form of the mixing matrices through the elements of the general mass matrices.

[1] The recent more conservative restrictions $m_H \leq 262$ GeV or $M_H \leq 300$ GeV at 95% C.L., respectively, from the first and second papers of Ref. [4] render the fourth chiral family only marginally possible.

2. Model independent analysis

The most general content of the SM families consisting of the $\mathrm{SU(2)_W \times U(1)_Y}$ doublets and singlets is illustrated in Table 1. The notations with a hat sign designate quarks in the symmetry/electroweak basis where, by definition, the SM symmetry structure is well stated. "Normal" in the row means the $n \geq 3$ chiral families, similar in their chiral and quantum number structure to three ordinary families of the minimal SM. "Mirror" means the $m \geq 0$ mirror conjugate families with the normal quantum numbers, or in other terms, the charge conjugate families with the normal chiral structure. We suppose, for definiteness, that $n > m$. "Chiral" in the column means the chiral notations, and "mixed" corresponds to the more traditional left-right notations[2].

Table 1. The general content of the SM families.

	#	Chiral	Mixed
Normal	n	$Q_L = (\hat{q}_L, \hat{u}^c_L, \hat{d}^c_L)$	$(\hat{q}_L, \hat{u}_R, \hat{d}_R)$
Mirror	m	$Q'_R = (\hat{q}'_R, \hat{u}'^c_R, \hat{d}'^c_R)$	$(\hat{q}'_R, \hat{u}'_L, \hat{d}'_L)$

In general, quarks gain masses from two different physical mechanisms: that of the SM Yukawa interactions and that of a New Physics resulting in the SM invariant direct mass terms. Being chirally unprotected the latter ones should naturally be characterized by a high mass scale M, $M \gg v$, with v being the SM Higgs vacuum expectation value. In the symmetry basis the kinetic, Yukawa and direct mass Lagrangian has the following most general form:

$$\begin{aligned}
\mathcal{L} \;=\; & i\overline{\hat{q}_L}\not{D}\hat{q}_L + i\overline{\hat{u}_R}\not{D}\hat{u}_R + i\overline{\hat{d}_R}\not{D}\hat{d}_R \\
& + i\overline{\hat{q}'_R}\not{D}\hat{q}'_R + i\overline{\hat{u}'_L}\not{D}\hat{u}'_L + i\overline{\hat{d}'_L}\not{D}\hat{d}'_L \\
& - \left(\overline{\hat{q}_L} Y^u \hat{u}_R \phi^c + \overline{\hat{q}_L} Y^d \hat{d}_R \phi + \overline{\hat{u}'_L} Y^{u\prime} \hat{q}'_R \phi^{c\dagger} + \overline{\hat{d}'_L} Y^{d\prime} \hat{q}'_R \phi^\dagger + \mathrm{h.c.}\right) \\
& - \left(\overline{\hat{q}_L} M \hat{q}'_R + \overline{\hat{u}'_L} M^{u\prime} \hat{u}_R + \overline{\hat{d}'_L} M^{d\prime} \hat{d}_R + \mathrm{h.c.}\right) , \qquad (1)
\end{aligned}$$

where $\not{D} \equiv \gamma^\mu D_\mu$ is the SM covariant derivative, ϕ is the Higgs doublet and ϕ^c is the charge conjugate one. In Eq. (1), Y and Y' are, respectively, the square $n \times n$ and $m \times m$ Yukawa matrices; M and M' are, respectively, the rectangular $n \times m$ and $m \times n$ direct mass matrices. the

Without loss of generality, the matrices M and M' can always be brought to the $m \times m$ triangular form with the rest being zero. Now, one can rewrite the Lagrangian (1) in terms of the m pairs of the Dirac families $Q = (Q_L, Q'_R)$, constituting the vector-like representations of the SM, and the $n - m$ chiral families Q_L. In neglect of the Yukawa couplings, the Lagrangian of the Dirac families is explicitly P invariant. Hence, of those

[2]To be as clear as possible, what we are talking about, say, in terms of the 15-plets of the GUT $SU(5)$ ($15 = 10 \oplus \overline{5}$) is $n15_L \oplus m15_R$, or $n15_L \oplus m\overline{15}_L$. Nevertheless, the scales we have in mind are much lower than those of the GUT's, typically $\mathcal{O}(1 - 100)$ TeV, i.e. rather those of the composite models.

initial $n+m$ chiral families, the $2m$ ones transform after mass diagonalization to m pairs of the heavy vector-like families (VLF's)[3]. This is to be expected according to the survival hypothesis [11] because the chirally conjugate families lose their chiral protection. The unbalanced $n-m$ families can be considered as the (approximate) pure chiral ones. In practice, we suppose that the net number of the chiral families is three and hence $n=3+m$.

We generalize the parameter counting for the chiral families of Ref. [8] to the case with extra VLF's. It goes as is shown in Table 2. Here G is the global symmetry of the kinetic part of the Lagrangian (1). It is broken explicitly by the mass terms, only the residual symmetry $H=U(1)$ of the baryon number being left in the general case we consider[4] Hence, the transformations of G/H can be used to absorb the spurious parameters in Eq. (1) . leaving only the physical set $\mathcal{M}_{phys}$ of them. The last four lines in Table 2 present the physical parameters for the minimal SM and for the three its simplest extensions: the traditional one with a normal family, the one with a mirror family and the one with of a pair of the normal and mirror families[5]. The last case will be considered in detail in the next section.

Table 2. Parameter counting in the symmetry/electroweak basis.

Couplings and symmetries	Moduli	Phases
$Y^u, Y^d, Y^{u\prime}, Y^{d\prime}$, $M, M^{u\prime}, M^{d\prime}$	$2\ (n^2+m^2)$ $+3\ nm$	$2\ (n^2+m^2)$ $+3\ nm$
$G=U(n)^3\times U(m)^3$	$-\frac{3}{2}[n(n-1)+m(m-1)]$	$-\frac{3}{2}[n(n+1)+m(m+1)]$
$H=U(1)$	0	1
$\mathcal{M}_{phys}(n,m)$	$\frac{1}{2}(n+m)(n+m-1)$ $+2\ nm+\ 2\ (n+m)$	$\frac{1}{2}(n+m-2)(n+m-1)$ $+2\ nm$
$\mathcal{M}^{SM}_{phys}(3,0)$	$6+3=9$	1
$\mathcal{M}_{phys}(4,0)$	$8+6=14$	3
$\mathcal{M}_{phys}(3,1)$	$8+12=20$	9
$\mathcal{M}_{phys}(4,1)$	$10+18=28$	14

Further, the kinetic part of the effective Lagrangian with the W, Z and Higgs bosons being integrated out is

$$\begin{aligned}\mathcal{L}_{eff} &= i\overline{u_L}\not{D}u_L + i\overline{d_L}\not{D}d_L + i\overline{u_R}\not{D}u_R + i\overline{d_R}\not{D}d_R \\ &\quad -(\overline{u_L}\mathcal{M}^u_{diag}u_R + \overline{d_L}\mathcal{M}^d_{diag}d_R + \text{h.c.})\ , \end{aligned} \tag{2}$$

where $\not{D}$ means the covariant derivatives w.r.t. the QED and QCD only; u_χ and d_χ ($\chi=L$, R) generically mean the quarks in the mass/flavour basis, and $\mathcal{M}^{u,d}_{diag}$ are the diagonal

[3]To be precise we call as VLF the family mass eigenstate which possesses the (approximate) left-right symmetric SM interactions.

[4]The degenerate cases leave more residual symmetries and require special consideration.

[5]The first two cases are practically excluded by the SM self-consistency requirements [2].

mass matrices defining the basis. The corresponding parameter counting is presented in Table 3. Due to the absence of the mutual quark transitions, the total residual symmetry of the mass matrices $\mathcal{M}^{u,d}_{diag}$ is here $H = U(1)^{2(n+m)}$. Table 3 clearly shows the breakdown of the moduli of $\mathcal{M}_{phys}$ in Table 2 on the physical masses and mixing angles.

Table 3. Parameter counting for the effective Lagrangian.

Couplings and symmetries	Moduli	Phases
$\mathcal{M}^u, \mathcal{M}^d$	$2\ (n+m)^2$	$2\ (n+m)^2$
$G = U(n+m)^4$	$-2(n+m)(n+m-1)$	$-2(n+m)(n+m+1)$
$H = U(1)^{2(n+m)}$	0	$2(n+m)$
$\mathcal{M}^u_{diag},\ \mathcal{M}^d_{diag}$	$2(n+m)$	0

Let us now redefine collectively quarks in the symmetry basis as $\hat{\kappa}_\chi = \hat{u}_\chi,\ \hat{d}_\chi$ and these in the mass basis, i.e. the quark eigenstates with $\mathcal{M}_{phys}$ being diagonal, as $\kappa_\chi = u_\chi,\ d_\chi$ ($\chi = L,\ R$). The bases are related by the unitary $(n+m) \times (n+m)$ transformations

$$\hat{\kappa}_{\chi A} = (U^\kappa_\chi)^F_A\, \kappa_{\chi F}\ , \tag{3}$$

with the ensuing bi-unitary mass diagonalization

$$U^{\kappa\dagger}_L \mathcal{M}^k U^\kappa_R = \mathcal{M}^\kappa_{diag} = \mathrm{diag}\,(\overline{m}^\kappa{}_f, \overline{M}^\kappa{}_4, \ldots, \overline{M}^\kappa{}_{n+m})\ . \tag{4}$$

In the equations above, the indices $A = A_L, A_R;\ A_L = 1, \ldots, n;\ A_R = n+1, \ldots, n+m$ are those in the symmetry basis, and $F = f, 4, \ldots, n+m;\ f = 1, 2, 3$ are indices in the mass basis. It is assumed that $\overline{m}^\kappa{}_f \ll \overline{M}^\kappa{}_4, \ldots, \overline{M}^\kappa{}_{n+m}$.

The matrices U^κ_χ satisfy the unitarity relations

$$U^\kappa_\chi\, U^{\kappa\dagger}_\chi = I \tag{5}$$

and

$$U^{\kappa\dagger}_\chi I_L U^\kappa_\chi + U^{\kappa\dagger}_\chi I_R U^\kappa_\chi = I\ , \tag{6}$$

were $I_L,\ I_R$ are the projectors onto the normal and mirror subspaces in the symmetry basis:

$$\begin{aligned} I_L &= \mathrm{diag}\,(\underbrace{1, \ldots, 1}_{n}; \underbrace{0, \ldots, 0}_{m})\ , \\ I_R &= \mathrm{diag}\,(\underbrace{0, \ldots, 0}_{n}; \underbrace{1, \ldots, 1}_{m}) \end{aligned} \tag{7}$$

with $I_L + I_R = I$ and $I^2_\chi = I_\chi$. Let us also introduce their transformation to the mass basis

$$X^\kappa_\chi = U^{\kappa\dagger}_\chi I_\chi U^\kappa_\chi\ . \tag{8}$$

($\kappa = u$, d and $\chi = L$, R). Clearly, X^κ_χ are Hermitian and satisfy the projector condition: $X^{\kappa 2}_\chi = X^\kappa_\chi$ (but note that $X^\kappa_L + X^\kappa_R \neq I$ in the notations adopted).

Now, the charged current Lagrangian is

$$-\mathcal{L}_W = \frac{g}{\sqrt{2}} W^+_\mu \sum_\chi \overline{u_\chi} \gamma^\mu V_\chi d_\chi + \text{h.c.} \tag{9}$$

and the neutral current one is

$$-\mathcal{L}_Z = \frac{g}{c} Z_\mu \sum_{\kappa,\chi} \overline{\kappa_\chi} \gamma^\mu N^\kappa_\chi \kappa_\chi \ , \tag{10}$$

where $c \equiv \cos\theta_W$, with θ_W being the Weinberg mixing angle. The corresponding quark mixing matrices for the charged currents are

$$V_\chi = U^{u\dagger}_\chi I_\chi U^d_\chi \ , \tag{11}$$

and for the neutral currents with the operator $T_3 - s^2 Q$

$$N^\kappa_\chi = T^\kappa_3 X^\kappa_\chi - s^2 Q^\kappa_\chi \ . \tag{12}$$

Here one has for the electroweak isospin: $T^\kappa_3 = 1/2$ at $\kappa = u$ and $-1/2$ at $\kappa = d$; for the electric charge: $Q^\kappa_{L,R} \equiv Q^\kappa I$ with $Q^\kappa = 2/3$ at $\kappa = u$ and $-1/3$ at $\kappa = d$; $s \equiv \sin\theta_W$.

The charged current mixing matrices V_L and V_R play the role of the generalized CKM matrices. But contrary to the minimal SM case, they as well as the neutral current mixing matrices N^κ_χ are non-unitary. Namely, one gets by the unitarity relations (5)

$$\begin{aligned} V_\chi V^\dagger_\chi &= X^u_\chi \ , \\ V^\dagger_\chi V_\chi &= X^d_\chi \ , \end{aligned} \tag{13}$$

where X^κ_χ ($X^\kappa_\chi \neq I$ in general) are given by Eq. (8). From the considerations above, the representations for the V_χ follow

$$V_\chi = X^u_\chi S_\chi = S_\chi X^d_\chi \tag{14}$$

with the unitary matrices $S_\chi = U^{u\dagger}_\chi U^d_\chi$ and the positive definite Hermitian matrices X^κ_χ, only one in a pair with fixed χ being independent, say, $X^d_\chi \equiv S^\dagger_\chi X^u_\chi S_\chi$. The decomposition (14) is known to be unique. In a case where there are only the normal families, one gets $X^\kappa_L = I$ and $X^\kappa_R = 0$, so that V_L is unitary, $V_L = S_L$, and $V_R = 0$.

It is seen that the neutral current matrices N^κ_χ are not independent of the charged current ones V_χ. In fact, one can get convinced that V_χ and the diagonal mass matrices $\mathcal{M}^\kappa_{diag}$ suffice to parametrize all the fermion interactions in a general class of the SM extensions by means of the arbitrary numbers of the vector-like isodoublets and isosinglets [6]. Indeed, in the case at hand, using the unitarity relations (6), one gets for the Yukawa Lagrangian in the unitary gauge

$$\begin{aligned} -\mathcal{L}_Y &= \frac{H}{v} \sum_\kappa \overline{\kappa_L} \Big(X^\kappa_L \mathcal{M}^\kappa_{diag} - 2 X^\kappa_L \mathcal{M}^\kappa_{diag} X^\kappa_R + \mathcal{M}^\kappa_{diag} X^\kappa_R \Big) \kappa_R \\ &\quad + \sum_\kappa \overline{\kappa_L} \mathcal{M}^\kappa_{diag} \kappa_R + \text{h.c.} \ , \end{aligned} \tag{15}$$

H being the physical Higgs boson. It follows from the above expression and Eqs. (10), (12) that all the flavour changing neutral currents are induced entirely by the lack of unitarity of the charged current mixing matrices V_χ. In the case with only the normal families ($X_L^\kappa = I$, $X_R^\kappa = 0$) the usual SM expressions for $\mathcal{L}_W$, $\mathcal{L}_Z$ and $\mathcal{L}_Y$ are recovered, the two latter ones being flavour conserving.

We propose the following prescription for the model independent parametrization of the V_χ. The problem is that they are non-unitary and thus are difficult to parametrize directly. So, the idea is to express them in terms of a set of the auxiliary unitary matrices. First of all, note that in the absence of any restrictions on the Lagrangian the unitary matrices U_χ^κ in Eq. (3) would be arbitrary. Now, an arbitrary $(n+m)\times(n+m)$ unitary matrix U can always be uniquely decomposed as $U = U|_{n\times n}\, U|_{m\times m}\, U|_{n\times m}$. Here $U|_{n\times n}$ is a unitary matrix in the $n\times n$ subspace. It is built of the n^2 generators. Similarly, $U|_{m\times m}$ is the restriction of U onto the $m\times m$ subspace, and it is built of the m^2 generators. And finally, $U|_{n\times m}$ means a unitary $(n+m)\times(n+m)$ matrix built of the $2nm$ generators which mix the two subspaces.

Now, by means of the symmetry basis transformations G of Table 2 one can always put, without loss of generality, the matrices U_χ^κ to the form

$$\begin{aligned} U_L^u &= U_L^u|_{n\times m}\ , \\ U_R^u &= U_R^u|_{n\times m}\ , \\ U_L^d &= U_L^d|_{n\times n}\, U_L^d|_{n\times m}\ , \\ U_R^d &= U_R^d|_{m\times m}\, U_R^d|_{n\times m}\ . \end{aligned} \tag{16}$$

This representation includes six auxiliary unitary matrices. Clearly, they depend on the $[n(n-1)/2 + m(m-1)/2 + 4mn]$ moduli and $[n(n+1)/2 + m(m+1)/2 + 4mn]$ phases, and these numbers are redundant. But the nm moduli and the same number of phases can be eliminated through the $n\times m$ matrix constraint

$$I_L U_L^u \mathcal{M}_{diag}^u U_R^{u\dagger} I_R = I_L U_L^d \mathcal{M}_{diag}^d U_R^{d\dagger} I_R\ . \tag{17}$$

The latter one follows from the equality of the direct mass matrices M in Eq. (1) for the up and down quarks, and it includes additionally the $2(n+m)$ independent moduli which enter $\mathcal{M}_{diag}^u$ and $\mathcal{M}_{diag}^d$. By means of Eq. (17) one can express, e.g., one of the $U_\chi^\kappa|_{n\times m}$ in terms of all other matrices. And finally, the $2(n+m)-1$ phases can be removed via the residual phase redefinition for the quarks in the mass basis. Putting all together, one can easily verify that the total number of the independent parameters is precisely as expected from Table 2.

Having parametrized the auxiliary unitary matrices, one gets for the V_χ

$$\begin{aligned} V_L &= U_L^{u\dagger}|_{n\times m}\, I_L\, U_L^d|_{n\times n}\, U_L^d|_{n\times m}\ , \\ V_R &= U_R^{u\dagger}|_{n\times m}\, I_R\, U_R^d|_{m\times m}\, U_R^d|_{n\times m} \end{aligned} \tag{18}$$

and for the X_χ^κ

$$X_\chi^\kappa = U_\chi^{\kappa\dagger}|_{n\times m}\, I_\chi\, U_\chi^\kappa|_{n\times m}\ . \tag{19}$$

When eliminating the $2(n+m)-1$ redundant phases, one can always make such a choice as to render the diagonal and above-the-diagonal elements of the V_L (or V_R) to be real and positive.

This gives a principal solution to the problem. When there are only the normal families ($m = 0$), the usual parametrization in terms of just one unitary matrix $U_L^d|_{n\times n}$ is readily recovered. For the case with a pair of VLF's ($n = 4$, $m = 1$) we got also the explicit expressions of all the relevant quantities in terms of a minimal common set of the independent arguments parametrizing the mass matrices (see the next section). It is of much use at the model independent parametrization to estimate the relative magnitudes of the various mixing elements in terms of a small quantity $\epsilon = v^2/M^2 \ll 1$. Otherwise, one has a priori no idea of this.

Finally, under small mixing it is useful to decompose

$$V_\chi = V_{0\chi} + \Delta V_\chi \ , \tag{20}$$

with the decoupling limit taken as the zeroth order approximation $V_{0\chi}$, and with corrections ΔV_χ vanishing at $M \gg v$. To illustrate the behavior in the limit, let us consider the aforementioned case with a pair of the VLF's. One gets here

$$V_{0L} = \begin{pmatrix} V_C & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{21}$$

and

$$V_{0R} = \mathrm{diag}\,(0,0,0,1,0) \ , \tag{22}$$

V_C being the usual 3×3 charged current matrix of the SM. Hence, for the X_χ^κ as given by Eq. (13), one has in the zeroth order

$$\begin{aligned} X_{0L}^{u,d} &= \mathrm{diag}\,(1,1,1,1,0) \ , \\ X_{0R}^{u,d} &= \mathrm{diag}\,(0,0,0,1,0) \ . \end{aligned} \tag{23}$$

It follows from Eqs. (21)–(23) that in the limit $M \gg v$ there are indeed two VLF's, the forth and the fifth ones, that interact in the left-right symmetric manner, one of the VLF's, the fifth one, being singlet under interactions with the W boson. Besides, as it follows from Eq. (15), both these families decouple from the Higgs boson in the leading order of $\mathcal{O}(M/v)$, only the Yukawa terms $\mathcal{O}(M^0)$ being left at most.

3. Explicit realization

The mass/flavour basis parameters, $\mathcal{M}_{diag}^{u,d}$ and $V_{L,R}$, are phenomenological by their very nature. They reflect an obscure mixture of contributions of quite a different physical origin. In particular, they shed no light on the mixing magnitudes. On the contrary, the parameters in the symmetry basis, i.e. Yukawa couplings and the direct mass terms M and $M^{u\prime}$, $M^{d\prime}$, have the straightforward theoretical meaning. So, we express the former

ones in terms of the latter ones. This permits us to expand upon the idea of the relative magnitude of the various mixing elements in terms of the small quantity v/M.

The asymptotic freedom requirement for the $\mathrm{SU(2)_W}$ electroweak interactions results in the restriction that the total number of the electroweak doublets should not exceed 21. The number of doublets in a chiral family being 4, this is equivalent to the restriction that the total number of the families is $(n+m) \leq 5$. Hence the maximum number of the extra VLF's allowed by the asymptotic freedom is two, the case we stick to in what follows[6].

Using here the global symmetries G of Table 2, one can bring, without loss of generality, the quark mass matrices in the symmetry basis to the following canonical form

$$\mathcal{M}^\kappa = \begin{pmatrix} m^{\kappa g}_{\ f} & \mu^{\kappa\prime}_{\ f} & 0 \\ \mu^{\kappa g} & m^\kappa{}_4 & M \\ 0 & M^{\kappa\prime} & m^\kappa{}_5 \end{pmatrix} , \tag{24}$$

where M, $M^{\kappa\prime}$ are the real scalars and $\mu^{\kappa f}$, $\mu^{\kappa\prime}_{\ f}$, $m^\kappa{}_4$, $m^\kappa{}_5$ are, in general, complex. Here the lower case characters generically mean the masses of the Yukawa origin ($\sim Yv$). Let us remind that M in Eq. (24) is common for both $\mathcal{M}^u$ and $\mathcal{M}^d$. The three-dimensional matrices m^κ are Hermitian and positive definite, and one of them, e.g. m^u, can always be chosen diagonal. Under such a choice one can simplify further:

$$\mathcal{M}^\kappa_0 = U^{\kappa\dagger}_0 \mathcal{M}^\kappa U^\kappa_0 , \tag{25}$$

where

$$\mathcal{M}^\kappa_0 = \begin{pmatrix} m^\kappa{}_1 & 0 & 0 & \mu^{\kappa\prime}_{\ 1} & 0 \\ 0 & m^\kappa{}_2 & 0 & \mu^{\kappa\prime}_{\ 2} & 0 \\ 0 & 0 & m^\kappa{}_3 & \mu^{\kappa\prime}_{\ 3} & 0 \\ \mu^{\kappa 1} & \mu^{\kappa 2} & \mu^{\kappa 3} & m^\kappa{}_4 & M \\ 0 & 0 & 0 & M^{\kappa\prime} & m^\kappa{}_5 \end{pmatrix} \tag{26}$$

with a redefinition of $\mu^{\kappa f}$ and $\mu^{\kappa\prime}_{\ f}$, and with the diagonal elements $m^\kappa{}_f$ being real and positive. The matrices $\mathcal{M}^\kappa_0$ have a lot of texture zeros and are easiest to operate. The corresponding unitary U^κ_0 are given by

$$\begin{aligned} U^u_0 &= I , \\ U^d_0 &= \begin{pmatrix} V_C & 0 \\ 0 & I_2 \end{pmatrix} , \end{aligned} \tag{27}$$

V_C being the 3×3 CKM matrix and I_2 the 2×2 identity matrix. The mass matrices of Eq. (26) possess the residual symmetry $U(1)^6$ which is reduced to $U(1)^5$ by the baryon number conservation. So, one can use the phase redefinitions for two of the light d quarks which leave just one complex phase in V_C in accordance with the decoupling limit requirement.

[6]This might be a landmark for the number of the extra families.

It is seen from Eqs. (26) and (27) that in this parametrization the total number of the physical moduli is $10 + 15 + 3 = 28$, as it should be according to Table 2. As for the phases, their number is in general $16 + 1 = 17$, i.e. three of them are spurious and can be removed. For example, by means of the residual phase redefinition for the three light u quarks, one can make μ^{uf} or $\mu^{u'}_f$ to be real, or put some other three relations on their phases. This exhausts the freedom of the phase redefinitions, leaving only the physical parameters.

The characteristic equations (see Appendix)

$$\det\left(\mathcal{M}_0^\kappa \mathcal{M}_0^{\kappa\dagger} - \lambda^\kappa I\right) = 0 \tag{28}$$

give for the roots in the first order (i.e. up to the relative corrections $\mathcal{O}(v^2/M^2)$ to the leading order):

$$\begin{aligned}
\lambda_f \equiv \overline{m}_f^2 &= m_f^2\Big(1 - \Big(\frac{|\mu^f|^2}{M^2} + \frac{|\mu'_f|^2}{M'^2}\Big)\Big) + \frac{m_f}{MM'}(m_5\mu^f\mu'_f + \text{h.c.}) \ , \\
\lambda_4 \equiv \overline{M}_4^2 &= M^2 + \Sigma|\mu^f|^2 + |m_4|^2 + |m_5|^2 \\
&\quad + \frac{M'^2}{M^2 - M'^2}\Big((|m_4|^2 + |m_5|^2) + \frac{M}{M'}(m_4 m_5 + \text{h.c.})\Big) \ , \\
\lambda_5 \equiv \overline{M}_5^2 &= M'^2 + \Sigma|\mu'_f|^2 + |m_4|^2 + |m_5|^2 \\
&\quad + \frac{M^2}{M'^2 - M^2}\Big((|m_4|^2 + |m_5|^2) + \frac{M'}{M}(m_4 m_5 + \text{h.c.})\Big)
\end{aligned} \tag{29}$$

with the superscripts $\kappa = u,\ d$ being suppressed[7]. Here it is supposed that one has, in general, $M \sim M'$ but $M \neq M'$ [8].

It is seen that corrections to m_f^2 are proportional to m_f themselves, i.e. the light quarks are still chirally protected. This property drastically reduces the otherwise dangerous corrections to the masses of the lightest u and d quarks at the moderate M. In the limit $m_f \to 0$ it naturally happens without any fine tuning beyond that of the SM. On the other hand, it means that within the perturbation theory the masses of the lightest quarks cannot entirely be induced by an admixture of the vector-like families: if $m_f = 0$ then $\overline{m}_f = 0$, too. But at the finite m_f one finds for the masses of the light quarks

$$\overline{m}_f = m_f\left(1 - \frac{1}{2}\Big(\frac{|\mu^f|^2}{M^2} + \frac{|\mu'_f|^2}{M'^2}\Big)\right) + \frac{1}{2}\Big(\frac{m_5\mu^f\mu'_f}{MM'} + \text{h.c.}\Big) \ , \tag{30}$$

and for the validity of perturbative expansion it could require some fine tuning for m_5 at the moderate M.

[7]Hence, the up and down quarks of the fourth family are always (almost) degenerate, whereas those of the fifth family are in general not. Nevertheless, because the fifth family does not couple to the W boson in the zeroth order (see Eqs. (21), (22)) this does not result in the strong coupling $\sim (M^{u'} - M^{d'})$ of the longitudinal W with the fifth heavy family, as well as with the fourth one.

[8]The degenerate case $M = M^{\kappa'}$ (for one or both $\kappa = u, d$) is to be studied separately. It modifies the results for heavy families, but fortunately does not influence the validity of those concerning the light quarks exclusively.

Once the physical masses are known, one can obtain the matrices U^{κ}_{1L} and U^{κ}_{1R} of the bi-unitary transformation

$$U^{\kappa\dagger}_{1L}\mathcal{M}^{\kappa}_{0}U^{\kappa}_{1R} = \mathcal{M}^{\kappa}_{diag}\ . \tag{31}$$

Obviously, they satisfy the relations

$$\begin{aligned}\mathcal{M}^{\kappa\dagger}_{0}\mathcal{M}^{\kappa}_{0}U^{\kappa}_{1R} &= U^{\kappa}_{1R}{\mathcal{M}^{\kappa}_{diag}}^{2}\ ,\\ \mathcal{M}^{\kappa}_{0}\mathcal{M}^{\kappa\dagger}_{0}U^{\kappa}_{1L} &= U^{\kappa}_{1L}{\mathcal{M}^{\kappa}_{diag}}^{2}\end{aligned} \tag{32}$$

which are to be considered as the sets of the independent linear equations for their columns. Having solved the equations, one can find the elements of $U^{\kappa}_{1\chi}$ which are given in Appendix. Finally, one has for the total matrices of the bi-unitary transformations of Eq. (4)

$$U^{\kappa}_{\chi} = U^{\kappa}_{0}\,U^{\kappa}_{1\chi}\ , \tag{33}$$

where U^{κ}_{0} are given by Eq. (27).

Hereof one gets the mixing matrices V_{χ} as given by Eqs. (A.9), (A.10) of Appendix and then the charged current Lagrangian $\mathcal{L}_{W}$ as given by Eq. (9). The Z-mediated neutral current Lagrangian $\mathcal{L}_{Z}$ is as given by Eqs. (10), (12) with X^{κ}_{χ} from Eqs. (A.11), (A.12). The neutral scalar current Lagrangian takes the general form

$$-\mathcal{L}_{H} = \frac{H}{v}\sum_{\kappa}\overline{\kappa_{L}}\,U^{\kappa\dagger}_{L}(\mathcal{M}^{\kappa}-\mathcal{M}^{\kappa}_{dir})U^{\kappa}_{R}\,\kappa_{R} + \text{h.c.} \tag{34}$$

with the direct mass matrices

$$\mathcal{M}^{\kappa}_{dir} = \begin{pmatrix} O_3 & 0 & 0\\ 0 & 0 & M\\ 0 & M^{\kappa\prime} & 0\end{pmatrix}\ , \tag{35}$$

where O_3 is the 3×3 zero matrix. As a consequence of the substraction of the direct mass terms, the total mass and Yukawa matrices are not diagonalizable simultaneously in the same basis, at variance with the SM case. In the mass basis, the Higgs interaction Lagrangian is non-diagonal

$$-\mathcal{L}_{H} = \frac{H}{v}\sum_{\kappa}\overline{\kappa_{L}}\,\mathcal{H}^{\kappa}\kappa_{R} + \text{h.c.} \tag{36}$$

with the explicit form of $\mathcal{H}^{\kappa}$ given by Eqs. (A.13), (A.14) of Appendix.

One should stress that for the light quarks all the off-diagonal components of the Lagrangian $\mathcal{L}_{W}$ (beyond that of the minimal SM), as well as those of the $\mathcal{L}_{Z}$ and $\mathcal{L}_{H}$ are suppressed by the ratio v^2/M^2, and it does not depend on the details of the mass matrices. Besides, it follows from the above that, among the off-diagonal interactions, the Higgs mediated interactions are the only ones that do not vanish in the decoupling limit. Hence, the heavy quarks are expected to decay mainly into the light ones and the Higgs boson with the natural decay width $\Gamma \sim |Y|^2/4\pi\ M$. As a result, all the leading loop corrections to the light quark processes with the internal heavy vector-like quarks are expected to be mediated by the Higgs boson exchanges. So, the modern SM physics, i.e. predominantly that of the light fermions and the gauge bosons, may be succeeded by that of the heavy vector-like fermions and the Higgs boson.

4. Conclusions

We have shown that the mere addition of a pair of the VLF's drastically changes all the characteristic features of the minimal SM. First of all, the generalized CKM matrix for the left-handed charged currents ceases to be unitary. Moreover, this non-unitarity takes place in the whole flavour space but not only in the light quark sector which would occur for adding only the normal families. Further, there appear the right-handed charged currents, the flavour changing neutral currents, both the vector and scalar ones, all with the non-unitary mixing matrices and with a number of CP violating phases.

Due to decoupling relative to the large direct mass terms M, the extended SM definitely does not contradict experiment in the limit $M \gg v$. But at the moderate $M > v$, the addition of a pair of the VLF's would make the model phenomenology, especially that of the flavour and CP violation, extremely rich. So, the extension opens new prospects for studying the deviations from the SM in the future experiments at high energies.

Appendix

One has generically (with the indices $\kappa = u,\, d$ being omitted)

$$\mathcal{M}_0\mathcal{M}_0^\dagger =$$

$$\begin{pmatrix} (m_1^2+|\mu_1'|^2) & \mu_1'\mu_2'^{\,*} & \mu_1'\mu_3'^{\,*} & (m_1\mu_1^*+\mu_1'm_4^*) & \mu_1'M' \\ \mu_2'\mu_1'^{\,*} & (m_2^2+|\mu_2'|^2) & \mu_2'\mu_3'^{\,*} & (m_2\mu_2^*+\mu_2'm_4^*) & \mu_2'M' \\ \mu_3'\mu_1'^{\,*} & \mu_3'\mu_2'^{\,*} & (m_3^2+|\mu_3'|^2) & (m_3\mu_3^*+\mu_3'm_4^*) & \mu_3'M' \\ (m_1\mu_1 & (m_2\mu_2 & (m_3\mu_3 & (M^2+|m_4|^2 & (m_4M' \\ +\mu_1'^{\,*}m_4) & +\mu_2'^{\,*}m_4) & +\mu_3'^{\,*}m_4) & +\Sigma|\mu^f|^2) & +Mm_5^*) \\ \mu_1'^{\,*}M' & \mu_2'^{\,*}M' & \mu_3'^{\,*}M' & (m_4^*M'+Mm_5) & (M'^2+|m_5|^2) \end{pmatrix}. \tag{A.1}$$

The characteristic equation

$$\det\left(\mathcal{M}_0\mathcal{M}_0^\dagger - \lambda I\right) = 0 \tag{A.2}$$

in the explicit form is

$$\begin{aligned} &\lambda^5 - \lambda^4\Big[M^2+M'^2+\Sigma\Big(m_f^2+|\mu^f|^2+|\mu_f'|^2\Big)+|m_4|^2+|m_5|^2\Big] \\ &+\lambda^3\Big[M^2M'^2+M^2\Sigma\Big(m_f^2+|\mu_f'|^2\Big)+M'^2\Sigma\Big(m_f^2+|\mu^f|^2\Big) \\ &-MM'(m_4m_5+\text{h.c.})+m_1^2m_2^2+m_1^2m_3^2+m_2^2m_3^2\Big] \\ &-\lambda^2\Big[M^2M'^2\Sigma m_f^2+M^2\Big(m_1^2m_2^2+m_1^2m_3^2+m_2^2m_3^2 \end{aligned}$$

$$
\begin{aligned}
&+m_1^2(|\mu_2'|^2+|\mu_3'|^2)+m_2^2(|\mu_1'|^2+|\mu_3'|^2)+m_3^2(|\mu_1'|^2+|\mu_2'|^2)\Big)\\
&+M'^2\Big(m_1^2m_2^2+m_1^2m_3^2+m_2^2m_3^2\\
&+m_1^2(|\mu_2|^2+|\mu_3|^2)+m_2^2(|\mu_1|^2+|\mu_3|^2)+m_3^2(|\mu_1|^2+|\mu_2|^2)\Big)\\
&+MM'\Big((-m_4\Sigma m_f^2+\Sigma m_f\mu^f\mu_f')m_5+\text{h.c.}\Big)+m_1^2m_2^2m_3^2\Big]\\
&+\lambda\Big[M^2M'^2\Big(m_1^2m_2^2+m_1^2m_3^2+m_2^2m_3^2\Big)\\
&+M^2\Big(m_1^2m_2^2m_3^2+m_2^2m_3^2|\mu_1'|^2+m_1^2m_3^2|\mu_2'|^2+m_1^2m_2^2|\mu_3'|^2\Big)\\
&+M'^2\Big(m_1^2m_2^2m_3^2+m_2^2m_3^2|\mu_1|^2+m_1^2m_3^2|\mu_2|^2+m_1^2m_2^2|\mu_3|^2\Big)\Big]\\
&-\Big[M^2M'^2m_1^2m_2^2m_3^2+MM'\Big(\big(-m_1^2m_2^2m_3^2m_4+m_1m_2^2m_3^2\mu_1\mu_1'\\
&+m_1^2m_2m_3^2\mu_2\mu_2'+m_1^2m_2^2m_3\mu_3\mu_3')m_5+\text{h.c.}\Big)\Big]+\ldots=0\ .
\end{aligned}
\tag{A.3}
$$

Let us rewrite it in terms of the dimensionless quantity $x\equiv\lambda/M^2$. Then, one can transform Eq. (A.3) as

$$
\Big[\prod_f(x-x_f^{(0)})\Big](x-x_4^{(0)})(x-x_5^{(0)})=\epsilon P_4(x)\ , \tag{A.4}
$$

where

$$
\epsilon=\frac{1}{M^2}\Big(\Sigma|\mu^f|^2+\Sigma|\mu_f'|^2+|m_4|^2+|m_5|^2\Big) \tag{A.5}
$$

is the small paremeter ($\epsilon=\mathcal{O}(v^2/M^2)$) and $x_f^{(0)}\equiv m_f^2/M^2=\mathcal{O}(\epsilon)$, $x_4^{(0)}=1$, $x_5^{(0)}=M'^2/M^2$ are the zeroth order roots. The fourth power polynomial $P_4(x)=(x^4+\ldots)$ has coefficients $\mathcal{O}(1)$ or less. The dropped out terms corresponding to dots in Eq. (A.3) result in the relative corrections $\mathcal{O}(\epsilon^2)$, and hence they can be omitted in our approximation. Iterating Eq. (A.4), one arrives at the roots of Eq. (29).

The elements of the U_{1L} matrix (with the indices $\kappa=u,\ d$ being suppressed) are as follows:

$$
\begin{aligned}
&U_{1L}{}_g^f=\delta_g^f\Big(1-\frac{1}{2M^2}n_f^f\Big)+(\delta_g^f-1)\frac{1}{M^2}p_g^f\ ,\\
&U_{1L}{}_4^f=\frac{1}{M^2}p_4^f\ \ ,\ \ U_{1L}{}_5^f=\frac{1}{M}p_5^f\ ,\\
&U_{1L}{}_f^4=\frac{1}{M^2}p_f^4\ \ ,\ \ U_{1L}{}_f^5=\frac{1}{M}p_f^5\ ,\\
&U_{1L}{}_5^4=\frac{1}{M}p_5^4\ \ ,\ \ U_{1L}{}_4^5=\frac{1}{M}p_4^5\ ,\\
&U_{1L}{}_4^4=1-\frac{1}{2M^2}n_4^4\ \ ,\ \ U_{1L}{}_5^5=1-\frac{1}{2M^2}n_5^5\ ,
\end{aligned}
\tag{A.6}
$$

and

$$
\begin{aligned}
&U_{1R}{}^f_g = \delta^f_g\Big(1-\frac{1}{2M'^2}n'^f_f\Big)+(\delta^f_g-1)\frac{1}{M'^2}p'^f_g\ ,\\
&U_{1R}{}^f_4 = \frac{1}{M'^2}p'^f_4\ \ ,\ \ U_{1R}{}^f_5 = \frac{1}{M'}p'^f_5\ ,\\
&U_{1R}{}^4_f = \frac{1}{M'^2}p'^5_f\ \ ,\ \ U_{1R}{}^5_f = \frac{1}{M'}p'^4_f\ ,\\
&U_{1R}{}^4_4 = \frac{1}{M'}p'^5_4\ \ ,\ \ U_{1R}{}^5_5 = \frac{1}{M'}p'^4_5\ ,\\
&U_{1R}{}^4_5 = 1-\frac{1}{2M'^2}n'^5_5\ \ ,\ \ U_{1R}{}^5_4 = 1-\frac{1}{2M'^2}n'^4_4\ ,
\end{aligned}
\tag{A.7}
$$

where

$$
\begin{aligned}
p^f_g &= \frac{\mu^f(m_f^2-|m_5|^2)(m_f\mu^{f^*}\mu'_g-m_g\mu^{g*}\mu'_f)+k_f(m_f\mu'_g-\frac{m_g}{m_f}\frac{M'}{M}\mu^{g*}m_5^*)}{(m_g^2-m_f^2)(m_f\mu'_f-\frac{M'}{M}m_5^*\mu^{f^*})}\ ,\\
p^f_4 &= -k_f\,\frac{\Big(k_f+|\mu^f|^2(m_f^2-|m_5|^2)\Big)\Big(\frac{M'}{M}m_5^*+\frac{1}{k_f}m_f\mu^f\mu'_f(m_f^2-|m_5|^2)\Big)}{m_f(m_f\mu'_f-\frac{M'}{M}m_5^*\mu^{f^*})}\ ,\\
p^4_f &= m_f\mu^{f^*}-\frac{\mu'_f(\rho+|m_5|^2)}{m_4+\frac{M'}{M}m_5^*}\ ,\\
p^f_5 &= \frac{\frac{M'}{M}(k_f+m_f^2|\mu^f|^2)-m_fm_5\mu^f\mu'_f}{m_f(m_f\mu'_f-\frac{M'}{M}m_5^*\mu^{f^*})}\ \ ,\ \ p^5_f=\frac{M}{M'}\mu'_f\ ,\\
p^4_5 &= \frac{m_4m_5-\frac{M'}{M}\rho}{m_4+\frac{M'}{M}m_5^*}\ \ ,\ \ p^5_4=\frac{MM'}{M'^2-M^2}\Big(m_4+\frac{M}{M'}m_5^*\Big)\ ,\\
n^f_f &= \Big|\frac{\frac{M'}{M}(k_f+m_f^2|\mu^f|^2)-m_fm_5\mu^f\mu'_f}{m_f(m_f\mu'_f-\frac{M'}{M}m_5^*\mu^{f^*})}\Big|^2\ ,\\
n^4_4 &= \Big|\frac{m_4m_5-\frac{M'}{M}\rho}{m_4+\frac{M'}{M}m_5^*}\Big|^2\ ,\\
n^5_5 &= \Big|\frac{M'^2}{M'^2-M^2}(m_4+\frac{M}{M'}m_5^*)\Big|^2+\Sigma|\mu'_f|^2
\end{aligned}
\tag{A.8}
$$

and $k_f = M^2(\overline{m}_f^2-m_f^2)$, $\rho = M^2+\Sigma|\mu^f|^2-\overline{M}_4^2$. The p', n' are obtained from p, n, respectively, by substituting $\mu^f \leftrightarrow \mu'^*_f$, $m_4 \leftrightarrow m_4^*$, $m_5 \leftrightarrow m_5^*$, $M \leftrightarrow M'$. All these auxiliary parameters are, in general, of order $\mathcal{O}(M^0)$. The elements of the matrix U_{1R} are obtained from those for U_{1L} by the same substitution followed by changing column indices $4 \leftrightarrow 5$ for the matrix elements $(U_{1L})^4_A$ and $(U_{1L})^5_A$.

Hereof one gets for the charged current matrix $V_L = V_{0L} + \Delta V_L$

$$\Delta V_L =$$

$$\left(\begin{array}{c|cc} -\frac{1}{M^2}\sum\left(p^{uf^*}_h V_C{}^g_h + V_C{}^h_f p^{dg}_h\right) & \frac{1}{M^2}\left(\sum V_C{}^h_f p^{d4}_h + p^{uf^*}_4\right) & \frac{1}{M}\sum V_C{}^h_f p^{d5}_h \\ -\frac{1}{2M^2}(n^{uf}_f + n^{dg}_g)V_C{}^g_f & & \\ \hline \frac{1}{M^2}\left(\sum p^{u4^*}_h V_C{}^g_h + p^{dg}_4\right) & -\frac{1}{2M^2}(n^{d4}_4 + n^{u4}_4) & \frac{1}{M}p^{d5}_4 \\ \frac{1}{M}\sum p^{u5^*}_h V_C{}^g_h & \frac{1}{M}p^{u5^*}_4 & \frac{1}{M^2}\left(\sum p^{u5^*}_h p^{d5}_k V_C{}^k_h + p^{u5^*}_4 p^{d5}_4\right) \end{array}\right), \tag{A.9}$$

with V_{0L} from Eq. (21) and similarly for $V_R = V_{0R} + \Delta V_R$ with $V_{0R} = \mathrm{diag}\ (0,0,0,1,0)$ and

$$\Delta V_R =$$

$$\left(\begin{array}{c|cc} \frac{1}{M^{u\prime}M^{d\prime}}p^{u\prime f^*}_5 p^{d\prime g}_5 & \frac{1}{M^{u\prime}}p^{u\prime f^*}_5 & \frac{1}{M^{u\prime}M^{d\prime}}p^{u\prime f^*}_5 p^{d\prime 4}_5 \\ \hline \frac{1}{M^{d\prime}}p^{d\prime g}_5 & -\frac{1}{2M^{u\prime 2}}n^{\prime u5}_5 - \frac{1}{2M^{d\prime 2}}n^{\prime d5}_5 & \frac{1}{M^{d\prime}}p^{d\prime 4}_5 \\ \frac{1}{M^{u\prime}M^{d\prime}}p^{u\prime 4^*}_5 p^{d\prime g}_5 & \frac{1}{M^{u\prime}}p^{u\prime 4^*}_5 & \frac{1}{M^{u\prime}M^{d\prime}}p^{u\prime 4^*}_5 p^{d\prime 4}_5 \end{array}\right). \tag{A.10}$$

For the neutral current matrices ($\kappa = u, d$ being suppressed everywhere below) one gets

$$X_L = X_{0L} - \begin{pmatrix} \frac{1}{M^2}p^{f^*}_5 p^g_5 & \frac{1}{M^2}p^{f^*}_5 p^4_5 & \frac{1}{M}p^{f^*}_5 \\ \frac{1}{M^2}p^{4^*}_5 p^g_5 & \frac{1}{M^2}|p^4_5|^2 & \frac{1}{M}p^{4^*}_5 \\ \frac{1}{M}p^g_5 & \frac{1}{M}p^4_5 & -\frac{1}{M^2}n^5_5 \end{pmatrix}, \tag{A.11}$$

and

$$X_R = X_{0R} + \begin{pmatrix} \frac{1}{M^{\prime 2}}p^{\prime f^*}_5 p^{\prime g}_5 & \frac{1}{M^\prime}p^{\prime f^*}_5 & \frac{1}{M^{\prime 2}}p^{\prime f^*}_5 p^{\prime 4}_5 \\ \frac{1}{M^\prime}p^{\prime g}_5 & -\frac{1}{M^{\prime 2}}n^{\prime 5}_5 & \frac{1}{M^\prime}p^{\prime 4}_5 \\ \frac{1}{M^{\prime 2}}p^{\prime 4^*}_5 p^{\prime g}_5 & \frac{1}{M^\prime}p^{\prime 4^*}_5 & \frac{1}{M^{\prime 2}}|p^{\prime 4}_5|^2 \end{pmatrix}, \tag{A.12}$$

with $X_{0L} = I_L$ and $X_{0R} = \mathrm{diag}\ (0,0,0,1,0)$.

Finally, for the Higgs mediated neutral current matrix $\mathcal{H} = \mathcal{H}_0 + \Delta\mathcal{H}$ one has

$$\mathcal{H}_0 = \begin{pmatrix} \overline{m}_f \delta_f^g & 0 & -\frac{M'}{M} p_5^{f\,*} \\ -\frac{M}{M'} p'^g_5 & 0 & -\frac{M'}{M} p_5^{4\,*} - \frac{M}{M'} p'^4_5 \\ 0 & -(p_4^{5\,*} + p'^5_4) & 0 \end{pmatrix} \tag{A.13}$$

and

$$\Delta\mathcal{H} =$$

$$\left(\begin{array}{c|cc} -\frac{1}{MM'}\left(p_4^{f\,*} p'^g_5 + p_5^{f\,*} p'^g_4\right) & -\frac{1}{M}\left(p_5^{f\,*} p'^5_4 + p_4^{f\,*}\right) & \frac{1}{MM'}\left(\frac{1}{2} p_5^{f\,*} n'^4_4 - p_4^{f\,*} p'^4_5\right) \\ \hline \frac{1}{MM'}\left(\frac{1}{2} n_4^4 p'^g_5 - p_5^{4\,*} p'^g_4\right) & -\frac{1}{2M}(\rho - \Sigma|\mu^f|^2) + \frac{1}{2M} n_4^4 + \frac{M}{2M'^2} n'^5_5 - \frac{1}{M} p_5^{4\,*} p'^5_4 & \frac{1}{2MM'}\left(n_4^4 p'^4_5 + n'^4_4 p_5^{4\,*}\right) \\ -\frac{1}{M'}\left(p_4^{5\,*} p'^g_5 + p'^g_4\right) & \frac{1}{2M'^2} n'^5_5 p_4^{5\,*} + \frac{1}{2M^2} n_5^5 p'^5_4 & -\frac{1}{2M'}(\rho' - \Sigma|\mu'_f|^2) + \frac{M'}{2M^2} n_5^5 + \frac{1}{2M'} n'^4_4 - \frac{1}{M'} p_4^{5\,*} p'^4_5 \end{array}\right), \tag{A.14}$$

where ρ is defined above in Appendix, and ρ' can be obtained from ρ by the usual substitutions $\mu^f \leftrightarrow \mu'^*_f$, $m_4 \leftrightarrow m_4^*$, $m_5 \leftrightarrow m_5^*$, $M \leftrightarrow M'$.

References

[1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.

[2] Yu.F. Pirogov and O.V. Zenin, IHEP preprint IHEP 98–50 (1998), to be publ. in Eur. Phys. J. C (1999), hep-ph/9808396; Report presented at the Int. Seminar "Quarks '98", Suzdal, May 17-24, 1998, hep-ph/9808414.

[3] E. Tournefier (ALEPH Collaboration), Report presented at the Int. Seminar "Quarks '98", Suzdal, May 17-24, 1998; www.inr.ac.ru/~q98/proc/index.html.

[4] F. Teubert, hep-ph/9811414; G. D'Agostini and G. Degrassi, hep-ph/9902226.

[5] G.C. Branco and L. Lavoura, Nucl. Phys B278 (1986) 738; Phys. Lett. B208 (1988) 123; B. Mukhopadhyaya and S. Nandi, Phys. Rev. Lett. 66 (1991) 285; Phys. Rev. D46 (1992) 5098; W.S. How, Phys. Rev. Lett. 69 (1992) 3587; T.P. Cheng and L.F. Li, Phys. Rev. D45 (1992) 1708; G.C. Branco, T. Morozumi, P. Parada and M.N. Rebelo, *ibid* D48 (1993) 1167; G.C. Branco, P. Parada and M.N. Rebelo, *ibid*

D52 (1995) 4217, hep-ph/9501347; G. Bhattacharyya, G.C. Branco and W.S. How, Phys. Rev. D54 (1996) 2114, hep-ph/9512239; L.T. Handoco and T. Morozumi, Mod. Phys. Lett. A10 (1995) 309, hep-ph/9409240; E. Ma, Phys. Rev. D53 (1996) 2276, hep-ph/9510289; T. Yoshikawa, Prog. Theor. Phys. 96 (1996) 269, hep-ph/9512251.

[6] L. Lavoura and J.P. Silva, Phys. Rev. D47 (1993) 1117.

[7] J. Shechter and J.W.F. Valle, Phys. Rev. D21 (1980) 309; D22 (1980) 2227; R. Mignani, Lett. Nuovo Cimento 28 (1980) 529; C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039; Z. Phys. C29 (1985) 491; Phys. Rev. D35 (1987) 1685; M. Gronau, R. Johnson and J. Shechter, *ibid* D32 (1985) 3062; M. Gronau, A. Kfir and R. Loewy, Phys. Rev. Lett. 56 (1986) 1538; J. Bernabeu, G.C. Branco and M. Gronau, Phys. Lett. B169 (1986) 243; H. Harari and M. Leurer, *ibid* B181 (1986) 123; H. Frietzsch and J. Plankl, Phys. Rev. D35 (1987) 1732; J.F. Nieves and P.B. Bal, *ibid* D36 (1987) 315; J.D. Bjorken and I. Dunitz, *ibid* D36 (1987) 2109.

[8] A. Santamaria, Phys. Lett. B305 (1993) 90, hep-ph/9302301.

[9] K. Fujikawa, Prog. Theor. Phys. 92 (1994) 1149; hep-ph/9604358; R.K. Babu, J. Pati and H. Stremnitzer, Phys. Rev. D51 (1995) 2451, hep-ph/9409381; H. Zheng, hep-ph/9602340.

[10] Yu.F. Pirogov and O.V. Zenin, IHEP preprint IHEP 99-12 (1999), hep-ph/9903344.

[11] H. Georgi, Nucl. Phys. B156 (1979) 126.

Received March 22, 1999

Ю.Ф.Пирогов, О.В.Зенин
Массы и смешивание кварков в стандартной модели с тяжелыми вектороподобными семействами.

Оригинал-макет подготовлен с помощью системы LaTeX.
Редактор Е.Н.Горина. Технический редактор Н.В.Орлова.

Подписано к печати 26.03.99. Формат 60 × 84/8. Офсетная печать.
Печ.л. 2,00. Уч.-изд.л. 1,536. Тираж 180. Заказ 80. Индекс 3649.
ЛР №020498 17.04.97.

ГНЦ РФ Институт физики высоких энергий
142284, Протвино Московской обл.

Индекс 3649

П Р Е П Р И Н Т 99-15, И Ф В Э, 1999