

## STATE RESEARCH CENTER OF RUSSIA INSTITUTE FOR HIGH ENERGY PHYSICS

IHEP 99-7

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# SENSITIVITY OF TIME-DEPENDENT CP-ASYMMETRY MEASUREMENT VERSUS TIME-INTEGRATED ONE IN B SYSTEM

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Protvino 1999

#### Abstract

Roinishvili V. Sensitivity of Time-Dependent CP-Asymmetry Measurement Versus Time-Integrated One in *B* System: IHEP Preprint 99-7. – Protvino, 1999. – p. 4, figs. 2, refs.: 4.

Simple expressions for the sensitivities of time-dependent and time-integrated measurements of CP-asymmetry in B system are obtained.

#### Аннотация

Ройнишвили В. Соотношения между точностями измерения зависящей от времени и интегрированной по ней СР-асимметрии при распаде нейтральных В-мезонов: Препринт ИФВЭ 99-7. – Протвино, 1999. – 4 с., 2 рис., библиогр.: 4.

Получены простые соотношения между точностями измерения параметров нарушения СР-четности при распаде нейтральных В-мезонов в случаях измерения асимметрии, зависящей от времени и интегрированной по ней.

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CP-violation in  $B^0$ -decay leads to the difference between the rates of  $B^0$  and  $\overline{B}{}^0$  decay into a CP eigenstate  $f_{CP}$ . If only a single diagram contributes to some decay, the decay rates, for example, for  $f_{CP} \equiv J/\psi K_s$ , are:

$$\Gamma_f(t) = e^{-\Gamma t} \times (1 - \sin 2\beta \times \sin \Delta m t) \quad and \quad \bar{\Gamma}_f(t) = e^{-\Gamma t} \times (1 + \sin 2\beta \times \sin \Delta m t),$$

where  $\Gamma(t)$  stands for  $B_d \to f_{CP}$ ,  $\overline{\Gamma}(t)$  for  $\overline{B}_d \to f_{CP}$ ,  $\Delta m$  is the mass difference between  $B_d$  and  $\overline{B}_d$  and  $\beta$  - the angle in the unitarity triangle of CKM matrix. In terms of  $\tau = \Gamma t$  and the mixing parameter  $x_d = \Delta m/\Gamma$  the time-dependent CP-asymmetry can be written as

$$A_{t-d}(\tau) = \frac{\Gamma(\tau) - \bar{\Gamma}(\tau)}{\Gamma(\tau) + \bar{\Gamma}(\tau)} = -\sin 2\beta \times \sin x_d \tau.$$
(1)

In Fig.1 the distribution of  $A_{t-d}(\tau)$  (points with error bars) is shown for a sample of  $N = 2.4 \cdot 10^6$  generated events of  $B_d$  decay into  $J/\psi K_s$  with  $\sin 2\beta = 0.5$  and  $x_d = 0.7$ . For any bin of  $\tau$  one can define the magnitude of  $\sin 2\beta$  as  $A_{t-d}(\tau)/\sin x_d\tau$  with the statistical error:

$$\delta_{\sin 2\beta}(\tau) \approx \frac{1}{\sqrt{\Delta N(\tau)} \times |\sin x_d \tau|},$$

where  $\Delta N(\tau) = N \times e^{-\tau} \times \Delta \tau$  is the number

of  $B_d$  events in the bin. It is convenient to introduce sensitivity — s (a quantity inverse to the statistical error) per event, whose square is an additive quantity. For the timedependent measurement

$$s_{t-d}^2(\tau) = e^{-\tau} \times \sin^2 x_d \tau.$$





The sensitivity  $s_{t-d}(\tau)$  is also presented in Fig.1 as a solid curve for  $x_d = 0.7$ . The total sensitivity for a full range of  $\tau$  per event is defined as

$$S_{t-d} = \left[\int_0^\infty s_{t-d}^2(\tau) d\tau\right]^{1/2} = \sqrt{\frac{2x_d^2}{1+4x_d^2}}.$$
(2)

The statistical error for the time-dependent CP-violation measurement with the total number of  $B_d(\bar{B}_d) \to f_{CP}$  events N will be

$$\delta_{t-d}[\sin 2\beta] = \frac{1}{S_{t-d} \times \sqrt{N}} = \sqrt{\frac{1+4x_d^2}{2x_d^2 N}}.$$
(3)

 $S_{t-d} = 0.575$  for  $x_d = 0.7$  and the error for  $\sin 2\beta$  in the above mentioned sample should be equal to 0.0011. This figure coincides, as it must be, with that obtained by fitting (MINUIT)  $A_{t-d}(\tau)$  in Fig.1.

In the case of time-integrated measurements the CP-asymmetry  $A_{int}$  is

$$A_{int} = \frac{\int_0^\infty \left[\Gamma(\tau) - \Gamma(\tau)\right] d\tau}{\int_0^\infty \left[\Gamma(\tau) + \bar{\Gamma}(\tau)\right] d\tau} = -\sin 2\beta \times \frac{x_d}{1 + x_d^2}.$$
(4)

The value of  $\sin 2\beta$  now can be obtained as  $A_{int} \times \frac{1+x_d^2}{x_d}$  with the sensitivity per event

$$S_{int} = \frac{x_d}{1 + x_d^2} \tag{5}$$

and with the statistical error for N events (a well known result)

$$\delta_{int}[\sin 2\beta] = \frac{1}{\sqrt{N} \times S_{int}} = \frac{1}{\sqrt{N}} \times \frac{1 + x_d^2}{x_d}.$$
(6)

Time-dependent measurements are statistically more precise than time-integrated ones, as it follows from (2) and (5), by a factor

$$R = \frac{S_{t-d}}{S_{int}} = (1 + x_d^2) \times \sqrt{\frac{2}{1 + 4x_d^2}}$$
(7)

which is equal to 1.22 for  $x_d = 0.7$ . Relation (7) holds if the efficiency of the  $B_d$  vertex reconstruction  $\varepsilon$  is equal to 1, otherwise it will be less by a factor of  $\sqrt{\varepsilon}$ .

Selection of reconstructed *B* events with the decay vertex separated from the primary one reduces the background under *B* signal. In this case the lower limits of the integrals in (4) -  $\tau_L \neq 0$  and the CP-asymmetry  $A_{int}(\tau_L)$  for  $\tau$  range  $\tau_L \leq \tau \leq \infty$  is equal to

$$A_{int}(\tau_L) = -\sin 2\beta \times \frac{\sin x_d \tau_L + x_d \cos x_d \tau_L}{1 + x_d^2}.$$
(8)

The sensitivity per event

$$S_{int}(\tau_L) = e^{-\frac{\tau_L}{2}} \times \frac{|\sin x_d \tau_L + x_d \cos x_d \tau_L|}{1 + x_d^2}$$
(9)

has the maximum at  $\tau_L^{max} = \frac{1}{x_d} \times \arctan \frac{x_d}{1+2x_d^2}$ .

For  $x_d = 0.7 \tau_L^{max} = 0.49$  and  $S_{int}(\tau_L^{max}) = 0.523$ . This figure can be increased up to 0.536 if the upper limits for the integrals in (4) are  $\tau_U = \tau_U^{max} = \frac{1}{x_d} \times [\arctan(-x_d) + \pi] = 3.615$  - the first value of  $\tau_L \ge 0$  at which  $A_{int}$  in (8) vanished. The sensitivities  $S_{int}(\tau_L)$  — solid curve and  $S_{int}(\tau_L^{max}, \tau_U)$  — dotted curve are plotted in Fig.2. Particularly, one can see from the figure that there is a range of  $\tau_L \le 1$  where  $S_{int}(\tau_L)$  is more sensitive than the time-integrated one over a full range of  $\tau - S_{int} = 0.47$ .

The time-dependent sensitivity for  $\tau$  range  $\tau \geq \tau_L^{max}$  decreases a little from 0.575 to 0.564 since the sensitivity for  $\tau \leq 0.5$  is small (see Fig.1). Now ratio R

$$R = \frac{S_{t-d}(\tau \ge \tau_L^{max})}{S_{int}(\tau_L^{max}, \tau_U^{max})} = 1.05$$
(10)

is rather close to 1.

Important note: In the above considerations it is assumed that  $B_d$  flavor at the pro-

0.6  $S_{int}(\tau_L), S_{int}(\tau_L^{mox}, \tau_U)$ 0.5 0.4 ')=0.536  $S_{int}(\tau_L^{max}) = 0.523$ 0.3 0.2 =0.49 =3.61 0.1 0 0.5 1.5 2 2.5 3 3.5 Lower(upper) limit of the integration  $\tau_{\rm L}(\tau_{\rm u})$ 

Fig. 2. Sensitivity for the range  $\tau > \tau_L$  solid curve and for the range  $\tau_L^{max} < \tau < \tau)_u$  — dotted curve.

duction stage is known. In practice some dilution effects will arise because of backgrounds and/or a wrong definition of  $B_d$  flavor. All the sensitivities will be decreased by a factor  $D = \frac{N_R - N_W}{N_R + N_W}$ , where  $N_R(N_W)$  is the number of events with right (wrong) definition of  $B_d$ flavor.

Magnitudes of D for different methods (and/or different experiments) of  $B_d$  flavor definition are different (for LHC experiments see, for example, /1/-/4/). However, it seems that there will be no significant difference in D between time-dependent and timeintegrated CP-asymmetry measurements and relation (7) and (10) for R will be valid.

#### Conclusion.

Simple expression (2) for the sensitivity of time-dependent CP-asymmetry measurements has been obtained. Time-dependent measurements are more sensitive than timeintegrated ones by a factor

$$R = (1 + x_q^2) \times \sqrt{\frac{2}{1 + 4x_q^2}},$$

which is equal to 1.22 for  $x_q = 0.7$ .

For the optimal range of integration  $\tau_L^{max} \leq \tau \leq \tau_U^{max}$  (see text), the factor R is 1.05 only. But in this case it is necessary to measure the decay time of  $B^0$  as for time-dependent methods.

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Received February 16, 1999

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Соотношения между точностями измерения зависящей от времени и интегрированной по ней СР-асимметрии при распаде нейтральных В-мезонов.

Оригинал-макет подготовлен с помощью системы ІАТ<sub>Е</sub>Х. Редактор Е.Н.Горина. Технический редактор Н.В.Орлова. Подписано к печати 18.02.99. Формат 60 × 84/8. Офсетная печать. Печ.л. 0,5. Уч.-изд.л. 0,39. Тираж 50. Заказ 56. Индекс 3649. ЛР №020498 17.04.97.

ГНЦ РФ Институт физики высоких энергий 142284, Протвино Московской обл.

Индекс 3649

 $\Pi P E \Pi P И H T 99-7,$   $И \Phi B Э,$  1999