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THE QCD RENORMALIZATION SCALE STABILITY OF HIGH TWISTS AND α_s IN DEEP INELASTIC SCATTERING

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Abstract

Alekhin S.I. The QCD renormalization scale stability of high twists and α_s in deep inelastic scattering: IHEP Preprint 99-55. – Protvino, 1999. – p. 9, figs. 9, refs.: 10.

A sensitivity of twist-4 and α_s values extracted in the NLO QCD analysis of nonsinglet SLAC-BCDMS-NMC deep inelastic scattering data to the choice of QCD renormalization scale (RS) is analysed. It is obtained that the high twist (HT) contribution to structure function F_2, is retuned with the change of RS. This retuning depends on the choice of starting QCD evolution point Q_0 and x. At $Q_0 \gtrsim 10$ GeV^2 the HT contribution to F_2 is retuned at small x and is not almost retuned at large x; at small Q_0 it exhibits approximate RS stability for all x in question. The HT contribution to F_L is RS stable for all Q_0 and x. The RS sensitivity of α_s also depends on the choice of Q_0: At large Q_0 this sensitivity is weaker than at small ones. For $Q_0^2 = 9$ GeV^2 the value $\alpha_s(M_Z) = 0.1183 \pm 0.0021(\mathrm{stat}+\mathrm{syst}) \pm 0.0013(\mathrm{RS})$ is obtained. Connection with the higher order QCD corrections is discussed.

Аннотация

Алёхин С.И. Устойчивость высших твистов и α_s по отношению к масштабу перенормировки QCD в глубоко-неупругом рассеянии: Препринт ИФВЭ 99-55. – Протвино, 1999. – 9 с., 9 рис., библиогр.: 10.

Рассмотрена чувствительность величины α_s и вклада высших твистов (ВТ), извлекемых в анализе несинглетных данных SLAC-BCDMS-NMC по глубоко неупругому рассеянию, к выбору масштаба перенормировки в рамках NLO QCD. Получено, что вклад ВТ в структурную функцию F_2 перенастраивается с изменением масштаба перенормировки. Величина этой перенастройки зависит от выбора начальной точки QCD эволюции Q_0 и x. При $Q^2 \gtrsim 10$ GeV^2 вклад ВТ в F_2 перенастраивается при малых x и почти не перенастраивается при больших x; при малых Q_0 он почти стабилен при всех рассматриваемых x. Вклад ВТ в структурную функцию F_L стабилен при всех Q_0 и x. Чувствительность величины α_s, извлекаемой в этом анализе, к масштабу перенормировки также зависит от выбора Q_0: при больших Q_0 эта чувствительность ниже, чем при малых. При $Q^2 = 9$ GeV^2 получена величина $\alpha_s(M_Z) = 0.1183 \pm 0.0021(\text{стат.}+\text{сист.}) \pm 0.0013(\text{ренорм.})$. Обсуждается связь полученных результатов с поправками от высших порядков QCD.

1. Interest to the quantitative description of high twist (HT) contribution to the deep inelastic scattering (DIS) cross sections has recently increased, in particular, due to the development of infrared renormalon (IRR) model (see e.g. review [1]). Within this model one can derive the x-shape of HT contribution from the x-shape of leading twist (LT) structure functions. This connection allows for obtaining precise predictions for the HT contribution since the LT contribution can be determined rather precisely from the experimental data. Meanwhile the experimental determination of HT contribution is not direct and is based on fitting a combination of log- and power-like terms to the data. If the data accuracy is not high enough, the correlation between these log- and power-like terms can be large, which was explicitly shown in the combined SLAC-BCDMS data analysis of Ref. [2]. If the large correlations occur, the separation of terms is unstable with respect to the various inputs of fit and thus, it becomes important to study the stability of HT separation. One of the poorly defined ansatz of a QCD analysis of DIS data is the choice of renormalization scale (RS). The uncertainty due to RS variation is connected with the account of higher order (HO) QCD corrections since in the analysis with complete account of HO terms, the RS dependence of fitted parameters should vanish and thus the observed RS dependence can be used for the estimation of HO terms effect. Earlier we reported the results of NLO QCD analysis of high x SLAC-BCDMS-NMC data [3]. A short communication on the RS dependence of HT contribution and $\alpha_{\rm s}$ obtained in this analysis was reported in Ref. [4]. In this paper a more detailed study of this dependence is given.

2. Our approach used for the study of RS stability of DGLAP evolution equation in NLO QCD is the same as described in Refs. [5,6]. Within this approach the RS of QCD evolution is changed from Q to $k_{\rm R}Q$, where $k_{\rm R}$ is an arbitrary parameter, conventionally varying from 1/2 to 2. This approach contains certain simplification since the change of scale can depend on x, but in our analysis this effect is not so essential due to a limited range of x. For the nonsinglet case the NLO DGLAP equation with an arbitrary choice of RS looks as follows:

$$Q\frac{\partial q^{\rm NS}(x,Q)}{\partial Q}=\frac{\alpha_{\rm s}\,(k_{\rm R}Q)}{\pi}P_{\rm qq}^{\rm NS,(0)}\otimes q^{\rm NS}+\frac{\alpha_{\rm s}^2(k_{\rm R}Q)}{2\pi^2}\left[P_{\rm qq}^{\rm NS,(1)}\otimes q^{\rm NS}+\ln(k_{\rm R})\beta_0P_{\rm qq}^{\rm NS,(0)}\otimes q^{\rm NS}\right], \quad (1)$$

where $P\otimes q=\int_x^1 dzP(z)q(x/z,Q)$ denotes the Mellin convolution; $q^{\rm NS}$ is the evolved distribution; $P^{\rm NS,(0)}$ and $P^{\rm NS,(1)}$ are the LO and NLO parts of splitting function; $\alpha_{\rm s}(Q)$ is the running strong coupling constant; and β_0 is the regular coefficient of renormgroup equation for $\alpha_{\rm s}$:

$$Q\frac{d\alpha_{\rm s}}{dQ}=-\frac{\beta_0}{2\pi}\alpha_{\rm s}^2-\frac{\beta_1}{4\pi^2}\alpha_{\rm s}^3.$$

Equation (1) was solved with the help of direct integration method implemented in the code used earlier [2]. In the analysis of Ref. [3] we used the combined SLAC-BCDMS-NMC proton-deuterium data [7] with the cuts $x \geq 0.3$ to reduce the QCD evolution to the nonsinglet case and $x \leq 0.75$ to reject the region where nuclear effects in deuterium may be significant. The initial scale of evolution was chosen equal to $Q_0^2 = 9$ GeV^2 to provide comparability with the earlier results of Ref. [2]. A complete account of point-to-point correlations due to systematic errors was made through a covariance matrix approach, similarly to our earlier analysis of Ref. [8]. The formula were fitted to the cross section data to allow for simultaneous and unbiased determination of a twist-4 contribution to structure functions $F_{\rm L}$ and F_2:

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2(s-M^2)}{Q^4}\left[\left(1-y-\frac{(Mxy)^2}{Q^2}\right)F_2^{\rm HT}(x,Q) + \left(1-2\frac{m_{\rm l}^2}{Q^2}\right)\frac{y^2}{2}2xF_1^{\rm HT}(x,Q)\right],$$

$$2xF_1^{\rm HT}(x,Q) = F_2^{\rm HT}(x,Q) - F_{\rm L}^{\rm HT}(x,Q),$$

$$F_{2,{\rm L}}^{\rm HT}(x,Q) = F_{2,{\rm L}}^{\rm TMC}(x,Q) + H_{2,{\rm L}}(x)\frac{1\ {\rm GeV}^2}{Q^2},$$

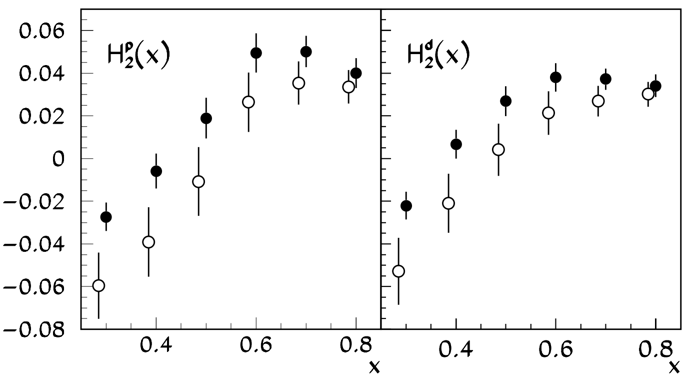

Fig. 1. The values of proton and deuterium $H_2(x)$ for different values of $k_{\rm R}$ (open circles: $k_{\rm R} = 1/2$; full circles: $k_{\rm R} = 1$.

where $F_{2,{\rm L}}^{\rm TMC}$ are the LT contributions obtained as a result of integration of Eq. (1) with the account of target mass corrections [9]; s is the total c.m.s. energy; $m_{\rm l}$ is the scattered lepton mass; and y is the regular lepton scattering variable. The values of functions $H_{2,{\rm L}}(x)$ at $x = 0.3, 0.4, 0, 5, 0.6, 0.7, 0.8$ were fitted, between these points $H_{2,{\rm L}}(x)$ were linearly interpolated. The functions H_2 for proton and deuterium were fitted independently, while the functions H_L for proton and deuterium due to a limited accuracy of data, turned out to be compatible within errors and all the fits were performed under constraint $H_{\rm L}^{\rm p}(x) = H_{\rm L}^{\rm d}(x)$.

3. The proton and deuterium $H_2(x)$ for $k_{\rm R} = 1$ and $k_{\rm R} = 1/2$ are given in Fig. 1. One can see that they depend on $k_{\rm R}$ at $x \sim 0.3$ and does not practically depend at $x \sim 0.8$. To give an explanation of this behaviour, recall the basic properties of solutions to the DGLAP evolution equations. After linearization on $\ln k_{\rm R}$, Eq. (1) can be analytically solved in the Mellin momentum space:

$$M^{\rm NS}(n,Q) = M^{\rm NS}(n,Q_0)M_{\rm NLO}(n,\alpha)\exp\left[g(n)\left(\alpha^2-\alpha_0^2\right)\ln(k_{\rm R})\right],$$

where $\alpha \equiv \alpha_{\rm s}(Q)$, $\alpha_0 \equiv \alpha_{\rm s}(Q_0)$; $M^{\rm NS}(n,Q)$ are the Mellin moments of $q^{\rm NS}$; $M_{\rm NLO}(n,\alpha)$ defines NLO evolution of these moments; $g(n)$ is the linear function of Mellin moments of the splitting functions $P_{\rm qq}^{\rm NS,(0)}$ and $P_{\rm qq}^{\rm NS,(1)}$. Introduce a function $\Delta q^{\rm NS}(k_{\rm R}) \equiv q^{\rm NS}(k_{\rm R}) - q^{\rm NS}(k_{\rm R}=1)$, which is convenient to study the RS dependence. The Mellin moments of this function $M^\Delta(n,Q)$ are given by

$$M^\Delta(n,Q) = M^{\rm NS}(n,Q_0)M_{\rm NLO}(n,\alpha)\left\{\exp\left[g(n)\left(\alpha^2-\alpha_0^2\right)\ln(k_{\rm R})\right]-1\right\}. \quad (2)$$

In Fig. 2 the precise dependence of proton $\Delta q^{\rm NS}$ on $\alpha_{\rm s}^2(Q)$ for different $k_{\rm R}$ obtained as the result of numerical integration of Eq. (1) at $Q_0^2 = 9$ GeV^2 is given[1]. The range of $\alpha_{\rm s}$ in the figure correspond to the variation of Q^2 from 1 to 9 GeV^2, i.e. the region where the HT contribution is most significant. It is evident that for all the values of x in question, $\Delta q^{\rm NS}$ is approximately proportional to $\ln k_{\rm R}$. This means that the linearization of Eq. (1) is justified and that in Eqn. (2) the part of exponent containing $\ln k_{\rm R}$ can be expanded, so that

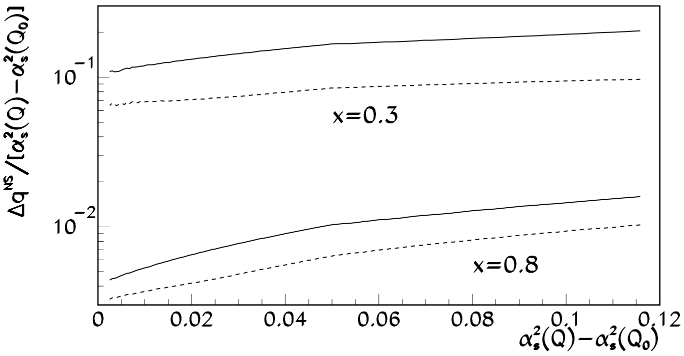

Fig. 2. The dependence of proton $\Delta q^{\rm NS}$ on $\alpha_{\rm s}^2$ at $Q_0^2 = 9$ GeV^2 (full lines: $\log_2 k_{\rm R} = -1$; dashed lines: $\log_2 k_{\rm R} = -1/2$).

$$\Delta q^{\rm NS}(x,Q) \approx \ln(k_{\rm R})\,(\alpha^2 - \alpha_0^2)\,\tilde{q}_{\rm NLO}(x,\alpha), \tag{3}$$

where $\tilde{q}_{\rm NLO}(x,\alpha)$ is the Mellin inverse of the product $M^{\rm NS}(n,Q_0)M_{\rm NLO}(n,\alpha)g(n)$.

The Q-behaviour of $\tilde{q}_{\rm NLO}$ is defined by $M_{\rm NLO}(n,\alpha)$. At $x = 0.3$ the function $\tilde{q}_{\rm NLO}$ depends weakly on α (see Fig. 2). This is consequence of the well known effect that the non-singlet QCD evolution has stationary point at $x \approx 0.1$ due to the fermion conservation. In the vicinity of stationary point scaling violation is small, but, due to at large n the function $M_{\rm NLO}(n,\alpha)$ rises with α faster than at low ones, at large x the scaling violation is more pronounced and the function $\Delta q^{\rm NS}$ rises with α significantly faster than α^2.

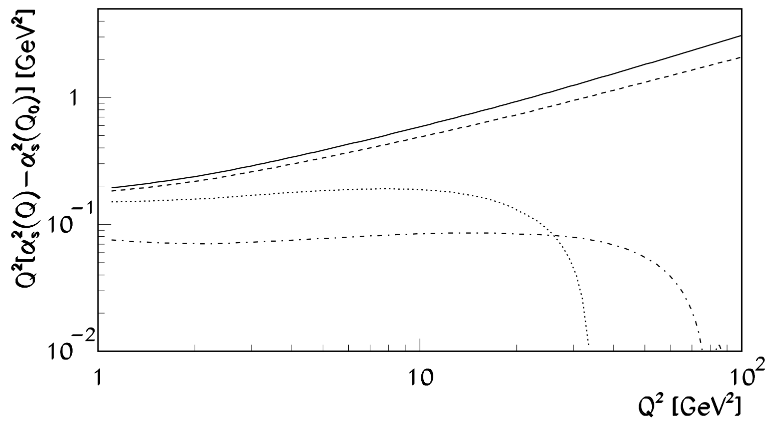

Fig. 3. The dependence of $Q^2\left[\alpha_{\rm s}^2(Q) - \alpha_{\rm s}^2(Q_0)\right]$ on Q^2 (full line: $\alpha_{\rm s}(Q_0) = 0$; dashed line: $\alpha_{\rm s}(Q_0) = 0.1$; dotted line: $\alpha_{\rm s}(Q_0) = 0.2$). The dashed-dotted line corresponds to $Q^2\left[\alpha_{\rm s}^3(Q) - \alpha_{\rm s}^3(Q_0)\right]$ for $\alpha_{\rm s}(Q_0) = 0.18$.

In the NLO non-singlet approximation the LT contribution to F_2 is given by

$$F_2^{\rm LT} = q^{\rm NS} + \frac{\alpha_{\rm s}}{2\pi} q^{\rm NS} \otimes C_2^{{\rm NS},(1)},$$

where $C_2^{{\rm NS},(1)}$ is the NLO coefficient function. Since the second term of above expression is suppressed with respect to the first one at moderate x, the Q-behaviour of function $\Delta F_2(k_{\rm R}) \equiv$

[1]Here and further on we do not give the results for deuterium since they are similar to the proton ones.

$F_2^{\rm LT}(k_{\rm R}) - F_2^{\rm LT}(k_{\rm R}=1)$ at $x=0.3$ coincides approximately with the ones of $\Delta q^{\rm NS}$. The Q-behaviour of the factor $[\alpha_{\rm s}^2(Q^2) - \alpha_{\rm s}^2(Q_0)]$ coming to Eqn. (3) depends on Q_0. If $Q \ll Q_0$ this factor is $\sim 1/\ln^2 Q$, i.e. falls with Q slower, than $1/Q^2$. Meanwhile in the vicinity of Q_0, where this factor vanishes, its Q-dependence is steeper and it can simulate the $1/Q^2$ behaviour in a rather wide range of Q (see Fig. 3). One can easily show that the value of $\alpha_{\rm s}$ at a starting evolution point $Q_0^{(n)}$, which provides the $1/Q^2$ behaviour simulation of the factor $[\alpha_{\rm s}^n(Q^2) - \alpha_{\rm s}^n(Q_0)]$ in the region from Q_1 to Q_2, is

$$\alpha_{\rm s}\left(Q_0^{(n)}\right) = \left[\frac{Q_2^2\alpha_{\rm s}^n(Q_2^2) - Q_1^2\alpha_{\rm s}^n(Q_1^2)}{Q_2^2 - Q_1^2}\right]^{1/n}. \tag{4}$$

The dependence of $\alpha_{\rm s}\left(Q_0^{(2)}\right)$ on Q_2 for $Q_1 = 1$ GeV^2 obtained with the help of Eq. (4) is given in Fig. 4. For the Q^2 interval from 1 to $\sim$ 10 GeV^2, where the HT are mostly significant, $\alpha_{\rm s}\left(Q_0^{(2)}\right) \sim 0.2$, which corresponds to $\left[Q_0^{(n)}\right]^2 \sim 40$ GeV^2. Due to the weak dependence of $\tilde{q}_{\rm NLO}$ on Q at small x, the function ΔF_2 also simulates the $1/Q^2$ behaviour at $x=0.3$, if the starting evolution point is chosen equal to $Q_0^{(2)}$. At a fixed Q_0 this simulation, in general, becomes less probable with the rise of x just due to the steeper rise of $\Delta q^{\rm NS}$ with α at large x.

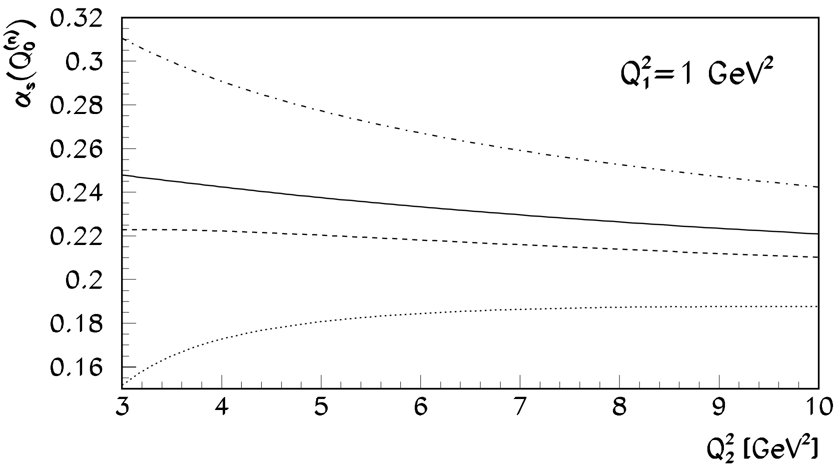

Fig. 4. The value of $\alpha_{\rm s}$ at starting evolution point $Q_0^{(n)}$ that provides the power-behaviour simulation of factors $\left[\alpha_{\rm s}^n(Q^2) - \alpha_{\rm s}^n(Q_0)\right]$ in the region from Q_1 to Q_2 (full line: $n=1$, dashed line: $n=2$, dotted line: $n=3$). The value of $\alpha_{\rm s}(Q_2)$ is also given for comparison (dashed-dotted line).

The behaviour of $Q^2\Delta F_2^{\rm p}$ at $k_{\rm R} = 1/2$ for different x and Q_0 is given in Fig. 5. At $Q_0^2 = 50$ GeV^2 and $x=0.3$ the function ΔF_2 simulates the $1/Q^2$ behaviour almost perfectly in the total Q region relevant for HT determination. This leads to the H_2 retuning in the fits with different $k_{\rm R}$, since ΔF_2 is compensated by the additional contribution to H_2 with the sign opposite to ΔF_2 (see Fig. 6). At $x=0.8$, due to the fall of $\tilde{q}_{\rm NLO}$ with Q, the simulation is much worse. If $Q_0^2 = 1$ GeV^2, the absolute value of $Q^2\Delta F_2$ steeply rises at $x=0.3$ and small Q due to the factor $[\alpha_{\rm s}^2(Q^2) - \alpha_{\rm s}^2(Q_0)]$. At $x=0.8$ this rise is not so steep because of the fall of $\tilde{q}_{\rm NLO}$ with Q, but it cannot suppress the general rise. As a consequence, at small Q_0 the function ΔF_2 cannot simulate the $1/Q^2$ behaviour at all x and it should be at least partially compensated by the change of Q-dependence of the LT contribution. The remnant HT retuning is still possible if the Q-dependence of ΔF_2 at some x is more similar to $1/Q^2$, than to the Q-dependence of

$\partial F_2/\partial\alpha_s(M_Z)^2$. The explicit tracing of the balance between the H_2 retuning and the change of LT contribution is not so simple due to $\alpha_s(M_Z)$ is determined by data at all x and Q. Anyway, as one can see from Fig. 6, at small Q_0 the function H_2 is retuned with the change of k_R significantly smaller than at large Q_0. At $Q_0^2 = 9$ GeV^2 the retuning effect is almost the same as for $Q_0^2 = 50$ GeV^2. This is natural since the ΔF_2 behaviour at small Q depends weakly on Q_0, if Q_0 is large, and the data are not very sensitive to the HT contribution at $Q^2 \gtrsim 10$ GeV^2. Thus, the choice of Q_0 has small effect on the fitted HT contribution if $Q_0^2 \gtrsim 10$ GeV^2.

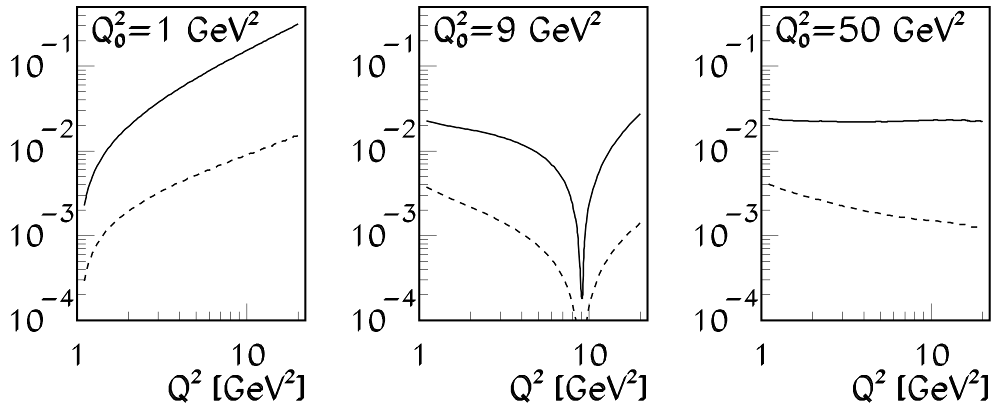

Fig. 5. The dependence of $\mid Q^2\Delta F_2^p \mid$ [GeV^2] on Q^2 at $k_R = 1/2$ and at different values of x and Q_0 (full lines: $x = 0.3$; dashed lines: $x = 0.8$).

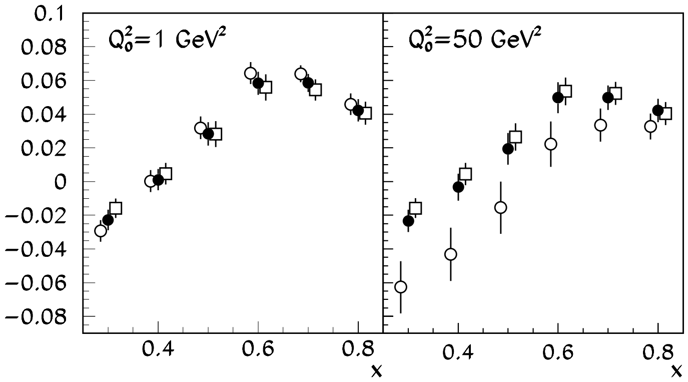

Fig. 6. The values of $H_2^p(x)$ for different choices of Q_0 and RS (open circles: $k_R = 1/2$; full circles: $k_R = 1$; squares: $k_R = 2$.

The RS stability of α_s also depends on the choice of Q_0. In the fit with HT fixed ΔF_2 is compensated by the change of LT contribution that leads to the shift of α_s. If H_2 is released in the fit, the shift of α_s can change due to the partial compensation of ΔF_2 by the change of H_2. Since the $1/Q^2$ simulation of ΔF_2 depends on Q_0, the α_s dependence on k_R changes with Q_0 as well as H_2 (see Fig. 7). At large Q_0 the ΔF_2 slope on k_R is negative at small Q. As a result, in the fit with HT released the H_2 slope on k_R is positive. Correspondingly the Q-dependence of LT contribution after HT releasing becomes weaker at large k_R and steeper at small ones, i.e.

[2]The Q-dependence of LT contribution is mainly driven by α_s and thus the balance between the absorbtion of ΔF_2 into the LT contribution and H_2 is defined by $\partial F_2/\partial\alpha_s(M_Z)$.

the slope of fitted $\alpha_s(M_Z)$ value on k_R decreases as compared to the fit with HT fixed. In the fit with HT fixed the $\alpha_s(M_Z)$ slope on k_R is positive and thus the RS uncertainty on α_s at large Q_0 becomes smaller. At small Q_0 the H_2 slope on k_R at $x \approx 0.5 \div 0.7$ is negative. The data from this region of x have the largest impact on the α_s determination and thus the H_2 releasing leads to the increase of RS error on α_s, although the scale of effect is smaller as compared with the fit at large Q_0 due to the weakness of HT retuning at small Q_0. In our analysis

$$\alpha_s(M_Z) = 0.1151 \pm 0.0015(\mathrm{stat} + \mathrm{syst}) \pm 0.0045(\mathrm{RS})$$

for $Q_0^2 = 1$ GeV^2 and

$$\alpha_s(M_Z) = 0.1183 \pm 0.0021(\mathrm{stat} + \mathrm{syst}) \pm 0.0013(\mathrm{RS})$$

for $Q_0^2 = 9$ GeV^2, where the RS error is estimated as half of $\alpha_s(M_Z)$ spread with the change of k_R from 1/2 to 2; the central value is shifted to the centre of this spread. The values of χ^2 are approximately the same for different k_R, but in view of that at small Q_0 the total $\alpha_s(M_Z)$ error is about two times larger, than at large one, we consider the α_s value determined from the fit with large Q_0 as more reliable.

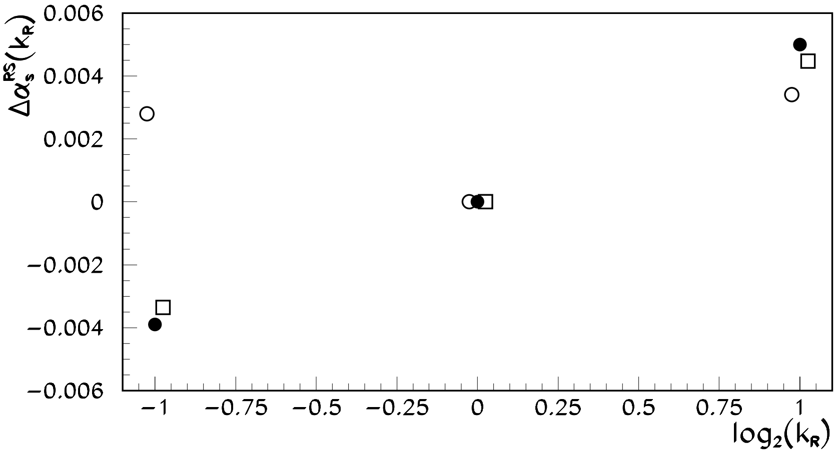

Fig. 7. The dependence of $\Delta\alpha_s^{\mathrm{RS}}(k_R) \equiv \alpha_s(M_Z)\,|_{k_R} - \alpha_s(M_Z)\,|_{k_R=1}$ on the choice of renormalization scale for different Q_0 (open circles: $Q_0^2 = 50$ GeV^2, full circles: $Q_0^2 = 1$ GeV^2). For comparison are also given the $\Delta\alpha_s^{\mathrm{RS}}(k_R)$ values obtained in the fits at $Q_0^2 = 50$ GeV^2 with HT fixed at the values obtained in fits with $k_R = 1$ (squares). Error bars are not given.

The function Δq^{NS}, by definition, is connected with the NNLO QCD corrections to evolved distributions. Natural assumption is that the exponent in the NNLO part of moment expression can be expanded, similarly to the exponent in Eqn. (2), and the NNLO contribution is $\sim [\alpha_s^2(Q^2) - \alpha_s^2(Q_0)]$ as well as Δq^{NS}. One can see from Fig. 3 that the factor $[\alpha_s^3(Q^2) - \alpha_s^3(Q_0)]$ coming to the N^{3}LO contribution can also simulate the $1/Q^2$ behaviour. Moreover, the region of Q, where the simulation is possible, widens from the lower orders to the higher ones. This gives an indication that the retuning of HT contribution after the accounting of HO QCD corrections to DGLAP kernel would exhibit the same properties as the RS retuning in NLO. Note that if $Q_2 \gg Q_1$, then $\alpha_s\left(Q_0^{(n)}\right) \sim \alpha_s(Q_2)$ for all n (see Eqn. 4 and Fig. 4). This means that in the analysis with simultaneous determination of α_s and HT at large Q_0 the contribution to F_2 due to the HO QCD corrections to DGLAP kernel can be merely fitted together with the HT, if $g(n)$

and the Mellin moments of HO splitting functions have the similar large n asymptotes. The fitting of HO corrections certainly leads to the increase of α_s error[3]. Meanwhile, the reduction of RS uncertainty, which is one of the dominant sources of α_s error, is larger and, as one can see from the above, the total α_s error becomes smaller.

One can see from Fig. 7 that at $Q_0^2 = 50$ GeV^2 the fitted $\alpha_s\,(M_Z)$ value is a nonlinear function of $\ln k_R$ contrary to the fits at $Q_0^2 = 1$ GeV^2. The reason for this difference is a large correlation between α_s and H_2 (c.f. Ref. [2]), which depends on how well $\partial F_2/\partial\alpha_s(M_Z)$ can simulate the $1/Q^2$ behaviour. With the rise of Q_0 this correlation increases. For example the correlation coefficient $\rho_{0.5}$ for $\alpha_s(M_Z)$ and $H_2(x = 0.5)$ at $k_R = 1/2$ is –0.82 for $Q_0^2 = 1$ GeV^2 and -0.97 for $Q_0^2 = 50$ GeV^2. As a consequence, at large Q_0 a small nonlinearity of q^{NS} on $\ln k_R$ manifests better and has unnegligible effect on the fitted parameters values (remind that the effective amplification of nonlinear effects in a fit is proportional to $1/(1-\rho^2)$). For comparison, the fits with HT fixed exhibit almost linear dependence of $\alpha_s\,(M_Z)$ on $\ln k_R$ (see Fig. 7). One of the reflections of this nonlinearity is that the difference between $H_2(x)$ at $k_R = 2$ and at $k_R = 1$ is small for $Q_0^2 = 50$ GeV^2 (see Fig. 6). This is an unpleasant feature of the analysis since the variation range of k_R is conventional and the nonlinearity does not allow one to rescale the RS uncertainty. One of the possible ways to suppress the nonlinear effects is to decrease the correlation between the HT contribution and α_s, e.g. adding more data to the analysis. On the another hand this correlation leads to the reducing of RS error on α_s and it is necessary keeping balance between the linearity and the size of RS error.

In the NLO QCD the LT contribution to the structure function F_L is proportional to the Mellin convolution of q^{NS} with the NLO coefficient function $C_L^{NS,(1)}$:

$$F_L^{LT} = \frac{\alpha_s}{2\pi} q^{NS} \otimes C_L^{NS,(1)}.$$

Due to the convolution smearing a function $\Delta F_L(k_R) \equiv F_L^{LT}(k_R) - F_L^{LT}(k_R = 1)$ depends on Q steeper, than ΔF_2. One can see that at $x = 0.3$ and large Q_0 the function $Q^2\Delta F_L(k_R = 1/2)$ falls about two times in the region of $Q^2 = 1 \div\ 3$ GeV^2 (see Fig. 8). At large x and low Q_0 it falls even more steeply. As a result, $H_L(x)$ exhibits only weak dependence on k_R for all x and Q_0 (see Fig. 9).

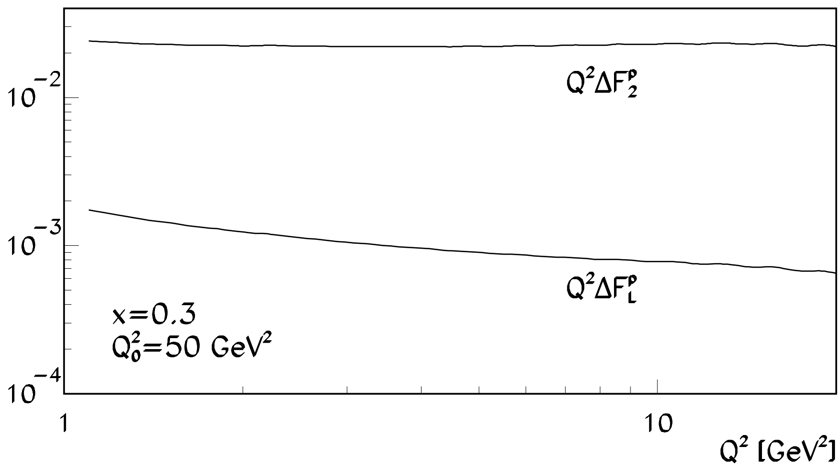

Fig. 8. The comparison of $Q^2\Delta F_2^p(k_R = 1/2)$ and $Q^2\Delta F_L^p(k_R = 1/2)$ dependence on Q^2.

[3]In the fit at $Q_0^2 = 50$ GeV^2 with HT fixed the $\alpha_s\,(M_Z)$ error is 0.0004 as compared with 0.0021 in the fit with HT released.

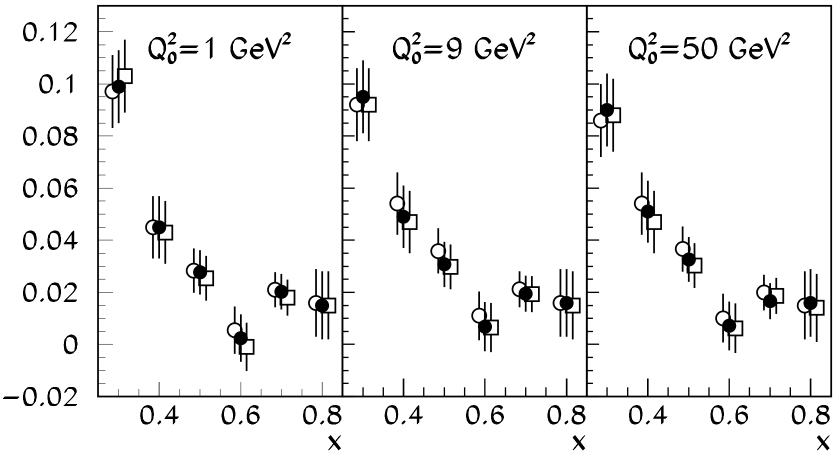

Fig. 9. The values of $H_{\rm L}^{\rm p}(x)$ for different choices of Q_0 and RS (open circles: $k_{\rm R} = 1/2$; full circles: $k_{\rm R} = 1$; squares: $k_{\rm R} = 2$).

4. In summary, we can conclude that the HT contribution to structure function F_2 extracted in the NLO QCD analysis of nonsinglet SLAC-BCDMS-NMC data is retuned with the RS change. This retuning depends on the choice of starting evolution point Q_0 and x. At $Q_0 \gtrsim 10$ GeV^2 the HT contribution to F_2 is retuned at small x and is not almost retuned at large x; at small Q_0 it exhibits approximate RS stability for all x in question. The RS sensitivity of $\alpha_{\rm s}$ also depends on the choice of Q_0: At large Q_0 this sensitivity is weaker, than at small ones. The HT contribution to $F_{\rm L}$ is RS stable for all Q_0 and x.

The RS stability of HT contribution is important for clarification of their nature: Due to both the HT and the HO corrections being the falling functions of Q, it was often claimed that the extracted HT terms can contain the contribution from HO. The HT absorbtion by NNLO correction was observed in the analysis of neutrino structure function xF_3 [10], although the effect was smashed due to a low accuracy of the data. Our results indicate that in the analysis of high statistical charged leptons DIS data an unambiguous separation of twist-4 contribution and NNLO QCD corrections to DGLAP kernel is possible if Q_0 is low. This conclusion is especially important because no complete NNLO calculation of the splitting functions is available at the moment and it is impossible to perform exact direct clarification of this point.

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