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Aleksey I. Alekseev<sup>1</sup>

# NONPERTURBATIVE POWER CORRECTIONS IN $\bar{\alpha}_s(q^2)$ OF TWO-LOOP ANALYTIZATION PROCEDURE

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<sup>1</sup>Electronic address: [alekseev@mx.ihep.su](mailto:alekseev@mx.ihep.su)

## Abstract

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The analytization procedure which allows one to remove nonphysical singularities of the QCD running coupling constant  $\bar{\alpha}_s(q^2)$  in the infrared region is applied to standard as well as to iterative solutions of the two-loop renormalization group equation. Non-leading at large momentum nonperturbative contributions in  $\bar{\alpha}_s(q^2)$  are obtained in an explicit form. The coefficients of nonperturbative contributions expansions in inverse powers of the squared Euclidean momentum are calculated. For both cases considered there appear convergent at  $q^2 > \Lambda^2$  power series of negative terms with different dependence on term numbers.

## Аннотация

Алексеев А.И. Степенные непертурбативные поправки в  $\bar{\alpha}_s(q^2)$  метода аналитизации на двухпетлевом уровне: Препринт ИФВЭ 99-23. – Протвино, 1999. – 8 с., библиогр.: 11.

Метод аналитизации, позволяющий избавиться от нефизических сингулярностей бегущей константы связи КХД  $\bar{\alpha}_s(q^2)$  в инфракрасной области, применен к стандартному, а также к итерационному решению двухпетлевого уравнения ренормгруппы. Получены явные выражения для нелогирующих при больших импульсах непертурбативных вкладов в  $\bar{\alpha}_s(q^2)$ . Найдены коэффициенты разложения непертурбативных вкладов по обратным степеням квадрата евклидова импульса. В обоих рассмотренных случаях получаются знакопостоянные степенные ряды, сходящиеся при  $q^2 > \Lambda^2$  с различной зависимостью коэффициентов разложения от порядкового номера.

## 1. Introduction

The nonperturbative contributions to observables as well as direct to the QCD running coupling constant  $\bar{\alpha}_s$  have been widely discussed in recent years. The evidence provided by very different approaches suggests the existence of power type nonperturbative corrections which reflect the complicated structure of the QCD vacuum.

The nonperturbative contributions arise quite naturally when one uses an analytization procedure. The main purpose of this procedure is to remove nonphysical singularities from approximate (perturbative) expressions for the Green functions of QFT. The idea of the procedure goes back to Refs. [1,2] devoted to the ghost pole problem in QED. The foundation of the procedure is the principle of summation of information derived from the perturbation theory under the sign of the Källen – Lehmann spectral integral. In recent papers [3,4] it is suggested to solve the ghost pole problem in QCD by demanding the  $\bar{\alpha}_s(q^2)$  to be analytical in  $q^2$  (to compare with dispersive approach [5]). As a result, instead of the one-loop expression  $\bar{\alpha}_s^{(1)}(q^2) = (4\pi/b_0)/\ln(q^2/\Lambda^2)$  which takes into account the leading logarithms and has the ghost pole at  $q^2 = \Lambda^2$  ( $q^2$  is the Euclidean momentum squared), one obtains the expression

$$\bar{\alpha}_{an}^{(1)}(q^2) = \frac{4\pi}{b_0} \left[ \frac{1}{\ln(q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - q^2} \right]. \quad (1)$$

Eq. (1) is an analytic function in the complex  $q^2$ -plane with a cut along the negative real semiaxis. The pole of the perturbative running coupling at  $q^2 = \Lambda^2$  is canceled by the nonperturbative contribution ( $\Lambda^2|_{g^2 \rightarrow 0} \simeq \mu^2 \exp\{-(4\pi)^2/(b_0 g^2)\}$ ) and the value  $\bar{\alpha}_{an}^{(1)}(0) = 4\pi/b_0$  appeared finite and independent of  $\Lambda$ . The nonperturbative contribution in Eq. (1) can be presented as convergent at  $q^2 > \Lambda^2$  constant signs series in the inverse powers of the momentum squared.

At the two-loop level one cannot succeed in finding the nonperturbative part of  $\bar{\alpha}_{an}^{(2)}(q^2)$  in an explicit form. There were obtained the approximate formulas [4,6] which, in spite of their accuracy, do not provide an exhaustive information on the behavior of the nonperturbative contributions. The most important feature of the analytization procedure discovered is the stability property of the value of the analytized coupling constant at zero with respect to high corrections,  $\bar{\alpha}_{an}^{(1)}(0) = \bar{\alpha}_{an}^{(2)}(0) = \bar{\alpha}_{an}^{(3)}(0)$ . This property provides a definite high corrections stability of  $\bar{\alpha}_{an}(q^2)$  in the infrared region in a hall.

In this paper we find out what happens with power corrections of nonperturbative ultraviolet "tail" of the analytized running coupling when going from the one-loop level to the two-loop level.

## 2. The extraction of nonperturbative contributions

Let us consider the two-loop  $\beta$ -function

$$\beta(g^2) = \beta_0 g^4 + \beta_1 g^6, \quad (2)$$

where the coefficients

$$\begin{aligned} \beta_0 &= -\frac{1}{(4\pi)^2} b_0, \quad b_0 = 11 - \frac{2}{3} n_f, \\ \beta_1 &= -\frac{1}{(4\pi)^4} b_1, \quad b_1 = 102 - \frac{38}{3} n_f \end{aligned} \quad (3)$$

do not depend on the renormalization scheme choice. We can present the Gell-Mann – Low equation

$$u \frac{\partial \bar{g}^2(u, g)}{\partial u} = \beta(\bar{g}^2), \quad (4)$$

$u = q^2/\mu^2$  in a "rationalized" form by introducing the function  $a(x)$  of  $x = q^2/\Lambda^2$  of the form

$$a(x) = \frac{b_0}{(4\pi)^2} \bar{g}^2(u, g) = \frac{b_0}{4\pi} \bar{\alpha}_s(q^2).$$

The two-loop differential equation for the running coupling constant can be written as follows:

$$x \frac{da(x)}{dx} = -a^2(x) - ba^3(x), \quad (5)$$

where

$$b = -\frac{\beta_1}{\beta_0^2} = \frac{102 - \frac{38}{3} n_f}{(11 - \frac{2}{3} n_f)^2}. \quad (6)$$

For the case of three active quark flavors  $n_f = 3$ ,  $b_0 = 9$ , and  $b = 64/81 \simeq 0.7901$ . Integrating Eq. (5) one obtains

$$\frac{1}{a(x)} - b \ln \left( 1 + \frac{1}{ba(x)} \right) = \ln x + \ln c, \quad (7)$$

where  $c$  being the integration constant. The transcendental Eq. (7) defines implicitly the function  $a(x)$ . The real solution of Eq. (7) has two branches, the former corresponds to a positive decreasing at  $x \rightarrow \infty$  function  $a(x)$ , whereas for the latter  $a(x)$  is negative and at  $x \rightarrow \infty$  it goes to  $-1/b$ . The positive decreasing at  $x \rightarrow \infty$  solution corresponds to the asymptotic freedom property, and this solution can be fixed further by the choice of the integration constant  $c$ , which means, in practice, the redefinition of  $\Lambda$ . The exact solution of Eq. (7) can be written [7] using the Lambert function  $W(y)$  defined by the equation  $y = W(y) \exp\{W(y)\}$ .

We shall consider the following two forms of approximate solutions of Eq. (7). The first one is the standard two-loop coupling

$$a^{(2)}(x) = \frac{1}{l} - \frac{b}{l^2} \ln l, \quad (8)$$

where  $l = \ln x$ , and the second one is the iterative solution [6] of the form

$$a^{it}(x) = \frac{1}{l + b \ln(1 + \frac{l}{b})}. \quad (9)$$

This solution, when expanding in inverse powers of logarithms apart from the terms Eq. (8), gives rise to the term  $b \ln b / l^2$  and additional terms of the form  $l^{-3} \ln^2 l + \dots$  which should be taken into account at the three-loop level. The exact solution of Eq. (7) also does not claim to be the description of the three-loop terms. At large  $x$  the functions (8), (9) behave similarly but for small  $x$  the behavior is different and at  $x = 1$  they have singularities of a different analytical structure. Namely, at  $x \simeq 1$

$$a^{(2)}(x) \simeq -\frac{b}{(x-1)^2} \ln(x-1), \quad a^{it}(x) \simeq \frac{1}{2(x-1)}. \quad (10)$$

These singularities are stronger than the integrable singularity of the exact solution of Eq. (7) of the form  $1/\sqrt{x-1/c}$ . But this is not an obstacle to the application of the analytization procedure.

According to the definition the analytized running coupling is obtained from the initial running coupling by the integral representation

$$a_{an}(x) = \frac{1}{\pi} \int_0^\infty \frac{d\sigma}{x + \sigma} \rho(\sigma), \quad (11)$$

where  $\rho(\sigma) = \text{Im } a_{an}(-\sigma - i0) = \text{Im } a(-\sigma - i0)$ . As initial expressions we will consider Eqs. (8), (9). By making the analytic continuation of these equations into the Minkowski space  $x = -\sigma - i0$ , one obtains, correspondingly

$$a^{(2)}(-\sigma - i0) = \frac{1}{\ln \sigma - i\pi} - \frac{b}{(\ln \sigma - i\pi)^2} \ln(\ln \sigma - i\pi), \quad (12)$$

$$\begin{aligned} a^{it}(-\sigma - i0) &= \frac{1}{\ln \sigma - i\pi + b \ln(1 + \frac{1}{b} \ln \sigma - \frac{i\pi}{b})} = \\ &= \frac{1}{\ln \sigma + b \ln \sqrt{(1 + \frac{1}{b} \ln \sigma)^2 + \frac{\pi^2}{b^2}} - i \left[ \pi + b \arctan \frac{\pi}{b + \ln \sigma} \right]}. \end{aligned} \quad (13)$$

For the spectral function  $\rho(\sigma)$  it follows that

$$\begin{aligned} \rho^{(2)}(\sigma) &= \frac{\pi}{\ln^2 \sigma + \pi^2} + \frac{b}{(\ln^2 \sigma + \pi^2)^2} \times \\ &\times \left[ (\ln^2 \sigma - \pi^2) \arctan \frac{\pi}{\ln \sigma} - 2\pi \ln \sigma \ln \sqrt{\ln^2 \sigma + \pi^2} \right], \end{aligned} \quad (14)$$

$$\rho^{it}(\sigma) = \frac{\pi + b \arctan \frac{\pi}{b + \ln \sigma}}{\left[ \ln \sigma + b \ln \sqrt{(1 + \frac{1}{b} \ln \sigma)^2 + \frac{\pi^2}{b^2}} \right]^2 + \left[ \pi + b \arctan \frac{\pi}{b + \ln \sigma} \right]^2}. \quad (15)$$

By the change of the variable of the form  $\sigma = \exp(t)$ , the analytized expressions are derived from (14), (15) as follows:

$$a_{an}^{(2)}(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} dt \frac{e^t}{x + e^t} \times \left\{ \frac{\pi}{t^2 + \pi^2} + \frac{b}{(t^2 + \pi^2)^2} \left[ (t^2 - \pi^2) \arctan \frac{\pi}{t} - 2\pi t \ln \sqrt{t^2 + \pi^2} \right] \right\}, \quad (16)$$

$$a_{an}^{it}(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} dt \frac{e^t}{x + e^t} \frac{\pi + b \arctan \frac{\pi}{b+t}}{\left[ t + b \ln \sqrt{\left(1 + \frac{t}{b}\right)^2 + \frac{\pi^2}{b^2}} \right]^2 + \left[ \pi + b \arctan \frac{\pi}{b+t} \right]^2}. \quad (17)$$

Eqs. (16), (17) can be used to study  $a_{an}(x)$  by numerical methods in particular. We are interested in nonperturbative contributions which decrease at  $x \rightarrow \infty$  much faster than the perturbative ones, therefore, their extraction by a direct comparison with the initial expressions is difficult at large  $x$ . We shall obtain an explicit analytic formulae for the nonperturbative contributions as a difference between output and input expressions of the analytization procedure.

Let us see what the singularities of the integrands of (16), (17) in the complex  $t$ -plane are. First of all, for both cases the integrands have the simple poles at  $t = \ln x \pm i\pi(1 + 2n)$ ,  $n = 0, 1, 2, \dots$ . All the residues of the function  $\exp(t)/(x + \exp(t))$  at these points are equal to unity. Making use of the formula

$$\arctan \frac{\pi}{t} = \frac{1}{2i} \ln \frac{t + i\pi}{t - i\pi} \quad (18)$$

one can obtain

$$\begin{aligned} \frac{1}{(t^2 + \pi^2)^2} \left[ (t^2 - \pi^2) \arctan \frac{\pi}{t} - 2\pi t \ln \sqrt{t^2 + \pi^2} \right] &= \\ &= \frac{i}{2} \left[ \frac{\ln(t - i\pi)}{(t - i\pi)^2} - \frac{\ln(t + i\pi)}{(t + i\pi)^2} \right]. \end{aligned} \quad (19)$$

This formula allows one to make transparent the singularities structure of the integrand in (16). We obtain

$$\begin{aligned} a_{an}^{(2)}(x) &= \frac{1}{2\pi i} \int_{-\infty}^{\infty} dt \frac{e^t}{x + e^t} \times \\ &\times \left\{ \frac{1}{t - i\pi} - \frac{1}{t + i\pi} - b \left[ \frac{\ln(t - i\pi)}{(t - i\pi)^2} - \frac{\ln(t + i\pi)}{(t + i\pi)^2} \right] \right\}. \end{aligned} \quad (20)$$

For the second case under consideration, it follows from (13), (17) that

$$\begin{aligned} a_{an}^{it}(x) &= \frac{1}{2\pi i} \int_{-\infty}^{\infty} dt \frac{e^t}{x + e^t} \times \\ &\times \left[ \frac{1}{t - i\pi + b \ln \left(1 + \frac{t}{b} - \frac{i\pi}{b}\right)} - \frac{1}{t + i\pi + b \ln \left(1 + \frac{t}{b} + \frac{i\pi}{b}\right)} \right]. \end{aligned} \quad (21)$$

The integrands in (20), (21) multiplied by  $t$  go to zero at  $|t| \rightarrow \infty$ . This allows one to append the integration by the arch of the "infinite" radius without affecting the value of the integral.

Closing the integration contour  $C$  in the upper half-plane of the complex variable  $t$ , by using the residue theorem for the one-loop part of (20), one readily obtains the well-known result (1)

$$a_{an}^{(1)}(x) = \frac{1}{2\pi i} \int_C dt f(t) = \sum_{n=0}^{\infty} \text{Res } f(t = \ln x + i\pi(1 + 2n)) + \\ + \text{Res } f(t = i\pi) = \frac{1}{\ln x} + \frac{1}{1 - x}, \quad (22)$$

where  $f(t)$  is the integrand of (20) at  $b = 0$ . The two-loop part of the integrand (20) apart from the simple poles at  $t = \ln x \pm i\pi(1 + 2n)$ ,  $n = 0, 1, 2, \dots$  has logarithmic branch points at  $t = \pm i\pi$  which coincide with the second order poles. Let us cut the complex  $t$ -plane in a standard way,  $t = \pm i\pi - \lambda$ , with  $\lambda$  being the real parameter varying from 0 to  $\infty$ . For the case of coincidence of the branch point and the pole of the second order the following theorem can be proved.

*Theorem. If  $f(z)$  analytic function inside the circle of the radius  $r$  with the center at zero, then*

$$\int_C dz \frac{\ln z}{z^2} f(z) = -2\pi i \left\{ \int_{-a}^0 \frac{dx}{x^2} [f(x) - f(0) - xf'(0)] - \frac{1}{a} f(0) - f'(0) \ln a \right\}, \quad (23)$$

where the contour  $C$  goes from  $z = -a - i0$  along the lower side of the cut till the circle of the radius  $\delta$  with the center at zero ( $r > a > \delta > 0$ ), then goes by this circle around  $z = 0$ , and then along the upper side of the cut it goes to  $z = -a + i0$ .

For the terms of (20) which are proportional to  $b$ , we also close the contour in the upper half-plane of the complex variable  $t$  excepting the singularity at  $t = i\pi$ . In this case an additional contribution emerges because of the integration along the sides of the cut and around the second order pole. The substitution  $z = t - i\pi$  brings this contribution to the form corresponding to the theorem (23) with  $f(z) = (b/2\pi i) \exp(z)/(x - \exp(z))$ . We shall call this contribution together with the one-loop contribution from the pole at  $t = i\pi$  as nonperturbative because the remaining contribution of the poles at  $t = \ln x + i\pi(1 + 2n)$ ,  $n = 0, 1, 2, \dots$  results exactly in perturbative expression (8). In fact, according to the residue theorem

$$a_{an}^{(2)pt}(x) = a_{an}^{(1)pt}(x) - b \sum_{n=0}^{\infty} \left[ \frac{\ln(\ln x + 2\pi i n)}{(\ln x + 2\pi i n)^2} - \frac{\ln(\ln x + 2\pi i(n + 1))}{(\ln x + 2\pi i(n + 1))^2} \right] = \\ = \frac{1}{\ln x} - b \frac{\ln(\ln x)}{(\ln x)^2}. \quad (24)$$

The nonperturbative part of the analytized coupling using Eqs. (20), (23), (22) can be derived with the result

$$a_{an}^{(2)npt}(x) = -\frac{1}{x - 1} - b \left\{ \int_0^1 \frac{dt}{t^2} \left[ \frac{1}{xe^t - 1} - \frac{1}{x - 1} + \frac{tx}{(x - 1)^2} \right] + \right. \\ \left. + \int_1^{\infty} \frac{dt}{t^2} \frac{1}{xe^t - 1} - \frac{1}{x - 1} \right\}. \quad (25)$$

Integrating by parts and making the change of variable  $\sigma = \exp(-t)$ , we can represent (25) in the form of one integral with finite limits

$$a_{an}^{(2) \text{ } npt}(x) = \frac{1}{1-x} + b \left\{ \frac{x}{(1-x)^2} + x \int_0^1 d\sigma \ln(-\ln \sigma) \frac{x+\sigma}{(x-\sigma)^3} \right\}. \quad (26)$$

This formula is convenient for the numerical study whereas formula (25) is suitable for finding the explicit power nonperturbative corrections at  $x \rightarrow \infty$ .

Let us now turn to the evaluation of the integral (21) for the analytized iterative coupling. The integrand has the simple poles at  $t = \ln x \pm i\pi(1+2n)$ ,  $n = 0, 1, 2, \dots$  and at  $t = \pm i\pi$ . It has also the logarithmic branch points at  $t = -b \pm i\pi$ . To provide the single-valuedness of the integrand, we draw two cuts beginning at  $t = -b \pm i\pi$  and going to infinity to the left in parallel to the real axis of the complex  $t$ -plane. We close the integration contour by the "infinite" semicircle in the upper half-plane with going around the branch point by the sides of the cut and integrate along this contour. As in the previous case, all the terms in the sum of residues at  $t = \ln x + \pi i(1+2n)$ ,  $n = 0, 1, 2, \dots$  mutually cancel apart from the term of  $n = 0$ . Integrating also along the sides of the cut, we obtain

$$a_{an}^{it}(x) = \frac{1}{\ln x + b \ln(1 + \frac{1}{b} \ln x)} + \frac{1}{2(1-x)} + \int_0^\infty d\xi \frac{1}{1 - x e^{b(1+\xi)}} \frac{1}{(1 + \xi - \ln \xi)^2 + \pi^2}. \quad (27)$$

Performing the change of variable  $\sigma = \exp(-\xi)$ , we come to the integral with finite limits. For additional "nonperturbative" output of the analytization procedure we obtain the following representation

$$\Delta a_{an}^{it}(x) = \frac{1}{2(1-x)} + \int_0^1 \frac{d\sigma}{\sigma} \frac{1}{1 - x(\sigma/e)^{-b}} \frac{1}{[1 - \ln(-\sigma \ln \sigma)]^2 + \pi^2}. \quad (28)$$

### 3. Nonperturbative contributions at large $q^2$ .

Consider the large  $x$  behavior of the nonperturbative contributions. It is seen from Eq. (25) that this behavior is regular. Expanding the integrands in powers of  $1/x$  and integrating, we find

$$a_{an}^{(2) \text{ } npt}(x) = \sum_{n=1}^{\infty} \frac{c_n}{x^n}, \quad (29)$$

where

$$c_n = -1 + bn \left[ n \int_0^\infty dt \ln t e^{-nt} + 1 \right]. \quad (30)$$

Performing the  $t$  integration in (30) yields the following simple formula for the coefficients of the ultraviolet expansion of the nonperturbative contributions

$$c_n = -1 + bn(1 - \gamma - \ln n), \quad (31)$$



where  $\gamma$  is the Euler constant,  $\gamma \simeq 0.5772$ . The leading term is

$$c_1 = -1 + b(1 - \gamma), \quad (32)$$

and the passing from the one-loop level to the two-loop one results in some compensation of the leading at large  $x$  term of the form  $1/x$ . For the next terms there is no compensation, instead they increase with  $n$  increasing. Estimating at  $n_f = 3$  the first three terms of the series (29), we obtain

$$a_{an}^{(2)np}(x) \simeq -\frac{0.666}{x} - \frac{1.427}{x^2} - \frac{2.602}{x^3} - \dots \quad (33)$$

Let us consider the large  $x$  behavior of the "nonperturbative" contribution in (27). Expanding the integrand in powers of  $1/x$ , we also obtain the infinite terms series

$$\Delta a_{an}^{it}(x) = \sum_{n=1}^{\infty} \frac{c_n^{it}}{x^n}, \quad (34)$$

where

$$c_n^{it} = -\frac{1}{2} - \int_0^{\infty} d\xi e^{-bn(1+\xi)} \frac{1}{[1 + \xi - \ln \xi]^2 + \pi^2}. \quad (35)$$

Expression (35) for the expansion coefficients is not so simple as (31) but it is explicit and can be calculated up to an arbitrary accuracy. We see from Eq. (35) that all  $c_n^{it}$  are also negative tending fast to  $-1/2$  when  $n$  increases. Substituting the denominator in the integrand of (35) by its minimal value  $\pi^2 + 4$ , one finds

$$|c_n^{it} + \frac{1}{2}| < \frac{1}{\pi^2 + 4} \frac{e^{-bn}}{bn}. \quad (36)$$

At  $n_f = 3$  it gives for the first coefficient  $|c_1^{it} + 1/2| < 0.0414$ . For the first three terms of the series (34), one can obtain

$$\Delta a_{an}^{it}(x) \simeq -\frac{0.535}{x} - \frac{0.508}{x^2} - \frac{0.502}{x^3} - \dots \quad (37)$$

Thus, passing from the one-loop level of the analytization procedure to the two-loop level leads to approximately two times decrease of each term of the expansion (34).

## 4. Conclusion

For both cases considered the nonperturbative contributions can be presented in the form of series in inverse powers of the momentum squared, all the terms are negative. Although the convergence radii of the series (29), (34) are the same and equal to unity, the dependence of the expansion coefficients on term numbers is different. This is defined by the expressions taken as an input for the analytization procedure. Choosing as an input the standard two-loop expression (8), we obtain the "standard nonperturbative corrections" of the form (33). For sufficiently large  $x = q^2/\Lambda^2$  the nonperturbative contributions are defined mainly by the leading terms in (33), (37). The partial compensation of the leading terms found for both cases considered when we pass from the one-loop level to the two-loop level can point to the tendency of a high-loop minimization of the nonperturbative contributions in the ultraviolet region.

The analytization procedure may be not an ultimate step of construction the "physical" running coupling constant. In terms of Ref. [8] the analytized coupling constant corresponds to the "regularized perturbative part" of the full coupling constant which contains also the "genuine nonperturbative" contribution. As a step in the direction of the full running coupling constant it has been suggested in [9,10] that the one-loop analytization output be modified in a minimal way by introducing the two additional pole type nonperturbative terms to provide the ultraviolet convergence of the gluon condensate. In this case the model for the running coupling constant arises with the enhancement at zero momentum and dynamical gluon mass  $m_g$ . This mass can be fixed [11] by the condition of minimum of the nonperturbative vacuum energy and is estimated  $m_g \simeq 0.6$  GeV for the "standard" value of the gluon condensate  $(0.33 \text{ GeV})^4$ . The next step of the model building for the full running coupling constant on the base of the two-loop analytized running coupling is under study.

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А.И.Алексеев

Степенные непертурбативные поправки в  $\bar{\alpha}_s(q^2)$  метода аналитизации на двухпетлевом уровне.

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ГНЦ РФ Институт физики высоких энергий  
142284, Протвино Московской обл.

